Risultati da richiamare

$L' = \mu r^2 \left(\frac{d\vartheta}{dt}\right)$ risp CM r distanza tra due part.

$E_K' = \frac{1}{2} \mu v^2$ risp CM v vel. rel. di part.

$L^2 = G \mu m M \varepsilon d$ per conica generica

$= G \mu m M a (1-\varepsilon^2)$ per ellisse

$E_T = - G m M \frac{1-\varepsilon^2}{2\varepsilon d}$ conica

$E_T = - \frac{G m M}{2a}$ ellisse

Eq. conica $\frac{1}{r} = \frac{1}{\varepsilon d} - \frac{\cos\vartheta}{d}$

ellisse $\begin{cases} \text{semiasse magg} & a = \frac{\varepsilon d}{1-\varepsilon^2} \\ \text{" min} & b = a\sqrt{1-\varepsilon^2} \end{cases}$

Visto $$\frac{dA}{dt} = \frac{L}{2m}$$

per $$L = m r^2 \frac{d\vartheta}{dt}$$

se uso $$L' = \mu r^2 \frac{d\vartheta}{dt}$$

allora $$\frac{dA}{dt} = \frac{L}{2\mu}$$

$$\frac{A}{T} = \frac{L}{2\mu} \qquad T = \frac{2\mu A}{L}$$ ellisse

$$A = \pi a b = \pi a^2 \sqrt{1-\varepsilon^2}$$

$$T^2 = \frac{4\mu^2 A^2}{L^2}$$

$$L^2 = G \mu m M a (1-\varepsilon^2)$$

$$T^2 = \frac{4\mu^2 \pi^2 a^4 (1-\varepsilon^2)}{G \mu m M a (1-\varepsilon^2)}$$

$$= \frac{4\pi^2 a^3 \mu}{G m M}$$

$$\boxed{T^2 = \frac{4\pi^2 a^3}{G(m+M)}}$$

Commentare risp. 3ª Kepl.

$$\frac{M_j}{M_\odot} = \frac{1}{1047}$$

$$\frac{M_{sat}}{M_\odot} = \frac{1}{3500}$$

per $m \ll M$

$$T^2 = \frac{4\pi^2 a^3}{G M}$$

- Cenni a principio equivalenza come fondamento di Rel. Gen.

All' epoca di Einstein

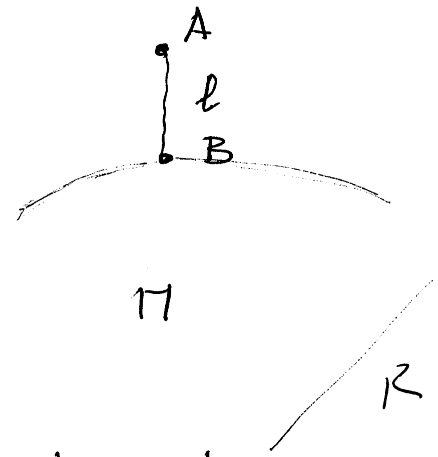
Eötvös $< 10^{-9}$

$\sim$1961 Dicke, Krotkov, Roll : $1.3 \pm 1.0 \times 10^{-11}$

2007 Eot-Wash : $0.8 \pm 1.3 \times 10^{-13}$

Hal 14.1

$m_A = m_B = m$



$T = G\dfrac{Mml}{R^3}$ dimostrare

$F_A = \dfrac{GMm}{(R+l)^2} + T$ (asse lungo $-\hat{u}_r$)

$F_B = \dfrac{GMm}{R^2} - T$

$a_A = a_B \Rightarrow \dfrac{GM}{(R+l)^2} + \dfrac{T}{m} = \dfrac{GM\cancel{m}}{R^2} - \dfrac{T}{m}$

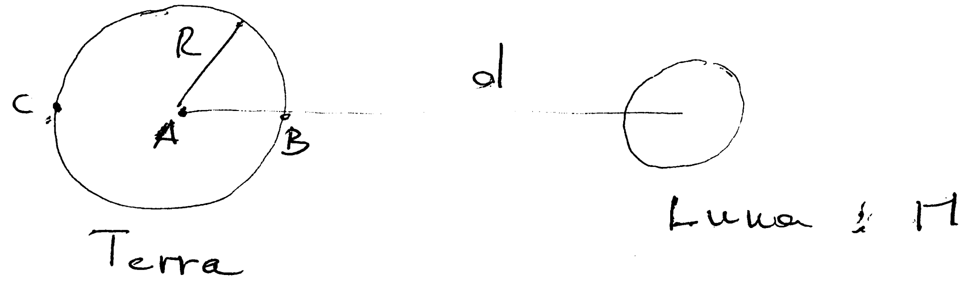
$\dfrac{2T}{m} = GM\cancel{m}\,\cancel{\dfrac{1}{(R+l)^2}}\left(\dfrac{1}{R^2} - \dfrac{1}{(R+l)^2}\right)$

$\dfrac{1}{(R+l)^2} = \dfrac{1}{R^2\left(1+\frac{l}{R}\right)^2} \simeq \dfrac{1-\frac{2l}{R}}{R^2}$

$\dfrac{2T}{m} = \dfrac{GM}{R^2}\dfrac{2l}{R}$

$\Rightarrow T = \dfrac{GMml}{R^3}$

Hal 14.5



$m_A = m_B = m_C = m$

$F_A = G \frac{Mm}{d^2}$ (verso luna)

$F_B = G \frac{Mm}{(d-R)^2} = G \frac{Mm}{d^2 (1-\frac{R}{d})^2} \sim G \frac{Mm}{d^2} (1 + 2\frac{R}{d})$

$F_B - F_A = 2 \frac{GMm}{d^3} = F_A - F_C$

nota: la terra "cade" verso la luna con accel. $\frac{GM}{d^2}$, come se tutta la sua massa fosse al centro

- la "forza apparente" agente su part A è $F_A' = -\frac{GMm}{d^2} = F_B' = F_C'$

quindi

$F_A + F_A' = 0$

$F_B + F_B' = + 2 \frac{GMmR}{d^3}$ } entrambe sono centrifughe risp. centro terra

$F_C + F_C' = - 2 \frac{GMmR}{d^3}$

La "forza di marea" esercitata da Luna su terra è (su part. m a sup)

$$F_{ML} = 2\,\frac{G\,M_L\,m\,R}{d_L^3}$$

per il sole

$$F_{MS} = \frac{2\,G\,M_S\,m\,R}{d_S^3}$$

$M_L = \frac{4}{3}\pi\, r_L^3\, \rho_L$ ρ_L : densità media luna

$M_S = \frac{4}{3}\pi\, r_S^3\, \rho_S$ ρ_S : densità media sole

$$\frac{F_{ML}}{F_{MS}} = \frac{M_L/d_L^3}{M_S/d_S^3} = \frac{(r_L/d_L)^3\,\rho_L}{(r_S/d_S)^3\,\rho_S}$$

ma r/d è il raggio apparente o raggio angolare e $r_L/d_L \simeq r_S/d_S$

da cui

$$\frac{F_{ML}}{F_{MS}} \simeq \frac{\rho_L}{\rho_S} = \frac{3.35 \times 10^3\ Kg/m^3}{1.41 \times 10^3\ Kg/m^3}$$

Sole

$M_\odot = 1.989 \times 10^{30}$ kg

$V_\odot = 1.41 \times 10^{18}$ km^3

$\langle \rho_\odot \rangle = 1.408 \cdot 10^3$ kg/m^3

$R_\odot$ (equatoriale) $= 6.963 \cdot 10^5$ km

Lumi $= 3.846 \cdot 10^{26}$ W

$\langle d_s \rangle = 1.496 \times 10^8$ km (semiasse magg. orbita terr)

Terra

$M_\oplus = 5.974 \cdot 10^{24}$ kg

$R_\oplus = 6371$ km (medio - $R_{eq} = 6378$ km)

$\langle \rho_\odot \rangle = 5.515 \cdot 10^3$ kg/m^3

Luna

$\langle d_L \rangle = 384\,399$ km (semiasse magg)

$R_L = 1737$ km

$M_L = 7.348 \times 10^{22}$ kg

$\langle \rho_L \rangle = 3.3464 \cdot 10^3$ kg/m^3

Hal 14.24

Grav.

due corpi di massa m, puntiformi, a riposo dist. d.

dim. $t_{inc} = \frac{\pi}{4}\sqrt{\frac{d^3}{Gm}}$

$\rightarrow$ t_{inc} è il semiperiodo per $m_1 = m_2 = m$ e semiasse magg $= d/2$

$$\Rightarrow t_{inc}^2 = \frac{\pi^2 (d/2)^3}{2Gm}$$

$$t_{inc} = \frac{\pi}{4}\sqrt{\frac{d^3}{Gm}}$$

Hal 14.10 Grav

(non fatto)

$\rho(r)$ $= 0$ per $r > R$

$dM(r) = 4\pi r^2 dr\, \rho(r)$

$M(r) = \int_0^r dM = 4\pi \int_0^r r^2 \rho(r)\, dr$

$F(r) = G \frac{m M(r)}{r^2}$

$\frac{dF}{dr} = -2GmM(r)r^{-3} + Gmr^{-2}\frac{dM}{dr}$

$\frac{dM}{dr} = 4\pi r^2 \rho(r)$

$M(r) = \frac{4}{3}\pi r^3 \langle\rho\rangle$ $\langle\rho\rangle$ è la densità media di $\rho(r)$ per $r' < r$

$\frac{dF}{dr} = -2Gm\frac{4}{3}\pi\langle\rho\rangle + Gm4\pi\rho(r)$

$\frac{dF}{dr} = 0 \implies \rho(r) = \frac{2}{3}\langle\rho\rangle_r$