

Lecture 1 : INTRODUCTION TO PHYSICS

(1)

* Why there are no mammals larger than elephants or whales and smaller than mice ?

— Perhaps this is not the question you asked yourself when you decided to follow this course. Yet, by addressing this question I will try to illustrate how science works and how you will learn to address questions through science.

• Fundamental questions requires fundamental understanding - In other words basic laws.
Let's use our basic laws (prior knowledge) and imagination to answer our question.

* However to answer a question precisely we need to formulate it precisely (precisely = quantitatively):

* Is there a relationship (math form) between the metabolic rate and the mass of mammals ?

②

A math. relationship ~~requires~~ relates
measurable quantities ← precise definitions needed

$$P \equiv \text{metabolic rate} = \frac{\text{Energy}}{\text{Unit Time}} \quad \leftarrow \begin{array}{l} \text{Food} \\ \text{procurement} \\ \text{rate} \end{array}$$

$M \equiv \text{mass}$

— You may have a feeling of what mass is (through weight). We'll come to the exact definition in a future lecture.

— Knowledge and Hypotheses:

① * Same cells and constituent material in all the mammals

H_{p.1}: Same density, ρ , in all mammals

$$\Rightarrow M = \rho V \quad V \equiv \text{volume}$$

Mass is proportional to the volume

② * Body temperature is constant in mammals
* Food is not converted into mass, but into energy (proteins, sugar, ... oxidation) spent to live (internal motion of organs), to move, etc.

* Movements → Friction → Heat
Heat dissipated through the SURFACE ($T_{\text{surf}} = \text{const.}$)

To keep ~~for~~ themselves alive

$$\text{Metabolic rate} \div \text{Surface}$$

[~~Already~~ quite some input from basic laws of physics here. \rightarrow Answers to fundamental questions requires fund. knowledge]

In summary :

$$\begin{aligned} \text{Mass} &\div \text{Volume} \\ \text{Metabolic Rate} &\div \text{Surface} \end{aligned}$$

* The relationship between mass and metabolic rate is related to the geometry of the space

- x Elegant!
- x Philosopher : So elegant, it must be true!
- x Scientist : Is it true?

Make the relationship a bit more quantitative - (4)

Hp. 3

spherical mammals
of radius R



PIG



COW



WHALE

$$\begin{aligned}\text{Surface} &= 4\pi R^2 \propto R^2 \\ \text{Volume} &= \frac{4\pi}{3} R^3 \propto R^3\end{aligned}$$

$$\Rightarrow \Phi \propto R^2 = (R^3)^{2/3} \propto M^{2/3}$$

We have found
a power law

$$\Phi \propto M^{2/3}$$

* There is a number of consequences to this law that we can list -

(consumption)
- Power per unit cell of Power/Mass

Cell metabolic rate in mammals $\propto \frac{P}{M} \propto M^{-1/3}$

$$P(\text{cell}) = \frac{1}{\sqrt[3]{M}}$$

Although the cells are all identical, the power consumption/cell is smaller in big mammals

* Big mammals \Rightarrow more efficient usage of a ~~cell~~ the energy

* Mammals too small cannot survive against more efficient ways of organizing life \rightarrow

- x Insects
- ~~Mammals~~
- x bacteria
- x viruses

WE HAVE FOUND A REASON FOR A LOWER LIMIT TO THE SIZE OF MAMMALS

There seems to be no upper limit, 6
though —

However: total power consumption
grows with mass $P \propto \sqrt[3]{M^2}$

Ex. g. Man $M = 100 \text{ kg}$
Elephant $M = 1000 \text{ kg}$

$$\frac{P_{\text{Man}}}{P_{\text{Elef}}} = \frac{\sqrt[3]{(1000)^2}}{\sqrt[3]{(100)^2}} = \sqrt[3]{10^2} = \sqrt[3]{100}$$

≈ 5

→ Elephant need to eat about
5 times what a man eat!

→ There is a problem ^{with} food
procurement - Big size
MAMMALS DISFAVORED

(AND IT WORKS FOR REPTILES TOO
(THINK OF DINOSAUR'S EXTINCTION))



An aside.

- The power consumption of human being is on the average $\sim 100 \text{ W}$ (about 1 bulb lamp) \rightarrow FOOD ONLY

- However: The total power used by "civilized" man is ~~about~~ \rightarrow 1 kW including - electric power for industry, services (hospitals), oil for transportation, etc.

- * Power consumption TERNA.IT ^(excludes fuels for transportation) $\sim 40 \text{ GW}$
- * Inhabitants in Italy $50 \times$

- * Human beings ~~then~~ ^{indirect} have a use resources as if their mass was:

$$\frac{M_{\text{civilized man}}}{M_{\text{man}}} = \left(\frac{P_{\text{civil.}}}{P_{\text{man}}} \right)^{3/2} = 10^{3/2}$$

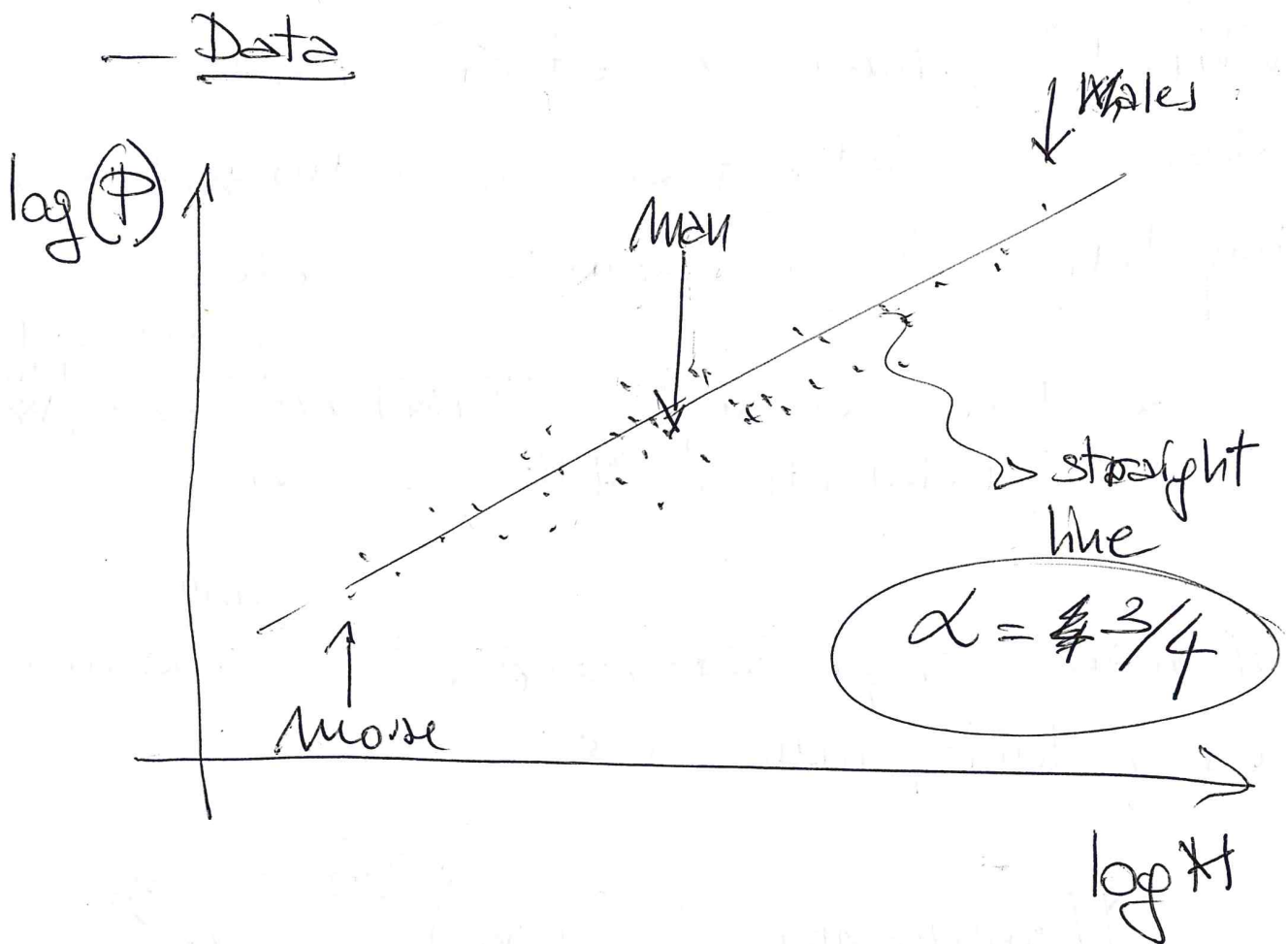
$$= \sqrt[2]{10^3} = \sqrt[2]{1000} \approx 30$$

- * Equivalent mass of a man ~ 3 tons !!!
- \Rightarrow Mammals of this size do not survive much!

So far, so ~~good~~ good.

but is this all true and consistent with observations?

[TEST OF ALL KNOWLEDGE IS EXPERIMENT]



$$P \propto M^{\frac{3}{4}}$$

$$\log(P) \approx \log(M^{\frac{3}{4}}) + \log \text{const}$$

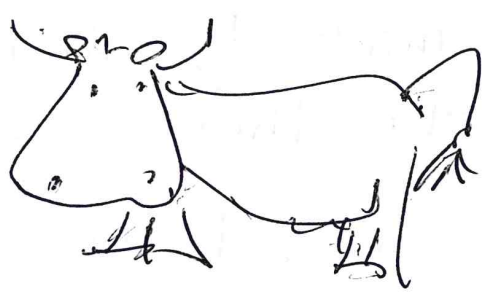
$$\log(P) \approx \frac{3}{4} \log M + \text{const} \quad \text{STRAIGHT LINE}$$

x Our model does not exactly fit data
($\alpha = 3/4$, while we found $\alpha = 2/3$)

- * The prediction of a power law is correct
- * General conclusions holds
- * Some hypothesis must be adjusted/reviewed

* Suspects ?

"Mammals are not spherical!", you may say



cow : SIDE VIEW



WHALE SIDE VIEW

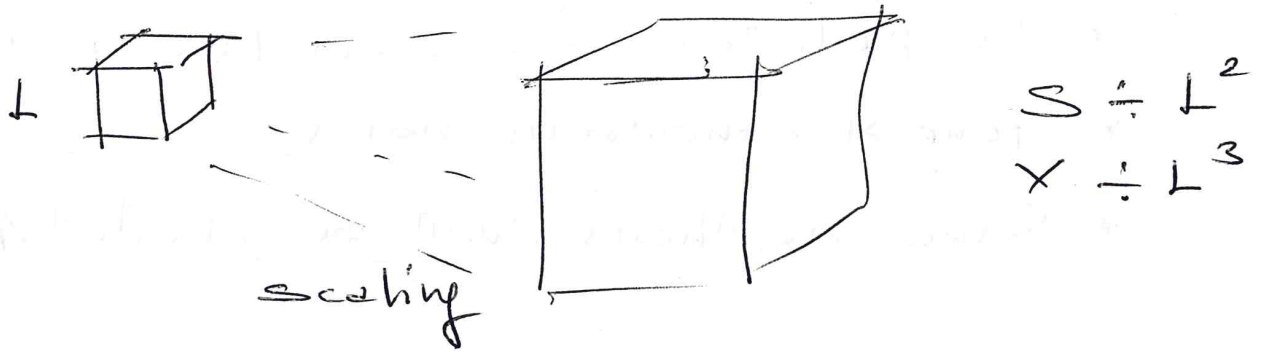
• However, as we said, the relationship we found is related to the geometry of the space

$$\text{Metabolic rate} \div \text{SURFACE}$$

$$\text{Mass} \div \text{VOLUME}$$

As long as an object has a characteristic scale length L (R in our simplified model), the surface scales with L^2 and the volume with L^3

Let's check this on a cube:



- It works for spheres and cubes, it must work for any shape! Indeed, we can think of a generic shape as made by many little cubes, one next to the other
- the application of a scale factor to the shape is equivalent to scale all the cubes

⇒ GENERAL VALIDITY OF SCALING LAWS

no

There are several applications of this

Example: Dose of DRUG to a child and an adult should scale as the height (linear dimension) to the 3rd power, if you wished to keep the same concentration of DRUG in the blood (volume)

~~what~~

* Can it be that we ~~live~~ live in a space with more than three spatial dimensions?

~~rat~~ $\frac{\text{surface}}{\text{Volume}} = \frac{3}{4}$

→ Almost true ←

The exact analysis cannot ignore the internal structure of mammals

- Energy distribute to cells by a network (cardio circulatory system)
- Complexity of the network grows with mammal size
- Efficiency gain with mammal size slightly lower than predicted by our simple geometry ~~to~~ (scaling) laws

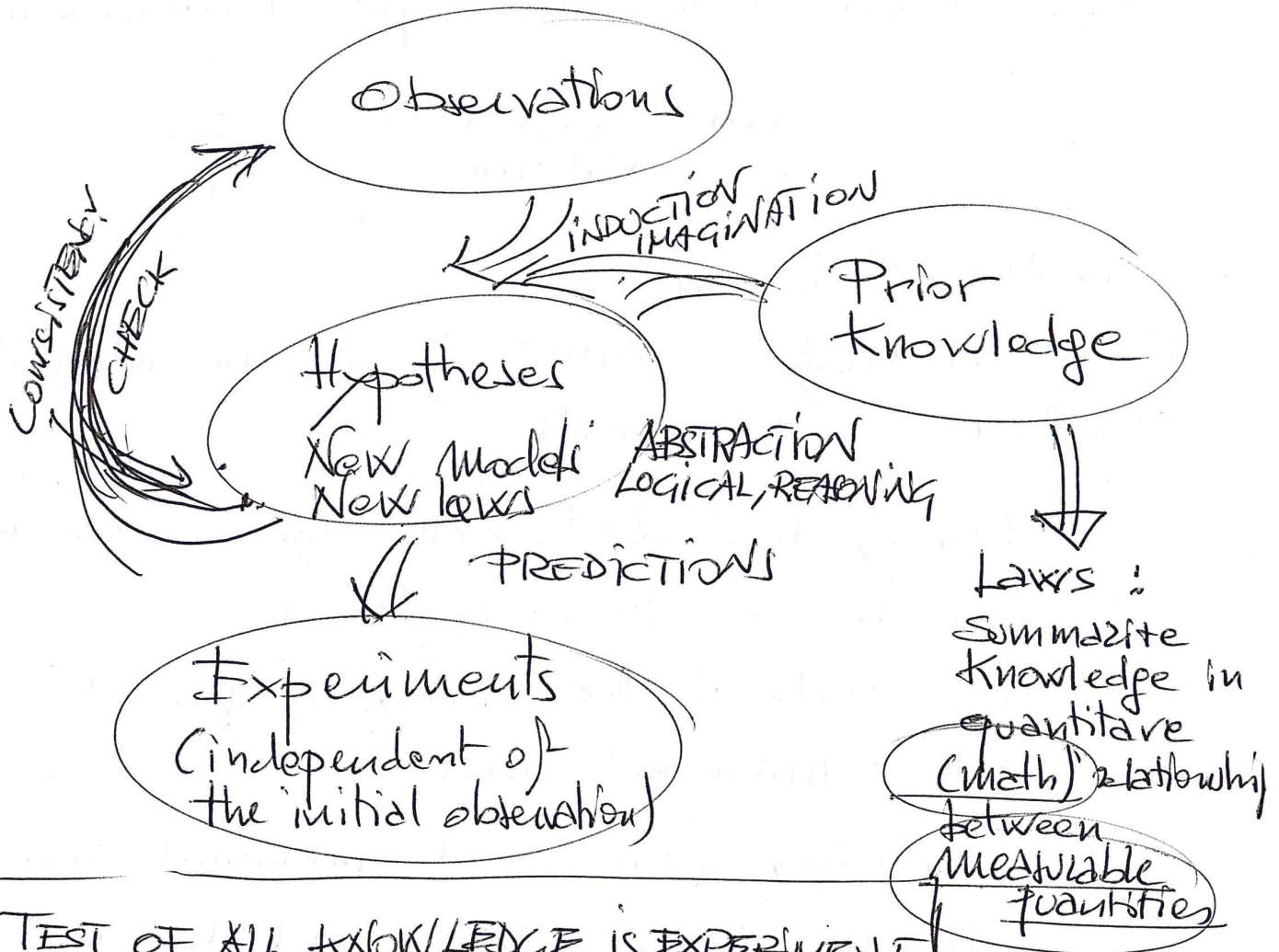
The internal structure adds one dimension to the problem

2 → 3
3 → 4

* INTIMATE $F = ma$

TAKE AWAY FROM THE EXAMPLE ! SCIENCE AT WORK

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TEST OF ALL KNOWLEDGE IS EXPERIMENT.

Needs:

* Math language [Keep it simple in this course: minimize use of calculus]

* Precise definition of measurable quantities

Final Note: Though the example is informative $P = M^2/3$ is not a physics law. It's a mathematical relationship derived from some laws. Physics laws do not simply relate things but contain dynamics