Reparameterization Invariance for Collinear Operators

Aneesh V. Manohar,[1] Thomas Mehen,[2,3] Dan Pirjol,[1] and Iain W. Stewart[1,4]

[1]*Department of Physics, University of California at San Diego, La Jolla, CA 92093* [*]

[2]*Department of Physics, Duke University, Durham NC 27708* [†]

[3]*Jefferson Laboratory 12000 Jefferson Ave. Newport News VA 23606*

[4]*Institute for Nuclear Theory, University of Washington, Seattle, WA 98195* [‡]

Abstract

We discuss restrictions on operators in the soft-collinear effective theory (SCET) which follow from the ambiguity in the decomposition of collinear momenta and the freedom in the choice of light-like basis vectors n and $\bar{n}$. Invariance of SCET under small changes in n and/or $\bar{n}$ implies a symmetry of the effective theory that constrains the form of allowed operators with collinear fields. The restrictions occur at a given order in the power counting as well as between different orders. As an example, we present the complete set of higher order operators that are related to the collinear quark kinetic term.

[*] Electronic address: manohar@wigner.ucsd.edu, pirjol@bose.ucsd.edu

[†] Electronic address: mehen@phy.duke.edu

[‡] Electronic address: iain@phys.washington.edu

Strong interaction processes involving highly energetic particles can be described within an effective field theory framework known as the Soft-Collinear Effective Theory (SCET) [1, 2, 3, 4]. SCET has been used to simplify proofs of classical factorization theorems [5], provide the first all-orders proof of factorization in $\bar{B} \to D\pi$ decays [6], and facilitate the resummation of Sudakov logarithms in B and Υ decays [1, 2, 7].

Suppose a set of light hadrons are created in a hard-scattering process. The hadrons are assumed to have a large energy $Q \gg \Lambda_{\text{QCD}}$, and invariant mass $\sim \Lambda_{\text{QCD}}$. To a good approximation each light hadron is composed of constituents nearly collinear to a light-like vector n. To decompose collinear momenta it is necessary to define an auxiliary light-like vector for an orthogonal direction, $\bar{n}$, such that $n \cdot \bar{n} = 2$. If for example, n is along the positive z-axis, $n^\mu = (1, 0, 0, 1)$, then one could choose $\bar{n}^\mu = (1, 0, 0, -1)$ or equally well $\bar{n}^\mu = (3, 2, 2, 1)$. Since the perpendicular size of the hadron is $\sim 1/\Lambda_{\text{QCD}}$ the momentum P^μ of a collinear constituent is $(n \cdot P, \bar{n} \cdot P, P_\perp) \sim Q(\lambda^2, 1, \lambda)$, where $\lambda \sim \Lambda_{\text{QCD}}/Q$. This scaling holds for either choice of $\bar{n}$ above. The SCET provides a systematic way of dealing with the disparate scales $Q \gg \Lambda_{\text{QCD}} \gg (\Lambda_{\text{QCD}})^2/Q$. The momentum P^μ of a fast particle is decomposed as the sum of a large momentum p^μ with $\bar{n} \cdot p \sim \lambda^0$, $p_\perp^\mu \sim \lambda$, and $n \cdot p = 0$ and a smaller momentum $k^\mu \sim \lambda^2$:

$$P^\mu = p^\mu + k^\mu = \frac{n^\mu}{2} \, \bar{n} \cdot (p + k) + \frac{\bar{n}^\mu}{2} \, n \cdot k + (p_\perp^\mu + k_\perp^\mu) . \tag{1}$$

The large momentum p is treated as a label on collinear fields, and the small residual momentum k is associated with the spatial variation of the fields. In this paper we show that requiring invariance under the ambiguity in the decomposition in Eq. (1) has important consequences for collinear operators in SCET. As examples, we show that this reparameterization invariance places important restrictions on the form of the leading order collinear quark action, and fixes the anomalous dimensions of an infinite class of subleading terms.

The decomposition in Eq. (1) is similar to the one in heavy quark effective theory (HQET). In HQET $P^\mu = mv^\mu + k^\mu$, where m is the heavy quark mass, the velocity v^μ labels HQET fields $h_v(x)$, and k^μ is a residual momentum picked out by derivatives on h_v. The ambiguity in the decomposition of P^μ leads to a reparameterization invariance [8]. This symmetry is the remnant of invariance under the Lorentz generators $v_\mu M^{\mu\nu}$ which were broken by the introduction of the vector v_μ (for the rest frame these generators are the boosts $M^{0i} = K^i$). Requiring that physics is invariant under the simultaneous change

$$v_\mu \to v_\mu + \frac{\Delta_\mu}{m} , \qquad k_\mu \to k_\mu - \Delta_\mu \qquad (v \cdot \Delta = 0) \tag{2}$$

gives useful constraints on the form of the HQET Lagrangian and currents [8, 9, 10, 11, 12, 13, 14].

In SCET reparameterization invariance is more involved, because the collinear momentum decomposition in Eq. (1) has a more complicated structure. In particular, HQET reparameterization invariance only connects operators appearing at different orders in the $1/m$ expansion, while we will see that the counterpart in SCET also constrains the form of operators appearing at any given order. From Eq. (1) two types of ambiguity are:

(a) The component decompositions, $\bar{n} \cdot (p + k)$ and $(p_\perp^\mu + k_\perp^\mu)$, are arbitrary by an order $Q\lambda^2$ amount, and any decomposition should yield an equivalent description.

(b) Any choice of the reference light-cone vectors n and $\bar{n}$ satisfying

$$n^2 = 0, \qquad \bar{n}^2 = 0, \qquad n \cdot \bar{n} = 2, \tag{3}$$

are equally good, and can not change physical predictions.

For type (b) the most general infinitesimal change in n and $\bar{n}$ which preserves Eq. (3) is a linear combination of

$$\text{(I)} \begin{cases} n_\mu \to n_\mu + \Delta_\mu^\perp \\ \bar{n}_\mu \to \bar{n}_\mu \end{cases} \qquad \text{(II)} \begin{cases} n_\mu \to n_\mu \\ \bar{n}_\mu \to \bar{n}_\mu + \varepsilon_\mu^\perp \end{cases} \qquad \text{(III)} \begin{cases} n_\mu \to (1+\alpha)\, n_\mu \\ \bar{n}_\mu \to (1-\alpha)\, \bar{n}_\mu \end{cases}, \tag{4}$$

where $\{\Delta_\mu^\perp, \epsilon_\mu^\perp, \alpha\}$ are five infinitesimal parameters, and $\bar{n}\cdot\varepsilon^\perp = n\cdot\varepsilon^\perp = \bar{n}\cdot\Delta^\perp = n\cdot\Delta^\perp = 0$. Invariance under subset (I) of these transformations has already been explored in Ref. [15], and used to derive important constraints on the next-to-leading order collinear Lagrangian and heavy-to-light currents. Here we explore the consequences of invariance under the full set of reparameterization transformations and extend the analysis of class (I) transformations to higher orders in λ. In particular we show that the transformations in classes (II) and (III) are necessary to rule out the possibility of additional operators in the lowest order collinear Lagrangian that are allowed by power counting and gauge invariance.

As might be expected the collinear reparameterization invariance is a manifestation of the Lorentz symmetry that was broken by introducing the vectors n and $\bar{n}$. Essentially reparameterization invariance restores Lorentz invariance to SCET order by order in λ. The five parameters in Eq. (4) correspond to the five generators of the Lorentz group which are "broken" by introducing the vectors n and $\bar{n}$, namely $\{n_\mu M^{\mu\nu}, \bar{n}_\mu M^{\mu\nu}\}$. If the perpendicular directions are $1, 2$ then the five broken generators are $Q_1^\pm = J_1 \pm K_2$, $Q_2^\pm = J_2 \pm K_1$, and K_3. The type (I) transformations are equivalent to the combined actions of an infinitesimal boost in the x (y) direction and a rotation around the y (x) axis, such that $\bar{n}_\mu$ is left invariant with generators (Q_1^-, Q_2^+). Type (II) transformations are similar but (Q_1^+, Q_2^-) leave n_μ invariant, while transformation (III) is a boost along the 3 direction (K_3).

In SCET one introduces three classes of fields: collinear, soft and ultrasoft (usoft), with momentum scaling as $Q(\lambda^2, 1, \lambda)$, $Q(\lambda, \lambda, \lambda)$ and $Q(\lambda^2, \lambda^2, \lambda^2)$, respectively. For our purposes the interesting fields are those for collinear quarks ($\xi_{n,p}$), collinear gluons ($A_{n,q}$), and usoft gluons (A_u). At tree level the transition from QCD to collinear quark fields can be achieved by a field redefinition [2]

$$\psi(x) = \sum_p e^{-ip\cdot x} \left[1 + \frac{1}{\bar{n}\cdot\mathcal{D}} \, \mathcal{D}^\perp \, \frac{\slashed{\bar{n}}}{2} \right] \xi_{n,p}, \tag{5}$$

where the two-component collinear quark field ξ_n satisfies [1]

$$\frac{\slashed{n}\slashed{\bar{n}}}{4} \, \xi_n = \xi_n, \qquad \slashed{n} \, \xi_n = 0. \tag{6}$$

The covariant derivatives are further decomposed into two parts, $\mathcal{D}^\mu = D_c^\mu + D_u^\mu$, where D_c^μ and D_u^μ involve collinear and usoft momenta and gauge fields respectively. To distinguish

the size of derivatives it is convenient to introduce label operators, $\bar{\mathcal{P}}$ and $\mathcal{P}_\perp$, which pick out the labels of collinear fields [3]. For instance, $\bar{\mathcal{P}}\xi_{n,p} = \bar{n}\cdot p\,\xi_{n,p}$. The label operators allow sums and phases involving label momenta to be suppressed. Under gauge transformations the usoft gluons act like a background field and collinear gauge invariance ensures that only the linear combinations [4]

$$i\bar{n} \cdot D_c = \bar{\mathcal{P}} + g\bar{n} \cdot A_{n,q}, \qquad iD_c^\perp = \mathcal{P}^\perp + gA_{n,q}^\perp, \qquad in \cdot \mathcal{D} = in \cdot D_u + gn \cdot A_{n,q} \qquad (7)$$

appear. Each term in Eq. (7) is the same order in λ. It is often convenient to swap the order λ^0 field $\bar{n} \cdot A_{n,q}(x)$ for the Wilson line

$$W = \Big[\sum_{\text{perms}} \exp\Big(-\frac{g}{\bar{\mathcal{P}}}\,\bar{n}\cdot A_{n,q}(x) \Big)\Big] = \Big[\frac{1}{i\bar{n}\cdot D_c}\,\bar{\mathcal{P}}\Big], \qquad (8)$$

where the differential operators do not act outside the square brackets, and $[i\bar{n} \cdot D_c\, W] = 0$.

Now consider the constraints imposed by reparameterization transformations of type (a) in Eq. (1). These act only on the components of label momenta, so

$$\xi_{n,p}(x) \to e^{i\beta\cdot x}\,\xi_{n,p+\beta}(x)\,, \qquad A_{n,q}(x) \to e^{i\beta\cdot x}\,A_{n,q+\beta}(x)\,, \qquad (9)$$

where the label and phase changes compensate each other. Expressed in terms of $\mathcal{P}^\mu = \frac{1}{2}n^\mu\bar{\mathcal{P}} + \mathcal{P}_\perp^\mu$, and $i\partial_\mu$ the transformation in Eq. (9) essentially causes

$$\mathcal{P}_\mu \to \mathcal{P}_\mu + \beta_\mu\,, \qquad i\partial_\mu \to i\partial_\mu - \beta_\mu\,, \qquad (10)$$

with $\beta_\mu = \bar{n}\cdot\beta\, n_\mu/2 + \beta_\mu^\perp$. This implies that reparameterization invariant operators must be built out of the linear combination $\mathcal{P}_\mu + i\partial_\mu$. Usoft gauge invariance forces $i\partial_\mu$ to appear along with an usoft gauge field in the combination $i\partial^\mu + gA_u^\mu$. Furthermore, from Eq. (7) collinear gauge invariance forces collinear fields to appear along with $\mathcal{P}_\mu$ and $n\cdot\partial$. Thus, reparameterization invariant operators should be built out of

$$\mathcal{D}^\mu \equiv D_c^\mu + D_u^\mu\,. \qquad (11)$$

Since the components of D_c and D_u scale in different ways we see that Eq. (11) connects operators at different orders in λ. It is important to note, however, that factors of D_u^μ can also appear by themselves (when the ∂_μ acts on non-collinear fields). An example is the $\slashed{D}_u$ in the usoft quark Lagrangian. Eq. (11) is analogous to the fact that reparameterization invariance in HQET forces $v^\mu + iD^\mu/m$ to appear together [8].

The ambiguities of type (b) give more interesting constraints on operators. The transformations (I,II,III) change the decomposition of any vector or tensor along the light-like directions. For example, for a vector

$$V^\mu = \frac{n^\mu}{2}\,\bar{n}\cdot V + \frac{\bar{n}^\mu}{2}\,n\cdot V + V_\perp^\mu\,, \qquad (12)$$

the components transform as

$$\Big(n\cdot V\,,\,\bar{n}\cdot V\,,\,V_\perp^\mu\Big) \xrightarrow{\text{I}} \Big(n\cdot V + \Delta^\perp\cdot V^\perp\,,\,\bar{n}\cdot V\,,\,V_\perp^\mu - \frac{\Delta_\perp^\mu}{2}\,\bar{n}\cdot V - \frac{\bar{n}^\mu}{2}\Delta^\perp\cdot V^\perp\Big)\,,$$

$$\Big(n\cdot V\,,\,\bar{n}\cdot V\,,\,V_\perp^\mu\Big) \xrightarrow{\text{II}} \Big(n\cdot V\,,\,\bar{n}\cdot V + \varepsilon_\perp\cdot V_\perp\,,\,V_\perp^\mu - \frac{\varepsilon_\perp^\mu}{2}\,n\cdot V - \frac{n^\mu}{2}\varepsilon_\perp\cdot V_\perp\Big)\,,$$

$$\Big(n\cdot V\,,\,\bar{n}\cdot V\,,\,V_\perp^\mu\Big) \xrightarrow{\text{III}} \Big(n\cdot V + \alpha n\cdot V\,,\,\bar{n}\cdot V - \alpha\bar{n}\cdot V\,,\,V_\perp^\mu\Big)\,. \qquad (13)$$

Type (I)	Type (II)	Type (III)
$n \to n + \Delta^\perp$	$n \to n$	$n \to n + \alpha\, n$
$\bar{n} \to \bar{n}$	$\bar{n} \to \bar{n} + \varepsilon^\perp$	$\bar{n} \to \bar{n} - \alpha\, \bar{n}$
$n \cdot \mathcal{D} \to n \cdot \mathcal{D} + \Delta^\perp \cdot \mathcal{D}^\perp$	$n \cdot \mathcal{D} \to n \cdot \mathcal{D}$	$n \cdot \mathcal{D} \to n \cdot \mathcal{D} + \alpha\, n \cdot \mathcal{D}$
$\mathcal{D}_\mu^\perp \to \mathcal{D}_\mu^\perp - \dfrac{\Delta_\mu^\perp}{2}\, \bar{n}\cdot\mathcal{D} - \dfrac{\bar{n}_\mu}{2}\Delta^\perp \cdot \mathcal{D}^\perp$	$\mathcal{D}_\mu^\perp \to \mathcal{D}_\mu^\perp - \dfrac{\varepsilon_\mu^\perp}{2}\, n\cdot\mathcal{D} - \dfrac{n_\mu}{2}\varepsilon^\perp \cdot \mathcal{D}^\perp$	$\mathcal{D}_\mu^\perp \to \mathcal{D}_\mu^\perp$
$\bar{n} \cdot \mathcal{D} \to \bar{n} \cdot \mathcal{D}$	$\bar{n} \cdot \mathcal{D} \to \bar{n} \cdot \mathcal{D} + \varepsilon^\perp \cdot \mathcal{D}^\perp$	$\bar{n}\cdot\mathcal{D} \to \bar{n}\cdot\mathcal{D} - \alpha\, \bar{n}\cdot\mathcal{D}$
$\xi_n \to \left(1 + \frac{1}{4}\,\slashed{\Delta}^\perp \slashed{\bar{n}}\right)\xi_n$	$\xi_n \to \left(1 + \frac{1}{2}\,\slashed{\varepsilon}^\perp \frac{1}{\bar{n}\cdot\mathcal{D}}\,\slashed{\mathcal{P}}^\perp\right)\xi_n$	$\xi_n \to \xi_n$
$W \to W$	$W \to \left[\left(1 - \frac{1}{\bar{n}\cdot\mathcal{D}}\,\epsilon^\perp \cdot \mathcal{D}^\perp\right)W\right]$	$W \to W$

TABLE I: Summary of infinitesimal type I, II, and III transformations.

These results are easily derived by demanding that the vector itself remains invariant, $V^\mu \to V^\mu$. These transformations apply to all vector components including those of the covariant derivative $\mathcal{D}^\mu$. Due to the projection operators in Eq. (6) the transformations also effect collinear spinors. To derive the spinor transformations we first equate the decomposition in Eq. (5) before and after the reparameterization transformation,

$$\sum_p e^{-ip\cdot x}\left[1 + \frac{1}{\bar{n}\cdot\mathcal{D}}\,\slashed{\mathcal{P}}^\perp\,\frac{\slashed{\bar{n}}}{2}\right]\xi_{n,p} = \sum_{p'} e^{-ip'\cdot x}\left[1 + \frac{1}{\bar{n}\cdot\mathcal{D}'}\,\slashed{\mathcal{P}}'_\perp\,\frac{\slashed{\bar{n}}'}{2}\right]\xi'_{n,p}, \qquad (14)$$

where the primes denote quantities transformed under (I), (II), or (III). In each case multiplying both sides by the transformed $\slashed{n}\slashed{\bar{n}}/4$ projection operator then gives an expression for $\xi'_{n,p}$ in terms of $\xi_{n,p}$. We find

$$\xi_n \overset{\mathrm{I}}{\longrightarrow} \left(1 + \frac{1}{4}\,\slashed{\Delta}^\perp \slashed{\bar{n}}\right)\xi_n\ , \qquad (15)$$

$$\xi_n \overset{\mathrm{II}}{\longrightarrow} \left(1 + \frac{1}{2}\,\slashed{\varepsilon}^\perp \frac{1}{\bar{n}\cdot\mathcal{D}}\,\slashed{\mathcal{P}}_\perp\right)\xi_n\,, \qquad (16)$$

$$\xi_n \overset{\mathrm{III}}{\longrightarrow} \xi_n\ . \qquad (17)$$

Finally, we consider the transformation of the Wilson line in Eq. (8). From this definition we see that W is invariant under type (I) and (III) transformations. The transformation of W under (II) can be derived by noting that $[\bar{n} \cdot D_c W] = 0$ implies $0 = \delta[\bar{n} \cdot D_c W] = [\delta(\bar{n} \cdot D_c)W] + [\bar{n} \cdot D_c\,\delta W]$ which we can solve for δW to find

$$W \overset{\mathrm{II}}{\longrightarrow} \left[\left(1 - \frac{1}{\bar{n}\cdot\mathcal{D}}\,\epsilon_\perp \cdot \mathcal{D}_\perp\right)W\right]. \qquad (18)$$

For easy reference a summary of the most commonly used transformations is given in Table I.

Requiring invariance under the transformations in Table I places important constraints on collinear operators. In particular some classes of operators are ruled out, while others

have coefficients whose matching and anomalous dimensions are related to all orders in perturbation theory.

For the purpose of power counting we take

$$\Delta_\perp \sim \lambda\,, \qquad \epsilon_\perp \sim \lambda^0\,, \qquad \alpha \sim \lambda^0\,, \tag{19}$$

and then demand reparameterization invariance order by order in λ. The scaling for the infinitesimal parameters is assigned to be the maximum power that leaves the power counting of the collinear momentum components intact. This means that only transformations that leave collinear fields collinear are considered. For example we do not consider large longitudinal boosts $\sim 1/\lambda$ that could turn a collinear momentum into a soft momentum. For $\Delta_\perp$ we see from Table I that the n and ξ_n transformations are suppressed by λ, while the transformation of $n \cdot \mathcal{D}$ and $\mathcal{D}^\perp$ include homogeneous terms. By homogeneous we mean the same order in λ as the untransformed operator. For $\mathcal{D}_\mu^\perp$ the term $\frac{1}{2}\bar{n}^\mu \Delta_\perp \cdot \mathcal{D}^\perp \sim \lambda^2$ is smaller than $\mathcal{D}_\mu^\perp \sim \lambda$. For $\varepsilon_\perp$ the transformations of ξ_n, W, and $\bar{n} \cdot \mathcal{D}$ are suppressed by λ. The $\mathcal{D}^\perp$ transformation involves a homogeneous term, as well as a term suppressed by a single λ. Finally, the type (III) transformations are completely homogeneous. Note that the homogeneous terms can induce constraints without mixing orders in the power counting. This occurs for all type (III) cases as well as for any leading order operator involving a quantity with a homogeneous term in its type (I) or (II) transformation.

In the remainder of this Letter we present the implications of the symmetry transformations (I)-(III) for the operators in the collinear quark Lagrangian. The leading order Lagrange density in the collinear quark sector can be obtained by tree level matching and is given by [2, 3]

$$\mathcal{L}_0 = \bar{\xi}_n \left\{ n \cdot iD_u + gn \cdot A_n + (\mathcal{P}_\perp + g\mathcal{A}_n^\perp)\frac{1}{\bar{\mathcal{P}} + g\bar{n} \cdot A_n}(\mathcal{P}_\perp + g\mathcal{A}_n^\perp) \right\} \frac{\bar{\slashed{n}}}{2}\,\xi_n\,, \tag{20}$$

where $\mathcal{L}_0 \sim \lambda^4$. As discussed above, invariance under collinear label reparameterization forces the collinear derivatives in SCET to appear with the ultrasoft derivatives. Therefore the Lagrangian (20) is just the first term in the expansion of the manifestly reparameterization invariant Lagrangian obtained by replacing the collinear derivatives with $i\mathcal{D}$

$$\mathcal{L} = \bar{\xi}_n \left\{ n \cdot i\mathcal{D} + i\slashed{\mathcal{D}}^\perp \frac{1}{\bar{n} \cdot i\mathcal{D}} i\slashed{\mathcal{D}}^\perp \right\} \frac{\bar{\slashed{n}}}{2}\,\xi_n\,. \tag{21}$$

Expanding the derivatives $i\mathcal{D}$ in powers of λ, one finds that $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \cdots$, where the terms $\mathcal{L}_i$ scale like λ^{4+i}. In particular, the next two subleading terms in the Lagrange density are

$$\mathcal{L}_1 = \bar{\xi}_n \left\{ i\slashed{\mathcal{D}}_u^\perp \frac{1}{\bar{n} \cdot iD_c} i\slashed{\mathcal{D}}_c^\perp + i\slashed{\mathcal{D}}_c^\perp \frac{1}{\bar{n} \cdot iD_c} i\slashed{\mathcal{D}}_u^\perp \right\} \frac{\bar{\slashed{n}}}{2}\xi_n \tag{22}$$

$$\mathcal{L}_2 = \bar{\xi}_n \left\{ i\slashed{\mathcal{D}}_u^\perp \frac{1}{\bar{n} \cdot iD_c} i\slashed{\mathcal{D}}_u^\perp - i\slashed{\mathcal{D}}_c^\perp \frac{1}{\bar{n} \cdot iD_c} \bar{n} \cdot iD_u \frac{1}{\bar{n} \cdot iD_c} i\slashed{\mathcal{D}}_c^\perp \right\} \frac{\bar{\slashed{n}}}{2}\xi_n\,.$$

The expressions in Eq. (22) agree with the tree level matching result. The reparameterization argument shows that these terms are connected to the leading order Lagrangian. Thus, their

structure is determined by $\mathcal{L}_0$ to all orders in perturbation theory and they have no non-trivial Wilson coefficients.[1] This result for $\mathcal{L}_1$ agrees with Ref. [15], while the structure of $\mathcal{L}_2$ (and all higher terms in the expansion of Eq. (21)) are new.

The observant reader will have noticed that the reasoning in Eqs. (20) through (22) relies on the fact that Eq. (20) is the unique lowest order Lagrangian. In fact using only collinear gauge invariance the operator appearing in Eq. (20) is not the most general allowed operator. For instance both

$$\mathcal{O}_1 = \bar{\xi}_n \, i\slashed{D}_c^\perp \frac{1}{\bar{n}\cdot iD_c} i\slashed{D}_c^\perp \frac{\slashed{\bar{n}}}{2} \xi_n \,, \qquad \mathcal{O}_2 = \bar{\xi}_n \, iD_{c\mu}^\perp \frac{1}{\bar{n}\cdot iD_c} \, iD_c^{\perp\mu} \frac{\slashed{\bar{n}}}{2} \xi_n \,, \qquad (23)$$

are collinear gauge invariant, but only $\mathcal{O}_1$ was included in Eq. (20). Even if only $\mathcal{O}_1$ is present at tree level, the other operator could in principle be induced through radiative corrections. The presence of $\mathcal{O}_2$ at leading order in λ would be a concern, since it would imply that the collinear quark kinetic Lagrangian can only be defined order by order in perturbation theory. However, it is easy to show that $\mathcal{O}_2$ is ruled out by invariance under type (II) reparameterization transformations in SCET. We will present this in some detail before extending the basis in Eq. (23) to the most general set of gauge invariant operators.

To see that class (I) is not sufficient to rule out $\mathcal{O}_2$ note that since $\delta_{(I)} [\slashed{\bar{n}} \xi_n] = 0$ we have

$$\delta_{(I)} \mathcal{O}_2 = -\bar{\xi}_n \, i\Delta^\perp \cdot D_c \frac{\slashed{\bar{n}}}{2} \xi_n = \delta_{(I)} \mathcal{O}_1 \,. \qquad (24)$$

Here $\delta_{(I)}\mathcal{O}_{1,2}$ and $\mathcal{O}_{1,2}$ are the same order in λ. Now at this order we also have

$$\delta_{(I)}\left(\bar{\xi}_n \, n \cdot iD \frac{\slashed{\bar{n}}}{2} \xi_n \right) = \bar{\xi}_n \, i\Delta^\perp \cdot D_c \frac{\slashed{\bar{n}}}{2} \xi_n \,. \qquad (25)$$

This shows that reparameterization invariance of type (I) ties the first two terms in Eq. (20) with the third one. However, as far as transformations of type (I) are concerned we could replace $\mathcal{O}_1$ in Eq. (20) by $\mathcal{O}_2$ and still leave $\mathcal{L}_0$ invariant.

For a type (II) transformation we begin by noting that for any scalar operator in the single collinear quark sector the terms homogeneous in λ vanish identically. This follows from the fact that the index on $D_\perp^\mu$ is contracted into another perpendicular vector, and that the transformation for $\bar{n}^\mu$ is subleading except for when it is contracted with γ_μ. As discussed in Ref. [5] a complete Dirac basis for these bilinears is $\slashed{\bar{n}}$, $\slashed{\bar{n}}\gamma_5$, and $\slashed{\bar{n}}\gamma_\perp^\mu$. However, if $\slashed{\bar{n}}$ in any of these Dirac structures is transformed the resulting structure vanishes between ξ_n fields. This implies that as far as the collinear Lagrangian is concerned we can impose invariance under the remaining type (II) transformations order by order in λ without worrying about connecting terms of different orders. In general this need not be true for currents.

Making a type (II) transformation one finds that the change in the first term of the effective Lagrangian in Eq. (20) is

$$\delta_{(II)} \, \bar{\xi}_n n \cdot iD \frac{\slashed{\bar{n}}}{2} \xi_n = \bar{\xi}_n \left(i\slashed{D}_c^\perp \frac{1}{\bar{n}\cdot iD_c} \frac{\slashed{\ell}^\perp}{2} n \cdot iD + n \cdot iD \frac{\slashed{\ell}^\perp}{2} \frac{1}{\bar{n}\cdot iD_c} i\slashed{D}_c^\perp \right) \frac{\slashed{\bar{n}}}{2} \xi_n \,. \qquad (26)$$

---

[1] In principle the above results do not rule out the possibility of additional terms in the power suppressed collinear Lagrangians which are reparameterization invariant all by themselves.

This variation is exactly canceled by the change in $\mathcal{O}_1$ coming from the $i\rlap{\,/}{\mathcal{D}}_c^\perp$ factors. The remaining variation in $\mathcal{O}_1$ is from the change in the ξ_n fields and in the $1/\bar{n}\cdot iD_c$ factor, thus

$$
\begin{aligned}
\delta_{\text{(II)}}\mathcal{O}_1 \;=\; & -\delta_{\text{(II)}}\,\bar{\xi}_n n\cdot iD\,\frac{\rlap{/}{\bar{n}}}{2}\xi_n + \bar{\xi}_n\left\{ i\rlap{\,/}{\mathcal{D}}_c^\perp\frac{1}{\bar{n}\cdot iD_c}\frac{\rlap{/}{\epsilon}^\perp}{2}i\rlap{\,/}{\mathcal{D}}_c^\perp\frac{1}{\bar{n}\cdot iD_c}i\rlap{\,/}{\mathcal{D}}_c^\perp \right. \\[2mm]
& \left. -i\rlap{\,/}{\mathcal{D}}_c^\perp\frac{1}{\bar{n}\cdot iD_c}\,\epsilon_\perp\!\cdot\! iD_c^\perp\frac{1}{\bar{n}\cdot iD_c}i\rlap{\,/}{\mathcal{D}}_c^\perp + i\rlap{\,/}{\mathcal{D}}_c^\perp\frac{1}{\bar{n}\cdot iD_c}i\rlap{\,/}{\mathcal{D}}_c^\perp\frac{\rlap{/}{\epsilon}^\perp}{2}\frac{1}{\bar{n}\cdot iD_c}i\rlap{\,/}{\mathcal{D}}_c^\perp \right\}\frac{\rlap{/}{\bar{n}}}{2}\xi_n \\[2mm]
\;=\; & -\delta_{\text{(II)}}\,\bar{\xi}_n n\cdot iD\,\frac{\rlap{/}{\bar{n}}}{2}\xi_n\,.
\end{aligned}
\tag{27}
$$

On the other hand, the variation of the $D_\perp$'s in $\mathcal{O}_2$ gives terms with $\epsilon^\perp\!\cdot\! D_c^\perp$ which can not cancel the variation of the $in\cdot\mathcal{D}$ term. Furthermore, for $\mathcal{O}_2$ the terms analogous to the ones in curly brackets in Eq. (27) do not vanish because the first and third terms from the variation of ξ do not cancel against the term from the $i\bar{n}\cdot D$ variation. Thus, $\mathcal{O}_2$ is not allowed in the leading order effective Lagrangian by reparameterization invariance of type (II).

We now proceed to a more general analysis of the complete set of gauge invariant operators in the collinear Lagrangian up to λ^4. A special feature of SCET is the presence of the dimensionful collinear covariant derivative $\bar{n}\cdot iD_c = \bar{\mathcal{P}} + g\bar{n}\cdot A_n$ which scales like λ^0. Because of this operator, dimensional analysis and gauge invariance are not sufficient to completely constrain the operators that appear at a given order in λ. Since we require two ξ_n fields the operators in the Lagrange density start at order λ^2. At this order gauge invariance allows

$$
\mathcal{L}_{-2} \;=\; \bar{\xi}_n\,\bar{n}\cdot iD\,\frac{\rlap{/}{\bar{n}}}{2}\,\xi_n\,.
\tag{28}
$$

However, invariance under transformation (III) requires that each time $\bar{n}$ appears in the numerator it is accompanied by either an n in the numerator or a factor of $\bar{n}$ in the denominator. Thus, the Lagrangian in Eq. (28) is not allowed by type (III) reparameterization invariance. At order λ^3 the only way to construct an operator is with one factor of $D_\perp^\mu$. Since the Lagrangian is a scalar this index must be contracted with an object that will not increase the power of λ, which leaves $\bar{\xi}_n\rlap{/}{\bar{n}}\rlap{\,/}{\mathcal{D}}_\perp\xi_n$. However, this operator is also ruled out by type III reparameterization invariance.

At order λ^4 there are infinitely many operators allowed by gauge invariance and invariance under (III):

$$
\mathcal{L}'_0 \;=\; \bar{\xi}_n\,\mathcal{O}\,\frac{\rlap{/}{\bar{n}}}{2}\,\xi_n\,,
\tag{29}
$$

where

$$
\mathcal{O} \;=\; \sum_a c_a\mathcal{O}^a + \sum_{a,b}(s_{ab}\mathcal{S}^{ab} + t_{ab}\mathcal{T}^{ab})\,,
\tag{30}
$$

and the operators $\mathcal{O}^a$, $\mathcal{S}^{ab}$ and $\mathcal{T}^{ab}$ are

$$
\begin{aligned}
\mathcal{O}^a \;=\; & N^a(n\cdot i\mathcal{D})\frac{1}{N^a} + \frac{1}{N^a}(n\cdot i\mathcal{D})N^a\,, \\[2mm]
\mathcal{S}^{ab} \;=\; & N^a i\rlap{\,/}{\mathcal{P}}_\perp\frac{1}{N^{a+b+1}}i\rlap{\,/}{\mathcal{P}}_\perp N^b + N^b i\rlap{\,/}{\mathcal{P}}_\perp\frac{1}{N^{a+b+1}}i\rlap{\,/}{\mathcal{P}}_\perp N^a\,, \\[2mm]
\mathcal{T}^{ab} \;=\; & N^a i\mathcal{D}_\perp^\mu\frac{1}{N^{a+b+1}}i\mathcal{D}_\mu^\perp N^b + N^b i\mathcal{D}_\perp^\mu\frac{1}{N^{a+b+1}}i\mathcal{D}_\mu^\perp N^a\,.
\end{aligned}
\tag{31}
$$

To simplify the formulae we have defined $N \equiv \bar{n} \cdot i\mathcal{D}$. The operators $\mathcal{O}^a, \mathcal{S}^{ab}$ and $\mathcal{T}^{ab}$ are Hermitian so the coefficients c_a, s_{ab} and t_{ab} are real numbers. Since $[iD_\perp^i, iD_\perp^j] = ig\, G_\perp^{ij}$ and $[N, iD_\perp^i] = ig\, \bar{n}_\mu G^{\mu i}$, any operator that is order λ^4 and contains a collinear gluon field strength can be reduced to a linear combination of those in Eq. (31).

The variation of Eq. (29) under the transformation (II) is given by

$$\delta_{\text{(II)}} \mathcal{L}_0' = \bar{\xi}_n \left[\delta_{\text{(II)}} \mathcal{O} + i\mathcal{P}_\perp \slashed{\epsilon}^\perp \frac{1}{2N} \mathcal{O} + \mathcal{O} \frac{1}{2N} \slashed{\epsilon}^\perp i\mathcal{P}_\perp \right] \frac{\slashed{\bar{n}}}{2} \xi_n \,. \tag{32}$$

Most terms appearing in Eq. (29) will not be invariant unless the operator coefficient is set to zero. First consider the variation of $\mathcal{S}^{ab}$:

$$\begin{aligned}
\delta_{\text{(II)}} \mathcal{S}^{ab} =\ & [\delta_{\text{(II)}} N^a] i\mathcal{P}_\perp \frac{1}{N^{a+b+1}} i\mathcal{P}_\perp N^b + N^a i\mathcal{P}_\perp \frac{1}{N^{a+b+1}} i\mathcal{P}_\perp [\delta_{\text{(II)}} N^b] \\
& - \frac{1}{2} N^a \slashed{\epsilon}_\perp n \cdot i\mathcal{D} \frac{1}{N^{a+b+1}} i\mathcal{P}_\perp N^b - \frac{1}{2} N^a \mathcal{P}_\perp \frac{1}{N^{a+b+1}} i\slashed{\epsilon}^\perp n \cdot i\mathcal{D} N^b \\
& + N^a i\mathcal{P}_\perp \left[\delta_{\text{(II)}} \frac{1}{N^{a+b+1}} \right] i\mathcal{P}_\perp N^b + (a \leftrightarrow b) \,,
\end{aligned} \tag{33}$$

where for positive a

$$\begin{aligned}
\delta_{\text{(II)}} N^a &= \epsilon^\perp \cdot i\mathcal{D}^\perp N^{a-1} + N\epsilon^\perp \cdot i\mathcal{D}^\perp N^{a-2} + \dots + N^{a-1} \epsilon^\perp \cdot i\mathcal{D}^\perp \,, \\
\delta_{\text{(II)}} \frac{1}{N^a} &= -\frac{1}{N} \epsilon^\perp \cdot iD^\perp \frac{1}{N^a} - \frac{1}{N^2} \epsilon^\perp \cdot i\mathcal{D}^\perp \frac{1}{N^{a-1}} - \dots - \frac{1}{N^a} \epsilon^\perp \cdot i\mathcal{D}^\perp \frac{1}{N} \,.
\end{aligned} \tag{34}$$

The vanishing of Eq. (32) requires $\delta_{\text{(II)}} \mathcal{O}$ to consist of operators in which $\mathcal{P}^\perp$ appears all the way to the left or all the way to the right. Except for the case $a = b = 0$, the operators appearing in $\delta_{\text{(II)}} \mathcal{S}^{ab}$ are not of this form. These operators cannot be canceled by variations of $\mathcal{O}^a$ or $\mathcal{T}^{ab}$ since the latter have a trivial Dirac structure, while the operators in $\delta_{\text{(II)}} \mathcal{S}^{ab}$ do not. Thus invariance under (II) requires $s_{ab} = 0$ except for s_{00}, which will be set to $1/2$ once the normalization of the free quark kinetic term is fixed.

The above analysis implies that

$$\mathcal{O} = i\mathcal{P}^\perp \frac{1}{N} i\mathcal{P}^\perp + \mathcal{O}' \,, \tag{35}$$

where $\mathcal{O}'$ contains contributions to $\mathcal{O}$ from the operators $\mathcal{O}^a$ and $\mathcal{T}^{ab}$. Inserting this into Eq. (32), one finds after some algebra:

$$\delta_{\text{(II)}} \mathcal{L}_0' = \bar{\xi}_n \left[\delta_{\text{(II)}} \mathcal{O}' + i\mathcal{P}^\perp \slashed{\epsilon}^\perp \frac{1}{2N} (\mathcal{O}' - n \cdot i\mathcal{D}) + (\mathcal{O}' - n \cdot i\mathcal{D}) \frac{1}{2N} \slashed{\epsilon}^\perp i\mathcal{P}^\perp \right] \frac{\slashed{\bar{n}}}{2} \xi_n \,. \tag{36}$$

The terms involving $\delta_{\text{(II)}} \mathcal{O}'$ and $\slashed{\epsilon}^\perp$ must vanish independently since they have different Dirac structures. Thus, $\mathcal{O}' = n \cdot iD$ and $\delta_{\text{(II)}} \mathcal{O}' = 0$ which together imply $\delta_{\text{(II)}} \mathcal{L}_0' = 0$. Thus, reparameterization invariance of type (II) and (III) require the Lagrangian to be $\mathcal{L}_0$ to all orders in perturbation theory. Since the free quark kinetic terms come from $\mathcal{L}_0$ the normalization of these terms is entirely fixed and they cannot acquire an anomalous dimension. This completes the proof that the result in Eq. (21) is the most general reparameterization invariant collinear quark Lagrangian whose expansion starts out at order λ^4.

In this paper we have used reparametrization invariance to constrain the form of the collinear SCET Lagrangian. An important result of this paper is that the form of the leading Lagrangian is uniquely determined by gauge invariance, reparameterization invariance, and tree level matching. Reparametrization invariance was also used to identify the complete set of subleading operators containing usoft gluon fields which are related to the leading order Lagrangian, and therefore are not renormalized to all orders in perturbation theory. We have not been able to construct subleading corrections to the two quark sector of the collinear Lagrangian which are by themselves reparameterization invariant. An important open question is whether such operators exist. If not, then reparameterization invariance would uniquely determine the SCET Lagrangian in the two collinear quark sector. It is of course possible to write down operators coupling to external currents which involve one or more collinear quark fields and receive nontrivial renormalization. Examples include the heavy to light currents [1, 2, 15] and deep inelastic scattering [5]. The anomalous dimensions of these operators can be used to sum logarithms involving ratios of the hard and infrared scales.

An application of reparameterization invariance which we have not considered in this paper is matching and anomalous dimension calculations for operators. For processes with energetic hadrons a local QCD current matches onto an infinite series of operators in SCET. Reparameterization invariance can be used to derive relations between Wilson coefficients of different operators in this expansion that are valid to all orders in perturbation theory much like the relations for heavy-to-heavy currents in HQET [9]. In SCET relations have been derived for heavy-to-light currents in Ref. [15] using the type (I) transformations. It would also be interesting to extend these calculations to include type (II) transformations as well as currents composed entirely of collinear fields. The latter should have applications for the calculation of power suppressed cross sections in high energy processes.

Acknowledgments

[1] C. W. Bauer, S. Fleming and M. Luke, Phys. Rev. D **63**, 014006 (2001).

[2] C. W. Bauer, S. Fleming, D. Pirjol and I. W. Stewart, Phys. Rev. D **63**, 114020 (2001).

[3] C. W. Bauer and I. W. Stewart, Phys. Lett. B **516**, 134 (2001).

[4] C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. D **65**, 054022 (2002).

[5] C. W. Bauer, S. Fleming, D. Pirjol, I. Z. Rothstein and I. W. Stewart, hep-ph/0202088.

[6] C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. Lett. **87**, 201806 (2001).

[7] C. W. Bauer, C. W. Chiang, S. Fleming, A. K. Leibovich and I. Low, Phys. Rev. D **64**, 114014 (2001).

[8] M. E. Luke and A. V. Manohar, Phys. Lett. B **286**, 348 (1992).

[9] M. Neubert, Phys. Lett. B **306**, 357 (1993).

[10] Y. Q. Chen, Phys. Lett. B **317**, 421 (1993).

[11] M. Finkemeier, H. Georgi, and M. McIrvin, Phys. Rev. D **55**, 6933 (1997).

[12] A. V. Manohar, Phys. Rev. D **56**, 230 (1997).

[13] R. Sundrum, Phys. Rev. D **57**, 331 (1998).

[14] F. Campanario and T. Mannel, arXiv:hep-ph/0201136.

[15] J. Chay and C. Kim, hep-ph/0201197.