Composizione di boost di Lorentz in direzioni non parallele [1]

1 Boost generico

$$B(\vec{v}) = \begin{bmatrix} \gamma & -\gamma v_1 & -\gamma v_2 & -\gamma v_3 \\ -\gamma v_1 & \dfrac{v_1^2(\gamma-1)}{v^2}+1 & \dfrac{v_1 v_2(\gamma-1)}{v^2} & \dfrac{v_1 v_3(\gamma-1)}{v^2} \\ -\gamma v_2 & \dfrac{v_1 v_2(\gamma-1)}{v^2} & \dfrac{v_2^2(\gamma-1)}{v^2}+1 & \dfrac{v_2 v_3(\gamma-1)}{v^2} \\ -\gamma v_3 & \dfrac{v_1 v_3(\gamma-1)}{v^2} & \dfrac{v_2 v_3(\gamma-1)}{v^2} & \dfrac{v_3^2(\gamma-1)}{v^2}+1 \end{bmatrix} \tag{1.1}$$

Composizione velocità: il sistema primato S' è in moto con velocità v lungo l'asse x rispetto al sistema S. Le velocità di una particella viste dai due sistemi sono $\vec{u} = (u_x, u_y, u_z)$ e $\vec{u'} = (u'_x, u'_y, u'_z)$

$$\begin{cases} u_x = \dfrac{u'_x + v}{1 + v u'_x} \\ u_y = \dfrac{u'_y \sqrt{1-v^2}}{1 + v u'_x} \\ u_z = \dfrac{u'_z \sqrt{1-v^2}}{1 + v u'_x} \end{cases} \tag{1.2}$$

2 Composizione di boost

Si consideri ora il tetravettore

$$p = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix} \tag{2.1}$$

[1] `http://virgilio.mib.infn.it/~oleari/`

2.1 Boost x, y

$$B_{1x} = \begin{bmatrix} 2/\sqrt{3} & -1/\sqrt{3} & 0 & 0 \\ -1/\sqrt{3} & 2/\sqrt{3} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.2}$$

$$B_{1y} = \begin{bmatrix} \sqrt{3/2} & 0 & -1/\sqrt{2} & 0 \\ 0 & 1 & 0 & 0 \\ -1/\sqrt{2} & 0 & \sqrt{3/2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.3}$$

$$B_1 = B_{1y}B_{1x} = \begin{bmatrix} \sqrt{2} & -1/\sqrt{2} & -1/\sqrt{2} & 0 \\ -1/\sqrt{3} & 2/\sqrt{3} & 0 & 0 \\ -\sqrt{2/3} & 1/\sqrt{6} & \sqrt{3/2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.4}$$

2.2 Boost y, x

$$B_{2y} = \begin{bmatrix} 2/\sqrt{3} & 0 & -1/\sqrt{3} & 0 \\ 0 & 1 & 0 & 0 \\ -1/\sqrt{3} & 0 & 2/\sqrt{3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.5}$$

$$B_{2x} = \begin{bmatrix} \sqrt{3/2} & -1/\sqrt{2} & 0 & 0 \\ -1/\sqrt{2} & \sqrt{3/2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.6}$$

$$B_2 = B_{2x}B_{2y} = \begin{bmatrix} \sqrt{2} & -1/\sqrt{2} & -1/\sqrt{2} & 0 \\ -\sqrt{2/3} & \sqrt{3/2} & 1/\sqrt{6} & 0 \\ -1/\sqrt{3} & 0 & 2/\sqrt{3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.7}$$

2.3 Boost singolo

$$B = \begin{bmatrix} \sqrt{2} & -1/\sqrt{2} & -1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2}+1/2 & 1/\sqrt{2}-1/2 & 0 \\ -1/\sqrt{2} & 1/\sqrt{2}-1/2 & 1/\sqrt{2}+1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.8}$$

2.4 Rotazioni

$$B_1^{-1} = B_{1x}^{-1}B_{1y}^{-1} = \begin{bmatrix} \sqrt{2} & 1/\sqrt{3} & \sqrt{2/3} & 0 \\ 1/\sqrt{2} & 2/\sqrt{3} & 1/\sqrt{6} & 0 \\ 1/\sqrt{2} & 0 & \sqrt{3/2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.9}$$

$$R_1 = BB_1^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{3}+1/\sqrt{6} & 1/\sqrt{3}-1/\sqrt{6} & 0 \\ 0 & 1/\sqrt{6}-1/\sqrt{3} & 1/\sqrt{3}+1/\sqrt{6} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.10}$$

$$B_2^{-1} = B_{2y}^{-1}B_{2x}^{-1} = \begin{bmatrix} \sqrt{2} & \sqrt{2/3} & 1/\sqrt{3} & 0 \\ 1/\sqrt{2} & \sqrt{3/2} & 0 & 0 \\ 1/\sqrt{2} & 1/\sqrt{6} & 2/\sqrt{3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.11}$$

$$R_2 = BB_2^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{3}+1/\sqrt{6} & 1/\sqrt{6}-1/\sqrt{3} & 0 \\ 0 & 1/\sqrt{3}-1/\sqrt{6} & 1/\sqrt{3}+1/\sqrt{6} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.12}$$