XI. The Collinear Anomaly

There are kinematical situations in which the construction of SCET as discussed so far in these lectures needs to be modified, because the ultra-soft modes do not contribute for some reason. This can happen, e.g.:

- in the presence of masses $m \sim \lambda$
- if cuts are imposed on transverse momenta
- if the ultra-soft scale is parametrically smaller than the QCD scale, e.g. if $\lambda \sim \frac{\Lambda_{QCD}}{Q}$

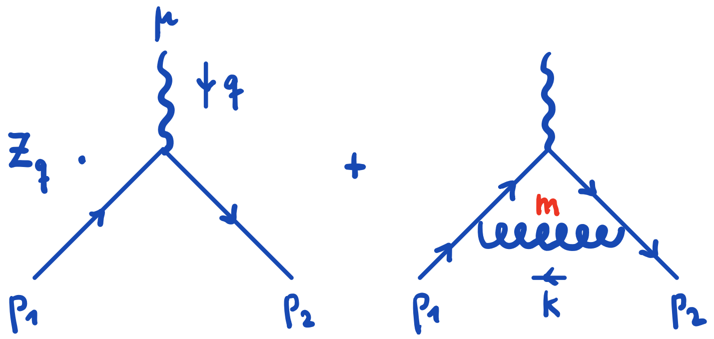
Important examples of these three cases are:

- electroweak Sudakov logarithms ($\sim \alpha_w \ln^2 Q^2/m_W^2$)
- transverse momentum distributions in Drell-Yan production ($pp \to Z + X,\ W + X,\ h + X,\ Z' + X, \dots$) and jet-veto cross sections
- exclusive B-meson decays, such as $B \to K^* \gamma$ or $B \to \pi\pi,\ \pi K, \dots$

In this lecture we first discuss a toy example and then consider the factorization formula for the Drell-Yan boson q_T distribution.

The massive Sudakov form factor

We reconsider the Sudakov form factor, but this time we put the external quarks on shell ($p_1^2 = p_2^2 = 0$) and introduce a gluon mass $m \sim \lambda$ as an IR regulator.
At one-loop order, the relevant contributions are:



$p_1^2 = p_2^2 = 0$

$Q^2 = -q^2 - i0$

$\lambda = \frac{m}{|Q|} \ll 1$

Working in a general R_ξ-gauge, I find that the sum of these contributions is gauge-invariant (at one-loop order). The answer can be written as:

$$\bar{u}(p_2)\, \gamma^\mu\, u(p_1) \cdot F\left(\frac{Q^2}{m^2}\right)$$

where:

$$F(x) = 2\left(1 - \frac{1}{x}\right)^2 \left[\mathrm{Li}_2(1-x) - \frac{\pi^2}{6} \right] + \left(3 - \frac{1}{x}\right) \ln x - \frac{7}{2} + \frac{2}{x}$$

In the limit $Q^2 = -q^2 \gg m^2$, we obtain:

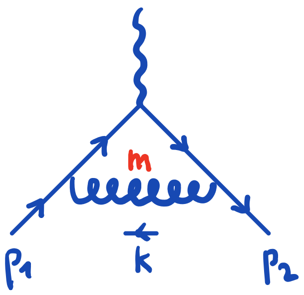
$$\boxed{F\left(\frac{Q^2}{m^2}\right) = -\ln^2 \frac{Q^2}{m^2} + 3 \ln \frac{Q^2}{m^2} - \frac{7}{2} - \frac{2\pi^2}{3} + \mathcal{O}\left(\frac{m^2}{Q^2}\right)}$$

It is instructive to compare this with our result for the off-shell (and massless) Sudakov form factor, for which we found on page 74 (setting $P_1^2 = P_2^2 \equiv P^2$ for simplicity):

$$F\left(\frac{Q^2}{P^2}\right) = -2\ln^2\frac{Q^2}{P^2} + 3\ln\frac{Q^2}{P^2} - c - \frac{2\pi^2}{3} + \mathcal{O}\left(\frac{P^2}{Q^2}\right)$$

Region analysis:

We now perform a region analysis of the massive Sudakov form factor. For simplicity, we work in Feynman gauge ($\xi = 1$). Then the vertex graph gives:



$$= -i\, C_F\, g_s^2\; \bar{u}(p_2)\, \gamma^\alpha \gamma^\xi \gamma^\mu \gamma^\eta \gamma_\alpha\, u(p_1)\; I_{\xi\eta}$$

with:

$$I_{\xi\eta} = \int \frac{d^Dk}{(2\pi)^D} \frac{(k+p_1)_\eta\,(k+p_2)_\xi}{(k^2-m^2+i0)\left[(k+p_1)^2+i0\right]\left[(k+p_2)^2+i0\right]}$$

As usual we assign the scaling $p_1^\mu \sim (\lambda^2, 1, \lambda)\,Q$ and $p_2^\mu \sim (1, \lambda^2, \lambda)\,Q$ to the external momenta. In principle, we could choose a reference frame where $p_1^\perp = -p_2^\perp = 0$, in which case $p_1^\mu \sim (0,1,0)$ and $p_2^\mu \sim (1,0,0)$. In either case, we define:

$$p_{1-}^\mu = \bar{n}\cdot p_1\,\frac{n^\mu}{2} \sim \lambda^0 \,, \qquad p_{2+}^\mu = n\cdot p_2\,\frac{\bar{n}^\mu}{2} \sim \lambda^0$$

Hard region: $k^\mu \sim (1,1,1)\,Q$

We find:

$$I^h_{g\eta} = \int \frac{d^Dk}{(2\pi)^D} \frac{(k+p_{1-})_\eta\,(k+p_{2+})_g}{(k^2+i0)\,[(k+p_{1-})^2+i0]\,[(k+p_{2+})^2+i0]} \sim \lambda^0$$

Clearly this is a leading-order ($\sim\lambda^0$) contribution. Evaluating the integral, I find the following contribution to the Sudakov form factor:

$$\delta F_h(Q^2,\mu) = \frac{C_F\alpha_s}{4\pi}\left(\frac{\mu^2}{Q^2}\right)^\epsilon \left[-\frac{2}{\epsilon^2}-\frac{3}{\epsilon}-8+\frac{\pi^2}{6}+\mathcal{O}(\epsilon)\right]$$

Collinear region: $k^\mu \sim (\lambda^2, 1, \lambda)\,Q$

The contribution from WFR is (like in full QCD):

$$\delta Z_q = \frac{C_F\alpha_s}{4\pi}\left(\frac{\mu^2}{m^2}\right)^\epsilon \left[-\frac{1}{\epsilon}+\frac{1}{2}+\mathcal{O}(\epsilon)\right]$$

Next consider the vertex graph in the collinear region. We obtain:

$$I^c_{g\eta} = \underbrace{\int \frac{d^Dk}{(2\pi)^D}}_{\lambda^4} \frac{\overbrace{\bar{n}\cdot(k+p_1)}^{\lambda^0}\;\overbrace{n\cdot p_2}^{\lambda^0}}{\underbrace{(k^2-m^2+i0)}_{\lambda^2}\,\underbrace{[(k+p_1)^2+i0]}_{\lambda^2}\,\underbrace{[2k\cdot p_{2+}+i0]}_{\lambda^0}}\,\frac{\bar{n}_g n_\eta}{4} \sim \lambda^0$$

This is indeed a leading-order contribution.

Anti-collinear region: $k^\mu \sim (1, \lambda^2, \lambda)\, Q$

This is given by an analogous expression:

$$I^{\bar c}_{\xi\eta} = \int \underset{\lambda^4}{\frac{d^Dk}{(2\pi)^D}} \frac{\overset{\lambda^0}{\bar n\cdot p_1}\;\overset{\lambda^0}{n\cdot(k+p_2)}}{\underset{\lambda^2}{(k^2-m^2+i0)}\,\underset{\lambda^0}{[2k\cdot p_{1-}+i0]}\,\underset{\lambda^2}{[(k+p_2)^2+i0]}}\,\frac{\bar n_\xi n_\eta}{4} \sim \lambda^0$$

Ultra-soft contribution: $k^\mu \sim (\lambda^2, \lambda^2, \lambda^2)\, Q$

In this region $m^2 \gg |k^2|$, and hence we must expand:

$$I^{us}_{\xi\eta} = \int \underset{\lambda^8}{\frac{d^Dk}{(2\pi)^D}} \frac{\overset{\lambda^0}{\bar n\cdot p_1}\;\overset{\lambda^0}{n\cdot p_2}}{\underset{\lambda^2}{(-m^2)}\,\underset{\lambda^2}{[2k\cdot p_{1-}+i0]}\,\underset{\lambda^2}{[2k\cdot p_{2+}+i0]}}\,\frac{\bar n_\xi n_\eta}{4} \sim \lambda^2$$

This is <u>not</u> a leading-order contribution, since the mass term "screens" the much smaller loop momentum. Due to the absence of a quadratic term $\sim k^2$ in the denominator, all poles in the complex k^0-plane lie below the real axis and hence the integral vanishes.

More generally, there are no pinch singularities, because the propagator carrying the ultra-soft loop momentum cannot go on shell. Consequently, the ultra-soft region does not exist!

Soft region: $k^\mu \sim (\lambda, \lambda, \lambda)\, Q$

Let us try to construct something like the ultra-soft region, but with a scaling such that $k^2 \sim m^2 \sim \lambda^2 Q^2$. Expanding the integral under this hypothesis gives:

$$I^{us}_{g\eta} = \underbrace{\int \frac{d^D k}{(2\pi)^D}}_{\lambda^4} \frac{\overbrace{\bar{n}\cdot p_1}^{\lambda^0}\, \overbrace{n \cdot p_2}^{\lambda^0}}{\underbrace{(k^2 - m^2 + i0)}_{\lambda^2} \underbrace{[2k\cdot p_{1-} + i0]}_{\lambda} \underbrace{[2k \cdot p_{2+} + i0]}_{\lambda}} \frac{\bar{n}_\xi n_\eta}{4} \sim \lambda^0$$

This is indeed a leading-order contribution. One can show that this completes the list of the leading regions.

First evaluation of the low-energy contributions:

Evaluating the scalar integrals (without the factor $\frac{\bar{n}_\xi n_\eta}{4}$) in the collinear and soft regions using Feynman parameters, one finds:

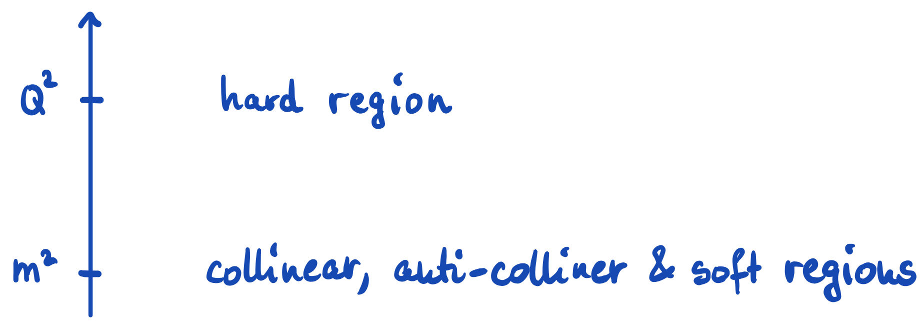
$$I_c = -\frac{i}{(4\pi)^{2-\epsilon}}\, \Gamma(\epsilon)\, m^{-2\epsilon} \int_0^1 dx\, x^{-1}\, (1-x)^{1-\epsilon}$$

$$I_s = -\frac{i}{(4\pi)^{2-\epsilon}}\, \Gamma(\epsilon)\, m^{-2\epsilon} \int_0^\infty d\lambda\, \lambda^{-1}$$

As expected, the low-energy contributions depend on the IR scale $m^2 \ll Q^2$. Surprisingly, however, they contain ill-defined parameter integrals! In other words, the standard dimensional regularization scheme does not allow us to calculate these integrals, even though the original loop integral in full QCD is well defined.

What is going on?

The absence of the ultra-soft region is very puzzling, since it implies that the massive Sudakov form factor is a <u>two-scale problem</u>:



But we have seen earlier that it is <u>impossible</u> to decompose a Sudakov double logarithm into two regions:

$$\ln^2 \frac{Q^2}{m^2} \neq H\left(\frac{Q^2}{\mu^2}\right) + S\left(\frac{m^2}{\mu^2}\right)$$

Indeed, we needed <u>three correlated scales</u> to do so! Somehow, the theory needs to find a way out of this dilemma.

$\boxed{\hookrightarrow \text{ collinear anomaly}}$

Analytic regularization:

To deal with the divergences in the parameter integrals, we introduce analytic regulator by raising the denominators of the Feynman propagators to a power $(1+\delta)$, where $\delta \ll 1$ is an infinitesimal parameter:

new scale parameter

$$\boxed{\frac{1}{p^2 - m^2 + i0} \to -\frac{\nu^{2\delta}}{(-p^2 + m^2 - i0)^{1+\delta}}}$$

(V. A. Smirnov: hep-ph/9703357)

This regulator:

- preserves Lorentz invariance
- breaks gauge invariance, since it corresponds to:

$$\phi^\dagger \left[(iD)^2 - m^2 \right] \phi = \phi^\dagger \left[(i\partial + gA)^2 - m^2 \right] \phi$$

$$\to \phi^\dagger (-\Box - m^2)^{1+\delta} \phi + g A_\mu \left(\phi^\dagger i \overleftrightarrow{\partial}^\mu \phi \right) + g^2 A^2 \phi^\dagger \phi$$

This is not catastrophic, because the regulator is only needed in the various regions, whereas loop integrals in the full theory (= sum of their regions) are always well defined in dimensional regularization.

It follows that we always must expand in δ <u>before</u> we expand in ϵ. The singular terms in the δ-regulator will then cancel in the sum of all regions (at fixed ϵ).

Second evaluation of the low-energy contributions:

It is straightforward to evaluate the scalar loop integrals with analytic regulators in place. I find:

$$I_c = (-1)^3 \int \frac{d^D k}{(2\pi)^D} \frac{\nu^{2(\delta_1+\delta_2+\beta)} \quad \bar{n}\cdot(k+p_1)\; n\cdot p_2}{(-k^2+m^2-i0)^{1+\beta}\,[-(k+p_1)^2-i0]^{1+\delta_1}\,[-2k\cdot p_{2+}-i0]^{1+\delta_2}}$$

$$= -\frac{i}{(4\pi)^{2-\epsilon}} \frac{\Gamma(\epsilon+\delta_1+\beta)}{\Gamma(1+\delta_1)\,\Gamma(1+\beta)}\; m^{-2\epsilon}$$

$$\times \boxed{\left(\frac{Q^2}{\nu^2}\right)^{-\delta_2}} \left(\frac{m^2}{\nu^2}\right)^{-\delta_1-\beta} \underbrace{\int_0^1 dx\; x^{-1+\delta_1-\delta_2}\,(1-x)^{1-\epsilon-\delta_1}}_{\dfrac{\Gamma(\delta_1-\delta_2)\,\Gamma(2-\epsilon-\delta_1)}{\Gamma(2-\epsilon-\delta_2)}}$$

dependence on hard scale Q^2!

singularity removed for $\delta_1 \neq \delta_2$

$I_{\bar{c}}$ = same expression with $\delta_1 \leftrightarrow \delta_2$

and:

$$I_\delta = (-1)^3 \int \frac{d^Dk}{(2\pi)^D} \frac{\nu^{2(\delta_1+\delta_2+\beta)} \quad \bar{n}\cdot p_1 \; n\cdot p_2}{(-k^2+m^2-i0)^{1+\beta}\,[-2k\cdot p_{1-}-i0]^{1+\delta_1}\,[-2k\cdot p_{2+}-i0]^{1+\delta_2}}$$

$$= -\frac{i}{(4\pi)^{2-\epsilon}} \frac{\Gamma(\epsilon+\delta_1+\beta)}{\Gamma(1+\delta_1)\,\Gamma(1+\beta)}\; m^{-2\epsilon}$$

$$\times \left(\frac{Q^2}{\nu^2}\right)^{-\delta_2} \left(\frac{m^2}{\nu^2}\right)^{-\delta_1-\beta} \int_0^\infty d\lambda\, \lambda^{-1+\delta_1-\delta_2}$$

well defined for $\delta_1 \neq \delta_2$, but scaleless

$$\left(\int_0^1 + \int_1^\infty\right) d\lambda\, \lambda^{-1+\delta_1-\delta_2}$$

$$= \frac{1}{\delta_1-\delta_2}\left(\lambda^{\delta_1-\delta_2}\Big|_0^1 + \lambda^{\delta_1-\delta_2}\Big|_1^\infty\right)$$

$(\delta_1>\delta_2)$ $\quad$ $(\delta_1<\delta)$

$$= \frac{1}{\delta_1-\delta_2}(1-1) = 0$$

by analytic continuation

$$= 0 \;!$$

The analytic regulators make the integrals well defined as long as we choose $\delta_1 \neq \delta_2$. But note the important effect that these regulators introduce an anomalous dependence of the low-energy contributions on the hard scale Q^2, which violates strict scale factorization!

$\boxed{\hookrightarrow \text{ collinear anomaly}}$

(Becher, MN: 1007.4005)

Let us see what happens when we expand in the analytic regulators. For the sum of all terms, it does not matter in which order we do this. For concreteness, we first take

$\beta \to 0$, then $\delta_2 \to 0$ and finally $\delta_1 \to 0$. This leaves us with:

$$I_c = -\frac{i}{(4\pi)^{2-\epsilon}} \frac{\Gamma(\epsilon+\delta_1)}{\Gamma(1+\delta_1)} m^{-2\epsilon} \left(\frac{m^2}{\nu^2}\right)^{-\delta_1} \frac{\Gamma(\delta_1)\,\Gamma(2-\epsilon-\delta_1)}{\Gamma(2-\epsilon)}$$

$$\overset{\delta_1 \to 0}{=} -\frac{i}{(4\pi)^{2-\epsilon}} \Gamma(\epsilon)\, m^{-2\epsilon} \left[\frac{1}{\delta_1} + \ln\frac{\nu^2}{m^2} + \psi(\epsilon) - \psi(2-\epsilon)\right]$$

$\psi(\epsilon) = \Gamma'(\epsilon)/\Gamma(\epsilon)$

$$I_{\bar{c}} = -\frac{i}{(4\pi)^{2-\epsilon}} \Gamma(\epsilon)\, m^{-2\epsilon} \left(\frac{Q^2}{\nu^2}\right)^{-\delta_1} \frac{\Gamma(-\delta_1)\,\Gamma(2-\epsilon)}{\Gamma(2-\epsilon-\delta_1)}$$

$$\overset{\delta_1 \to 0}{=} -\frac{i}{(4\pi)^{2-\epsilon}} \Gamma(\epsilon)\, m^{-2\epsilon} \left[-\frac{1}{\delta_1} - \ln\frac{\nu^2}{Q^2} - \gamma_E - \psi(2-\epsilon)\right]$$

Because of the presence of the unphysical (and not gauge-invariant) regulator δ_1, only the sum of these contributions is well defined:

$$\boxed{I_c + I_{\bar{c}} = -\frac{i}{(4\pi)^{2-\epsilon}} \Gamma(\epsilon)\, m^{-2\epsilon} \left[\ln\frac{Q^2}{m^2} + \psi(\epsilon) - \gamma_E - 2\psi(2-\epsilon)\right]}$$

anomalous logarithm

Note:

In cases where the soft contribution is not scaleless, only the sum $I_c + I_{\bar{c}} + I_s$ of all low-scale contributions is well defined.

Using the above result, expanding in ϵ, and adding the WFR contribution, we obtain for the low-scale contributions to the Sudakov form factor:

$$\delta F_{c+\bar{c}}(Q^2,\mu) = \frac{C_F \alpha_s}{4\pi} \left(\frac{\mu^2}{m^2}\right)^{\epsilon} \left[\frac{2}{\epsilon^2} - \frac{2}{\epsilon} \ln\frac{Q^2}{m^2} + \frac{3}{\epsilon} + \frac{9}{2} - \frac{5\pi^2}{6} + O(\epsilon) \right]$$

Adding to this the hard contribution (see page 100)

$$\delta F_h(Q^2,\mu) = \frac{C_F \alpha_s}{4\pi} \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \frac{\pi^2}{6} + O(\epsilon) \right]$$

and the tree-level result, we find:

$$\begin{aligned} F(Q^2, m^2) = 1 + \frac{C_F \alpha_s}{4\pi} \Big[& -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - \frac{2}{\epsilon} \ln\frac{\mu^2}{Q^2} \\ & - \ln^2\frac{\mu^2}{Q^2} - 3\ln\frac{\mu^2}{Q^2} - 8 + \frac{\pi^2}{6} \qquad \text{hard} \\ \text{(anti-) collinear} \qquad & + \frac{2}{\epsilon^2} + \frac{3}{\epsilon} - \frac{2}{\epsilon}\ln\frac{Q^2}{m^2} + \frac{2}{\epsilon}\ln\frac{\mu^2}{m^2} \\ & + \ln^2\frac{\mu^2}{m^2} - 2\ln\frac{Q^2}{m^2}\ln\frac{\mu^2}{m^2} + 3\ln\frac{\mu^2}{m^2} + \frac{9}{2} - \frac{5\pi^2}{6} \Big] \end{aligned}$$

We see that all poles in ϵ as well as all μ-dependent terms cancel out. When the dust settles, we recover exactly the result obtained in the full theory and given on page 98:

$$F\left(\frac{Q^2}{m^2}\right) = -\ln^2\frac{Q^2}{m^2} + 3\ln\frac{Q^2}{m^2} - \frac{7}{2} - \frac{2\pi^2}{3}$$

The above discussion shows how a Sudakov doube log is decomposed into regions in the case of a two-scale problem: the presence of the collinear anomaly spoils a clean scale separation.

Expressed as a matching relation for the QCD vector current, we find in this case:

soft, ~~not~~ ultra-soft!

anomaly

$$\bar{\psi}\gamma^\mu\psi \longrightarrow C_V(Q^2,\mu)\underbrace{\left[(\bar{\xi}_{\bar{n}} W_{\bar{c}})\,\gamma_\perp^\mu\,(S_{\bar{n}}^\dagger S_n)\,(W_c^\dagger \xi_n)\right]}_{\text{no interactions between different sectors, but requires analytic regularization}}{}_{Q^2}(\mu)$$

if present

The three sectors are decoupled in SCET, because:

$c + \bar{c} \sim (\lambda^2, 1, \lambda) + (1, \lambda^2, \lambda) \sim (1,1,\lambda)$ hard

$c + s \sim (\lambda^2, 1, \lambda) + (\lambda, \lambda, \lambda) \sim (\lambda, 1, \lambda)$ "hard-collinear" $(\lambda, 1, \lambda^{1/2})$

Both have virtualities $\gg \lambda^2 Q^2$ and are thus integrated out in the low-energy EFT.

Two-step matching:

integrate out hard collinear modes (if present) or "lower the virtuality"

$$\mathrm{QCD} \longrightarrow \underbrace{\mathrm{SCET}(hc, \overline{hc}, s)}_{\text{SCET with } \lambda \to \lambda^{1/2}} \longrightarrow \boxed{\mathrm{SCET\text{-}2}(c, \bar{c}, s)}$$

integrate out hard modes

(see above)

Note that in our example the hard matching coefficient C_V is simply inherited from SCET, and indeed we find after renormalization:

$$\boxed{C_V(Q^2,\mu) = 1 + \frac{C_F\alpha_s}{4\pi}\left[-\ln^2\frac{Q^2}{\mu^2} + 3\ln\frac{Q^2}{\mu^2} - 8 + \frac{\pi^2}{6}\right]}$$

which coincides with the result we found on page 75. It follows that, as before:

$$\mu\frac{d}{d\mu}C_V(Q^2,\mu) = \left[\Gamma_{cusp}(\alpha_s)\ln\frac{Q^2}{\mu^2} + \gamma_V(\alpha_s)\right]C_V(Q^2,\mu)$$

which in turn implies that:

$$\mu\frac{d}{d\mu}V^\mu_{SCET-2}(\mu) = -\left[\Gamma_{cusp}(\alpha_s)\ln\frac{Q^2}{\mu^2} + \gamma_V(\alpha_s)\right]V^\mu_{SCET-2}(\mu)$$

collinear anomaly

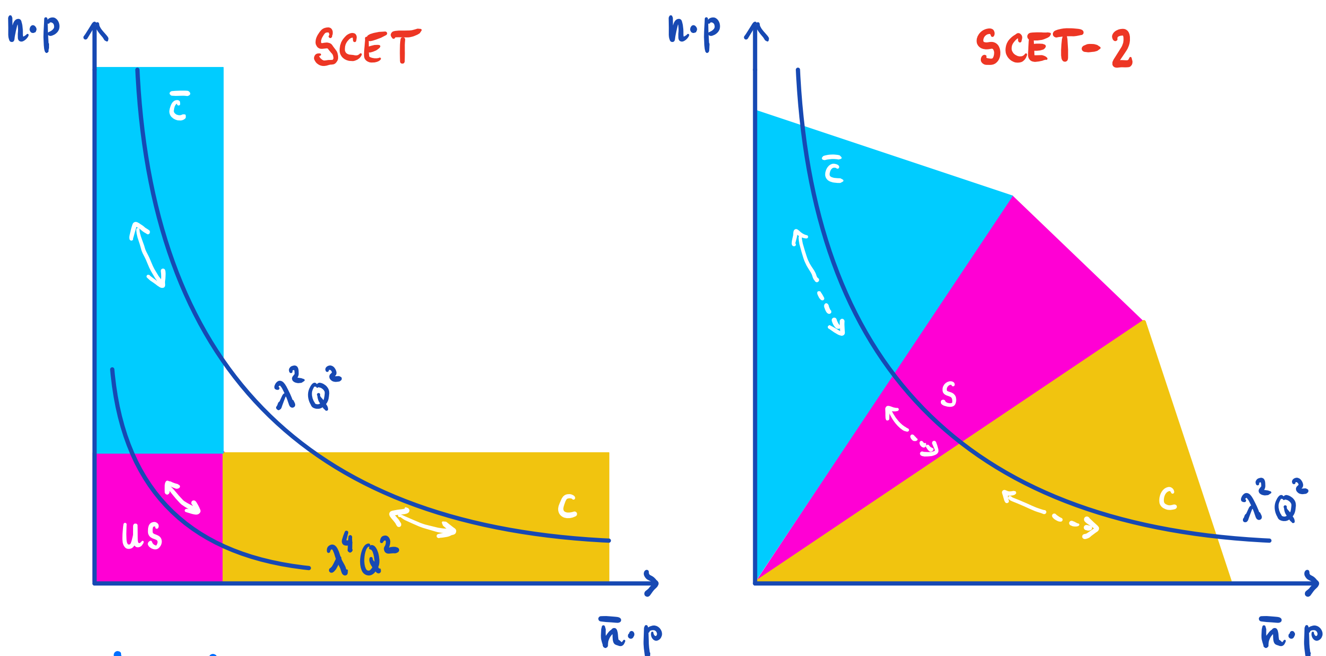
Why "anomaly"?

After decoupling, the different sectors of SCET are each equivalent to full QCD, and they are therefore invariant under separate Lorentz boosts in each sector:

$$\begin{aligned} p^0 &\xrightarrow{LT} p^0\cosh\eta - p^3\sinh\eta \\ p^3 &\longrightarrow p^3\cosh\eta - p^0\sinh\eta \end{aligned} \quad\Rightarrow\quad \begin{aligned} n\cdot p &\xrightarrow{LT} e^{\eta}\, n\cdot p \\ \bar n\cdot p &\longrightarrow e^{-\eta}\,\bar n\cdot p \end{aligned}$$

rapidity

In SCET-2, this separate boost invariance is a classical symmetry of the effective Lagrangian, which is broken by quantum effects (analytic regularization). This is an anomaly in the usual sense of QFT, but in the context of SCET. Overall Lorentz invariance is, of course, not broken, but the same Lorentz boosts must be performed in all sectors of SCET-2.



invariance under separate Lorentz boosts

invariance under global Lorentz boosts

Redefining the auxilliary scale $\nu^2 \to \bar{n}\cdot p_1 \tilde{\nu}$, we have from page 107:

not boost invariant

$$I_c \sim \left(\frac{\bar{n}\cdot p_1 \tilde{\nu}}{m^2}\right)^{\delta_1}$$ depends on collinear scales m^2 and $\bar{n}\cdot p_1$

$$I_{\bar{c}} \sim \left(\frac{\tilde{\nu}}{n\cdot p_2}\right)^{\delta_1}$$ depends on collinear scales m^2 and $n\cdot p_2$

$\tilde{\nu}$-independent ratio

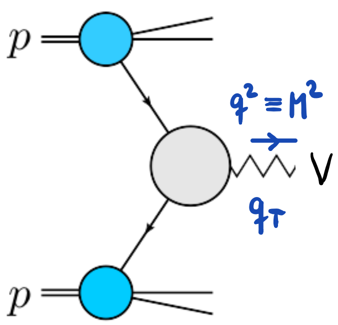
$$\frac{\bar{n}\cdot p_1\, n\cdot p_2}{m^2} = \frac{Q^2}{m^2}$$

is boost invariant!

XII. Drell-Yan Production at Small q_T

(Becher, MN: 1007.4005)

Consider the production of a Drell-Yan boson at small transverse momentum:



$q_T = |\vec{q}_\perp| \ll M$

$(\lambda = q_T/M)$

Interesting cases:

$V = \gamma^* \to \ell^+\ell^-$

$W^\pm, Z^0$, also Higgs scalar (gg initiated)

$Z' \to \mu^+\mu^-$

...

We consider the case of a virtual photon for concreteness.

(↪ see Becher, MN, Wilhelm: 1212.2621 for $pp \to h$ case)

Using SCET technology, one finds the same regions as in the case of the massive Sudakov form factor: collinear, anti-collinear, and no soft region (scaleless). The ultra-soft region cancels out to all orders. In the limit where $q_T \ll M$, one can derive the factorization formula:

$$\boxed{\begin{aligned}\frac{d^3\sigma}{dM^2\, dq_T^2\, dy} &= \frac{4\pi\alpha^2}{3N_c M^2 s}\, |C_V(-M^2,\mu)|^2\, \frac{1}{4\pi} \int d^2x_\perp\, e^{-iq_\perp\cdot x_\perp} \\ &\times \sum_q e_q^2 \left[\mathcal{B}_{q/N_1}(\xi_1, x_T^2,\mu)\, \mathcal{B}_{\bar q/N_2}(\xi_2, x_T^2,\mu) + (q \leftrightarrow \bar q)\right] + \mathcal{O}\left(\frac{q_T^2}{M^2}\right)\end{aligned}}$$

↪ (naive) factorization in x_T space

Here C_V is the hard matching coefficient of the vector current (given above), and

$$\mathcal{B}_{q/N}(z,x_T^2,\mu) = \frac{1}{2\pi}\int dt\, e^{-izt\bar{n}\cdot p}\,\langle N(p)|\,\bar{\chi}(t\bar{n}+x_\perp)\,\frac{\bar{\not{n}}}{2}\,\overset{(W_c^\dagger \xi_n)}{\chi(0)}\,|N(p)\rangle \quad \text{(naively!)}$$

are generalized, x_T-dependent PDFs. (For $x_T = |\vec{x}_\perp| = 0$ one recovers the standard PDFs.) The relevant kinematic variables in the expression for the cross section are:

y: rapidity of V in the LAB frame

$\xi_1 = \sqrt{\tau}\, e^{y}$, $\xi_2 = \sqrt{\tau}\, e^{-y}$ with $\tau = \dfrac{M^2 + q_T^2}{s}$

As defined above, the x_T-dependent PDFs appear to be universal (i.e. process-independent) functions characterizing the nucleon N. However, this would be in conflict with the RG invariance of the cross section! In order to cancel the μ-dependence of $|C_V|^2$, the product of the two x_T-dependent PDFs must depend on the hard scale M^2, due to the collinear anomaly:

$$\left[\mathcal{B}_{q/N_1}(z_1,x_T^2,\mu)\,\mathcal{B}_{\bar{q}/N_2}(z_2,x_T^2,\mu)\right] = \left[\mathcal{B}_{q/N_1}(z_1,x_T^2,\mu)\,\mathcal{B}_{\bar{q}/N_2}(z_2,x_T^2,\mu)\right]_{q^2}$$

anomalous dependence on $q^2 = M^2$

Only this product of PDFs is unambiguously defined, and it carries a <u>process dependence</u> via the anomalous dependence on the hard scale q^2. The cross-section formula shown above thus does not achieve a complete scale separation.

Can we control the q^2-dependence of the product of PDFs?

Introducing analytic regulators in the same way as for the Sudakov form factor, one can show that:

$$\left[\ln \mathcal{B}_{q/N_1}(z_1, x_T^2, \mu)\, \mathcal{B}_{\bar{q}/N_2}(z_2, x_T^2, \mu)\right]_{q^2}$$

$$= \ln \mathcal{B}_{q/N_1}(\ln \nu^2 x_T^2; z_1, x_T^2, \mu) + \ln \mathcal{B}_{\bar{q}/N_2}\left(\ln \frac{\nu^2}{q^2}; z_2, x_T^2, \mu\right)$$

This is analogous to the results on p.107, once we substitute $m^2 \to x_T^{-2}$ and $Q^2 \to q^2$. The condition that the unphysical regulators drop out requires that, to all orders in perturbation theory, the two terms must be <u>linear</u> in $\ln \nu^2$, i.e.:

$$\left[\ln \mathcal{B}_{q/N_1}(z_1, x_T^2, \mu)\, \mathcal{B}_{\bar{q}/N_2}(z_2, x_T^2, \mu)\right]_{q^2}$$

$$= \ln \tilde{\mathcal{B}}_{q/N_1}(z_1, x_T^2, \mu) + \ln \tilde{\mathcal{B}}_{\bar{q}/N_2}(z_2, x_T^2, \mu) + \ln(x_T^2 q^2)\, F_{q\bar{q}}(x_T^2, \mu)$$

$$\left(\ln \nu^2 x_T^2 - \ln \frac{\nu^2}{q^2}\right)$$

We thus obtain the exact refactorization relation:

$$\boxed{\left[\mathcal{B}_{q/N_1}(z_1,x_T^2,\mu)\,\mathcal{B}_{\bar q/N_2}(z_2,x_T^2,\mu)\right]_{q^2} = \left(\frac{x_T^2 q^2}{4e^{-2\gamma_E}}\right)^{-F_{q\bar q}(x_T^2,\mu)} \widetilde{B}_{q/N_1}(z_1,x_T^2,\mu)\,\widetilde{B}_{\bar q/N_2}(z_2,x_T^2,\mu)}$$

The properly factorized cross section now takes the form:

$$\boxed{\begin{aligned}\frac{d^3\sigma}{dM^2\,dq_T^2\,dy} &= \frac{4\pi\alpha^2}{3N_cM^2 s}\left|C_V(-M^2,\mu)\right|^2 \frac{1}{4\pi}\int d^2x_\perp\, e^{-iq_\perp\cdot x_\perp}\left(\frac{x_T^2 M^2}{4e^{-2\gamma_E}}\right)^{-F_{q\bar q}(x_T^2,\mu)} \\ &\quad\times \sum_q e_q^2\left[\widetilde{B}_{q/N_1}(\xi_1,x_T^2,\mu)\,\widetilde{B}_{\bar q/N_2}(\xi_2,x_T^2,\mu) + (q\leftrightarrow\bar q)\right] + \mathcal{O}\!\left(\frac{q_T^2}{M^2}\right)\end{aligned}}$$

anomalous dependence on hard scale M^2

The scale independence of the cross section implies the exact RG evolution equations (cf. page 110):

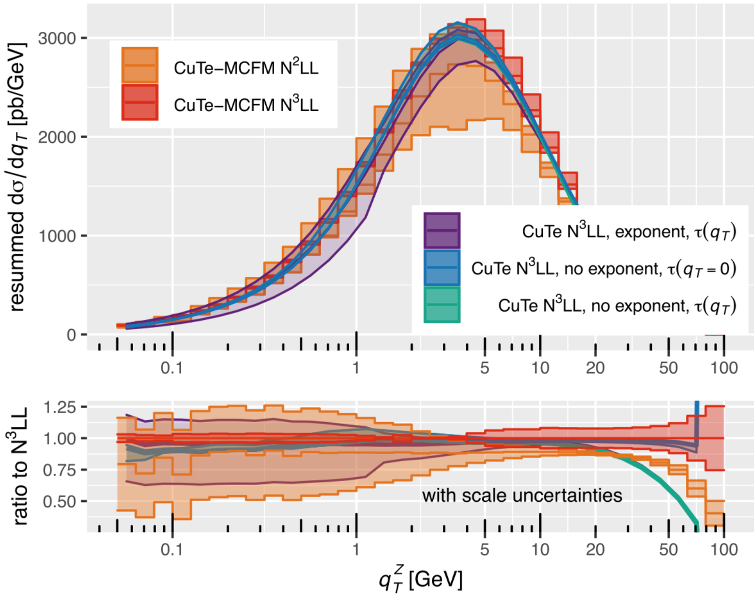
$$\boxed{\begin{aligned}\frac{dF_{q\bar q}(x_T^2,\mu)}{d\ln\mu} &= 2\Gamma^F_{\text{cusp}}(\alpha_s)\,, \\ \frac{d}{d\ln\mu}\widetilde{B}_{q/N}(z,x_T^2,\mu) &= \left[\Gamma^F_{\text{cusp}}(\alpha_s)\ln\frac{x_T^2\mu^2}{4e^{-2\gamma_E}} - \overbrace{2\gamma^q(\alpha_s)}^{\gamma_V(\alpha_s)}\right]\widetilde{B}_{q/N}(z,x_T^2,\mu)\end{aligned}}$$

Note that the new functions $\widetilde{B}_{q/N}(z,x_T^2,\mu)$ are now independent of the hard scale q^2. They can be defined to be the (universal) x_T-dependent PDFs. For $x_T \ll \Lambda_{QCD}^{-1}$, these functions can be obtained from conventional PDFs using perturbation theory:

$$\widetilde{B}_{q/N}(\xi,x_T^2,\mu) = \sum_j \int_0^1 \frac{dz}{z}\, \underbrace{I_{q\leftarrow j}(z,x_T^2,\mu)}_{\text{perturbative}}\; \underbrace{\phi_{j/N}(\xi/z,\mu)}_{\text{PDFs}}$$

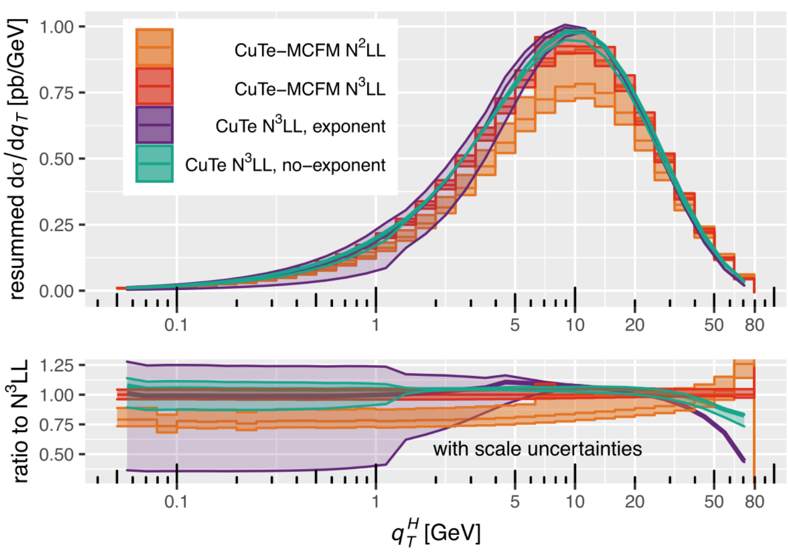
State-of-the-art predictions:

(Becher, Neumann: 2009.11437) (Becher, MN: 1007.4005)



Z

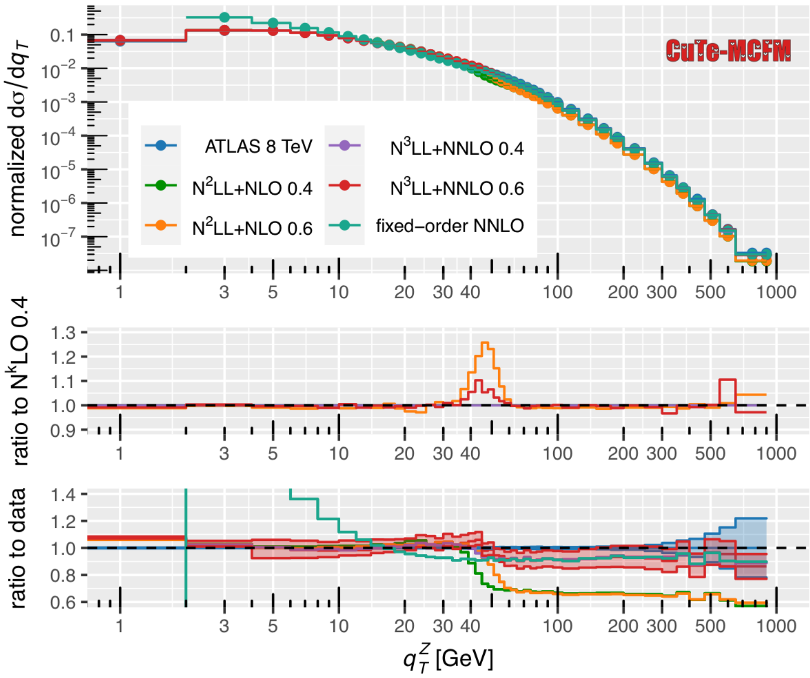
Resummed results without matching for inclusive Z production at 13 TeV obtained using CuTe-MCFM and CuTe at different logarithmic orders. For CuTe we show results in two schemes: expanded in the exponent or on the level of the cross section. We furthermore present results with two different treatments of power-suppressed terms in the phase space related to the choice of $\tau(q_T)$, see text. The shaded band displays scale uncertainties. The bottom panel shows the ratio to the N^{3}LL result in CuTe-MCFM.



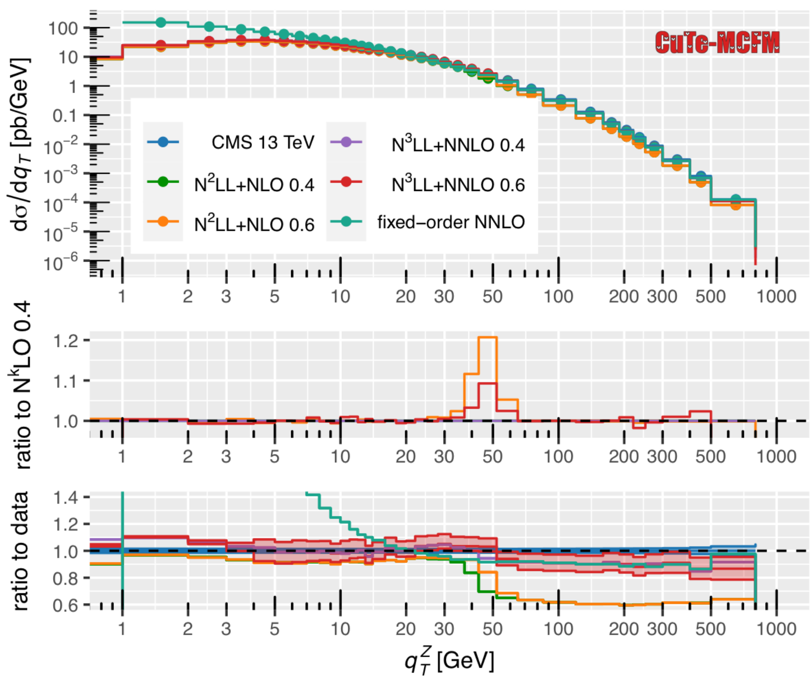
h

Resummed results without matching for inclusive Higgs production at 13 TeV obtained using CuTe-MCFM and CuTe at different logarithmic orders. For CuTe we show results in two schemes: expanded in the exponent or on the level of the cross section. The shaded bands display scale uncertainties. The bottom panel shows the ratio to the N^{3}LL result in CuTe-MCFM.

Z

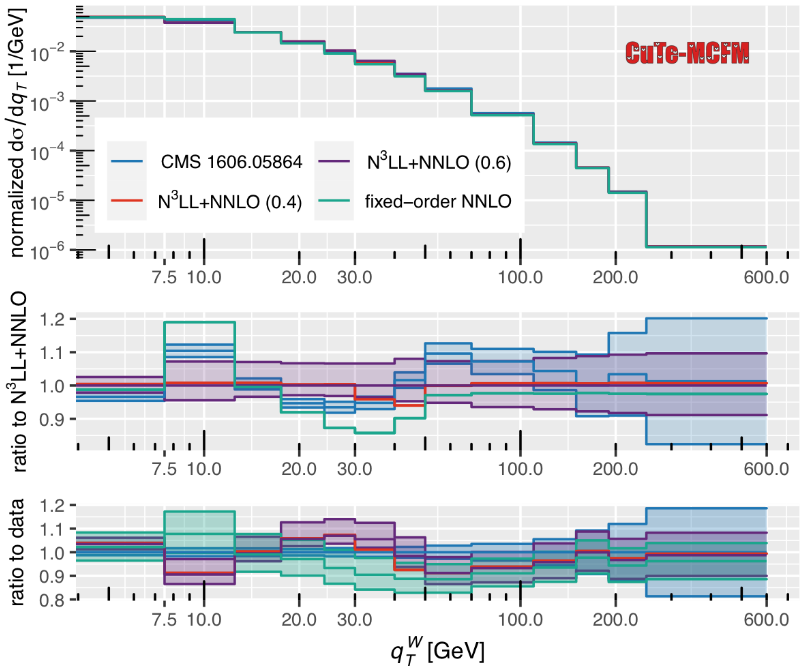


Predicted and measured normalized transverse-momentum distribution of the Z boson with fiducial cuts as in the ATLAS study at 8 TeV in ref. [93]. The middle panel shows the effect of varying the transition function, while the bottom panel shows the ratio to data with estimated scale uncertainties.



Predicted and measured transverse-momentum distribution of the Z boson with fiducial cuts as in the CMS study [94] at 13 TeV. The middle panel shows the effect of varying the transition function, while the bottom panel shows the ratio to data with estimated scale uncertainties.

W



11: Comparison to normalized W transverse-momentum data from CMS at 8 TeV with predictions at N^{3}LL+NNLO including uncertainties associated with scale variation.

The End