PROBLEMS

WEEK 1

SCALING LAWS

Exercise 1

- A man is 1.8 m tall and has a mass of 70 kg. Estimate the mass of 2.0 m tall man. Assume that the two men have the same body mass index.

Exercise 2

- The strength of tendons, and bones scales with their cross-sectional area. How does the strength to mass ratio scale for geometrically similar animals? Does this pose a structural limit to the size of land animals?

Exercise 3 (may be somewhat challenging)

- In mammals the heat loss is proportional to their surface; the energy gained is proportional to the amount of blood flowing through the lungs (the source of oxygen for the body). At constant body temperature, these two energies are identical. Derive from this information a scaling law between the beat rate and the mass of mammals *[Hint: the amount of blood flowing through the lungs is given by the product of the stroke volume and the beat rate]*
- Compare your scaling law to data
  - Rat 0.2 kg 420 bpm (beats per minute)
  - Rabbit 2 kg 205 bpm
  - Dog 5 kg 120 bpm
  - Man 70 kg 72 bpm
  - Horse 450 kg 38 bpm

VECTORS

Exercise 1

- Find the unit of measure of the vector **b** = Δt **a**, where Δt is a scalar with units 1 s, and **a** is a vector with units 1 m/s^2

Exercise 2

- Given the points A = (2, -1), B = (-1, 3) and C = (0, 1) in a Cartesian coordinate system, express each of the following vectors as a linear combination of the standard basis unit vectors (or *versors*) **i** and **j** :
  - **AB, BC**, and **AC**;
  - **AB**+**BC** and 2**AC** − 3**CB**
  - A unit vector (versor) in the direction of **AB**.

Exercise 3:

- Draw the vectors A=(3,2) and B=(−1,1) and perform their sum **A+B** and their difference **A−B**, both component wise and geometrically. Find their magnitudes and their direction relative to the x-axis

KINEMATICS

Exercise 1

- The average height of vertical jumps from standing still is about 0.6 m for volleyball defenders. Find their initial vertical velocity, i.e. when their foot leave the ground.
[*hint: Motion with constant acceleration g=9.8 m/s*^{2}]

Exercise 2

- The height vertical jumps is monitored by using a pressure pad to measure the time it takes for an athlete to complete a jump. What time would last a vertical jump of an athlete reaching a maximum height of 1 m?

Exercise 3

- A centrifuge is operated at 5×10^5 turns per minute. Find the angular velocity. If the radius of the centrifuge is R=0.2 m, find the centripetal acceleration

Exercise 4

- A dive with three complete rotations in the tuck position is performed from a platform of 10 m.
  1. Use the equation of free-fall motion and derive the duration of the flight from the platform to the water (d=10 m).
  2. Derive the mean angular velocity of the rotational motion.
  3. Assuming that during the rotation the head describes a circle of radius R=0.5 m around the centre-of-gravity of the diver, derive the centripetal acceleration of the head relative to the centre-of-gravity.

SOLUZIONI

- The metabolic rate is proportional to the surface, which scales as L^2, where L is a scale parameter of the linear size of the mammal
- The blood flow rate is $\Delta V/\Delta t = R V_s$, where R is the beat rate and the stroke volume V_s scales with L^3.
- From the hypotheses, the metabolic rate (energy per unit time) is proportional to the flow rate of oxygen through the lungs, which is proportional to the total volume of blood through the lungs (1), i.e.:
  - $L^2 \div \Delta V/\Delta t = R V_s \div R L^3$
- From this relation we find (remember that at constant density $M \div L^3$)
  - $L^2 \div R L^3 \rightarrow R \div 1/L \div 1/\sqrt[3]{M}$ or $\mathbf{R \div M^{-1/3}}$
- This relationship is presented in the Fig. at the right (blue line), together with the data give in the table for some mammals.