I. Motivation

Multi-scale problems are abundant in physics but difficult to deal with → effective theories reduce them to problems involving fewer scales

EFT: modern tool to achieve scale separation in QFT

→ reduce multi-scale problems to a sequence of single-scale problems

→ RGEs allow for systematic resummation of large logarithms of scale ratios; particularly important in QCD, where running of $\alpha_s(\mu)$ is significant and $\alpha_s \ln(Q_1/Q_2)$ can be large if $Q_1 \gg Q_2$

Scale separation is the basis of factorization theorems

→ crucial for separation of short-distance from long-distance physics in QCD ($\sigma_{LHC} = \sigma_{partons} \otimes \text{PDFs}$)

→ in strongly interacting theories, long-distance dofs can be different from short-distance dofs, so scale separation becomes a necessity (quarks & gluons vs. hadrons, electrons vs. Cooper pairs, etc.)

In a perturbative context, use of EFTs to resum large logarithms is often an important focus

→ having a systematic approach can be an advantage over diagram-based resummation techniques

→ allows for RG-based resummations even in very complicated cases (e.g. NGLs, SLLs; see 1508.06645, 1605.02737 and 2107.01212)

EFT provides important new perspectives on QFTs

→ "renormalizability" no longer a paradigm, but a consequence (Wilsonian viewpoint)

→ importance of "naturalness"

→ once we realize that a physical theory is not the "theory of everything", we should consider it as an EFT, e.g.:

$$\mathrm{SM} \longrightarrow \mathrm{SMEFT}$$

↳ same dofs, but SMEFT contains more physics: neutrino masses and mixings, many more (weak) interaction ($D \geq 5$), less global symmetries ($\not{B}, \not{L}$)

Why should you learn about SCET?

- SCET is the EFT for high-energy processes involving light particles → relevant for collider physics and heavy-quark physics

- very powerful, but probably the most complex EFT ever developed → full of subtleties, but physics can be subtle (e.g. "collinear anomaly")

- originally developed to understand factorization theorems in B physics, e.g.:

  $B \to X_s \gamma$, $B \to X_u \ell \bar{\nu}$ (inclusive decays to light part.)

  $B \to D\pi$, $B \to \pi\pi$ (exclusive nonleptonic decays)

  ↳ QCD factorization approach (BBNS: hep-ph/9905312, 0006124, 0104110) solved a problem that was intractable before

- Later, SCET found many applications outside flavor physics, in particular: collider physics, dense QCD matter, hadron physics, scattering amplitudes, DM phenomenology, SYM theory, BSM physics, ...

↳ series of annual workshops since 2003

II. Effective Field Theory in a Nutshell

Consider a QFT with a large fundamental scale M (e.g. the mass of a heavy particle, but also a large Euclidean momentum transfer). Suppose we are interested in experiments at energies $E \ll M$. How can we systematically expand S-matrix elements in powers of E/M?

Step 1: choose cutoff $\mu \lesssim M$ ("threshold of ignorance") and split up fields into high-frequency and low-frequency modes (in Fourier space):

$$\underbrace{\phi = \phi_L}_{\omega < \mu} + \underbrace{\phi_H}_{\omega > \mu}$$

→ physics at energies below μ can be described in terms of correlation functions of fields ϕ_L:

$$\langle 0 | T\{\phi_L(x_1) \ldots \phi_L(x_n)\} | 0 \rangle$$

$$= \frac{1}{Z[0]} \left(-i \frac{\delta}{\delta J_L(x_1)}\right) \ldots \left(-i \frac{\delta}{\delta J_L(x_n)}\right) Z[J_L] \bigg|_{J_L = 0}$$

where:

$$Z[J_L] = \int \mathcal{D}\phi_L \, \mathcal{D}\phi_H \; e^{\,i \overbrace{S[\phi_L, \phi_H]}^{\text{action } \int d^Dx \, \mathcal{L}(x)} + i \int d^Dx \, J_L(x) \, \phi_L(x)}$$

no need for sources J_H

Step 2: "integrate out" the high-frequency fields by performing the functional integral over ϕ_H:

$$Z[J_L] \equiv \int \mathcal{D}\phi_L \; e^{i S_\mu[\phi_L] + i \int d^D x \; J_L(x)\, \phi_L(x)}$$

where:

$$\boxed{e^{i S_\mu[\phi_L]} = \int \mathcal{D}\phi_H \; e^{i S[\phi_L, \phi_H]}}$$

"Wilsonian effective action"

- by construction, $S_\mu[\phi_L]$ depends on the choice of μ
- $S_\mu[\phi_L]$ is non-local on scales $\Delta x_\mu \sim \frac{1}{\mu}$ due to propagators of high-frequency modes
- in addition to heavy particles with masses $m_i > \mu$, also the high-frequency modes of light particles are integrated out!

Step 3: expand the non-local action functional in terms of local operators composed of the fields ϕ_L

(possible since $E \ll \mu \longrightarrow$ resolution $\delta x \sim \frac{1}{E} \gg \Delta x_\mu$):

$$S_\mu[\phi_L] \equiv \int d^D x \; \mathcal{L}_\mu^{eff}(x)$$

"Wilsonian OPE"

with: $$\boxed{\mathcal{L}_\mu^{eff}(x) = \sum_i g_i(\mu)\, Q_i(\phi_L(x), \mu)}$$

"effective Lagrangian"

→ infinite sum over local operators Q_i multiplied by couplings g_i called "Wilson coefficients"

→ in general, all operators allowed by the symmetries of the problem (e.g. Poincaré invariance, gauge invariance, global symmetries) can appear in the sum

How can such an effective Lagrangian be predictive?

Step 4: derive EFT power counting in $\lambda = E/M \ll 1$

→ in simplest case (but not in general), this follows from naive dimensional analysis (NDA)

NDA counting rules:

- let $[g_i] = -\gamma_i$ denote the mass dimension of g_i, i.e.:

$$g_i = C_i \, M^{-\gamma_i}$$

dimensionless, "naturalness" suggests $C_i = \mathcal{O}(1)$ (otherwise there should be a reason why $|C_i| \ll 1$ or $|C_i| \gg 1$)

- matrix elements $\langle Q_i \rangle$ in the EFT scale with powers of the characteristic energy E; hence, for an observable O with $[O] = \Delta$ we get:

$$O = \sum_i g_i \langle Q_i \rangle \sim E^\Delta \sum_i c_i \left(\frac{E}{M}\right)^{\gamma_i}$$

→ individual terms in the sum are:

$$c_i \left(\frac{E}{M}\right)^{\gamma_i} = \begin{cases} \mathcal{O}(1) & ; \quad \gamma_i = 0 \\ \ll 1 & ; \quad \gamma_i > 0 \\ \gg 1 & ; \quad \gamma_i < 0 \end{cases}$$

- since $[S] = 0$, it follows that $[\mathcal{L}] = D$ ($= 4$ in most cases) and hence:

$$[Q_i] = D + \gamma_i \equiv \delta_i$$

→ only operators with mass dimension $\delta_i \leq D$ give unsuppressed contributions at low energies!

→ this makes EFTs useful, because for a given theory only a relatively small number of such operators exist

→ in general: the larger $\delta_i = [Q_i]$ is, the more suppressed is the contribution of Q_i

Dimension δ_i	Importance for $E \to 0$	Terminology
$< D$	grows	relevant operator (super-renormalizable)
$= D$	~ constant (logarithmic)	marginal operator (renormalizable)
$> D$	falls	irrelevant operator (non-renormalizable)

Comments:

- "relevant" operators are usually forbidden by means of a symmetry, otherwise they give rise to a naturalness problem (cosmological constant problem, hierarchy problem)
- "marginal operators are all there is in "renormalizable" QFTs
- "irrelevant" operators are the most interesting ones, because they teach us something about the physics at the scale M (Fermi constant, neutrino masses, SMEFT fits ...)

- in practise, one truncates the sum over operators at some maximum value of δ_i set by the precision goal (typically $\delta_{max} = 5$ or 6 for $D=4$)

  → finite operator basis, which can be constructed in a straightforward way

  → an EFT with such a truncation is always renormalizable!

- if the theory above μ is perturbative, the Wilson coefficients $C_i(\mu)$ can be derived from a perturbative matching procedure, by requiring a finite set of matrix elements in the full theory and in the EFT to agree up to higher-order power corrections, which are neglected in the EFT

- above construction also works if the full theory is unknown, like for SMEFT

  → treat $C_i(\mu)$ as unknown parameters, which must be extracted from a fit to exp. data

Final comment:

The construction of an EFT becomes significantly more complicated in cases where the large scale M remains as a parameter in the EFT, characterizing the large energies of light particles. This is precisely what happens in SCET.

In this case, as we will see, the operators in the effective Lagrangian (step 3) are non-local on large scales. These non-localities are along light-like directions. They are a characteristic feature of SCET.