Physics. — "*Simplified Theory of Electrical and Optical Phenomena in Moving Systems*". By Prof. H. A. LORENTZ.

§ 1. In former investigations I have assumed that, in all electrical and optical phenomena, taking place in ponderable matter, we have to do with small charged particles or ions, having determinate positions of equilibrium in dielectrics, but free to move in conductors except in so far as there is a resistance, depending on their velocities. According to these views an electric current in a conductor is to be considered as a progressive motion of the ions, and a dielectric polarization in a non-conductor as a displacement of the ions from their positions of equilibrium. The ions were supposed to be perfectly permeable to the aether, so that they can move while the aether remains at rest. I applied to the aether the ordinary electromagnetic equations, and to the ions certain other equations which seemed to present themselves rather naturally. In this way I arrived at a system of formulae which were found sufficient to account for a number of phenomena.

In the course of the investigation some artifices served to shorten the mathematical treatment. I shall now show that the theory may be still further simplified if the fundamental equations are immediately transformed in an appropriate manner.

§ 2. I shall start from the same hypotheses and introduce the same notations as in my „Versuch einer Theorie der electrischen und optischen Erscheinungen in bewegten Körpern". Thus, $\mathfrak{d}$ and $\mathfrak{H}$ will represent the dielectric displacement and the magnetic force, ϱ the density to which the ponderable matter is charged, $\mathfrak{v}$ the velocity of this matter, and $\mathfrak{E}$ the force acting on it per unit charge (electric force). It is only in the interior of the ions that the density ϱ differs from 0; for simplicity's sake I shall take it to be a continuous function of the coordinates, even at the surface of the ions. Finally, I suppose that each element of an ion retains its charge while it moves.

If, now, V be the velocity of light in the aether, the fundamental equations will be

$$Div\ \mathfrak{d} = \varrho\,, \quad \ldots\ldots\ldots \quad (I_a)$$

$$Div\ \mathfrak{H} = 0\,, \quad \ldots\ldots\ldots \quad (II_a)$$

$$Rot\ \mathfrak{H} = 4\pi\varrho\mathfrak{v} + 4\pi\dot{\mathfrak{d}}\,, \quad \ldots\ldots \quad (III_a)$$

$$4\pi V^2\ Rot\ \mathfrak{d} = -\dot{\mathfrak{H}}\,, \quad \ldots\ldots\ldots \quad (IV_a)$$

$$\mathfrak{E} = \frac{1}{4\pi V^2}\mathfrak{d} + [\mathfrak{v}.\mathfrak{H}]\,. \quad \ldots\ldots\ldots \quad (V_a)$$

We shall apply these equations to a system of bodies, a common velocity of translation $\mathfrak{p}$, of constant direction ıgnitude, the aether remaining at rest, and we shall hence- enote by $\mathfrak{v}$, not the whole velocity of a material element, velocity it may have in addition to $\mathfrak{p}$.

it is natural to use a system of axes of coordinates, which s of the translation $\mathfrak{p}$. If we give to the axis of x the direc- the translation, so that $\mathfrak{p}_y$ and $\mathfrak{p}_z$ are 0, the equations $_a$) will have to be replaced by

$$Div\, \mathfrak{d} = \varrho\,, \quad \ldots\ldots\ldots \quad (\mathrm{I_b})$$

$$Div\, \mathfrak{H} = 0\,, \quad \ldots\ldots\ldots \quad (\mathrm{II_b})$$

$$\left.\begin{aligned} \frac{\partial \mathfrak{H}_z}{\partial y} - \frac{\partial \mathfrak{H}_y}{\partial z} &= 4\pi\varrho(\mathfrak{p}_x + \mathfrak{v}_x) + 4\pi\left(\frac{\partial}{\partial t} - \mathfrak{p}_x\frac{\partial}{\partial x}\right)\mathfrak{d}_x\,,\\ \frac{\partial \mathfrak{H}_x}{\partial z} - \frac{\partial \mathfrak{H}_z}{\partial x} &= 4\pi\varrho\,\mathfrak{v}_y + 4\pi\left(\frac{\partial}{\partial t} - \mathfrak{p}_x\frac{\partial}{\partial x}\right)\mathfrak{d}_y\,,\\ \frac{\mathfrak{H}_y}{\partial x} - \frac{\partial \mathfrak{H}_x}{\partial y} &= 4\pi\varrho\,\mathfrak{v}_z + 4\pi\left(\frac{\partial}{\partial t} - \mathfrak{p}_x\frac{\partial}{\partial x}\right)\mathfrak{d}_z\,, \end{aligned}\right\} \quad (\mathrm{III_b})$$

$$\left.\begin{aligned} 4\pi V^2\left(\frac{\partial \mathfrak{d}_z}{\partial y} - \frac{\partial \mathfrak{d}_y}{\partial z}\right) &= -\left(\frac{\partial}{\partial t} - \mathfrak{p}_x\frac{\partial}{\partial x}\right)\mathfrak{H}_x\,,\\ 4\pi V^2\left(\frac{\partial \mathfrak{d}_x}{\partial z} - \frac{\partial \mathfrak{d}_z}{\partial x}\right) &= -\left(\frac{\partial}{\partial t} - \mathfrak{p}_x\frac{\partial}{\partial x}\right)\mathfrak{H}_y\,,\\ 4\pi V^2\left(\frac{\partial \mathfrak{d}_y}{\partial x} - \frac{\partial \mathfrak{d}_x}{\partial y}\right) &= -\left(\frac{\partial}{\partial t} - \mathfrak{p}_x\frac{\partial}{\partial x}\right)\mathfrak{H}_z\,, \end{aligned}\right\} \quad (\mathrm{IV_b})$$

$$\mathfrak{E} = 4\pi V^2\mathfrak{d} + [\mathfrak{p}\,.\,\mathfrak{H}] + [\mathfrak{v}\,.\,\mathfrak{H}] \quad \ldots\ldots \quad (\mathrm{V_b})$$

ıese formulae the sign Div, applied to a vector $\mathfrak{A}$, has still ıning defined by

$$Div\, \mathfrak{A} = \frac{\partial \mathfrak{A}_x}{\partial x} + \frac{\partial \mathfrak{A}_y}{\partial y} + \frac{\partial \mathfrak{A}_z}{\partial z}\,.$$

ıas already been said, $\mathfrak{v}$ is the relative velocity with regard moving axes of coordinates. If $\mathfrak{v} = 0$, we shall speak of a at rest; this expression therefore means relative rest with ıo the moving axes.

In most applications $\mathfrak{p}$ would be the velocity of the earth in its yearly motion.

§ 4. Now, in order to simplify the equations, the following quantities may be taken as independent variables

$$x' = \frac{V}{\sqrt{V^2 - \mathfrak{p}_x^2}}\, x,\quad y' = y,\quad z' = z,\quad t' = t - \frac{\mathfrak{p}_x}{V^2 - \mathfrak{p}_x^2}\, x \quad . \quad . \quad (1)$$

The last of these is the time, reckoned from an instant that is not the same for all points of space, but depends on the place we wish to consider. We may call it the *local time*, to distinguish it from the *universal time* t.

If we put

$$\frac{V}{\sqrt{V^2 - \mathfrak{p}_x^2}} = k \ ,$$

we shall have

$$\frac{\partial}{\partial x} = k\frac{\partial}{\partial x'} - k^2\,\frac{\mathfrak{p}_x}{V^2}\,\frac{\partial}{\partial t'} \ , \quad \frac{\partial}{\partial y} = \frac{\partial}{\partial y'},\quad \frac{\partial}{\partial z} = \frac{\partial}{\partial z'},\quad \frac{\partial}{\partial t} = \frac{\partial}{\partial t'} \ .$$

The expression

$$\frac{\partial\, \mathfrak{A}_x}{\partial x'} + \frac{\partial\, \mathfrak{A}_y}{\partial y'} + \frac{\partial\, \mathfrak{A}_z}{\partial z'}$$

will be denoted by

$$Div'\ \mathfrak{A} \ .$$

We shall also introduce, as new dependent variables instead of the components of $\mathfrak{d}$ and $\mathfrak{H}$, those of two other vectors $\mathfrak{F}'$ and $\mathfrak{H}$, which we define as follows

$$\mathfrak{F}'_x = 4\pi V^2 \mathfrak{d}_x,\quad \mathfrak{F}'_y = 4\pi k V^2 \mathfrak{d}_y - k\mathfrak{p}_x \mathfrak{H}_z,\quad \mathfrak{F}'_z = 4\pi k V^2 \mathfrak{d}_z + k\mathfrak{p}_x \mathfrak{H}_y,$$

$$\mathfrak{H}'_x = k\mathfrak{H}_x,\qquad \mathfrak{H}'_y = k^2 \mathfrak{H}_y + 4\pi k^2 \mathfrak{p}_x \mathfrak{d}_z,\quad \mathfrak{H}'_z = k^2 \mathfrak{H}_z - 4\pi k^2 \mathfrak{p}_x \mathfrak{d}_y.$$

In this way I find by transformation and mutual combination of the equations (Ib)—(Vb):

$$Div'\,\mathfrak{F}' = \frac{4\pi}{k}\,V^2\,\varrho - 4\,\pi\,k\,\mathfrak{p}_x\,\varrho\,\mathfrak{v}_x\,, \quad \ldots\ldots \quad (\mathrm{I_c})$$

$$Div'\,\mathfrak{H}' = 0\,, \quad \ldots\ldots\ldots\ldots \quad (\mathrm{II_c})$$

$$\left.\begin{aligned} \frac{\partial \mathfrak{H}'_z}{\partial y'} - \frac{\partial \mathfrak{H}'_y}{\partial z'} &= 4\,\pi\,k^2\,\varrho\,\mathfrak{v}_x + \frac{k^2}{V^2}\frac{\partial \mathfrak{F}'_x}{\partial t'} \\ \frac{\partial \mathfrak{H}'_x}{\partial z'} - \frac{\partial \mathfrak{H}'_z}{\partial x'} &= 4\,\pi\,k\,\varrho\,\mathfrak{v}_y + \frac{k^2}{V^2}\frac{\partial \mathfrak{F}'_y}{\partial t'} \\ \frac{\partial \mathfrak{H}'_y}{\partial x'} - \frac{\partial \mathfrak{H}'_x}{\partial y'} &= 4\,\pi\,k\,\varrho\,\mathfrak{v}_z + \frac{k^2}{V^2}\frac{\partial \mathfrak{F}'_z}{\partial t'} \end{aligned}\right\}, \quad \ldots \quad (\mathrm{III_c})$$

$$\left.\begin{aligned} \frac{\partial \mathfrak{F}'_z}{\partial y'} - \frac{\partial \mathfrak{F}'_y}{\partial z'} &= -\frac{\partial \mathfrak{H}'_x}{\partial t'} \\ \frac{\partial \mathfrak{F}'_x}{\partial z'} - \frac{\partial \mathfrak{F}'_z}{\partial x'} &= -\frac{\partial \mathfrak{H}'_y}{\partial t'} \\ \frac{\partial \mathfrak{F}'_y}{\partial x'} - \frac{\partial \mathfrak{F}'_x}{\partial y'} &= -\frac{\partial \mathfrak{H}'_z}{\partial t'} \end{aligned}\right\}, \quad \ldots\ldots\ldots \quad (\mathrm{IV_c})$$

$$\left.\begin{aligned} \mathfrak{E}_x &= \mathfrak{F}'_x + k\,\frac{\mathfrak{p}_x}{V^2}\,(\mathfrak{v}_y\,\mathfrak{F}'_y + \mathfrak{v}_z\,\mathfrak{F}'_z) + (\mathfrak{v}_y\,\mathfrak{H}'_z - \mathfrak{v}_z\,\mathfrak{H}'_y) \\ \mathfrak{E}_y &= \frac{1}{k}\,\mathfrak{F}'_y - k\,\frac{\mathfrak{p}_x}{V^2}\,\mathfrak{v}_x\,\mathfrak{F}'_y + \left(\frac{1}{k}\,\mathfrak{v}_z\,\mathfrak{H}'_x - \mathfrak{v}_x\,\mathfrak{H}'_z\right) \\ \mathfrak{E}_z &= \frac{1}{k}\,\mathfrak{F}'_z - k\,\frac{\mathfrak{p}_x}{V^2}\,\mathfrak{v}_x\,\mathfrak{F}'_z + \left(\mathfrak{v}_x\,\mathfrak{H}'_y - \frac{1}{k}\,\mathfrak{v}_y\,\mathfrak{H}'_x\right) \end{aligned}\right\} \ldots \quad (\mathrm{V_c})$$

Putting $\mathfrak{v} = 0$ in the three last equations we see that

$$\mathfrak{F}'_x\,,\ \frac{1}{k}\,\mathfrak{F}'_y\,,\ \frac{1}{k}\,\mathfrak{F}'_z$$

are the components of the electric force that would act on a particle at rest.

§ 5. We shall begin with an application of the equations to

electrostatic phenomena. In these we have $v = 0$ and $\mathfrak{F}'$ independent of the time. Hence, by (II_c) and (III_c)

$$\mathfrak{H}' = 0,$$

and by (IV_c) and (I_c)

$$\frac{\partial \mathfrak{F}'_z}{\partial y'} - \frac{\partial \mathfrak{F}'_y}{\partial z'} = 0, \frac{\partial \mathfrak{F}'_x}{\partial z'} - \frac{\partial \mathfrak{F}'_z}{\partial x'} = 0, \frac{\partial \mathfrak{F}'_y}{\partial x'} - \frac{\partial \mathfrak{F}'_x}{\partial y'} = 0,$$

$$Div' \mathfrak{F}' = \frac{4\pi}{k} V^2 \varrho.$$

These equations show that $\mathfrak{F}'$ depends on a potential ω, so that

$$\mathfrak{F}'_x = -\frac{\partial \omega}{\partial x'}, \mathfrak{F}'_y = -\frac{\partial \omega}{\partial y'}, \mathfrak{F}'_z = -\frac{\partial \omega}{\partial z'}$$

and

$$\frac{\partial^2 \omega}{\partial x'^2} + \frac{\partial^2 \omega}{\partial y'^2} + \frac{\partial^2 \omega}{\partial z'^2} = -\frac{4\pi}{k} V^2 \varrho \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Let S be the system of ions with the translation $\mathfrak{p}_x$, to which the above formulae are applied. We can conceive a second system S_0 with no translation and consequently no motion at all; we shall suppose that S is changed into S_0 by a dilatation in which the dimensions parallel to OX are changed in ratio of 1 to k, the dimensions perpendicular to OX remaining what they were. Moreover we shall attribute equal charges to corresponding volume-elements in S and S_0; if then ϱ_0 be the density in a point P of S, the density in the corresponding point P_0 of S_0 will be

$$\varrho_0 = \frac{1}{k} \varrho.$$

If x, y, z are the coordinates of P, the quantities x', y', z', determined by (1), may be considered as the coordinates of P_0.

In the system S_0, the electric force, which we shall call $\mathfrak{E}_0$ may evidently be derived from a potential ω_0, by means of the equations

$$\mathfrak{E}_{0x} = -\frac{\partial \omega_0}{\partial x'}, \mathfrak{E}_{0y} = -\frac{\partial \omega_0}{\partial y'}, \mathfrak{E}_{0z} = -\frac{\partial \omega_0}{\partial z'},$$

and the function ω_0 itself will satisfy the condition

$$\frac{\partial^2 \omega_0}{\partial x'^2} + \frac{\partial^2 \omega_0}{\partial y'^2} + \frac{\partial^2 \omega_0}{\partial z'^2} = -4\pi V^2 \varrho_0 = -\frac{4\pi}{k} V^2 \varrho.$$

Comparing this with (2), we see that in corresponding points

$$\omega = \omega_0,$$

and consequently

$$\mathfrak{F}'_x = \mathfrak{E}_{0x}, \quad \mathfrak{F}'_y = \mathfrak{E}_{0y}, \quad \mathfrak{F}'_z = \mathfrak{E}_{0z}.$$

In virtue of what has been remarked at the end of § 4, the components of the electric force in the system S will therefore be

$$\mathfrak{E}_{0x}, \quad \frac{1}{k}\mathfrak{E}_{0y}, \quad \frac{1}{k}\mathfrak{E}_{0z}.$$

Parallel to OX we have the same electric force in S and S_0, but in a direction perpendicular to OX the electric force in S will be $\frac{1}{k}$ times the electric force in S_0.

By means of this result every electrostatic problem for a moving system may be reduced to a similar problem for a system at rest; only the dimensions in the direction of translation must be slightly different in the two systems. If, e.g., we wish to determine in what way innumerable ions will distribute themselves over a moving conductor C, we have to solve the same problem for a conductor C_0, having no translation. It is easy to show that if the dimensions of C_0 and C differ from each other in the way that has been indicated, the electric force in one case will be perpendicular to the surface of C, as soon as, in the other case, the force $\mathfrak{E}_0$ is normal to the surface of C_0.

Since

$$k = \left(1 - \frac{p_x^2}{V^2}\right)^{-\frac{1}{2}}$$

exceeds unity only by a quantity of the second order — if we call $\frac{p_x}{V}$ of the first order — the influence of the earth's yearly motion on electrostatic phenomena will likewise be of the second order.

§ 6. We shall now shew how our general equations (I_c)—(V_c) may be applied to optical phenomena. For this purpose we consider a system of ponderable bodies, the ions in which are capable of vibrating about determinate positions of equilibrium. If the system be traversed by waves of light, there will be oscillations of the ions, accompanied by electric vibrations in the aether. For convenience of treatment we shall suppose that, in the absence of light-waves, there is no motion at all; this amounts to ignoring all molecular motion.

Our first step will be to omit all terms of the second order. Thus, we shall put $k = 1$, and the electric force acting on ions at rest will become $\mathfrak{F}'$ itself.

We shall further introduce certain restrictions, by means of which we get rid of the last term in (J_c) and of the terms containing $\mathfrak{v}_x$, $\mathfrak{v}_y$, $\mathfrak{v}_z$ in (V_c).

The first of these restrictions relates to the magnitude of the displacements $\mathfrak{a}$ from the positions of equilibrium. We shall suppose them to be exceedingly small, even relatively to the dimensions of the ions and we shall on this ground neglect all quantities which are of the second order with respect to $\mathfrak{a}$.

It is easily seen that, in consequence of the displacements, the electric density in a fixed point will no longer have its original value ϱ_0, but will have become

$$\varrho = \varrho_0 - \frac{\partial}{\partial x}(\varrho_0\, \mathfrak{a}_x) - \frac{\partial}{\partial y}(\varrho_0\, \mathfrak{a}_y) - \frac{\partial}{\partial z}(\varrho_0\, \mathfrak{a}_z).$$

Here, the last terms, which evidently must be taken into account, have the order of magnitude $\frac{c\,\varrho_0}{a}$, if c denotes the amplitude of the vibrations; consequently, the first term of the right-hand member of (I_c) will contain quantities of the order

$$\frac{V^2\, c\, \varrho_0}{a} \quad \ldots\ldots\ldots \quad (3)$$

On the other hand, if T is the time of vibration, the last term in (I_c) will be of the order

$$\frac{\mathfrak{p}_x\, \varrho_0\, c}{T}, \quad \ldots\ldots\ldots \quad (4)$$

Dividing this by (3), we get

$$\frac{\mathfrak{p}_x}{V} \cdot \frac{a}{VT},$$

an extremely small quantity, because the diameter of the ions is a very small fraction of the wave-length. This is the reason why we may omit the last term in (I_c).

As to the equations (V_c), it must be remarked that, if the displacements are infinitely small, the same will be true of the velocities and, in general, of all quantities which do not exist as long as the system is at rest and are entirely produced by the motion. Such are $\mathfrak{H}'_x$, $\mathfrak{H}'_y$, $\mathfrak{H}'_z$. We may therefore omit the last terms in (V_c), as being of the second order.

The same reasoning would apply to the terms containing $\frac{\mathfrak{p}_x}{V^2}$, if we could be sure that in the state of equilibrium there are no electric forces at all. If, however, in the absence of any vibrations, the vector $\mathfrak{F}'$ has already a certain value $\mathfrak{F}_0'$, it will only be the difference $\mathfrak{F}'-\mathfrak{F}_0'$, that may be called infinitely small; it will then be permitted to replace $\mathfrak{F}'_y$ and $\mathfrak{F}'_z$ by $\mathfrak{F}'_{0y}$ and $\mathfrak{F}'_{0z}$.

Another restriction consists in supposing that an ion is incapable of any motion but a translation as a whole, and that, in the position of equilibrium, though its parts may be acted on by electric forces, as has just been said, yet the whole ion does not experience a resultant electric force. Then, if $d\tau$ is an element of volume, and the integrations are extended all over the ion,

$$\int \varrho_0 \mathfrak{F}'_{0y}\, d\tau = \int \varrho_0 \mathfrak{F}'_{0z}\, d\tau = 0 . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Again, in the case of vibrations, the equations (V_c) will only serve to calculate the resultant force acting on an ion. In the direction of the axis of y e. g. this force will be

$$\int \varrho\, \mathfrak{E}_y\, d\tau .$$

Its value may be found, if we begin by applying the second of the three equations to each point of the ion, always for the same universal time t, and then integrate. From the second term on the right-hand side we find

$$-\frac{\mathfrak{p}_x}{V^2}\int \varrho\, \mathfrak{v}_x\, \mathfrak{F}'_y\, d\tau\,,$$

or, since we may replace $\mathfrak{F}'_y$ by $\mathfrak{F}'_{0y}$ and ϱ by ϱ_0,

$$-\frac{\mathfrak{p}_x}{V^2}\,\mathfrak{v}_x\int \varrho_0\, \mathfrak{F}'_{0y}\, d\tau\,,$$

which vanishes on account of (5).

Hence, as far as regards the resultant force, we may put $\mathfrak{E} = \mathfrak{F}'$, that is to say, we may take $\mathfrak{F}'$ as the electric force, acting not only on ions at rest, but also on moving ions.

The equations will be somewhat simplified, if, instead of $\mathfrak{F}'$, we introduce the already mentioned difference $\mathfrak{F}'-\mathfrak{F}'_0$. In order to do this, we have only twice to write down the equations (I_c)—(IV_c), once for the vibrating system and a second time for the same system in a state of rest; and then to subtract the equations of the second system from those of the first. In the resulting equations, I shall, for the sake of brevity, write $\mathfrak{F}'$ instead of $\mathfrak{F}'-\mathfrak{F}'_0$, so that henceforth $\mathfrak{F}'$ will denote not the total electric force, but only the part of it that is due to the vibrations. At the same time we shall replace the value of ϱ, given above, by

$$\varrho_0 - a_x\frac{\partial\varrho_0}{\partial x'} - a_y\frac{\partial\varrho_0}{\partial y'} - a_z\frac{\partial\varrho_0}{\partial z'}\,.$$

We may do so, because we have supposed a_x, a_y, a_z to have the same values all over an ion, and because ϱ_0 is independent of the time, so that

$$\frac{\partial\varrho_0}{\partial x} = \frac{\partial\varrho_0}{\partial x'}\,.$$

Finally we have

$$Div'\,\mathfrak{F}' = -4\pi V^2\left(a_x\frac{\partial\varrho_0}{\partial x'} + a_y\frac{\partial\varrho_0}{\partial y'} + a_z\frac{\partial\varrho_0}{\partial z'}\right), \quad (I_d)$$

$$Div'\,\mathfrak{H}' = 0\,, \quad \ldots\ldots\ldots \quad (II_d)$$

$$\frac{\partial\mathfrak{H}'_z}{\partial y'} - \frac{\partial\mathfrak{H}'_y}{\partial z'} = 4\pi\varrho_0\frac{\partial a_x}{\partial t'} + \frac{1}{V^2}\frac{\partial\mathfrak{F}'_x}{\partial t'}, \text{ etc.} \quad \ldots \quad (III_d)$$

$$\frac{\partial\mathfrak{F}'_z}{\partial y'} - \frac{\partial\mathfrak{F}'_y}{\partial z'} = -\frac{\partial\mathfrak{H}'_x}{\partial t'}, \text{ etc.} \quad \ldots\ldots \quad (IV_d)$$

Since these equations do no longer explicitly contain the velocity $\mathfrak{p}_x$, they will hold, without any change of form, for a system that has no translation, in which case, of course, t' would be the same thing as the universal time t.

Yet, strictly speaking, there would be a slight difference in the formulae, when applied to the two cases. In the system without a translation $\mathfrak{a}_x$, $\mathfrak{a}_y$, $\mathfrak{a}_z$ would be, in all points of an ion, the same functions of t', i. e. of the universal time, whereas, in the moving system, these components would not depend in the same way on t' in different parts of the ion, just because they must everywhere be the same functions of t.

However, we may ignore this difference, of the ions are so small, that we may assign to each of them a single local time, applicable to all its parts.

The equality of form of the electromagnetic equations for the two cases of which we have spoken will serve to simplify to a large extent our investigation. However, it should be kept in mind, that, to the equations (I_d)—(IV_d), we must add the equations of motion for the ions themselves In establishing these, we have to take into account, not only the electric forces, but also all other forces acting on the ions. We shall call these latter the molecular forces and we shall begin by supposing them to be sensible only at such small distances, that two particles of matter, acting on each other, may be said to have the same local time.

§ 7. Let us now imagine two systems of ponderable bodies, the one S with a translation, and the other one S_0 without such a motion, but equal to each other in all other respects. Since we neglect quantities of the order $\mathfrak{p}_x^2/V^2$, the electric force will, by § 5 be the same in both systems, as long as there are no vibrations.

After these have been excited, we shall have for both systems the equations (I_d)—(IV_d).

Further we shall imagine motions of such a kind, that, if in a point (x', y', z') of S_0 we find a certain quantity of matter or a certain electric charge at the universal time t', an equal quantity of matter or an equal charge will be found in the corresponding point of S at the local time t'. Of course, this involves that at these corresponding times we shall have, in the point (x', y', z') of both systems, the same electric density, the same displacement $\mathfrak{a}$, and equal velocities and accelerations.

Thus, some of the dependent variables in our equations (I_d)—(IV_d) will be represented in S_0 and S by the same functions of x', y', z', t',

whence we conclude that the equations will be satisfied by values of $\mathfrak{H}'_x$, $\mathfrak{H}'_y$, $\mathfrak{H}'_z$, $\mathfrak{F}'_x$, $\mathfrak{F}'_y$, $\mathfrak{F}'_z$, which are likewise in both cases the same functions of x', y', z', t'. By what has been said at the beginning of this §, not only $\mathfrak{F}'$, but also the total electric force will be the same in S_0 and S, always provided that corresponding ions at corresponding times (i. e. for equal values of t') be considered.

As to the molecular forces, acting on an ion, they are confined to a certain small space surrounding it, and by what has been said in § 6, the difference of local times within this space may be neglected. Moreover, if equal spaces of this kind are considered in S_0 and S, there will be, at corresponding times, in both the same distribution of matter. This is a consequence of what has been supposed concerning the two motions.

Now, the simplest assumption we can make on the molecular forces is this, that they are *not* changed by the translation of the system. If this be admitted, it appears from the above considerations that corresponding ions in S_0 and S will be acted on by the same molecular forces, as well as by the same electric forces. Therefore, since the masses and accelerations are the same, the supposed motion in S will be possible as soon as the corresponding motion in S_0 can really exist. In this way we are led to the following theorem.

If, in a body or a system of bodies, without a translation, a system of vibrations be given, in which the displacements of the ions and the components of $\mathfrak{F}'$ and $\mathfrak{H}'$ are certain functions of the coordinates and the time, then, if a translation be given to the system, there can exist vibrations, in which the displacements and the components of $\mathfrak{F}'$ and $\mathfrak{H}'$ are the same functions of the coordinates and the *local* time. This is the theorem, to which I have been led in a much more troublesome way in my „Versuch einer Theorie, etc.", and by which most of the phenomena, belonging to the theory of aberration may be explained.

§ 8. In what precedes, the molecular forces have been supposed to be confined to excessively small distances. If two particles of matter were to act upon each other at such a distance that the difference of their local times might not be neglected, the theorem would no longer be true in the case of molecular forces that are not altered at all by the translation. However, one soon perceives that the theorem would again hold good, if these forces were changed by the translation in a definite way, in such a way namely that the action between two quantities of matter were determined, not by the *simultaneous* values of their coordinates, but by their values

at *equal local times*. If therefore, we should meet with phenomena, in which the difference of the local times for mutually acting particles might have a sensible influence, and in which yet observation showed the above theorem to be true, this would indicate a modification, like the one we have just specified, of the molecular forces by the influence of a translation. Of course, such a modification would only be possible, if the molecular forces were no direct actions at a distance, but were propagated by the aether in a similar way as the electromagnetic actions. Perhaps the rotation of the plane of polarization in the so-called active bodies will be found to be a phenomenon of the kind, just mentioned.

§ 9. Hitherto all quantities of the order p^2_x/V^2 have been neglected. As is well known, these must be taken into account in the discussion of MICHELSON's experiment, in which two rays of light interfered after having traversed rather long paths, the one parallel to the direction of the earth's motion, and the other perpendicular to it. In order to explain the negative result of this experiment FITZGERALD and myself have supposed that, in consequence of the translation, the dimensions of the solid bodies serving to support the optical apparatus, are altered in a certain ratio.

Some time ago, M. LIÉNARD [1]) has emitted the opinion that, according to my theory, the experiment should have a positive result, if it were modified in so far, that the rays had to pass through a solid or a liquid dielectric.

It is impossible to say with certainty what would be observed in such a case, for, if the explication of MICHELSON's result which I have proposed is accepted, we must also assume that the mutual distances of the molecules of transparent media are altered by the translation.

Besides, we must keep in view the possibility of an influence, be it of the second order, of the translation on the molecular forces.

In what follows I shall shew, not that the result of the experiment must necessarily be negative, but that this might very well be the case. At the same time it will appear what would be the theoretical meaning of such a result.

Let us return again to the equations (I_c)—(V_c). This time we shall not put in them $k = 1$, but the other simplifications of which we have spoken in § 6 will again be introduced. We shall now have to distinguish between the vectors $\mathfrak{E}$ and $\mathfrak{F}$, the former alone

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[1]) L'Éclairage Électrique, 20 et 27 août 1898.

being the electric force. By both signs I shall now denote, not the whole vector, but the part that is due to the vibrations.

The equations may again be written in a form in which the velocity of translation does not explicitly appear. For this purpose, it is necessary to replace the variables x', y', z', t', $\mathfrak{F}'$, $\mathfrak{H}'$, $\mathfrak{a}$ and ϱ_0 by new ones, differing from the original quantities by certain constant factors.

For the sake of uniformity of notation all these new variables will be distinguished by double accents. Let ε be an indeterminate coefficient, differing from unity by a quantity of the order $\mathfrak{p}_x^2/V^2$, and let us put

$$x = \frac{\varepsilon}{k} x'', \quad y = \varepsilon y'', \quad z = \varepsilon z'' \ , \ . \ . \ . \ . \ . \ . \quad (6)$$

$$\mathfrak{a}_x = \frac{\varepsilon}{k} \mathfrak{a}''_x, \quad \mathfrak{a}_y = \varepsilon \mathfrak{a}''_y, \quad \mathfrak{a}_z = \varepsilon \mathfrak{a}'_z \ , \ . \ . \ . \ . \quad (7)$$

$$\varrho_0 = \frac{k}{\varepsilon^3} \varrho_0'' \ , \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (8)$$

$$\mathfrak{F}'_x = \frac{1}{\varepsilon^2} \mathfrak{F}''_x \ , \quad \mathfrak{F}'_y = \frac{1}{\varepsilon^2} \mathfrak{F}''_y \ , \quad \mathfrak{F}'_z = \frac{1}{\varepsilon^2} \mathfrak{F}''_z \ ,$$

$$\mathfrak{H}_x = \frac{k}{\varepsilon^2} \mathfrak{H}''_x \ , \quad \mathfrak{H}'_y = \frac{k}{\varepsilon^2} \mathfrak{H}''_y \ , \quad \mathfrak{H}'_z = \frac{k}{\varepsilon^2} \mathfrak{H}''_z \ ,$$

$$t' = k \varepsilon t'' \ , \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (9)$$

so that t'' is a modified local time; then we find

$$Div'' \mathfrak{F}'' = 4 \pi V^2 \left(- \mathfrak{a}''_x \frac{\partial \varrho_0''}{\partial x''} - \mathfrak{a}''_y \frac{\partial \varrho_0''}{\partial y''} - \mathfrak{a}''_z \frac{\partial \varrho_0''}{\partial z''} \right), \quad (\mathrm{I_c})$$

$$Div'' \mathfrak{H}'' = 0 \ , \ . \ . \ . \ . \ . \ . \ . \ . \quad (\mathrm{II_c})$$

$$\frac{\partial \mathfrak{H}''_z}{\partial y''} - \frac{\partial \mathfrak{H}''_y}{\partial z''} = 4 \pi \varrho_0'' \frac{\partial \mathfrak{a}''_x}{\partial t''} + \frac{1}{V^2} \frac{\partial \mathfrak{F}''_x}{\partial t''}, \text{ etc.} \ . \quad (\mathrm{III_c})$$

$$\frac{\partial \mathfrak{F}''_z}{\partial y''} - \frac{\partial \mathfrak{F}''_y}{\partial z''} = - \frac{\partial \mathfrak{H}''_x}{\partial t''}, \text{ etc.} \ . \ . \ . \ . \ . \quad (\mathrm{IV_c})$$

$$\mathfrak{E}_x = \frac{1}{\varepsilon^2} \mathfrak{F}''_x \ , \quad \mathfrak{E}_y = \frac{1}{k \varepsilon^2} \mathfrak{F}''_y \ , \quad \mathfrak{E}_z = \frac{1}{k \varepsilon^2} \mathfrak{F}''_z \ . \ . \ . \quad (\mathrm{V_c})$$

These formulae will also hold for a system without translation; only, in this case we must take $k = 1$, and we shall likewise take $\varepsilon = 1$, though this is not necessary. Thus, x'', y'', z'' will then be the coordinates, t'' the same thing as t, i. e. the universal time, $\mathfrak{a}''$ the displacement, ϱ_0'' the electric density, $\mathfrak{H}''$ and $\mathfrak{F}''$ the magnetic and electric forces, the last in so far as it is due to the vibrations.

Our next object will be to ascertain under what conditions, now that we retain the terms with $\mathfrak{p}_x^2/V^2$, two systems S and S_0, the first having a translation, and the second having none, may be in vibratory states that are related to each other in some definite way. This investigation resembles much the one that has been given in § 7; it may therefore be expressed in somewhat shorter terms.

To begin with, we shall agree upon the degree of similarity there shall be between the two systems in their states of equilibrium. In this respect we define S by saying that the system S_0 may be changed into it by means of the dilatations indicated by (6); we shall suppose that, in undergoing these dilatations, each element of volume retains its ponderable matter, as well as its charge. It is easily seen that this agrees with the relation (8).

We shall not only suppose that the system S_0 *may* be changed in this way into an imaginary system S, but that, as soon as the translation is given to it, the transformation *really* takes place, of itself, i. e. by the action of the forces acting between the particles of the system, and the aether. Thus, after all, S will be the *same* material system as S.

The transformation of which I have now spoken, is precisely such a one as is required in my explication of MICHELSON's experiment. In this explication the factor ε may be left indeterminate. We need hardly remark that for the real transformation produced by a translatory motion, the factor should have a definite value. I see, however, no means to determine it.

Before we proceed further, a word on the electric forces in S and S_0 in their states of equilibrium. If $\varepsilon = 1$, the relation between these forces will be given by the equations of § 5. Now ε indicates an alteration of all dimensions in the same ratio, and it is very easy to see what influence this will have on the electric forces. Thus, it will be found that, in passing from S_0 to S, the electric force in the direction of OX will be changed in the ratio of 1 to $\frac{1}{\varepsilon^2}$, and that the corresponding ratio for the other components will be as 1 to $\frac{1}{k\varepsilon^2}$.

As to the corresponding vibratory motions, we shall require that at corresponding times, i. e. for equal values of t'', the configuration of S may always be got from that of S_0 by the above mentioned dilatations. Then, it appears from (7) that $\mathfrak{a}''_x$, $\mathfrak{a}''_y$, $\mathfrak{a}''_z$ will be, in both systems, the same functions of x'', y'', z'', t'', whence we conclude that the equations (I_e)—(IV_e) can be satisfied by values of $\mathfrak{F}''_x$, $\mathfrak{H}''_x$, etc., which are likewise, in S_0 and in S, the same functions of x'', y'', z'', t''.

Always provided that we start from a vibratory motion in S_0 that can really exist, we have now arrived at a motion in S, that is possible in so far as it satisfies the electromagnetic equations. The last stage of our reasoning will be to attend to the molecular forces. In S_0 we imagine again, around one of the ions, the same small space, we have considered in § 7 and to which the molecular forces acting on the ion are confined; in the other system we shall now conceive the corresponding small space, i. e. the space that may be derived from the first one by applying to it the dilatations (6). As before, we shall suppose these spaces to be so small that in the second of them there is no necessity to distinguish the local times in its different parts; then we may say that in the two spaces there will be, at corresponding times, corresponding distributions of matter.

We have already seen that, in the states of equilibrium, the electric forces parallel to OX, OY, OZ, existing in S differ from the corresponding forces in S_0 by the factors

$$\frac{1}{\varepsilon^2}, \frac{1}{k\varepsilon^2} \text{ and } \frac{1}{k\varepsilon^2}.$$

From (V_e) it appears that the same factors come into play when we consider the part of the electric forces that is due to the vibrations. If, now, we suppose that the molecular forces are modified in quite the same way in consequence of the translation, we may apply the just mentioned factors to the components of the *total* force acting on an ion. Then, the imagined motion in S will be a possible one, provided that these same factors to which we have been led in examining the forces present themselves again, when we treat of the product of the masses and the accelerations.

According to our suppositions, the accelerations in the directions of OX, OY, OZ in S are resp. $\frac{1}{k^3\varepsilon}$, $\frac{1}{k^2\varepsilon}$ and $\frac{1}{k^2\varepsilon}$ times what they

are in S_0. If therefore the required agreement is to exist with regard to the vibrations parallel to OX, the ratio of the masses of the ions in S and S_0 should be $\frac{k^3}{\varepsilon}$; on the contrary we find for this ratio $\frac{k}{\varepsilon}$, if we consider in the same way the forces and the accelerations in the directions of OY and OZ.

Since k is different from unity, these values cannot both be 1; consequently, states of motion, related to each other in the way we have indicated, will only be possible, if in the transformation of S_0 into S the masses of the ions change; even, this must take place in such a way that the same ion will have different masses for vibrations parallel and perpendicular to the velocity of translation.

Such a hypothesis seems very startling at first sight. Nevertheless we need not wholly reject it. Indeed, as is well known, the *effective* mass of an ion depends on what goes on in the aether; it may therefore very well be altered by a translation and even to different degrees for vibrations of different directions.

If the hypothesis might be taken for granted, MICHELSON's experiment should always give a negative result, whatever transparent media were placed on the path of the rays of light, and even if one of these went through air, and the other, say through glass. This is seen by remarking that the correspondence between the two motions we have examined is such that, if in S_0 we had a certain distribution of light and dark (interference-bands) we should have in S a similar distribution, which might be got from that in S_0 by the dilatations (6), provided however that in S the time of vibration be $k\varepsilon$ times as great as in S_0. The necessity of this last difference follows from (9). Now the number $k\varepsilon$ would be the same in all positions we can give to the apparatus; therefore, if we continue to use the *same* sort of light, while rotating the instruments, the interference-bands will never leave the parts of the ponderable system, e. g. the lines of a micrometer, with which they coincided at first.

We shall conclude by remarking that the alteration of the molecular forces that has been spoken of in this § would be one of the second order, so that we have not come into contradiction with what has been said in § 7.