Relatività

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28/01/2022

On the first sheet, please write clearly your **name, surmame, student ID, and signature**. If you need more sheets, please at least report your **name and surname** on each of them.

Exam duration: 3 hours.

Solve the problems taking into account that only results where the final answers have been simplified as much as possible will be given full marks.

Please write your results and intermediate steps with as clear as possible handwriting. If needed, perform your calculations/attempts on a draft, and report them afterwards on the sheet you hand in.

Problema 1

Consider a $2 \to 2$ scattering process, where 4-momentum conservation reads

$$p_1^\mu + p_2^\mu = p_3^\mu + p_4^\mu$$

a) Show that, if

$$p_1^2 = p_2^2 = 0, \quad p_3^2 = m_3^2, \quad p_4^2 = m_4^2$$

then one has

$$s + t + u = m_3^2 + m_4^2$$

where $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$, $u = (p_1 - p_4)^2$

b) Let's now assume that $m_3 = 0$.

- In the center-of-mass reference frame (i.e. the frame where $\vec{p}_1 + \vec{p}_2 = 0$), and assuming that $t = 0$, compute the angle between the 3-vectors $\vec{p}_1$ and $\vec{p}_3$.
  (hint: it's useful to choose the componentes of $\vec{p}_1$ and $\vec{p}_2$ so that $\vec{p}_1$ and $\vec{p}_2$ are parallel to a coordinate axis)
- Always assuming that $t = 0$, does the value of the angle depend on the reference frame? Motivate your answer.

Problema 2

Obtain the Euler-Lagrange equations for a scalar field ϕ whose Lagrangian density not only depends on ϕ and $\partial_\mu \phi$, but also on the second and third derivatives of the field ϕ, i.e.

$$\mathcal{L} = \mathcal{L}(\phi,\ \partial_\mu \phi,\ \partial_\mu \partial_\nu \phi,\ \partial_\mu \partial_\nu \partial_\rho \phi)$$

Problema 3

In an inertial reference frame there is a charged particle with charge q and mass m, an electric field $\bar{\mathbf{E}}$ and a magnetic field $\bar{\mathbf{B}}$. The fields are uniform, constant and parallel. Compute the component of the velocity of the particle on the direction of the fields as a function of the particle's proper time τ, assuming that at the initial time such component is 0.

Problema 4

An antiparticle with energy E and mass m hits a particle (with mass m) at rest in the laboratory frame. After the annhilation, in the final state there are only two photons. Each of them travels along the same direction of the incoming antiparticle.

a) Compute the energies of the two photons in the lab frame S.

b) By explicitly using a Lorentz transformation, compute the energies of the two photons in the reference frame S', where S' is the frame where the incoming antiparticle is at rest.