Armoniche sferiche [1]

$$Y_{l\,-m} = (-1)^m Y^*_{lm} \tag{0.1}$$

$$\begin{aligned}
Y_{00} &= \frac{1}{2}\sqrt{\frac{1}{\pi}} && (0.2)\\
Y_{1-1} &= \frac{1}{4}\sqrt{\frac{6}{\pi}}\sin\theta\exp(-i\,\phi) && (0.3)\\
Y_{10} &= \frac{1}{2}\sqrt{\frac{3}{\pi}}\cos\theta && (0.4)\\
Y_{11} &= -\frac{1}{4}\sqrt{\frac{6}{\pi}}\sin\theta\exp(i\,\phi) && (0.5)\\
Y_{2-2} &= \frac{1}{8}\sqrt{\frac{30}{\pi}}\sin^2\theta\exp(-2i\,\phi) && (0.6)\\
Y_{2-1} &= \frac{1}{4}\sqrt{\frac{30}{\pi}}\sin\theta\cos\theta\exp(-i\,\phi) && (0.7)\\
Y_{20} &= \frac{1}{4}\sqrt{\frac{5}{\pi}}\left(3\cos^2\theta - 1\right) && (0.8)\\
Y_{21} &= -\frac{1}{4}\sqrt{\frac{30}{\pi}}\sin\theta\cos\theta\exp(i\,\phi) && (0.9)\\
Y_{22} &= \frac{1}{8}\sqrt{\frac{30}{\pi}}\sin^2\theta\exp(2i\,\phi) && (0.10)\\
Y_{3-3} &= \frac{1}{8}\sqrt{\frac{35}{\pi}}\sin^3\theta\exp(-3i\,\phi) && (0.11)\\
Y_{3-2} &= \frac{1}{8}\sqrt{\frac{210}{\pi}}\sin^2\theta\cos\theta\exp(-2i\,\phi) && (0.12)\\
Y_{3-1} &= \frac{1}{8}\sqrt{\frac{21}{\pi}}\sin\theta\left(5\cos^2\theta - 1\right)\exp(-i\,\phi) && (0.13)\\
Y_{30} &= \frac{1}{4}\sqrt{\frac{7}{\pi}}\cos\theta\left(5\cos^2\theta - 3\right) && (0.14)
\end{aligned}$$

[1] `http://virgilio.mib.infn.it/~oleari/`

$$Y_{31} = -\frac{1}{8}\sqrt{\frac{21}{\pi}}\sin\theta\left(5\cos^2\theta - 1\right)\exp(i\,\phi) \tag{0.15}$$

$$Y_{32} = \frac{1}{8}\sqrt{\frac{210}{\pi}}\sin^2\theta\cos\theta\exp(2i\,\phi) \tag{0.16}$$

$$Y_{33} = -\frac{1}{8}\sqrt{\frac{35}{\pi}}\sin^3\theta\exp(3i\,\phi) \tag{0.17}$$

$$Y_{00} = \frac{1}{2}\sqrt{\frac{1}{\pi}} \tag{0.18}$$

$$Y_{1-1} = \frac{1}{4}\sqrt{\frac{6}{\pi}}(x - i\,y)/r \tag{0.19}$$

$$Y_{10} = \frac{1}{2}\sqrt{\frac{3}{\pi}}z/r \tag{0.20}$$

$$Y_{11} = -\frac{1}{4}\sqrt{\frac{6}{\pi}}(x + i\,y)/r \tag{0.21}$$

$$Y_{2-2} = \frac{1}{8}\sqrt{\frac{30}{\pi}}(x - i\,y)^2/r^2 \tag{0.22}$$

$$Y_{2-1} = \frac{1}{4}\sqrt{\frac{30}{\pi}}z/r^2(x - i\,y) \tag{0.23}$$

$$Y_{20} = \frac{1}{4}\sqrt{\frac{5}{\pi}}(3z^2/r^2 - 1) \tag{0.24}$$

$$Y_{21} = -\frac{1}{4}\sqrt{\frac{30}{\pi}}z/r^2(x + i\,y) \tag{0.25}$$

$$Y_{22} = \frac{1}{8}\sqrt{\frac{30}{\pi}}(x + i\,y)^2/r^2 \tag{0.26}$$

$$Y_{3-3} = \frac{1}{8}\sqrt{\frac{35}{\pi}}(x - i\,y)^3/r^3 \tag{0.27}$$

$$Y_{3-2} = \frac{1}{8}\sqrt{\frac{210}{\pi}}z/r^3(x - i\,y)^2 \tag{0.28}$$

$$Y_{3-1} = \frac{1}{8}\sqrt{\frac{21}{\pi}}(5z^2/r^2 - 1)(x - i\,y)/r \tag{0.29}$$

$$Y_{30} = \frac{1}{4}\sqrt{\frac{7}{\pi}}z/r(5z^2/r^2 - 3) \tag{0.30}$$

$$Y_{31} = -\frac{1}{8}\sqrt{\frac{21}{\pi}}(5z^2/r^2 - 1)(x + i\,y)/r \tag{0.31}$$

$$Y_{32} = \frac{1}{8}\sqrt{\frac{210}{\pi}}z/r^3(x + i\,y)^2 \tag{0.32}$$

$$Y_{33} = -\frac{1}{8}\sqrt{\frac{35}{\pi}}(x + i\,y)^3/r^3 \tag{0.33}$$

$$f(x, y, z) = f(r, \theta, \phi) = \sum_{l,m} c_{lm}\,Y_{lm} \tag{0.34}$$

$$1 \tag{0.35}$$

$$c_{00} = 2\sqrt{\pi} \tag{0.36}$$

$$x/r \tag{0.37}$$

$$c_{1-1} = \frac{1}{3}\sqrt{6\pi} \tag{0.38}$$

$$c_{11} = -\frac{1}{3}\sqrt{6\pi} \tag{0.39}$$

$$y/r \tag{0.40}$$

$$c_{1-1} = \frac{1}{3}i\sqrt{6\pi} \tag{0.41}$$

$$c_{11} = \frac{1}{3}i\sqrt{6\pi} \tag{0.42}$$

$$z/r \tag{0.43}$$

$$c_{10} = \frac{2}{3}\sqrt{3\pi} \tag{0.44}$$

$$x^2/r^2 \tag{0.45}$$

$$c_{00} = \frac{2}{3}\sqrt{\pi} \tag{0.46}$$

$$c_{2-2} = \frac{1}{15}\sqrt{30\pi} \tag{0.47}$$

$$c_{20} = -\frac{2}{15}\sqrt{5\pi} \tag{0.48}$$

$$c_{22} = \frac{1}{15}\sqrt{30\pi} \tag{0.49}$$

$$y^2/r^2 \tag{0.50}$$

$$c_{00} = \frac{2}{3}\sqrt{\pi} \tag{0.51}$$

$$c_{2-2} = -\frac{1}{15}\sqrt{30\pi} \tag{0.52}$$

$$c_{20} = -\frac{2}{15}\sqrt{5\pi} \tag{0.53}$$

$$c_{22} = -\frac{1}{15}\sqrt{30\pi} \tag{0.54}$$

$$z^2/r^2 \tag{0.55}$$

$$c_{00} = \frac{2}{3}\sqrt{\pi} \tag{0.56}$$

$$c_{20} = \frac{4}{15}\sqrt{5\pi} \tag{0.57}$$

$$xy/r^2 \tag{0.58}$$

$$c_{2-2} = \frac{1}{15}i\sqrt{30\pi} \tag{0.59}$$

$$c_{22} = -\frac{1}{15}i\sqrt{30\pi} \tag{0.60}$$

$$xz/r^2 \tag{0.61}$$

$$c_{2-1} = \frac{1}{15}\sqrt{30\pi} \tag{0.62}$$

$$c_{21} = -\frac{1}{15}\sqrt{30\pi} \tag{0.63}$$

$$yz/r^2 \tag{0.64}$$

$$c_{2-1} = \frac{1}{15}i\sqrt{30\pi} \tag{0.65}$$

$$c_{21} = \frac{1}{15}i\sqrt{30\pi} \tag{0.66}$$

$$x^3/r^3 \tag{0.67}$$

$$c_{1-1} = \frac{1}{5}\sqrt{6\pi} \tag{0.68}$$

$$c_{11} = -\frac{1}{5}\sqrt{6\pi} \tag{0.69}$$

$$c_{3-3} = \frac{1}{35}\sqrt{35\pi} \tag{0.70}$$

$$c_{3-1} = -\frac{1}{35}\sqrt{21\pi} \tag{0.71}$$

$$c_{31} = \frac{1}{35}\sqrt{21\pi} \tag{0.72}$$

$$c_{33} = -\frac{1}{35}\sqrt{35\pi} \tag{0.73}$$

$$y^3/r^3 \tag{0.74}$$

$$c_{1-1} = \frac{1}{5}i\sqrt{6\pi} \tag{0.75}$$

$$c_{11} = \frac{1}{5}i\sqrt{6\pi} \tag{0.76}$$

$$c_{3-3} = -\frac{1}{35}i\sqrt{35\pi} \tag{0.77}$$

$$c_{3-1} = -\frac{1}{35}i\sqrt{21\pi} \tag{0.78}$$

$$c_{31} = -\frac{1}{35}i\sqrt{21\pi} \tag{0.79}$$

$$c_{33} = -\frac{1}{35}i\sqrt{35\pi} \tag{0.80}$$

$$z^3/r^3 \tag{0.81}$$

$$c_{10} = \frac{2}{5}\sqrt{3\pi} \tag{0.82}$$

$$c_{30} = \frac{4}{35}\sqrt{7\pi}$$

$$xy^2/r^3 \tag{0.83}$$

$$c_{1-1} = \frac{1}{15}\sqrt{6\pi} \tag{0.84}$$

$$c_{11} = -\frac{1}{15}\sqrt{6\pi} \tag{0.85}$$

$$c_{3-3} = -\frac{1}{35}\sqrt{35\pi} \tag{0.86}$$

$$c_{3-1} = -\frac{1}{105}\sqrt{21\pi} \tag{0.87}$$

$$c_{31} = \frac{1}{105}\sqrt{21\pi} \tag{0.88}$$

$$c_{33} = \frac{1}{35}\sqrt{35\pi} \tag{0.89}$$

$$xz^2/r^3 \tag{0.90}$$

$$c_{1-1} = \frac{1}{15}\sqrt{6\pi} \tag{0.91}$$

$$c_{11} = -\frac{1}{15}\sqrt{6\pi} \tag{0.92}$$

$$c_{3-1} = \frac{4}{105}\sqrt{21\pi} \tag{0.93}$$

$$c_{31} = -\frac{4}{105}\sqrt{21\pi} \tag{0.94}$$

$$yz^2/r^3 \tag{0.95}$$

$$c_{1-1} = \frac{1}{15}i\sqrt{6\pi} \tag{0.96}$$

$$c_{11} = \frac{1}{15}i\sqrt{6\pi} \tag{0.97}$$

$$c_{3-1} = \frac{4}{105}i\sqrt{21\pi} \tag{0.98}$$

$$c_{31} = \frac{4}{105}i\sqrt{21\pi} \tag{0.99}$$

$$x^2y/r^3 \tag{0.100}$$

$$c_{1-1} = \frac{1}{15}i\sqrt{6\pi} \tag{0.101}$$

$$c_{11} = \frac{1}{15}i\sqrt{6\pi} \tag{0.102}$$

$$c_{3-3} = \frac{1}{35}i\sqrt{35\pi} \tag{0.103}$$

$$c_{3-1} = -\frac{1}{105}i\sqrt{21\pi} \tag{0.104}$$

$$c_{31} = -\frac{1}{105} i \sqrt{21\pi} \tag{0.105}$$

$$c_{33} = \frac{1}{35} i \sqrt{35\pi} \tag{0.106}$$

$$x^2 z / r^3 \tag{0.107}$$

$$c_{10} = \frac{2}{15} \sqrt{3\pi} \tag{0.108}$$

$$c_{3-2} = \frac{1}{105} \sqrt{210\pi} \tag{0.109}$$

$$c_{30} = -\frac{2}{35} \sqrt{7\pi} \tag{0.110}$$

$$c_{32} = \frac{1}{105} \sqrt{210\pi} \tag{0.111}$$

$$y^2 z / r^3 \tag{0.112}$$

$$c_{10} = \frac{2}{15} \sqrt{3\pi} \tag{0.113}$$

$$c_{3-2} = -\frac{1}{105} \sqrt{210\pi} \tag{0.114}$$

$$c_{30} = -\frac{2}{35} \sqrt{7\pi} \tag{0.115}$$

$$c_{32} = -\frac{1}{105} \sqrt{210\pi} \tag{0.116}$$

$$xyz / r^3 \tag{0.117}$$

$$c_{3-2} = \frac{1}{105} i \sqrt{210\pi} \tag{0.118}$$

$$c_{32} = -\frac{1}{105} i \sqrt{210\pi} \tag{0.119}$$

$$\tag{0.120}$$