

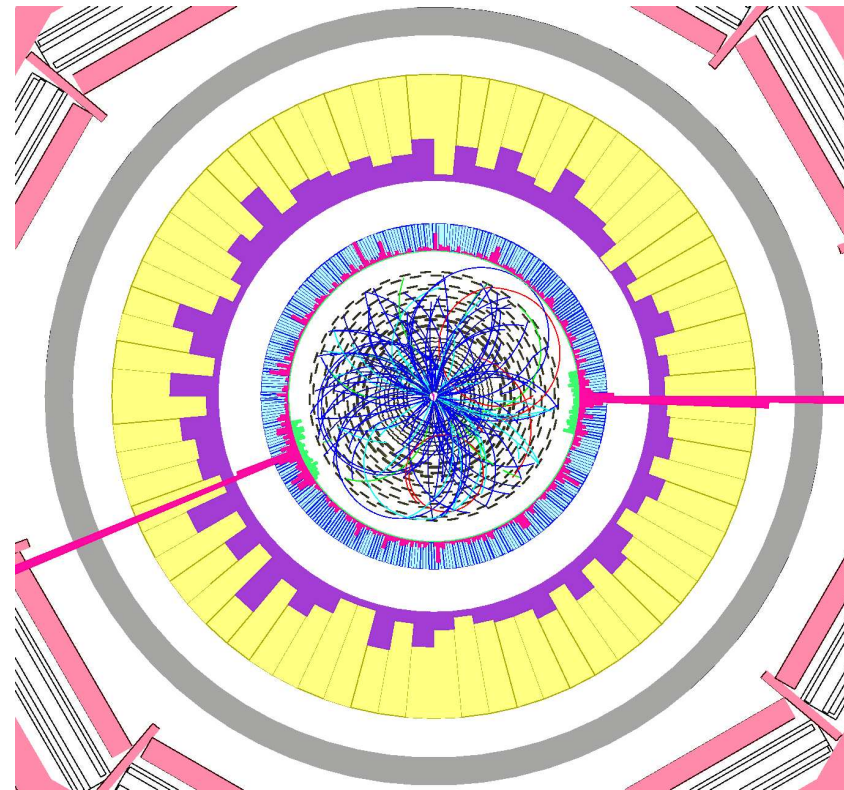
# INTRODUCTION TO ELECTROWEAK THEORY AND HIGGS-BOSON PHYSICS AT THE LHC

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GGI, Firenze, September 2007

- Theoretical introduction
- Constraints on the Higgs boson
- Higgs boson signals at the LHC



# The Standard Model (SM)

- A **quick introduction** to non-Abelian **gauge theories**: many formulae but **they will look familiar!**
  - QED
  - Yang-Mills theories
  - Electroweak interactions
- **Spontaneous symmetry breaking** and mass generation: the Higgs boson
- **Theoretical bounds** on the mass of the Higgs boson
- **Experimental bounds** on the mass of the Higgs boson



**Exercise:** Please, do the exercises! You will be given all the elements to solve them.

## Abelian gauge theory: QED

We start with a Lagrangian (density)

$$\mathcal{L}_0 = \bar{\psi}(x) (i\not{\partial} - m) \psi(x)$$

invariant under a **GLOBAL** U(1) **symmetry** ( $\theta$  is constant)

$$\begin{aligned}\psi(x) &\rightarrow e^{iq\theta} \psi(x) \\ \partial_\mu \psi(x) &\rightarrow e^{iq\theta} \partial_\mu \psi(x)\end{aligned}$$

From **Noether's theorem**, there is a **conserved current**:

$$J_\mu(x) = q\bar{\psi}(x)\gamma_\mu\psi(x) \quad \Longrightarrow \quad \partial^\mu J_\mu(x) = 0$$

To **gauge** this theory, we promote the **GLOBAL** U(1) symmetry to **local symmetry**:

$$\begin{aligned}\psi(x) &\rightarrow e^{iq\theta(x)} \psi(x) \\ \partial_\mu \psi(x) &\rightarrow e^{iq\theta(x)} \partial_\mu \psi(x) + iq e^{iq\theta(x)} \psi(x) \partial_\mu \theta(x)\end{aligned}$$

## Covariant derivative

Invent a **new derivative**  $D_\mu$  such that

$$\begin{aligned}\psi(x) &\rightarrow e^{iq\theta(x)}\psi(x) = U(x)\psi(x) \\ D_\mu\psi(x) &\rightarrow e^{iq\theta(x)}D_\mu\psi(x) = U(x)D_\mu\psi(x)\end{aligned}$$

i.e. **both**  $\psi(x)$  and  $D_\mu\psi(x)$  transform the **same way** under the U(1) local symmetry

$$D_\mu \equiv \partial_\mu + iqA_\mu$$

where  $A_\mu$  transforms under the local gauge symmetry as

$$A_\mu \rightarrow A_\mu - \partial_\mu\theta(x)$$

The **commutator** of the covariant derivatives gives the electric and the magnetic fields, i.e. the **field strength tensor**

$$F_{\mu\nu}(x) = \frac{1}{iq} [D_\mu, D_\nu] = \frac{1}{iq} [\partial_\mu + iqA_\mu, \partial_\nu + iqA_\nu] = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$$

$F_{\mu\nu}$  is **invariant** under a **gauge transformation**.

## From global to local symmetry

From

$$\mathcal{L}_0 = \bar{\psi}(x) (i\not{\partial} - m) \psi(x)$$

invariant under **GLOBAL** U(1), to

$$\begin{aligned}\mathcal{L}_1 &= \bar{\psi}(x) (i\not{D} - m) \psi(x) \\ &= \bar{\psi}(x) (i\not{\partial} - m) \psi(x) - q\bar{\psi}(x)\gamma_\mu\psi(x)A^\mu(x)\end{aligned}$$

invariant under **LOCAL** U(1) and interpret  $A^\mu(x)$  as the **photon field** and  $J_\mu = q\bar{\psi}\gamma_\mu\psi$  as the **electromagnetic current**. The only missing ingredient is the kinetic term for the photon field

$$\mathcal{L}_2 = \mathcal{L}_1 - \frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)$$

$\mathcal{L}_2$  **cannot** contain a term proportional to  $A_\mu A^\mu$  (a **mass term** for the photon field) since this term is **not gauge invariant** under the local U(1)

$$A_\mu \rightarrow A_\mu - \partial_\mu\theta(x)$$

## Non-Abelian (Yang-Mills) gauge theories

The starting point is a Lagrangian of **free** or **self-interacting** fields, that is symmetric under a **GLOBAL symmetry**

$$\mathcal{L}_\psi(\psi, \partial_\mu \psi)$$

where

$$\psi = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_n \end{pmatrix} = \text{multiplet of a compact Lie group } G$$

The Lagrangian is symmetric under the transformation

$$\psi \rightarrow \psi' = U(\theta)\psi \quad U(\theta) = \exp(igT^a\theta_a) \quad \text{unitary matrix} \quad UU^\dagger = U^\dagger U = 1$$

If  $U$  is unitary, the  $T^a$  are **hermitian**, and are called **group generators** (they “generate” infinitesimal transformation around the unity)

$$U(\theta) = 1 + igT^a\theta_a + \mathcal{O}(\theta^2)$$

If  $U \in \text{SU}(N)$  matrix (unitary and  $\det U = 1$ ), then there are  $N^2 - 1$  traceless, **hermitian** generators  $T^a = \lambda^a/2$ .



**Exercise:** Show this.

## Gauging the symmetry

The generators satisfy the relation

$$[T^a, T^b] = i f^{abc} T^c$$

and the  $f^{abc}$  are called the **structure functions** of the group  $G$ . The starting hypothesis is that  $\mathcal{L}$  is invariant under  $G$

$$\mathcal{L}_\psi(\psi, \partial_\mu \psi) = \mathcal{L}_\psi(\psi', \partial_\mu \psi') \quad \psi' = U(\theta)\psi$$

**Gauging the symmetry** means to allow the parameters  $\theta^a$  to be function of the space-time coordinates  $\theta^a \rightarrow \theta^a(x)$  so that  $\implies U \rightarrow U(x)$

$$U(x) = 1 + igT^a \theta_a(x) + \mathcal{O}(\theta^2)$$

## From $\partial_\mu \rightarrow D_\mu$

We obtain a **LOCAL** invariant Lagrangian if we make the substitution

$$\mathcal{L}_\psi(\psi, \partial_\mu \psi) \rightarrow \mathcal{L}_\psi(\psi, D_\mu \psi) \quad D_\mu = \partial_\mu - ig A_\mu^a(x) T_a \equiv \partial_\mu - ig A_\mu(x)$$

with the transformation properties

$$\begin{aligned} \psi(x) &\rightarrow U(x) \psi(x) = (1 + ig \theta^a T_a + \mathcal{O}(\theta^2)) \psi(x) \\ D_\mu &\rightarrow U(x) D_\mu \psi(x) = U(x) D_\mu U^{-1}(x) U(x) \psi(x) \end{aligned}$$

i.e. the covariant derivative must transform as

$$D_\mu \rightarrow U(x) D_\mu U^{-1}(x) \quad \text{implying} \quad A_\mu^a \rightarrow A_\mu^a + \partial_\mu \theta^a(x) + gf^{abc} A_\mu^b \theta^c + \mathcal{O}(\theta^2)$$

We can build a kinetic term for the  $A_\mu^a$  fields from

$$F_{\mu\nu} = F_{\mu\nu}^a T^a = \frac{i}{g} [D_\mu, D_\nu] \quad \text{with} \quad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c$$

which transforms homogeneously under a local gauge transformation

$$F_{\mu\nu} \rightarrow U F_{\mu\nu} U^{-1} \quad \Longrightarrow \quad F_{\mu\nu}^a F_a^{\mu\nu} \equiv 2 \text{Tr} F_{\mu\nu} F^{\mu\nu} \rightarrow \text{Tr} U F_{\mu\nu} U^{-1} U F^{\mu\nu} U^{-1} = \text{Tr} F_{\mu\nu} F^{\mu\nu}$$

where  $F_{\mu\nu}^a F_a^{\mu\nu}$  is **gauge invariant** ( $F_{\mu\nu}^a$  in **not** gauge-invariant by itself).



## The Lagrangian for gauge and matter field

Gauge invariant Yang-Mills (YM) Lagrangian for **gauge** and **matter** fields

$$\mathcal{L}_{YM} = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} + \mathcal{L}_\psi(\psi, D_\mu\psi)$$

where

$$\begin{aligned} D_\mu &= \partial_\mu - igA_\mu^a T_a \\ F_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c \\ [T^a, T^b] &= if^{abc} T^c \end{aligned}$$

## Remarks on Yang-Mills theories

- **Mass terms**  $A_\mu^a A_\mu^a$  for the gauge bosons are **NOT** gauge invariant!

No mass term is allowed in the Lagrangian.

**Gauge bosons** of (unbroken) YM theories are **massless**.

- From the  $F_{\mu\nu}^a F_a^{\mu\nu} = (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c)(\partial^\mu A_a^\nu - \partial^\nu A_a^\mu + gf_{abc} A_b^\mu A_c^\nu)$  part of the Lagrangian, we have **cubic** and **quartic** gauge boson **self interactions**
- **gauge invariance**, **Lorentz structure** and **renormalizability** (absence of higher powers of fields and covariant derivatives in  $\mathcal{L}$ ) determines gauge-boson/matter couplings and gauge-boson self interaction
- if  $G = \text{SU}_c(N = 3)$  and the fermion are in triplets,

$$\psi = \begin{pmatrix} \psi_{\text{red}} \\ \psi_{\text{blue}} \\ \psi_{\text{green}} \end{pmatrix} = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$$

we have the **QCD** Lagrangian and  $N^2 - 1 = 8$  gauge bosons = gluons.



**Exercise:** Derive the form of the three- and four-gluon vertex starting from gauge invariance, Lorentz structure and renormalizability of the Lagrangian.

## Electroweak sector

From experimental facts (charged currents couple only with left-handed fermions, the existence of a massless photon and a neutral  $Z$ ...), the electroweak group is chosen to be  $\text{SU}(2)_L \times \text{U}(1)_Y$ .

$$\psi_L \equiv \frac{1}{2}(1 - \gamma_5)\psi \quad \psi_R \equiv \frac{1}{2}(1 + \gamma_5)\psi \quad \psi = \psi_L + \psi_R$$

$$L_L \equiv \frac{1}{2}(1 - \gamma_5) \begin{pmatrix} \nu_e \\ e \end{pmatrix} = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \quad \nu_{eR} \equiv \frac{1}{2}(1 + \gamma_5)\nu_e \quad e_R \equiv \frac{1}{2}(1 + \gamma_5)e$$

- $\text{SU}(2)_L$ : weak isospin group. Three generators  $\implies$  three gauge bosons:  $W^1$ ,  $W^2$  and  $W^3$ .

The generators for doublets are  $T^a = \sigma^a/2$ , where  $\sigma^a$  are the 3 Pauli matrices (when acting on the gauge singlet  $e_R$  and  $\nu_R$ ,  $T^a \equiv 0$ ), and they satisfy  $[\sigma^a, \sigma^b] = i\epsilon^{abc}\sigma^c$ .

The gauge coupling will be indicated with  $g$ .

- $\text{U}(1)_Y$ : weak hypercharge  $Y$ . One gauge boson  $B$  with gauge coupling  $g'$ .

One generator (charge)  $Y(\psi)$ , whose value depends on the corresponding field.

## Gauging the symmetry: fermionic Lagrangian

Following the gauging recipe (for one generation of leptons. [Quarks](#) work the [same way](#))

$$\mathcal{L}_\psi = i \bar{L}_L \not{D} L_L + i \bar{\nu}_{eR} \not{D} \nu_{eR} + i \bar{e}_R \not{D} e_R$$

where

$$D^\mu = \partial^\mu - ig W_i^\mu T^i - ig' \frac{Y(\psi)}{2} B^\mu \quad T^i = \frac{\sigma^i}{2} \quad \text{or} \quad T^i = 0 \quad i = 1, 2, 3$$

$$\mathcal{L}_\psi \equiv \mathcal{L}_{kin} + \mathcal{L}_{CC} + \mathcal{L}_{NC}$$

$$\mathcal{L}_{kin} = i \bar{L}_L \not{\partial} L_L + i \bar{\nu}_{eR} \not{\partial} \nu_{eR} + i \bar{e}_R \not{\partial} e_R$$

$$\mathcal{L}_{CC} = g W_\mu^1 \bar{L}_L \gamma^\mu \frac{\sigma_1}{2} L_L + g W_\mu^2 \bar{L}_L \gamma^\mu \frac{\sigma_2}{2} L_L = \frac{g}{\sqrt{2}} W_\mu^+ \bar{L}_L \gamma^\mu \sigma^+ L_L + \frac{g}{\sqrt{2}} W_\mu^- \bar{L}_L \gamma^\mu \sigma^- L_L$$

$$= \frac{g}{\sqrt{2}} W_\mu^+ \bar{\nu}_L \gamma^\mu e_L + \frac{g}{\sqrt{2}} W_\mu^- \bar{e}_L \gamma^\mu \nu_L$$

$$\mathcal{L}_{NC} = \frac{g}{2} W_\mu^3 [\bar{\nu}_{eL} \gamma^\mu \nu_{eL} - \bar{e}_L \gamma^\mu e_L] + \frac{g'}{2} B_\mu \left[ Y(L) (\bar{\nu}_{eL} \gamma^\mu \nu_{eL} + \bar{e}_L \gamma^\mu e_L) \right. \\ \left. + Y(\nu_{eR}) \bar{\nu}_{eR} \gamma^\mu \nu_{eR} + Y(e_R) \bar{e}_R \gamma^\mu e_R \right]$$

with

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp i W_\mu^2) \quad \sigma^\pm = \frac{1}{2} (\sigma^1 \pm i \sigma^2)$$

## Electroweak unification

$$\mathcal{L}_{NC} = \frac{g}{2} W_\mu^3 [\bar{\nu}_{eL} \gamma^\mu \nu_{eL} - \bar{e}_L \gamma^\mu e_L] + \frac{g'}{2} B_\mu \left[ Y(L) (\bar{\nu}_{eL} \gamma^\mu \nu_{eL} + \bar{e}_L \gamma^\mu e_L) + Y(\nu_{eR}) \bar{\nu}_{eR} \gamma^\mu \nu_{eR} + Y(e_R) \bar{e}_R \gamma^\mu e_R \right]$$

Neither  $W_\mu^3$  nor  $B_\mu$  can be interpreted as the photon field  $A_\mu$ , since they couple to neutral fields.

$$\Psi \equiv \begin{pmatrix} \nu_{eL} \\ e_L \\ \nu_{eR} \\ e_R \end{pmatrix} \quad \mathcal{T}_3 \equiv \begin{pmatrix} 1/2 & & & \\ & -1/2 & & \\ & & 0 & \\ & & & 0 \end{pmatrix} \quad \mathcal{Y} \equiv \begin{pmatrix} Y(L) & & & \\ & Y(L) & & \\ & & Y(\nu_{eR}) & \\ & & & Y(e_R) \end{pmatrix}$$

$$\mathcal{L}_{NC} = g \bar{\Psi} \gamma^\mu \mathcal{T}_3 \Psi W_\mu^3 + g' \bar{\Psi} \gamma^\mu \frac{\mathcal{Y}}{2} \Psi B_\mu$$

## Weak mixing angle

We perform a rotation of an angle  $\theta_W$ , the **Weinberg angle**, in the space of the two neutral gauge fields ( $W_\mu^3$  and  $B_\mu$ ). We use an **orthogonal transformation** in order to keep the kinetic terms diagonal in the vector fields

$$\begin{aligned} B_\mu &= A_\mu \cos \theta_W - Z_\mu \sin \theta_W \\ W_\mu^3 &= A_\mu \sin \theta_W + Z_\mu \cos \theta_W \end{aligned}$$

so that

$$\mathcal{L}_{NC} = \bar{\Psi} \gamma^\mu \left[ g \sin \theta_W \mathcal{T}_3 + g' \cos \theta_W \frac{\mathcal{Y}}{2} \right] \Psi A_\mu + \bar{\Psi} \gamma^\mu \left[ g \cos \theta_W \mathcal{T}_3 - g' \sin \theta_W \frac{\mathcal{Y}}{2} \right] \Psi Z_\mu$$

We can identify  $A_\mu$  with the photon field provided

$$e\mathcal{Q} = g \sin \theta_W \mathcal{T}_3 + g' \cos \theta_W \frac{\mathcal{Y}}{2} \quad \mathcal{Q} = \text{electromagnetic charge}$$

The weak hypercharges  $\mathcal{Y}$  appear only through the combination  $g' \mathcal{Y}$ . We use this freedom to fix

$$Y(L) = -1$$

## Weak mixing angle

With this choice, the doublet of left-handed leptons gives ( $eQ = g \sin \theta_W \mathcal{T}_3 + g' \cos \theta_W \frac{Y}{2}$ )

$$\begin{aligned} 0 &= \frac{g}{2} \sin \theta_W - \frac{g'}{2} \cos \theta_W \\ -e &= -\frac{g}{2} \sin \theta_W - \frac{g'}{2} \cos \theta_W \end{aligned}$$

so that

$$g \sin \theta_W = g' \cos \theta_W = e$$

and

$$Q = \mathcal{T}_3 + \frac{Y}{2} \quad \text{Gell-Mann–Nishijima formula.}$$

From this formula we have  $Y(\nu_{eR}) = 0$  and  $Y(e_R) = -2$ .

Notice that the **right-handed neutrino** has zero charge, zero hypercharge and it is in a SU(2) singlet: it does **not** take part in electroweak interactions.



**Exercise:** Verify that, with the previous hypercharge assignments, one can generate the correct electromagnetic current.

## The neutral current

$$\begin{aligned}
 \mathcal{L}_{NC} &= \bar{\Psi} \gamma^\mu \left[ g \sin \theta_W \mathcal{T}_3 + g' \cos \theta_W \frac{\mathcal{Y}}{2} \right] \Psi A_\mu + \bar{\Psi} \gamma^\mu \left[ g \cos \theta_W \mathcal{T}_3 - g' \sin \theta_W \frac{\mathcal{Y}}{2} \right] \Psi Z_\mu \\
 &= e \bar{\Psi} \gamma^\mu \mathcal{Q} \Psi A_\mu + \bar{\Psi} \gamma^\mu \mathcal{Q}_Z \Psi Z_\mu
 \end{aligned}$$

where  $\mathcal{Q}_Z$  is a diagonal matrix given by

$$\mathcal{Q}_Z = \frac{e}{\cos \theta_W \sin \theta_W} (\mathcal{T}_3 - \mathcal{Q} \sin^2 \theta_W)$$



**Exercise:** Show this.

We can proceed, in a similar way, with quarks (see more later)

$$\mathcal{Q}_L^i = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \qquad \begin{aligned} u_R^i &= u_R, c_R, t_R \\ d_R^i &= d_R, s_R, b_R \end{aligned}$$



# Fermion fields of the SM and gauge quantum numbers

				<u><math>SU(3)</math></u>	<u><math>SU(2)</math></u>	<u><math>U(1)_Y</math></u>	<u><math>Q = T_3 + \frac{Y}{2}</math></u>
$Q_L^i =$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	3	2	$\frac{1}{3}$	$\frac{2}{3}$ $-\frac{1}{3}$
$u_R^i =$	$u_R$	$c_R$	$t_R$	3	1	$\frac{4}{3}$	$\frac{2}{3}$
$d_R^i =$	$d_R$	$s_R$	$b_R$	3	1	$-\frac{2}{3}$	$-\frac{1}{3}$
$L_L^i =$	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	1	2	-1	0 -1
$e_R^i =$	$e_R$	$\mu_R$	$\tau_R$	1	1	-2	-1
$\nu_R^i =$	$\nu_{eR}$	$\nu_{\mu R}$	$\nu_{\tau R}$	1	1	0	0

## Electroweak gauge-boson sector

Gauge invariance and renormalizability completely determine the kinetic terms for the gauge bosons

$$\mathcal{L}_{YM} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W_{\mu\nu}^a W_a^{\mu\nu}$$

$$B^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu$$

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g \epsilon^{abc} W_{b,\mu} W_{c,\nu}$$

The gauge symmetry does NOT allow any mass terms for  $W^\pm$  and  $Z$ .

Mass terms for gauge bosons

$$\mathcal{L}_{mass} = \frac{1}{2} m_A^2 A_\mu A^\mu$$

are not invariant under a gauge transformation

$$A^\mu \rightarrow U(x) \left( A^\mu + \frac{i}{g} \partial^\mu \right) U^{-1}(x)$$

However, the gauge bosons of weak interactions are massive (short range of weak interactions).

## Symmetries and conservation laws

To any continuous symmetry of the Lagrangian we can associate a conservation law and a conserved current.

**Noether's theorem:** if, **without** using the **equation of motion**, one can show that the Lagrangian density changes by a total divergence under an infinitesimal transformation

$$\begin{aligned} \phi \rightarrow \phi + \delta\phi \sim \phi + i\delta\theta\phi & \quad (\phi_j \rightarrow \phi_j + i\delta\theta^a t_{jk}^a \phi_k) & \quad \delta\theta \ll 1 \\ \delta\mathcal{L}(\phi, \partial\phi) = \delta\theta \partial^\mu K_\mu & \quad \delta S = 0 \end{aligned}$$

then

$$J^\mu = \frac{\delta\mathcal{L}}{\delta\partial_\mu\phi} \delta\phi - K^\mu \quad \text{is conserved} \quad \partial_\mu J^\mu = 0$$

### Important consequences



$$Q = \int d^3x J^0(\vec{x}, t)$$

is **conserved** ( $dQ/dt = 0$ ) and is a **Lorentz scalar**

- ✓ After canonical quantization,  $i\delta\theta [Q, \phi] = \delta\phi$ , hence  $Q$  **generates the symmetry** acting on the fields

## Symmetries in quantum field theories

**Two ways** of realizing symmetries in a QFT. Suppose we have a charge  $Q$  (obtained from Noether's theorem) that commutes with the Hamiltonian  $[Q, H] = 0$ . Then

- **Wigner–Weyl**

$$[Q, H] = 0 \quad Q|0\rangle = 0$$

The spectrum falls in explicit multiplets of the symmetry group (the vacuum  $|0\rangle$  is the state of lowest energy)

- **Nambu–Goldstone**

$$[Q, H] = 0 \quad Q|0\rangle \neq 0$$

The symmetry is **not manifest** in the spectrum.

There is a **third way** too: **the anomalous symmetries**. In this case, the classical theory respects the symmetry, that is violated by quantum fluctuations

$$\partial^\mu J_\mu = 0 + \mathcal{O}(\hbar)$$

As we have stressed up to now, another important distinction is between **global** and **local** symmetries.

## Spontaneous symmetry breaking

A symmetry is said to be **spontaneously broken** when the vacuum state is not invariant

$$\exp(i \delta\theta^a t^a) |0\rangle \neq |0\rangle \quad \implies \quad Q^a |0\rangle \neq 0$$

This condition is equivalent to the existence of some set of fields operators  $\phi_k$  with non-trivial transformation property under that symmetry transformation, and non-vanishing vacuum expectation values

$$\langle 0 | \phi_k | 0 \rangle = v_k \neq 0$$

### Proof

If the set of fields  $\phi_j$  transforms non-trivially

$$\phi_j \rightarrow \left( e^{i \delta\theta^a t^a} \right)_{jk} \phi_k \sim \phi_j + \underbrace{i \delta\theta^a t_{jk}^a \phi_k}_{\delta\phi_j} = \phi_j + i \delta\theta^a [Q^a, \phi_j]$$

Taking the expectation value on the vacuum

$$t_{jk}^a \langle 0 | \phi_k | 0 \rangle = \langle 0 | [Q^a, \phi_j] | 0 \rangle \neq 0 \quad \iff \quad Q^a |0\rangle \neq 0$$

# Spontaneous symmetry breaking

## Observations

- Experimentally, the **space is isotropic**, so  $\phi_k$  must be a **scalar**, otherwise its vacuum expectation value would be frame-dependent.
- Experimentally, the **space is homogeneous**, so that  $\langle 0|\phi_k|0\rangle$  is a **constant**.  
In fact, if the vacuum state is invariant under translations

$$\langle 0|\phi_k(x)|0\rangle = \langle 0|e^{iPx} \phi_k(0)e^{-iPx}|0\rangle = \langle 0|\phi_k(0)|0\rangle$$

- $\phi_k$  is **not necessarily** an elementary field

## Spontaneous symmetry breaking in the SM

- ✓ Experimentally, the **weak bosons** have **masses**.
- ✓ The only way to introduce masses for the  $W$  and  $Z$  vector bosons, without spoiling unitarity and renormalizability, is spontaneous breaking of the gauge symmetry.
- ✓ The **simplest way** is through the (minimal) **Higgs mechanism**.

## Spontaneous symmetry breaking in the SM

We give mass to the gauge bosons through the **Higgs mechanism**: generate mass terms from the **kinetic energy** term of a **scalar doublet** field  $\Phi$  that undergoes a broken-symmetry process.

Introduce a complex scalar doublet: **four scalar real fields** (why will become clear later)

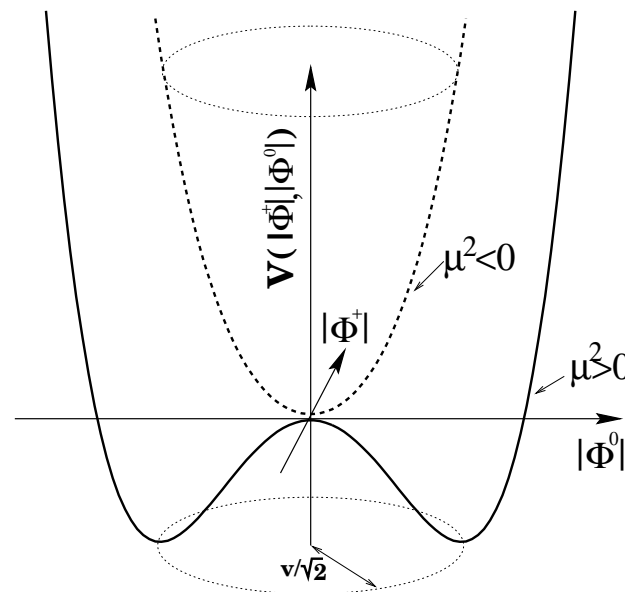
$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad Y(\Phi) = 1$$

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi^\dagger \Phi)$$

$$D^\mu = \partial^\mu - igW_i^\mu \frac{\sigma^i}{2} - ig' \frac{Y(\Phi)}{2} B^\mu$$

$$V(\Phi^\dagger \Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2, \quad \mu^2, \lambda > 0$$

Notice the **“wrong”** mass sign.



$V(\Phi^\dagger \Phi)$  is  **$SU(2)_L \times U(1)_Y$  symmetric**.

- The reason why  $Y(\Phi) = 1$  is **not** to break electric-charge conservation.
- Charge assignment for the Higgs doublets is done according to  $Q = T_3 + Y/2$ .



## Spontaneous symmetry breaking

The potential has a minimum in correspondence of

$$|\Phi|^2 = \frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2}$$

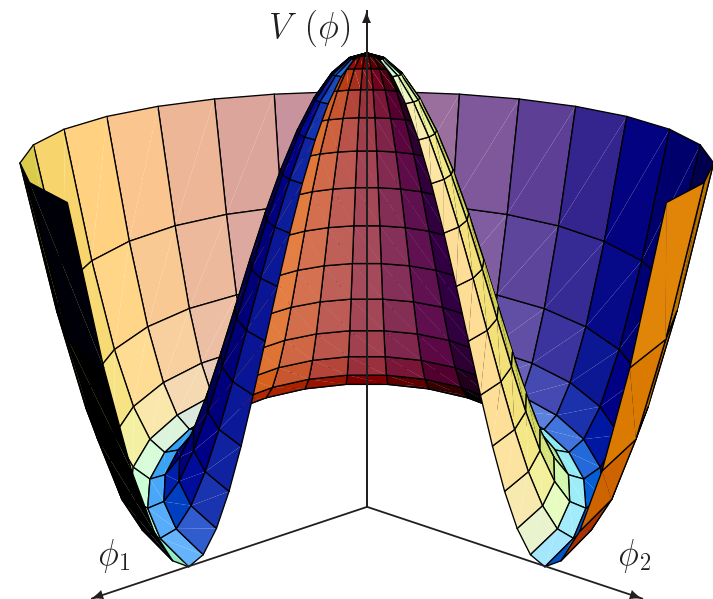
All these minimum configurations (ground states) are connected by gauge transformations, that change the phase of the complex field  $\Phi$ , without affecting its modulus.

$v$  is called the **vacuum expectation value (VEV)** of the neutral component of the Higgs doublet.

When the system chooses one of the minimum configurations, **this configuration is no longer symmetric** under the the gauge symmetry.

This is called **spontaneous symmetry breaking**.

The **Lagrangian** is still **gauge invariant** and all the properties connected with that (such that current conservation) are still there!



## Expanding $\Phi$ around the minimum

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}} [v + H(x) + i\chi(x)] \end{pmatrix} = \frac{1}{\sqrt{2}} \exp \left[ \frac{i\sigma_i \theta^i(x)}{v} \right] \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

We can **rotate away** the fields  $\theta^i(x)$  by an  $SU(2)_L$  gauge transformation

$$\Phi(x) \rightarrow \Phi'(x) = U(x)\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

where  $U(x) = \exp \left[ -\frac{i\sigma_i \theta^i(x)}{v} \right]$ .

This gauge choice is called **unitary gauge**, and is equivalent to **absorbing the Goldstone modes**  $\theta^i(x)$ . **Three would-be Goldstone bosons** “eaten up” by **three vector bosons** ( $W^\pm, Z$ ) that **acquire mass**. This is why we introduced a complex scalar doublet (four elementary fields).

The **vacuum state** can be chosen to correspond to the vacuum expectation value

$$\Phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\text{SU}(2)_L \times \text{U}(1)_Y \rightarrow \text{U}(1)_{\text{em}}$$

We can easily verify that the vacuum state **breaks** the gauge symmetry.

A state  $\tilde{\Phi}$  is invariant under a symmetry operation  $\exp(igT^a\theta_a)$  if

$$\exp(igT^a\theta_a)\tilde{\Phi} = \tilde{\Phi}$$

This means that a state is invariant if (just expand the exponent)

$$T^a\tilde{\Phi} = 0$$

For the  $\text{SU}(2)_L \times \text{U}(1)_Y$  case we have

$$\begin{aligned} \sigma_1\Phi_0 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 && \text{broken} \\ \sigma_2\Phi_0 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -iv/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 && \text{broken} \end{aligned}$$

$$\text{SU}(2)_L \times \text{U}(1)_Y \rightarrow \text{U}(1)_{\text{em}}$$

$$\sigma_3 \Phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken}$$

$$Y \Phi_0 = Y(\Phi) \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = +1 \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken}$$

But, if we examine the effect of the **electric charge operator**  $\hat{Q} = Y/2 + T_3$  on the (electrically neutral) vacuum state, we have ( $Y(\Phi) = 1$ )

$$\hat{Q} \Phi_0 = \frac{1}{2} (\sigma_3 + Y) \Phi_0 = \frac{1}{2} \begin{pmatrix} Y(\Phi) + 1 & 0 \\ 0 & Y(\Phi) - 1 \end{pmatrix} \Phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

the **electric charge symmetry** is **unbroken!**

## Consequences for the scalar field $H$

The scalar potential

$$V(\Phi^\dagger\Phi) = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$$

expanded around the vacuum state

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

becomes

$$V = \frac{1}{2} (2\lambda v^2) H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4 - \frac{\lambda}{4} v^4$$

- the scalar field  $H$  gets a mass

$$m_H^2 = 2\lambda v^2$$

- there is a term of cubic and quartic self-coupling.
- a constant term: the cosmological constant (irrelevant in the Standard Model)

$$\rho_H \equiv \frac{\lambda}{4} v^4 = \frac{v^2 m_H^2}{8}$$

## Cosmological constant

Up to now, we don't have a theory of gravitation. Gravitational interactions are commonly introduced by replacing  $\partial_\mu$  by an appropriate derivative  $D_\mu$ , containing the gravitation field

$$g_{\mu\nu} \equiv \eta_{\mu\nu} + \kappa h_{\mu\nu} \quad \eta_{\mu\nu} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Furthermore, the **Lagrangian** must be given the overall factor  $\sqrt{-\det(g_{\mu\nu})}$ . At this point, the **addition of a constant** to the Lagrangian is of **physical consequence**.

The coefficient of the term that contains no other field dependence other than  $\sqrt{-\det(g_{\mu\nu})}$  is the **cosmological constant**.

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \mathcal{C}g_{\mu\nu} = -8\pi G_N T_{\mu\nu}$$

where  $R_{\mu\nu}$  is the curvature tensor, and  $T_{\mu\nu}$  is the energy-matter tensor. A non-zero value implies that a curved Universe in the absence of energy-matter. The cosmological constant defines the **curvature of the vacuum**.

## Cosmological constant

Experimentally the Universe is known to be very flat, with a very tiny vacuum energy density

$$\rho_{\text{vac}} \leq 10^{-46} \text{ GeV}^4$$

Inserting the current experimental lower bound for the Higgs boson mass,  $m_H \geq 114 \text{ GeV}$ , and the value of  $v = 246.22 \text{ GeV}$  (see more later), we find

$$\rho_H \geq 10^8 \text{ GeV}^4$$

some 54 order of magnitude larger than the upper bound inferred from the cosmological constant!

The smallness of the cosmological constant needs to be explained.

Either we must find a separate principle to zero the vacuum energy density of the Higgs field, or we may suppose that a proper quantum theory of gravity, in combination with the other interactions, will resolve the puzzle of the cosmological constant.

The vacuum energy problem must be an important clue. But to what?

## Kinetic terms

$$\begin{aligned}
 D^\mu \Phi &= \left( \partial^\mu - igW_i^\mu \frac{\sigma^i}{2} - ig' \frac{1}{2} B^\mu \right) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \\
 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \partial^\mu H \end{pmatrix} - \frac{i}{2\sqrt{2}} \left[ g \begin{pmatrix} W_3^\mu & W_1^\mu - iW_2^\mu \\ W_1^\mu + iW_2^\mu & -W_3^\mu \end{pmatrix} + g' B^\mu \right] \begin{pmatrix} 0 \\ v + H \end{pmatrix} \\
 &= \frac{1}{\sqrt{2}} \left[ \begin{pmatrix} 0 \\ \partial^\mu H \end{pmatrix} - \frac{i}{2} (v + H) \begin{pmatrix} g(W_1^\mu - iW_2^\mu) \\ -gW_3^\mu + g'B^\mu \end{pmatrix} \right] \\
 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \partial^\mu H \end{pmatrix} - \frac{i}{2} \left( 1 + \frac{H}{v} \right) \begin{pmatrix} gvW^{\mu+} \\ -v\sqrt{(g^2 + g'^2)/2} Z^\mu \end{pmatrix}
 \end{aligned}$$

$$(D^\mu \Phi)^\dagger D_\mu \Phi = \frac{1}{2} \partial^\mu H \partial_\mu H + \left[ \left( \frac{gv}{2} \right)^2 W^{\mu+} W_\mu^- + \frac{1}{2} \frac{(g^2 + g'^2) v^2}{4} Z^\mu Z_\mu \right] \left( 1 + \frac{H}{v} \right)^2$$



**Exercise:** Show this.



## Consequences

- The  $W$  and  $Z$  gauge bosons have acquired masses

$$m_W^2 = \frac{g^2 v^2}{4} \qquad m_Z^2 = \frac{(g^2 + g'^2) v^2}{4} = \frac{m_W^2}{\cos^2 \theta_W}$$

From the measured value of the Fermi constant  $G_F$

$$\frac{G_F}{\sqrt{2}} = \left( \frac{g}{2\sqrt{2}} \right)^2 \frac{1}{m_W^2} \qquad \Longrightarrow \qquad v = \sqrt{\frac{1}{\sqrt{2}G_F}} \approx 246.22 \text{ GeV}$$

- the photon stays massless
- $HWW$  and  $HZZ$  couplings from  $2H/v$  term (and  $HHWW$  and  $HHZZ$  couplings from  $H^2/v^2$  term)

$$\mathcal{L}_{HVV} = \frac{2m_W^2}{v} W_\mu^+ W^{-\mu} H + \frac{m_Z^2}{v} Z^\mu Z_\mu H \equiv gm_w W_\mu^+ W^{-\mu} H + \frac{1}{2} \frac{gm_Z}{\cos \theta_W} Z^\mu Z_\mu H$$

Higgs coupling proportional to mass

- tree-level  $HVV$  ( $V =$  vector boson) coupling requires VEV!  
Normal scalar couplings give  $\Phi^\dagger \Phi V$  or  $\Phi^\dagger \Phi VV$  couplings only.

## Fermion mass generation

A **direct mass term** is **not** invariant under  $SU(2)_L$  or  $U(1)_Y$  gauge transformation

$$m_f \bar{\psi}\psi = m_f (\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R)$$

Generate fermion masses through Yukawa-type interactions terms

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} = & -\Gamma_d^{ij} \bar{Q}'_L{}^i \Phi d'_R{}^j - \Gamma_d^{ij*} \bar{d}'_R{}^i \Phi^\dagger Q'_L{}^j \\ & -\Gamma_u^{ij} \bar{Q}'_L{}^i \Phi_c u'_R{}^j + \text{h.c.} \\ & -\Gamma_e^{ij} \bar{L}_L{}^i \Phi e_R{}^j + \text{h.c.} \\ & -\Gamma_\nu^{ij} \bar{L}_L{}^i \Phi_c \nu_R{}^j + \text{h.c.} \end{aligned} \quad \Phi_c = i\sigma_2 \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

where  $Q'$ ,  $u'$  and  $d'$  are quark fields that are generic linear combination of the mass eigenstates  $u$  and  $d$  and  $\Gamma_u$ ,  $\Gamma_d$  and  $\Gamma_e$  are  $3 \times 3$  complex matrices in generation space, spanned by the indices  $i$  and  $j$ .

$\mathcal{L}_{\text{Yukawa}}$  is **Lorentz invariant**, **gauge invariant** and **renormalizable**, and therefore it can (actually it **must**) be included in the Lagrangian.

**Notice:** neutrino masses can be implemented via the  $\Gamma_\nu$  term. Since  $m_\nu \approx 0$ , we neglect it.

## Expanding around the vacuum state

In the unitary gauge we have

$$\begin{aligned}\bar{Q}'^i_L \Phi d'^j_R &= (\bar{u}'^i_L \ \bar{d}'^i_L) \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix} d'^j_R = \frac{v+H}{\sqrt{2}} \bar{d}'^i_L d'^j_R \\ \bar{Q}'^i_L \Phi_c u'^j_R &= (\bar{u}'^i_L \ \bar{d}'^i_L) \begin{pmatrix} \frac{v+H}{\sqrt{2}} \\ 0 \end{pmatrix} u'^j_R = \frac{v+H}{\sqrt{2}} \bar{u}'^i_L u'^j_R\end{aligned}$$

and we obtain

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} &= -\Gamma_d^{ij} \frac{v+H}{\sqrt{2}} \bar{d}'^i_L d'^j_R - \Gamma_u^{ij} \frac{v+H}{\sqrt{2}} \bar{u}'^i_L u'^j_R - \Gamma_e^{ij} \frac{v+H}{\sqrt{2}} \bar{e}^i_L e^j_R + \text{h.c.} \\ &= -\left[ M_u^{ij} \bar{u}'^i_L u'^j_R + M_d^{ij} \bar{d}'^i_L d'^j_R + M_e^{ij} \bar{e}^i_L e^j_R + \text{h.c.} \right] \left( 1 + \frac{H}{v} \right)\end{aligned}$$

$$M^{ij} = \Gamma^{ij} \frac{v}{\sqrt{2}}$$

## A little help from linear algebra

**Theorem:** For any generic complex square matrix  $C$ , there exist two unitary matrices  $U, V$  such that

$$D = U^\dagger C V$$

is diagonal with real positive entries

## Diagonalizing $M_f$

Using the previous theorem, we know that we can diagonalize the matrix  $M_f$  ( $f = u, d, e$ ) with the help of two unitary matrices,  $U_L^f$  and  $U_R^f$

$$\left(U_L^f\right)^\dagger M_f U_R^f = \text{diagonal with real positive entries}$$

For example:

$$\left(U_L^u\right)^\dagger M_u U_R^u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \quad \left(U_L^d\right)^\dagger M_d U_R^d = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

## Mass terms

We can make the following change of fermionic fields

$$f'_{Li} = \left( U_L^f \right)_{ij} f_{Lj} \quad f'_{Ri} = \left( U_R^f \right)_{ij} f_{Rj}$$

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} &= - \sum_{f', i, j} \bar{f}'_L{}^i M_f^{ij} f'_R{}^j \left( 1 + \frac{H}{v} \right) + \text{h.c.} \\ &= - \sum_{f, i, j} \bar{f}_L{}^i \left[ \left( U_L^f \right)^\dagger M_f U_R^f \right]_{ij} f_R{}^j \left( 1 + \frac{H}{v} \right) + \text{h.c.} \\ &= - \sum_f m_f (\bar{f}_L f_R + \bar{f}_R f_L) \left( 1 + \frac{H}{v} \right) \end{aligned}$$

- We succeed in producing **fermion masses** and we got a **fermion-antifermion-Higgs coupling** proportional to the **fermion mass**.
- Notice that the fermionic field redefinition **preserves** the form of the **kinetic terms** in the Lagrangian ( $\bar{\psi} \not{\partial} \psi = \bar{\psi}_R \not{\partial} \psi_R + \bar{\psi}_L \not{\partial} \psi_L$  invariant for left and right field unitary transformation).
- The Higgs Yukawa couplings are flavor diagonal: **no flavor changing** Higgs interactions.

## Mass diagonalization and charged current interaction

The charged current interaction is given by

$$\frac{e}{\sqrt{2} \sin \theta_W} \bar{u}'_L{}^i \mathcal{W}^+ d'_L{}^i + \text{h.c.}$$

After the mass diagonalization described previously, this term becomes

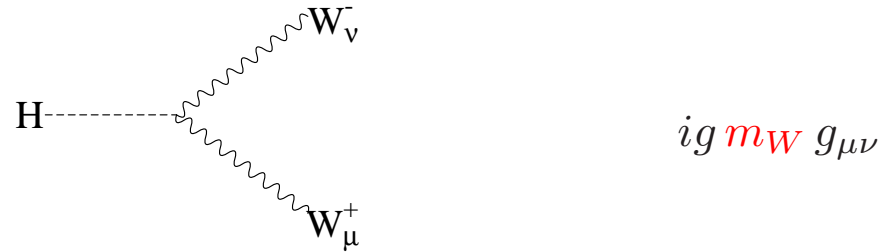
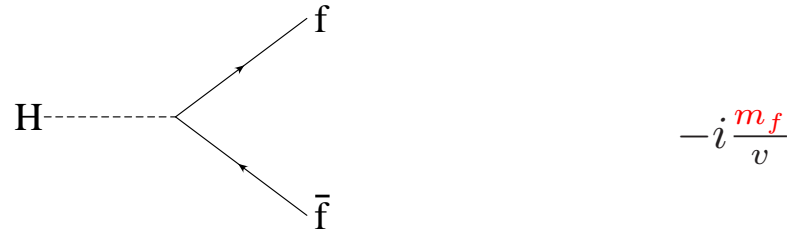
$$\frac{e}{\sqrt{2} \sin \theta_W} \bar{u}_L{}^i \left[ (U_L^u)^\dagger U_L^d \right]_{ij} \mathcal{W}^+ d_L{}^j + \text{h.c.}$$

and we define the Cabibbo-Kobayashi-Maskawa matrix  $V_{CKM}$

$$V_{CKM} = (U_L^u)^\dagger U_L^d$$

- $V_{CKM}$  is a complex **not diagonal** matrix and then it **mixes** the **flavors** of the different quarks.
- For  $N$  flavour families,  $V_{CKM}$  depends on  $(N - 1)^2$  parameters.  $(N - 1)(N - 2)/2$  of them are complex phases. For  $N = 3$  there is **one complex phase** and this implies **violation** of the **CP symmetry** (first observed in the  $K^0$ - $\bar{K}^0$  system in 1964).
- It is a **unitary** matrix and the values of its entries must be determined from experiments.

## Feynman rules for Higgs couplings



Within the Standard Model, the Higgs couplings are almost completely constrained. The only **free** parameter (not yet measured) is the **Higgs mass**

$$m_H^2 = 2\lambda v^2$$



## Constraints on the Higgs boson mass

We have found that the Higgs boson mass is related to the value of the quartic Higgs coupling  $\lambda$

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi^\dagger \Phi) \qquad V(\Phi^\dagger \Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

leads to

$$m_H^2 = 2\lambda v^2$$

So far we have measured **neither  $m_H$  nor  $\lambda$**   $\implies$  **no direct** experimental information.

This raises several questions

- Can we **get rid** of the **Higgs boson** by setting  $m_H = \infty$  and  $\lambda = \infty$ ? Can we eliminate the Higgs boson from the SM?
- Does **consistency of the SM** as a renormalizable field theory **provide constraints**?
- Is there **indirect information** on  $m_H$ , e.g. from precision observables and loop effects?

## The perturbative unitary bound

A very severe constraint on the Higgs boson mass comes from **unitarity** of the scattering amplitude.

**unitarity**  $\iff$  **probability**

and **probability** is the **link** between the theoretical calculations and reality!

Considering the elastic scattering of longitudinally polarized  $Z$  bosons

$$Z_L Z_L \rightarrow Z_L Z_L$$

$$\mathcal{M} = -\frac{m_H^2}{v^2} \left[ \frac{s}{s - m_H^2} + \frac{t}{t - m_H^2} + \frac{u}{u - m_H^2} \right] \quad \text{in the } s \gg m_Z^2 \text{ limit}$$

where  $s$ ,  $t$  and  $u$  are the usual Mandelstam variables.

The **perturbative unitary bound** on the  $J = 0$  partial amplitude takes the form

$$|\mathcal{M}_0|^2 = \left[ \frac{3}{16\pi} \frac{m_H^2}{v^2} \right]^2 < 1 \quad \implies \quad m_H < \sqrt{\frac{16\pi}{3}} v \approx 1 \text{ TeV}$$

More restrictive bounds ( $\sim 800$  GeV) are obtained with other scattering processes, such as  $Z_L W_L \rightarrow Z_L W_L$

## The perturbative unitary bound

If the bound is respected, weak interactions remain weak at all energies, and perturbation theory is everywhere reliable.

If the bound is violated, perturbation theory breaks down, and weak interactions among  $W^\pm$ ,  $Z$  and  $H$  become strong on the 1 TeV scale.

## Running of $\lambda$

The one-loop renormalization group equation (RGE) for  $\lambda(\mu)$  is

$$\frac{d\lambda(\mu)}{d\log\mu^2} = \frac{1}{16\pi^2} \left[ 12\lambda^2 + \frac{3}{8}g^4 + \frac{3}{16}(g^2 + g'^2)^2 - 3h_t^4 - 3\lambda g^2 - \frac{3}{2}\lambda(g^2 + g'^2) + 6\lambda h_t^2 \right]$$

where

$$m_t = \frac{h_t v}{\sqrt{2}} \quad m_H^2 = 2\lambda v^2$$

This equation must be solved together with the one-loop RGEs for the gauge and Yukawa couplings, which, in the Standard Model, are given by

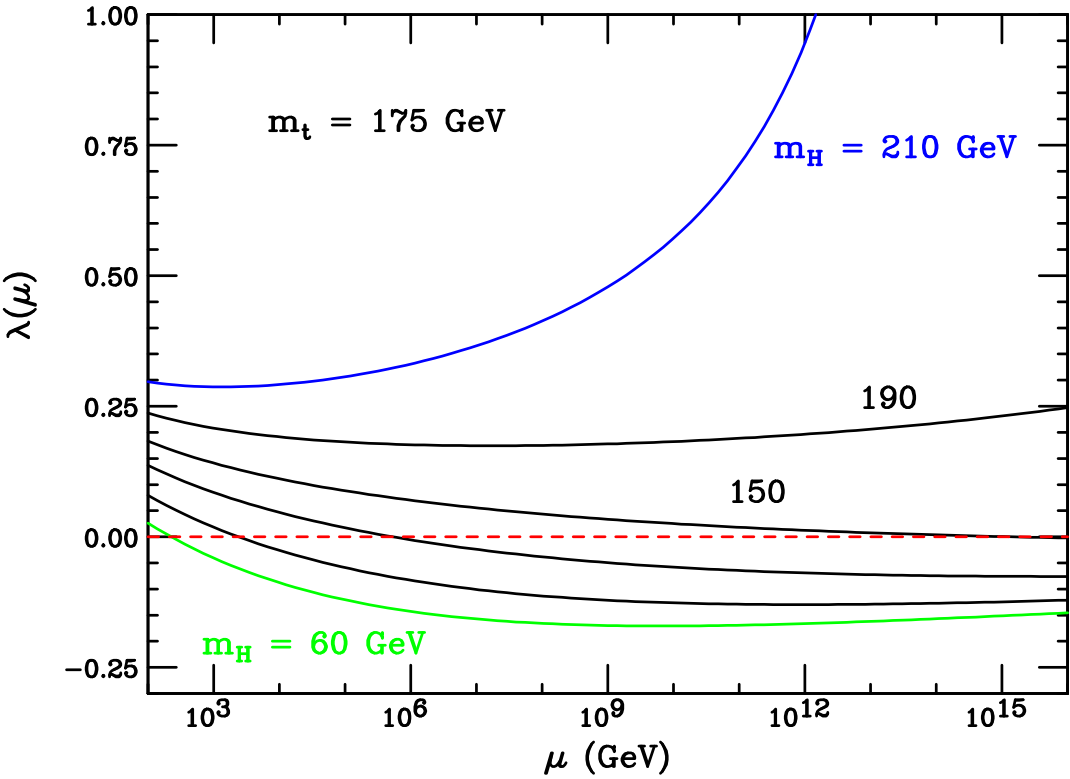
$$\begin{aligned} \frac{dg(\mu)}{d\log\mu^2} &= \frac{1}{32\pi^2} \left( -\frac{19}{6}g^3 \right) \\ \frac{dg'(\mu)}{d\log\mu^2} &= \frac{1}{32\pi^2} \frac{41}{6}g'^3 \\ \frac{dg_s(\mu)}{d\log\mu^2} &= \frac{1}{32\pi^2} (-7g_s^3) \\ \frac{dh_t(\mu)}{d\log\mu^2} &= \frac{1}{32\pi^2} \left[ \frac{9}{2}h_t^3 - \left( 8g_s^2 + \frac{9}{4}g^2 + \frac{17}{12}g'^2 \right) h_t \right] \end{aligned}$$

here  $g_s$  is the strong interaction coupling constant, and the  $\overline{\text{MS}}$  scheme is adopted.

# Solutions for $\lambda(\mu)$

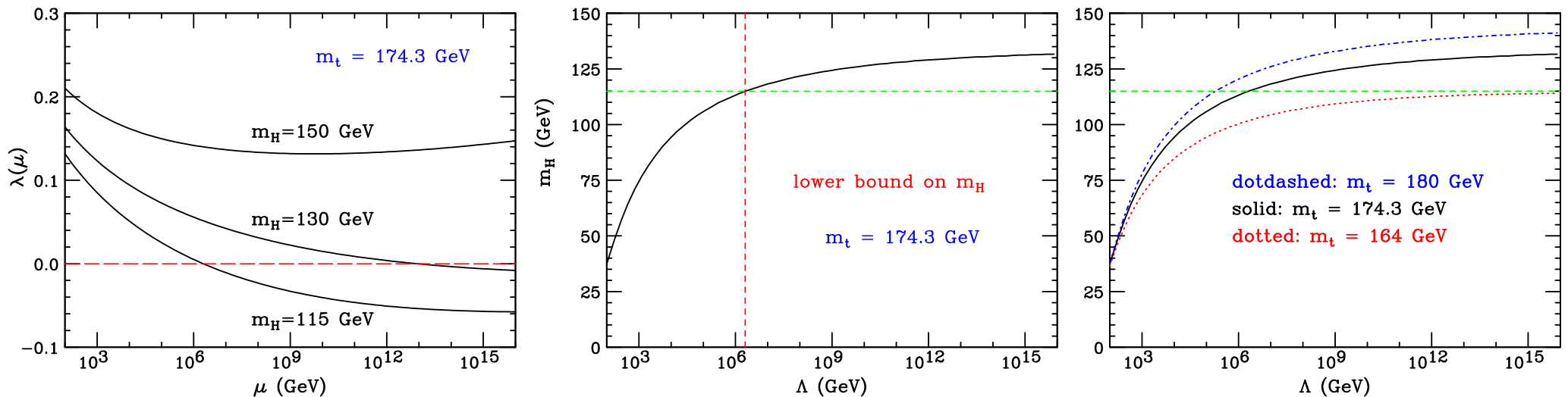
Solving this system of coupled equations with the **initial condition**

$$\lambda(m_H) = \frac{m_H^2}{2v^2}$$



## Lower bound for $m_H$ : vacuum stability

It can be shown that the requirement that the Higgs potential be bounded from below, even after the inclusion of radiative corrections, is fulfilled if  $\lambda(\mu)$  stays positive, at least up to a certain scale  $\mu \approx \Lambda$ , the maximum energy scale at which the theory can be considered reliable (use effective action).



✗ This limit is extremely sensitive to the top-quark mass.

✓ The stability lower bound can be relaxed by allowing metastability

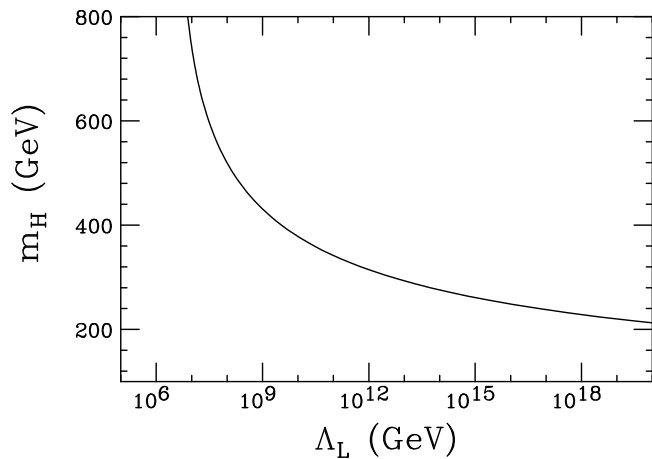
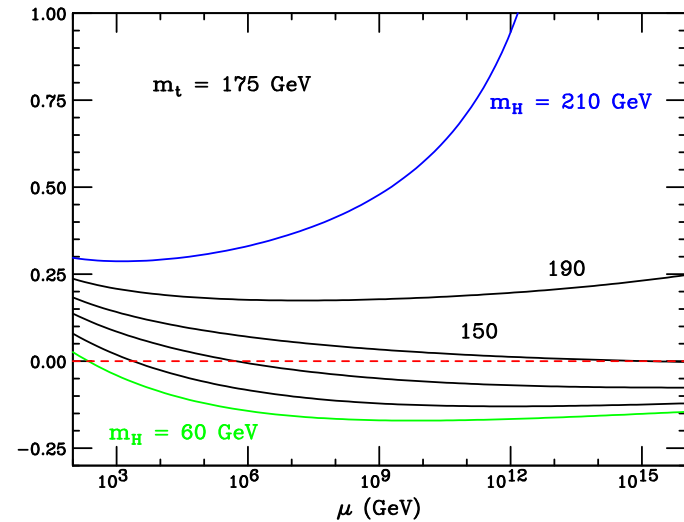
# Upper bound for $m_H$ : triviality bound

For large values of the Higgs boson mass, the coupling  $\lambda(\mu)$  grows with increasing  $\mu$ , and eventually **leaves the perturbative domain** ( $\lambda \lesssim 1$ ): the solution has a singularity in  $\mu$ , known as the **Landau singularity**.

For the theory to make sense up to a scale  $\Lambda$ , we must ask  $\lambda(\mu) \lesssim 1$  (or something similar), for  $\mu \leq \Lambda$ .

Neglecting gauge and Yukawa coupling, we have

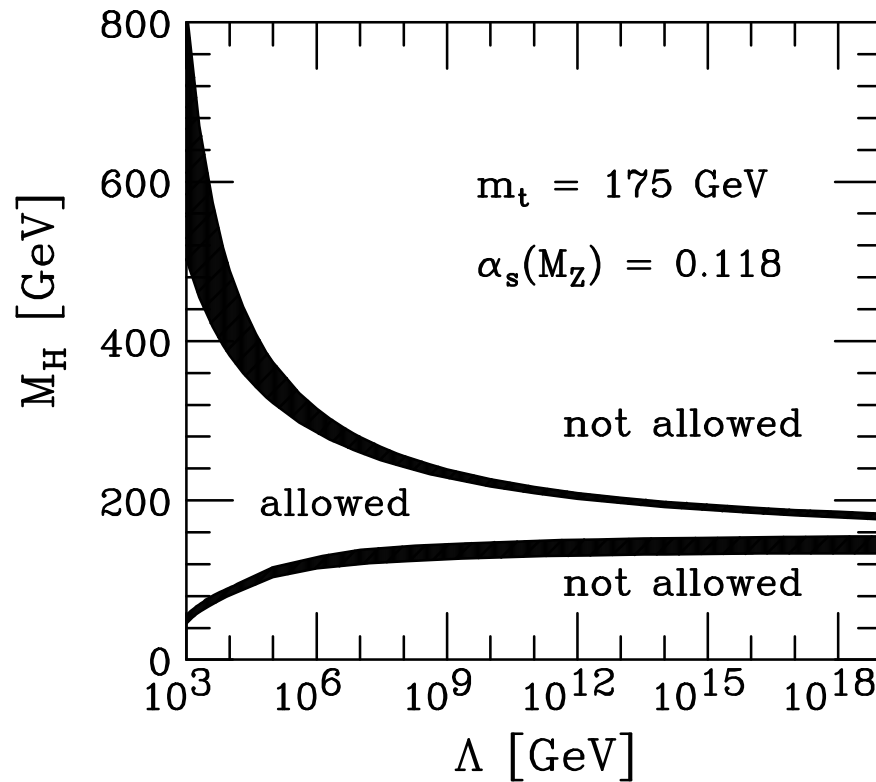
$$\lambda(\mu^2) = \frac{\lambda(m_H^2)}{1 - \frac{3}{4\pi^2} \lambda(m_H^2) \log \frac{\mu^2}{m_H^2}} \quad \text{singular when} \quad \mu^2 \approx \Lambda_L^2 \equiv m_H^2 \exp \left[ \frac{4\pi^2}{3\lambda(m_H^2)} \right]$$



- For **any value** of  $\lambda(m_H^2)$  the theory has an **upper scale**  $\Lambda$  of **validity**.
- $\Lambda \rightarrow \infty$  for pure scalar theory possible only if  $\lambda(m_H^2) \equiv 0$ , i.e. **no scalar self-coupling**  $\implies$  free or **“trivial” theory**

## Higgs boson mass bounds

Riesselmann, hep-ph/9711456

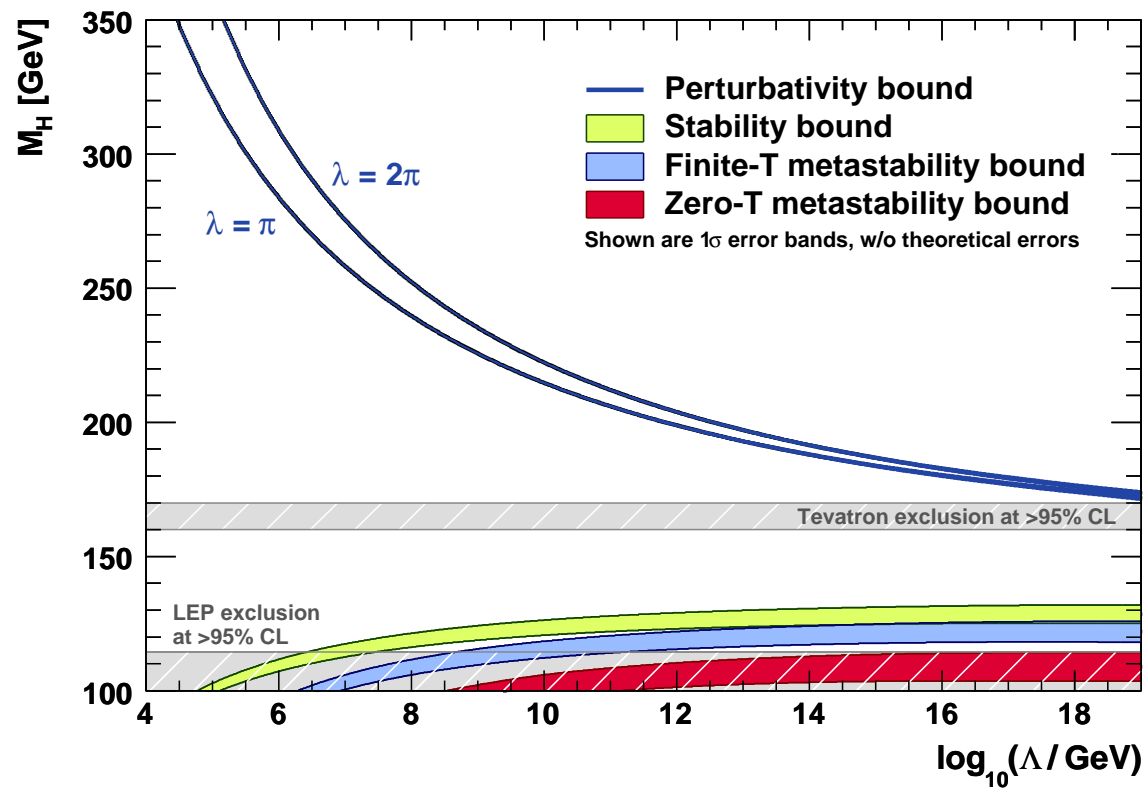


Notice the **small window**  $150 \text{ GeV} < m_H < 180 \text{ GeV}$ , where the theory is valid up to the Planck scale  $M_{\text{Planck}} = (\hbar c / G_{\text{Newton}})^{1/2} \approx 1.22 \times 10^{19} \text{ GeV}$ .



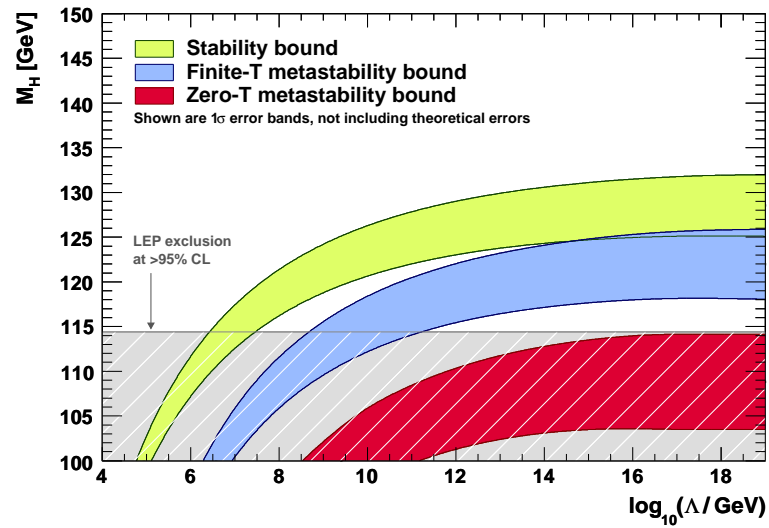
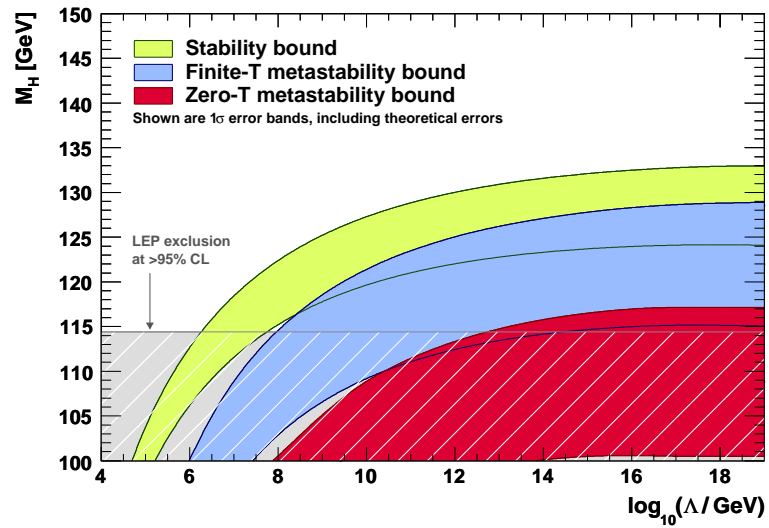
# Higgs boson mass bounds

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# Higgs boson mass bounds

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## Hierarchy, naturalness and fine tuning

Apart from the considerations made up to now, the SM must be considered as an **effective low-energy theory**: at very high energy new phenomena take place that are not described by the SM (gravitation is an obvious example)  $\implies$  **other scales** have to be **considered**.

Why the weak scale ( $\sim 10^2$  GeV) is much smaller than other relevant scales, such as the Planck mass ( $\approx 10^{19}$  GeV) or the unification scale ( $\approx 10^{16}$  GeV) (or why the Planck scale is so high with respect to the weak scale  $\implies$  extra dimensions)?

This is the **hierarchy problem**.

And this problem is especially difficult to solve in the SM because of the un-naturalness of the Higgs boson mass.

As we have seen and as the experimental data suggest, the Higgs boson mass is of the same order of the weak scale. However, it's **not naturally small**, in the sense that there is **no approximate symmetry** that prevent it from receiving **large radiative corrections**.

As a consequence, it **naturally** tends to become as **heavy** as the **heaviest degree of freedom** in the underlying theory (Planck mass, unification scale).

## Toy model

Two scalars interacting through the potential

$$V(\varphi, \Phi) = \frac{m^2}{2}\varphi^2 + \frac{M^2}{2}\Phi^2 + \frac{\lambda}{4!}\varphi^4 + \frac{\sigma}{4!}\Phi^4 + \frac{\delta}{4}\varphi^2\Phi^2$$

which is the **most general** renormalizable potential, if we require the symmetry under  $\varphi \rightarrow -\varphi$  and  $\Phi \rightarrow -\Phi$ . We assume that  $M^2 \gg m^2$ . Let's check if this **hierarchy** is conserved at the quantum level. Compute the one-loop radiative corrections to the pole mass  $m^2$

$$m_{\text{pole}}^2 = m^2(\mu^2) + \frac{\lambda m^2}{32\pi^2} \left( \log \frac{m^2}{\mu^2} - 1 \right) + \frac{\delta M^2}{32\pi^2} \left( \log \frac{M^2}{\mu^2} - 1 \right)$$

where the running mass  $m^2(\mu^2)$  obeys the RGE

$$\frac{dm^2(\mu^2)}{d \log \mu^2} = \frac{1}{32\pi^2} (\lambda m^2 + \delta M^2)$$

**Corrections** to  $m^2$  proportional to  $M^2$  appear at one loop. One can choose  $\mu^2 \approx M^2$  to get rid of them, but they reappear through the running of  $m^2(\mu^2)$ .

## Toy model, cont'd

The only way to preserve the hierarchy  $m^2 \ll M^2$  is **carefully choosing the coupling constants**

$$\lambda m^2 \approx \delta M^2$$

and this requires fixing the renormalized coupling constants with an **unnaturally high accuracy**

$$\frac{\lambda}{\delta} \approx \frac{m^2}{M^2}$$

This is what is usually called the **fine tuning** of the parameters.

The same happens if the theory is spontaneously broken ( $m^2 < 0$ ,  $M^2 \gg |m^2| > 0$ ).

Therefore, without a suitable fine tuning of the parameters, the **mass** of the scalar **Higgs** boson **naturally** becomes as **large** as the largest energy scale in the theory. And this is related to the fact that **no extra symmetry** is recovered when the scalar masses vanish, in **contrast** to what happens to **fermions**, where the **chiral symmetry** prevents the dependence from powers of higher scales, and gives a typical **logarithmic dependence**.

## Solutions to the naturalness problem?

Leaving the toy model and back to the Standard Model, the **corrections** to  $m_H^2$  due to a **top-quark loop** is given by

$$\delta m_H^2 = \frac{3G_F m_t^2}{\sqrt{2}\pi^2} \Lambda^2 \approx (0.27\Lambda)^2$$

where we are assuming that the scale  $\Lambda$  that characterizes **non-standard physics** acts as a **cut-off** for the loop momentum.

So, how can we prevent these large corrections to the Higgs boson mass?

- **SUperSYmmetry** offers a solution to the **naturalness** problem: exploiting the fact that fermion loops contribute with an overall minus sign (because of Fermi statistics), SUSY balances the contributions of fermion and boson loops.

In the limit of **unbroken SUSY**, in which the masses of bosons are degenerate with those of their fermion counterparts, the **cancellation is exact**.

If the supersymmetry is broken (as it must be in our world), the contribution of the integrals may still be acceptably small if the fermion-boson mass splittings  $\Delta M$  are not too large. The condition that  $g^2 \Delta M^2$  be “small enough” leads to the requirement that superpartner masses be less than about 1 TeV.

## Solutions to the naturalness problem?

- A second solution is offered by theories of **dynamical symmetry breaking** such as **technicolor**. In technicolor models, the Higgs boson is **composite**, and new physics arises on the scale of its binding,  $\Lambda_{\text{TC}} \simeq \mathcal{O}(1 \text{ TeV})$ . Thus the effective range of integration is cut off, and mass shifts are under control.
- A third possibility is that the gauge sector becomes **strongly interacting**. This would give rise to  $WW$  resonances, multiple production of gauge bosons, and other new phenomena at energies of 1 TeV or so.

## Constraints from precision data

$$\begin{aligned}\alpha &= \frac{1}{4\pi} \frac{g^2 g'^2}{g^2 + g'^2} = \frac{1}{137.03599976(50)} \\ G_F &= \frac{1}{\sqrt{2}v^2} = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2} \\ m_Z &= \frac{1}{2} \sqrt{g^2 + g'^2} v = 91.1875(21) \text{ GeV} ,\end{aligned}$$

where the uncertainty is given in parentheses. The value of  $\alpha$  is extracted from **low-energy experiments**,  $G_F$  is extracted from the **muon lifetime**, and  $m_Z$  is measured from  $e^+e^-$  **annihilation** near the  $Z$  mass.

We can express  $m_W$  as

$$m_W^2 = \frac{1}{\sin^2 \theta_W} \frac{\pi\alpha}{\sqrt{2}G_F}$$

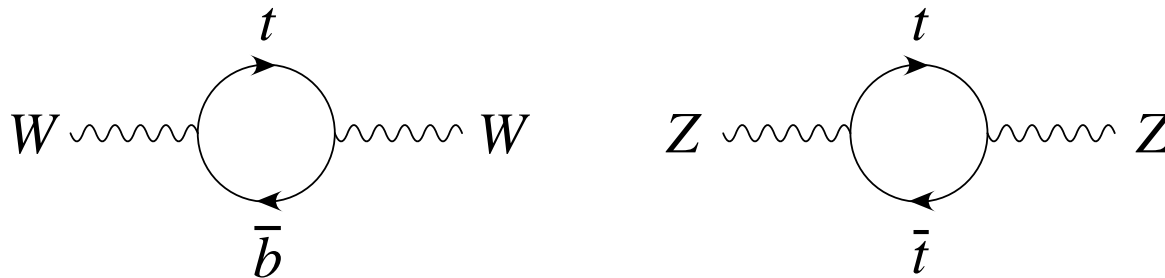
where

$$\sin^2 \theta_W = 1 - \frac{m_W^2}{m_Z^2}$$



## Clues to the Higgs boson mass

From the **sensitivity of electroweak observables** to the mass of the top quark, we are able to measure its mass, even **without directly producing** it

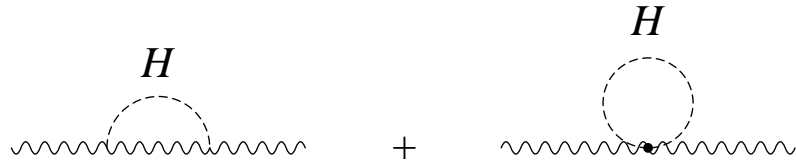


These quantum corrections alter the link between  $W$  and  $Z$  boson masses

$$m_W^2 = \frac{1}{\sin^2 \theta_W} \frac{\pi \alpha}{\sqrt{2} G_F} \quad \Delta \rho_{(\text{top})} \approx -\frac{3 G_F}{8 \pi^2 \sqrt{2}} \frac{1}{\tan^2 \theta_W} m_t^2$$

The **strong dependence** on  $m_t^2$  accounts for the precision of the top-quark mass estimates derived from electroweak observables.

The Higgs boson quantum corrections are typically smaller than the top-quark corrections, and exhibit a more subtle dependence on  $m_H$  than the  $m_t^2$  dependence of the top-quark corrections.



$$\Delta\rho_{(\text{Higgs})} = \frac{11G_F m_Z^2 \cos^2 \theta_W}{24\sqrt{2}\pi^2} \log\left(\frac{m_H^2}{m_W^2}\right)$$

Since  $m_Z$  has been determined at LEP to 23 ppm, it is interesting to examine the dependence of  $m_W$  upon  $m_t$  and  $m_H$ .

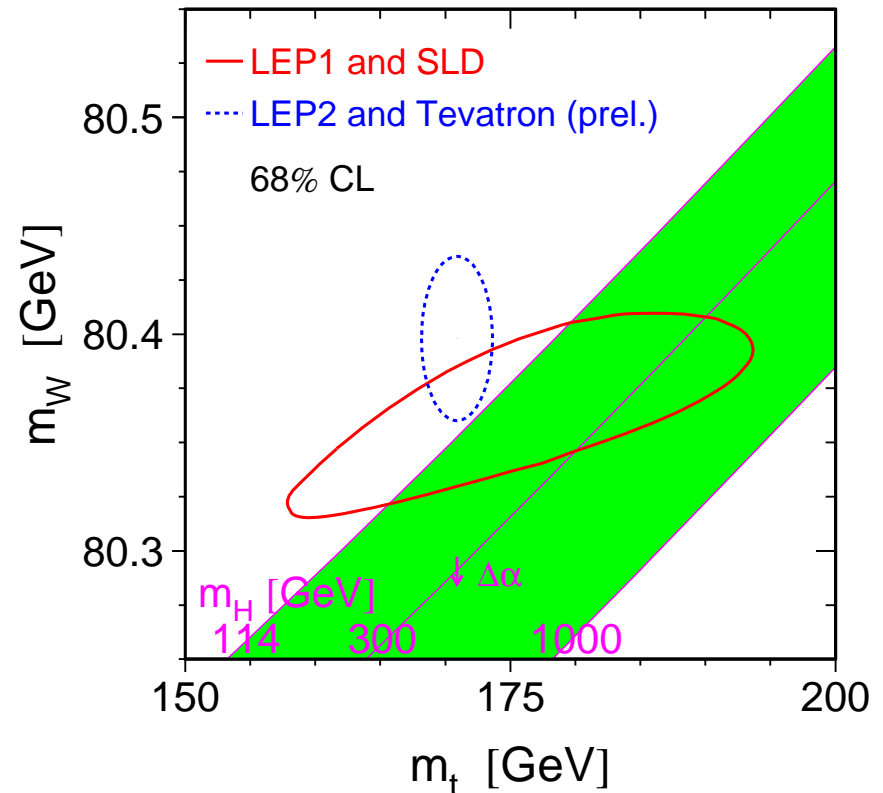
Indirect measurements of  $m_W$  and  $m_t$  (solid line)

Direct measurements of  $m_W$  and  $m_t$  (dotted line)

$$m_t = 170.9 \pm 1.8 \text{ GeV}$$

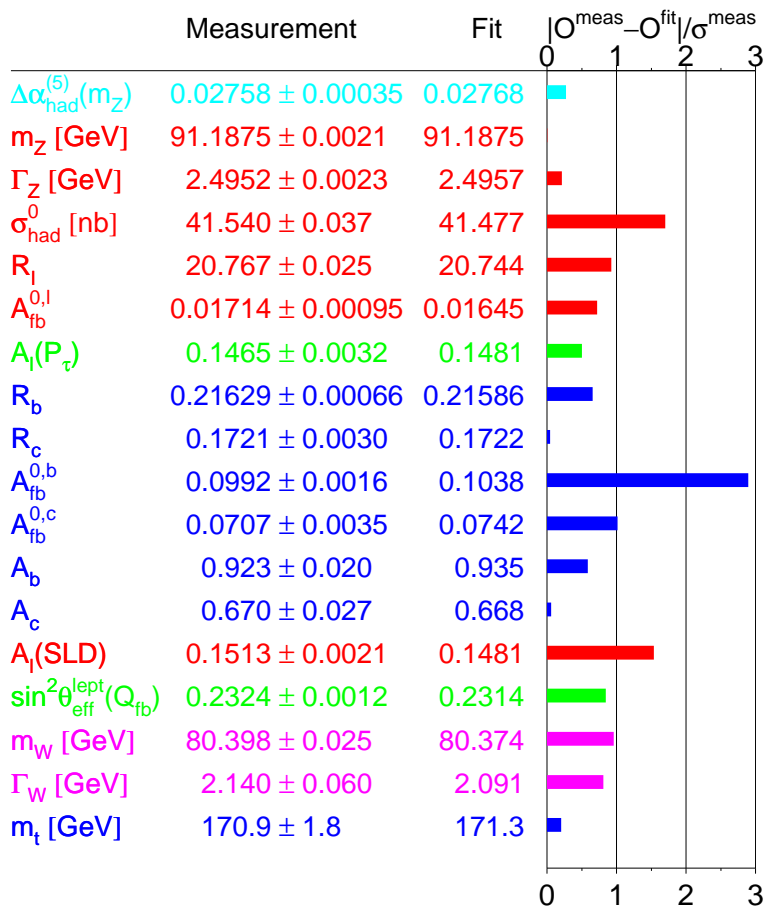
$$m_W = 80.398 \pm 0.025 \text{ GeV}$$

both shown as one-standard-deviation regions.



The indirect and direct determinations are in reasonable agreement and both favor a light Higgs boson, within the framework of the SM.

# Summary of EW precision data



Better estimates of the SM Higgs boson mass are obtained by combining all available data.

Summary of electroweak precision measurements (status winter 2007) are given on LEP-EWWG page

<http://lepewwg.web.cern.ch/LEPEWWG>



**Exercise:** Derive the slope of the lines of constant Higgs mass of the previous slide and compare numerically with the plot.

## Blue band plot

The indication for a **light Higgs** boson becomes somewhat stronger when all the electroweak observables are examined.

$$m_H = 76^{+33}_{-24} \text{ GeV}$$

Including theory uncertainty

$$m_H < 144 \text{ GeV} \quad (95\% \text{CL})$$

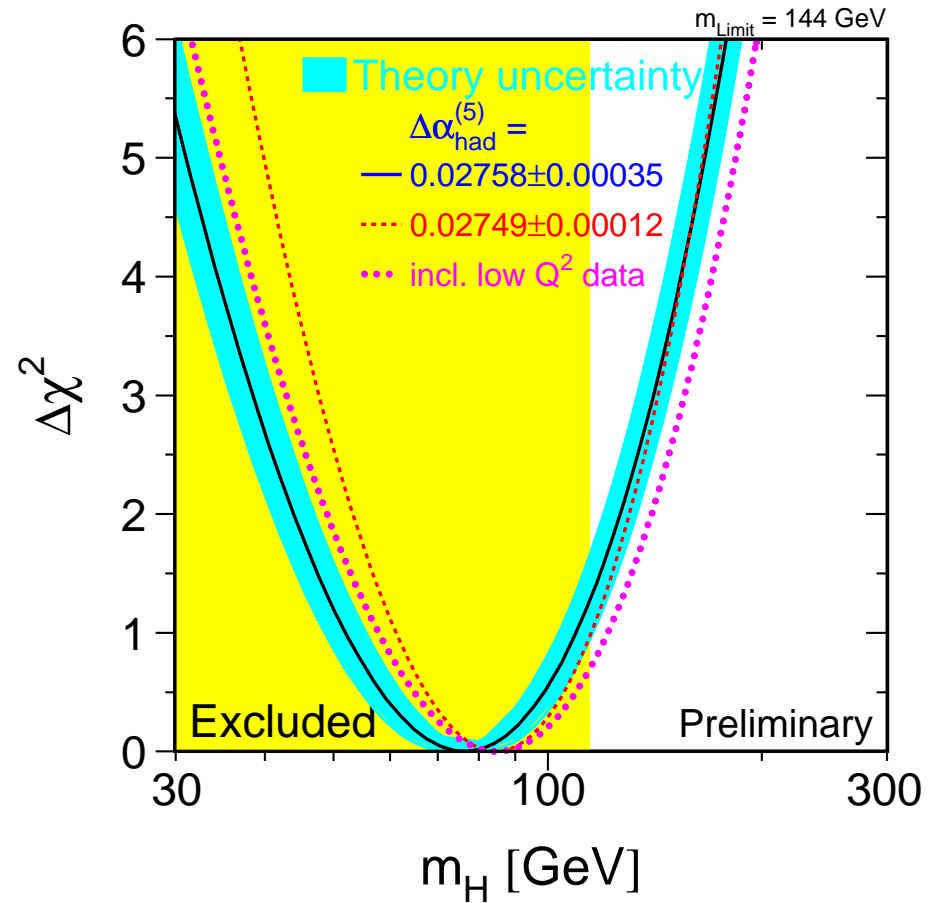
Direct search limit from LEP

$$m_H > 114.4 \text{ GeV} \quad (95\% \text{CL})$$

But the  $\chi^2$  of the fit is **very bad!**

$$\chi^2/\text{dof} = 25.4/15$$

$$\chi^2/\text{dof} = 16.8/14 \quad \text{without NuTeV}$$



Up to now...



Peter W. Higgs, University of Edinburgh

← Only unambiguous example of  
observed Higgs

(D. Froidevaux, HCP School, 2007)

## Final remarks

The **Standard Model** is **not the whole story**

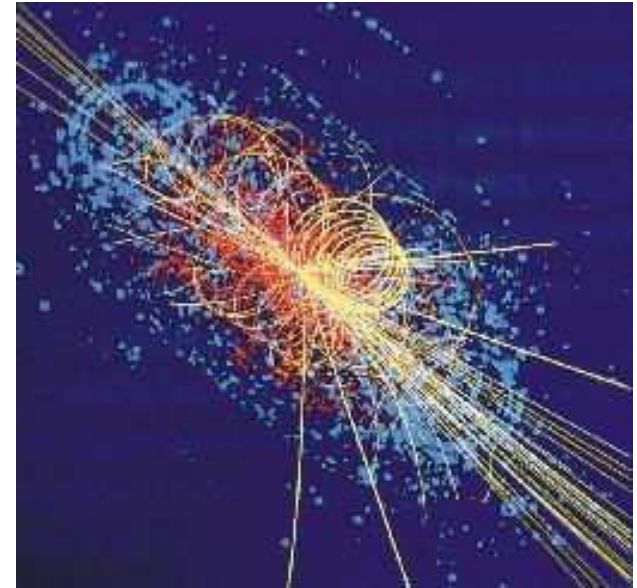
### Open questions

- ✗ gravity
- ✗ neutrino masses and oscillations (heavy sterile neutrinos + see-saw mechanism)
- ✗ dark matter/dark energy
- ✗ baryogenesis

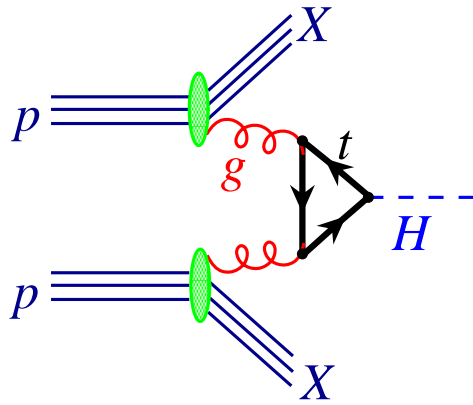
# Higgs boson at the LHC

Two steps

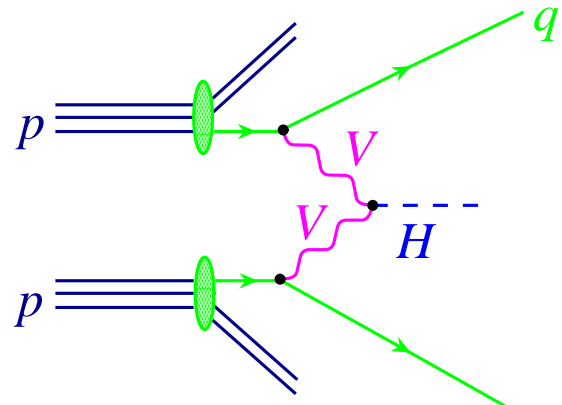
- Production of the Higgs boson
- Detection of the decay products of the Higgs boson and identification of the events



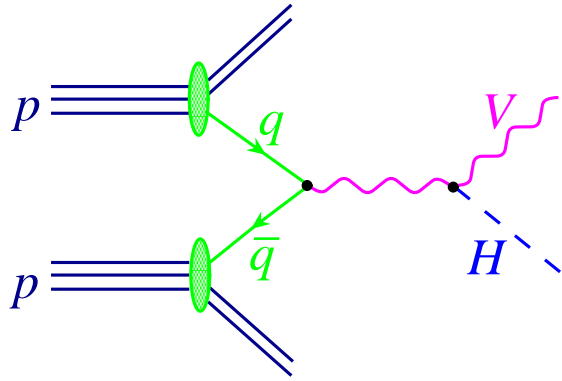
# Production Modes



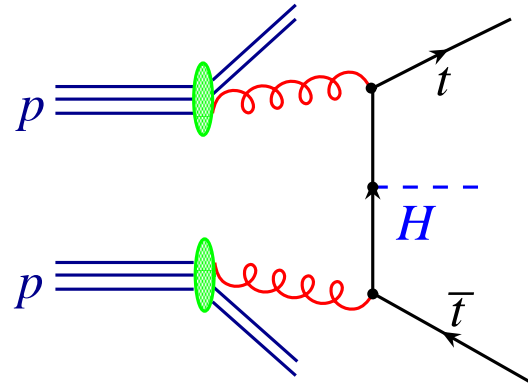
Gluon fusion



Weak-Boson Fusion



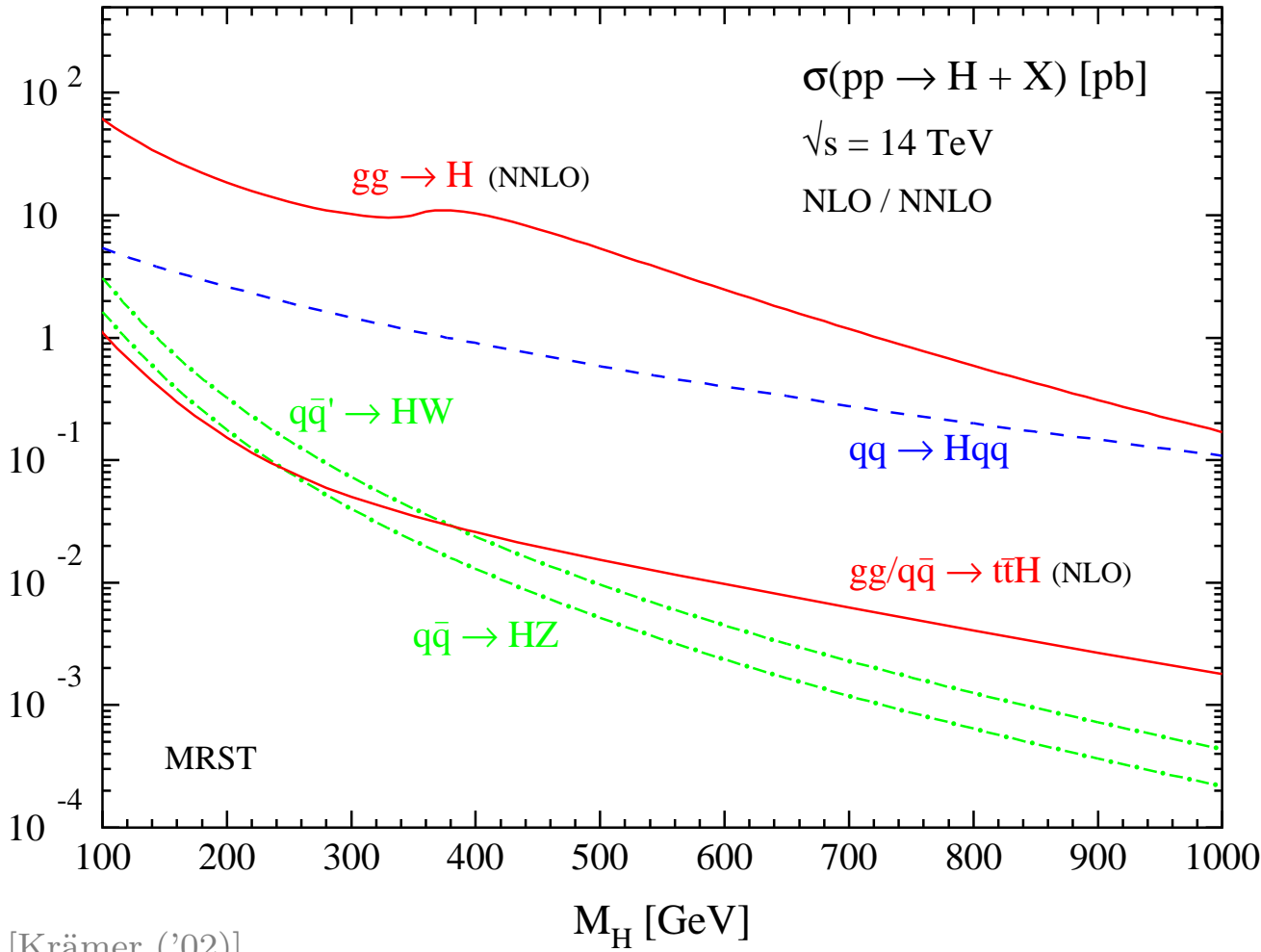
Higgs Strahlung



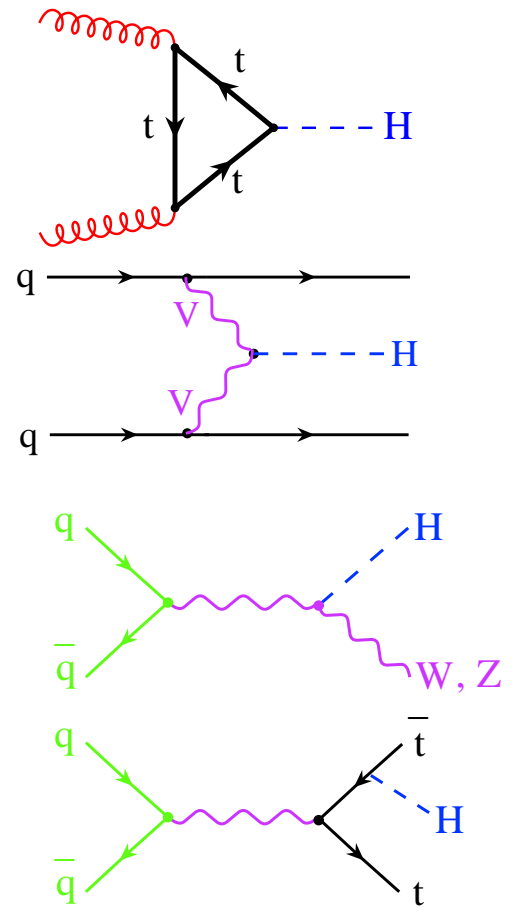
$t\bar{t}H$



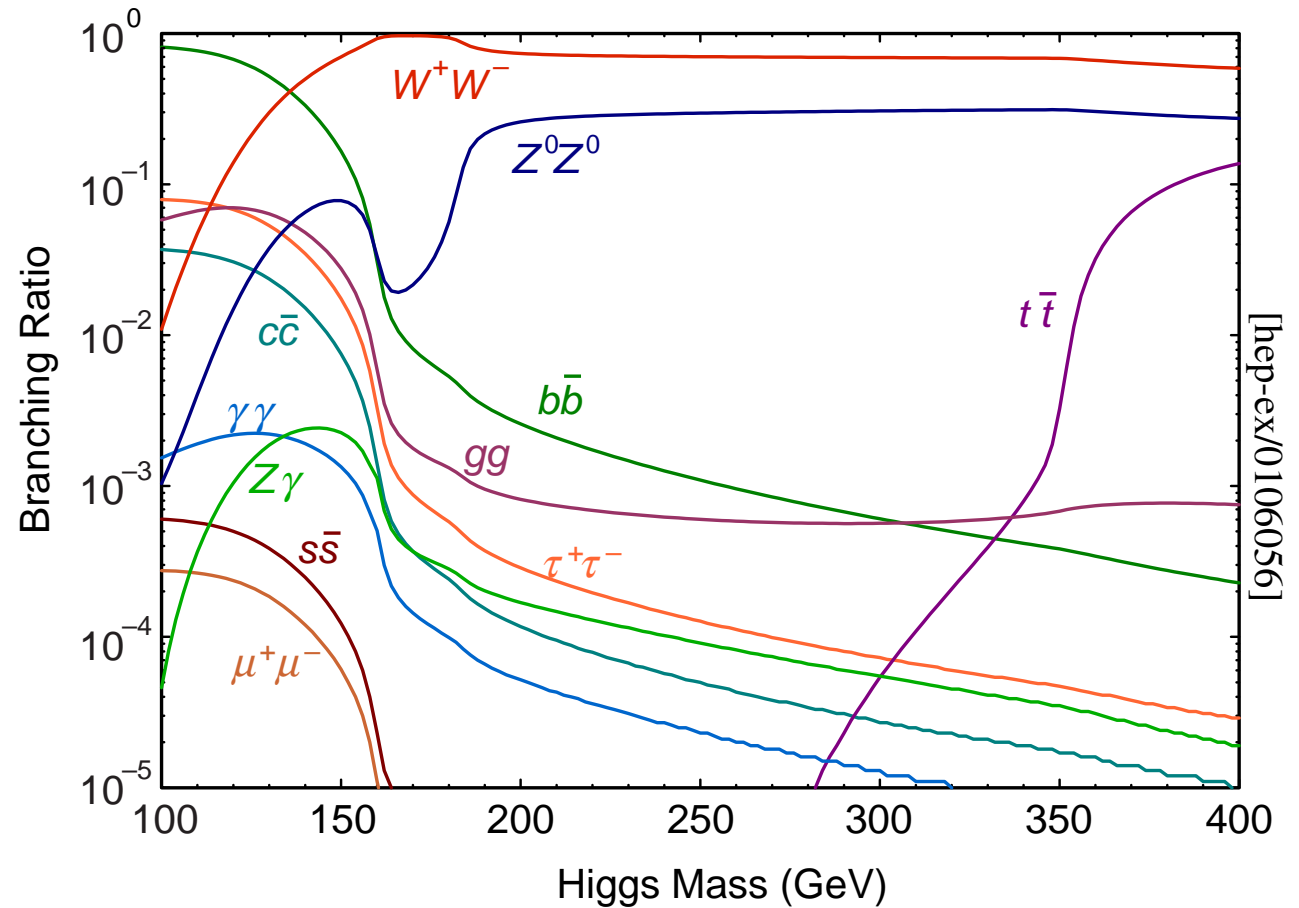
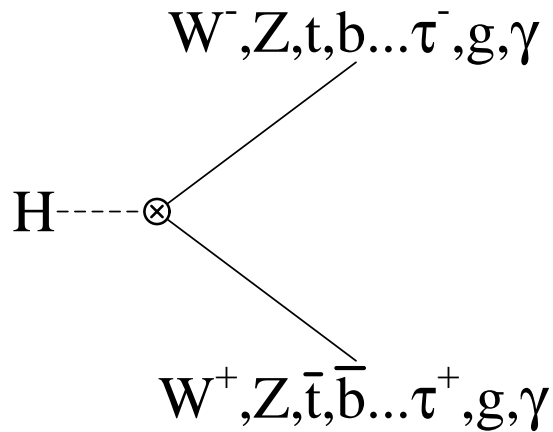
# Total cross sections at the LHC



[Krämer ('02)]

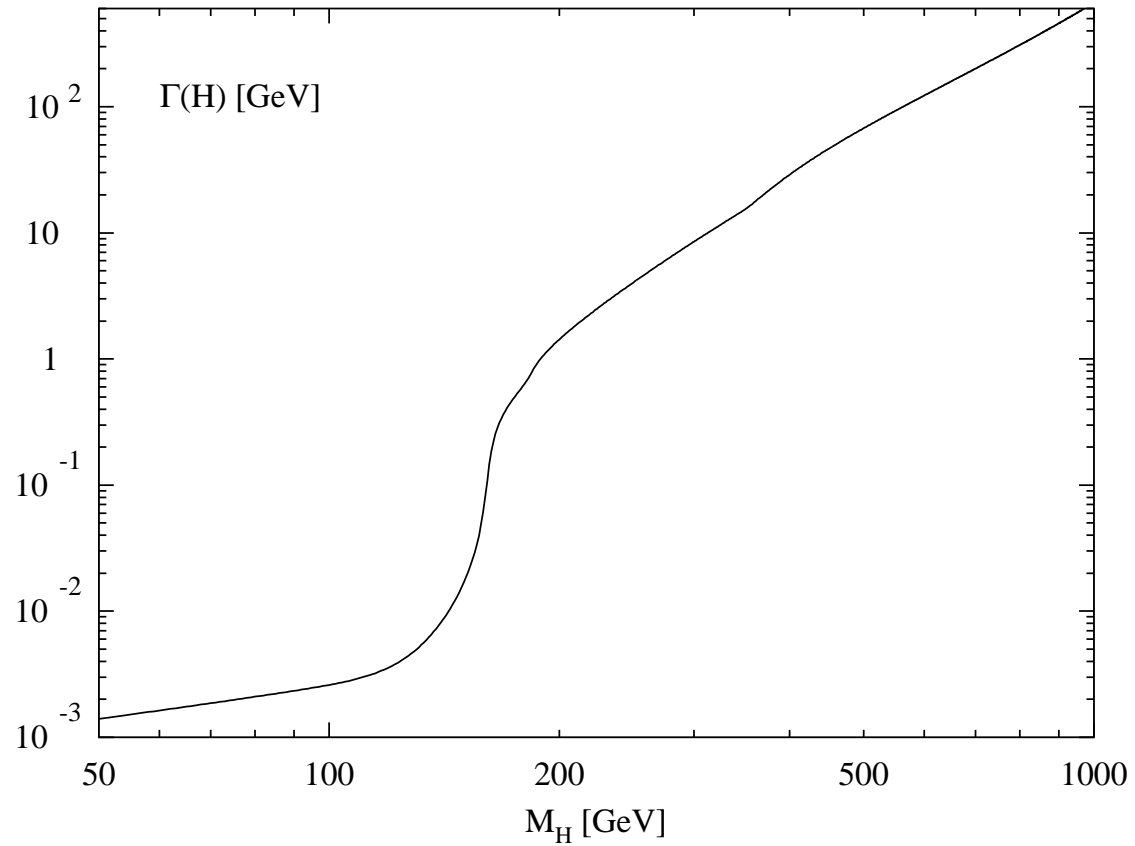


# Branching fractions of the SM Higgs



**Exercise:** compute, at leading order,  $\Gamma(H \rightarrow f\bar{f})$  and  $\Gamma(H \rightarrow VV)$ . More challenging (one-loop integral)  $\Gamma(H \rightarrow gg)$  and  $\Gamma(H \rightarrow \gamma\gamma)$ . [Spira (hep-ph/9705337)]

## Total decay width



[Spira and Zerwas]

## Inclusive search channels

- inclusive search for

$$H \rightarrow \gamma\gamma$$

invariant-mass peak, for  $m_H < 150$  GeV

- inclusive search for

$$H \rightarrow ZZ^* \rightarrow \ell^+ \ell^- \ell^+ \ell^-$$

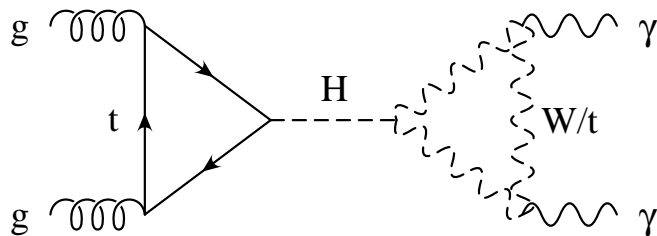
for  $m_H \geq 130$  GeV and  $m_H \neq 2m_W$ .

- inclusive search for

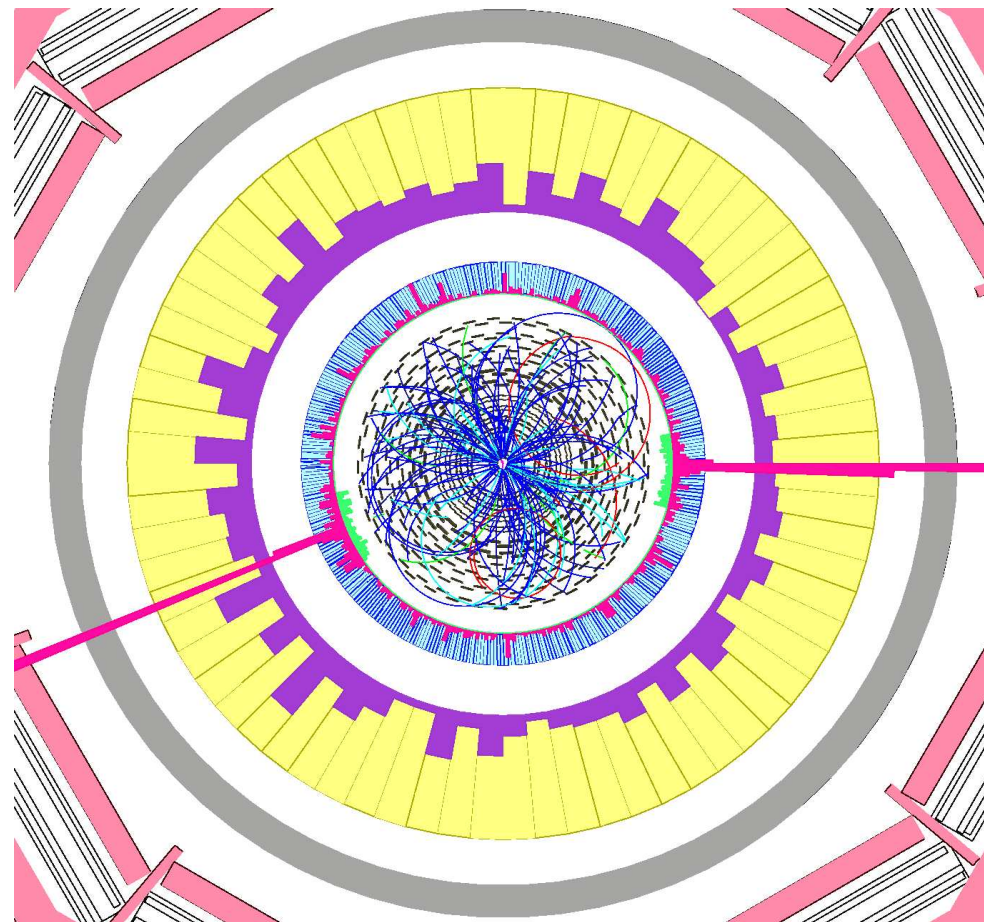
$$H \rightarrow W^+ W^- \rightarrow \ell^+ \bar{\nu} \ell^- \nu$$

for  $140 \text{ GeV} \leq m_H \leq 200 \text{ GeV}$

# $H \rightarrow \gamma\gamma$



- ✗  $\text{BR}(H \rightarrow \gamma\gamma) \approx 10^{-3}$
- ✗ large backgrounds from  $q\bar{q} \rightarrow \gamma\gamma$  and  $gg \rightarrow \gamma\gamma$
- ✓ but CMS and ATLAS will have excellent photon-energy resolution (order of 1%)

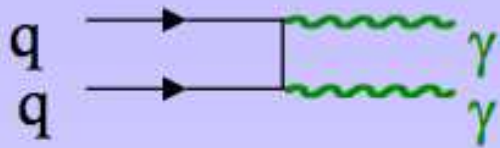


Look for **two isolated** photons.

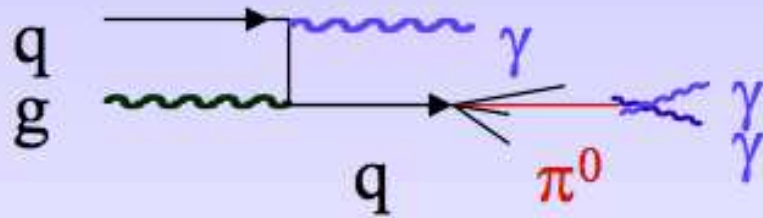
# $H \rightarrow \gamma\gamma$

## Main backgrounds:

$\gamma\gamma$  irreducible background



$\gamma$ -jet and jet-jet (reducible)

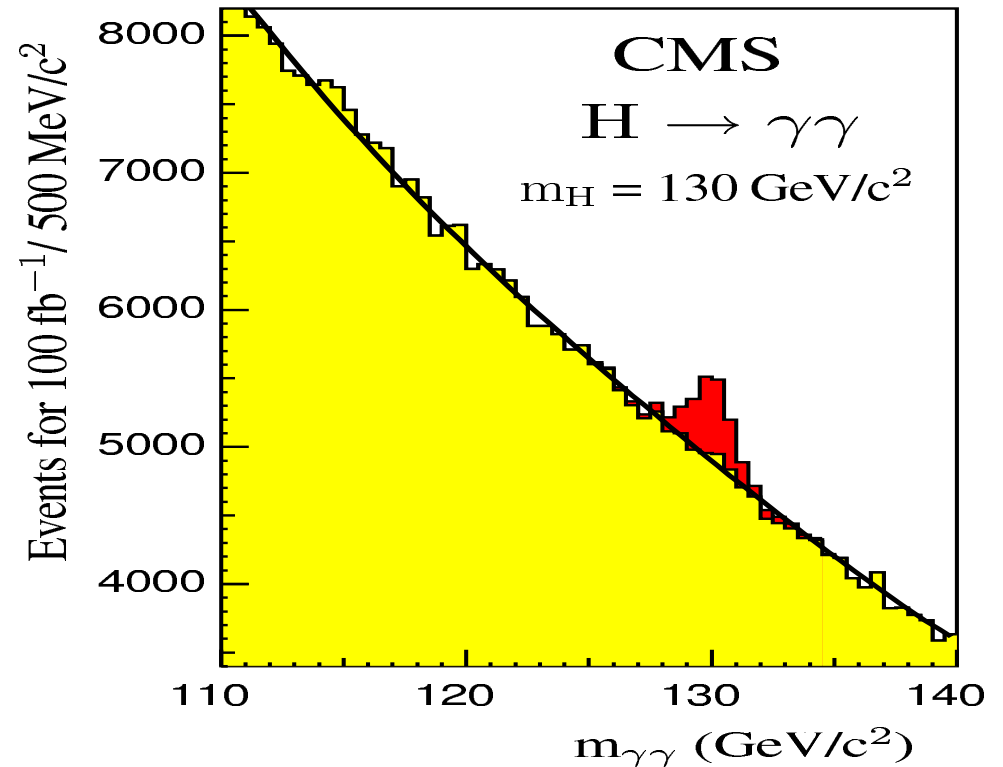


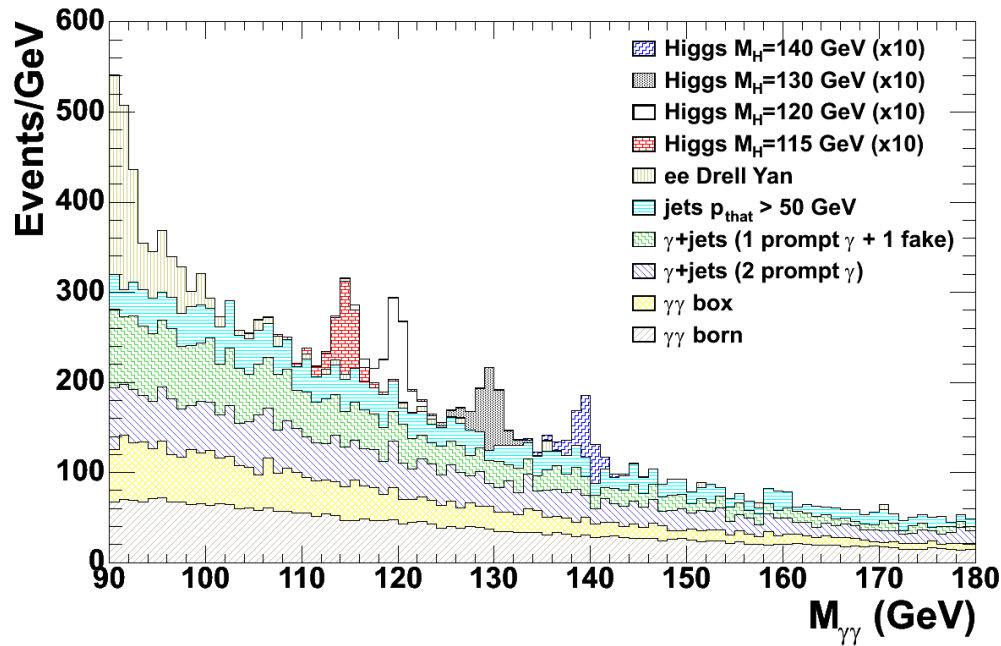
- ✓  $\sigma_{\gamma j} \sim 10^6 \sigma_{\gamma\gamma}$  with large uncertainties
- ✓ we can at most misidentify 1 jet in  $10^3$
- ✓ we need an efficiency  $\epsilon_\gamma \sim 80\%$  to get  $\sigma_{\gamma j + jj} \ll \sigma_{\gamma\gamma}$

K. Jakobs, CSS07

$$H \rightarrow \gamma\gamma$$

- ✓ Look for a **narrow**  $\gamma\gamma$  invariant mass peak
- ✓ extrapolate background into the signal region from sidebands.

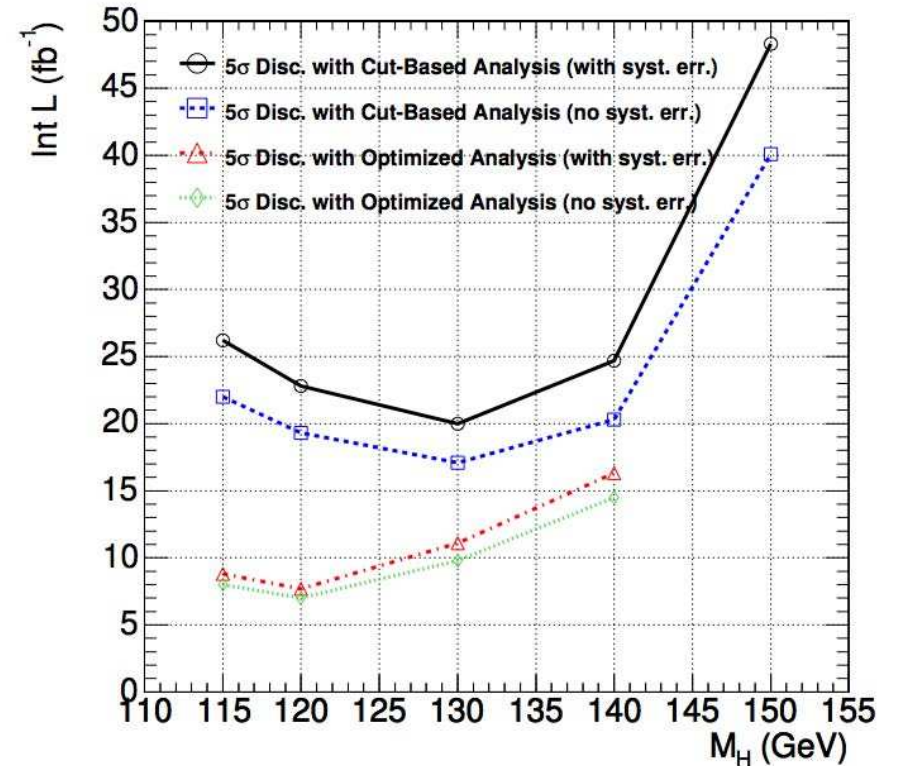




✓  $1 \text{ fb}^{-1}$

✓ cut-based analysis

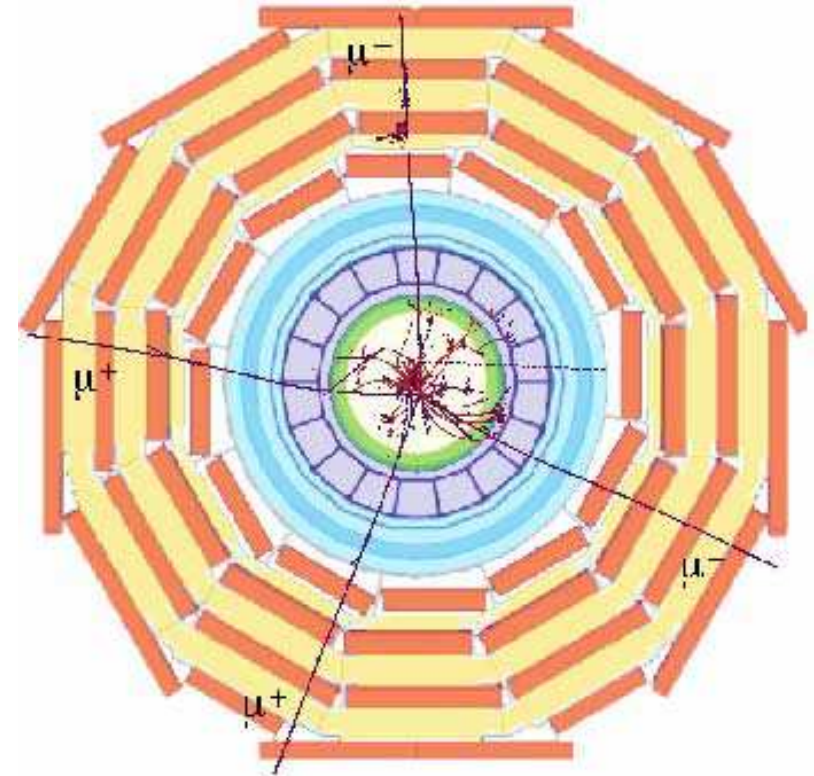
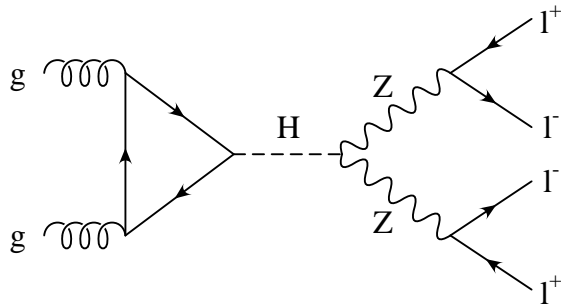
- ✓ discovery with less than  $30 \text{ fb}^{-1}$
- ✓ assumes ECAL intercalibration, for which  $10 \text{ fb}^{-1}$  are needed
- ✓ optimized analysis: assumes perfect understanding of detector. Uses Neural Net





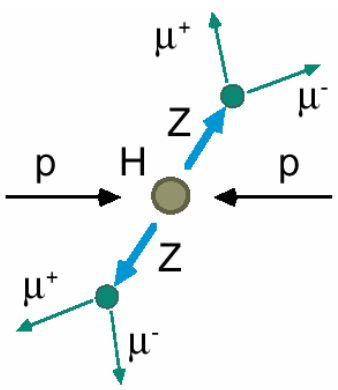
$$H \rightarrow ZZ \rightarrow \ell^+ \ell^- \ell^+ \ell^-$$

The **gold-plated** mode

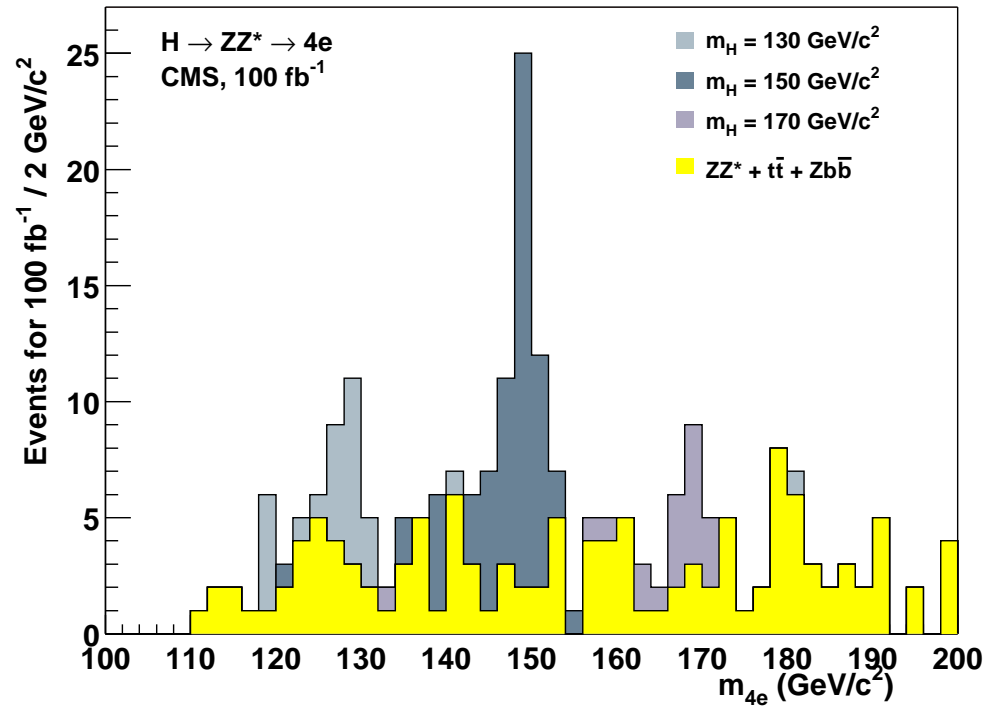


- ✓ This is the **most important** and **clean** search mode for  $2m_Z < m_H < 600$  GeV.
- ✓ **continuum, limited, irreducible background** from  $q\bar{q} \rightarrow ZZ$
- ✗ **small BR** ( $H \rightarrow \ell^+ \ell^- \ell^+ \ell^-$ )  $\approx 0.15\%$   
(even smaller when  $m_H < 2m_Z$ )

$H \rightarrow ZZ \rightarrow l^+l^-l^+l^-$



✓ invariant mass of the charged leptons fully reconstructed

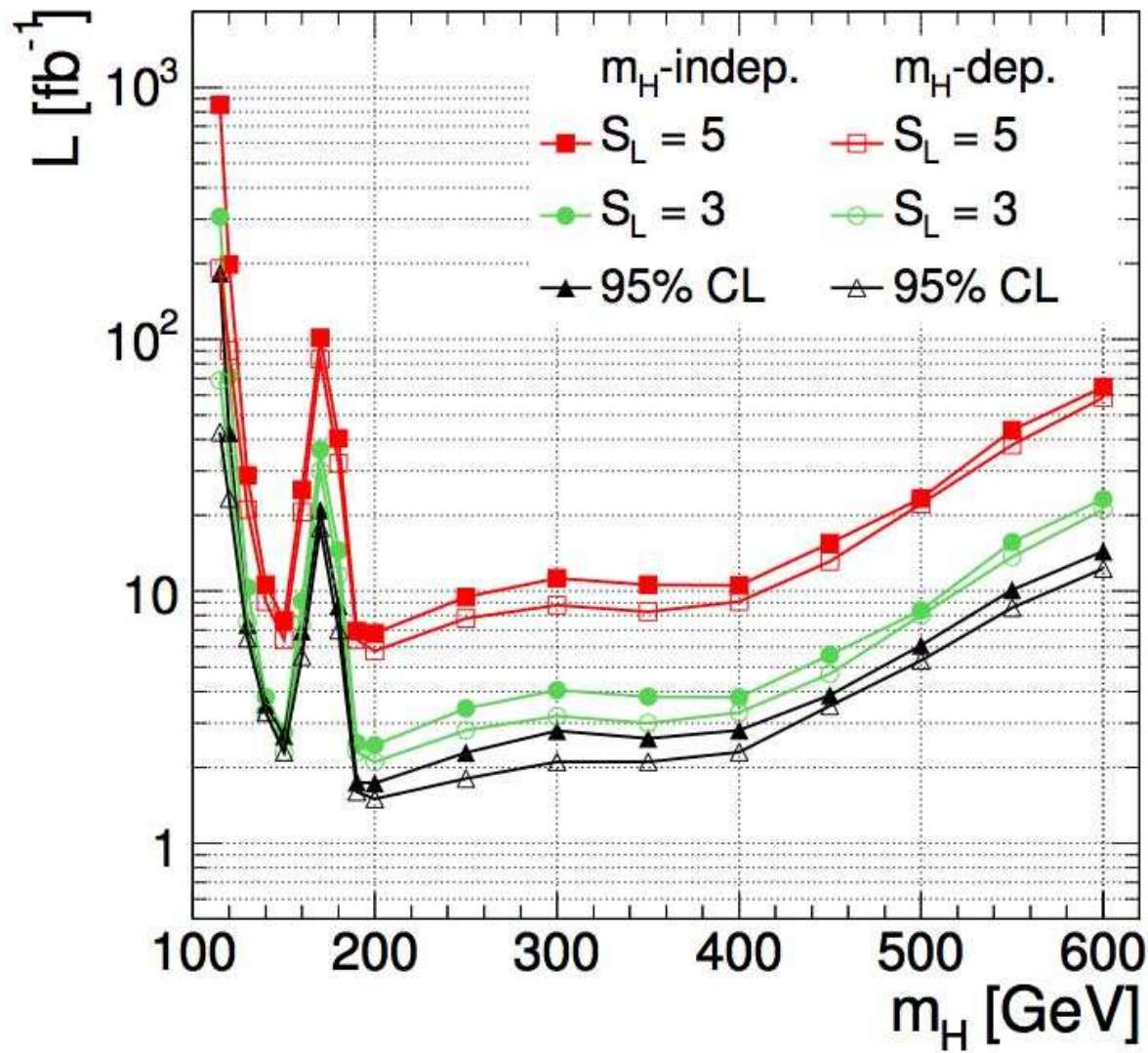


For  $m_H \approx 0.6-1$  TeV, use the “silver-plated” mode  $H \rightarrow ZZ \rightarrow \nu\bar{\nu}l^+l^-$

✓  $BR(H \rightarrow \nu\bar{\nu}l^+l^-) = 6 BR(H \rightarrow l^+l^-l^+l^-)$

✓ the large  $E_T$  missing allows a measurement of the transverse mass

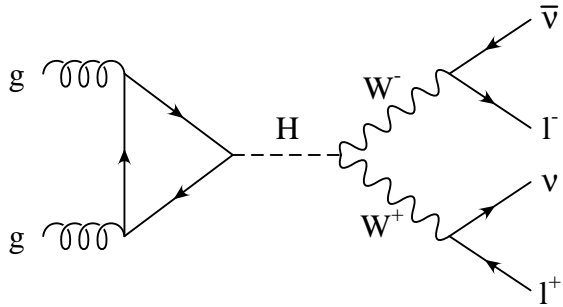
$$H \rightarrow ZZ \rightarrow \mu^+ \mu^- \mu^+ \mu^-$$



with  $30 \text{ fb}^{-1}$

- ✓  $m_H$  measured with  $0.1 \div 5\%$  precision
- ✓ production cross section known at 30% precision

$H \rightarrow WW \rightarrow \ell^+ \bar{\nu} \ell^- \nu$

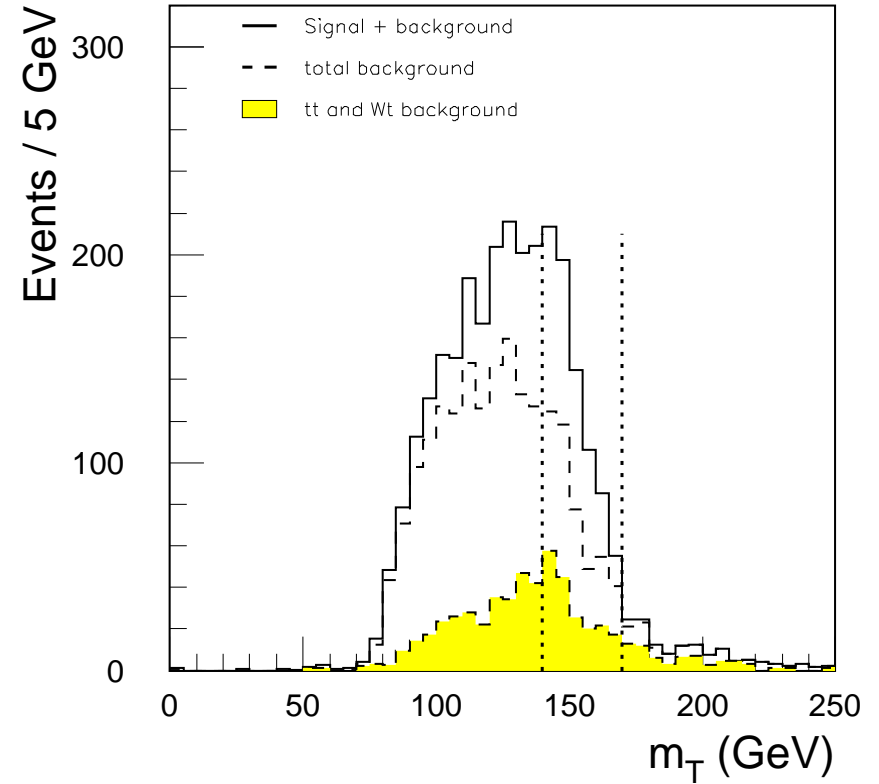


- ✓ No reconstruction of clear mass peak. Measure the transverse mass with a Jacobian peak at  $m_H$

$$m_T = \sqrt{2 p_T^{\ell\ell} \cancel{E}_T (1 - \cos(\Delta\Phi))}$$

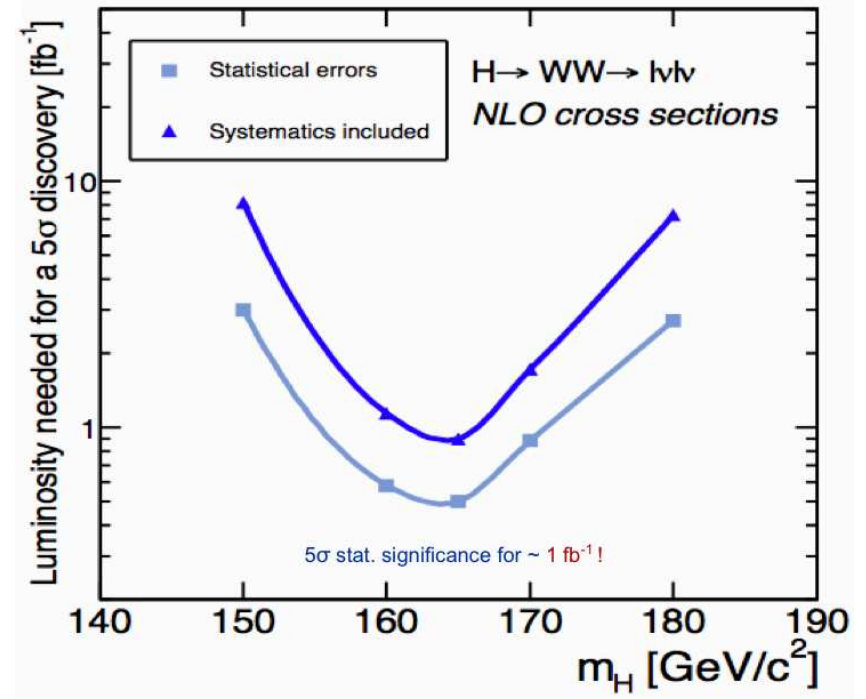
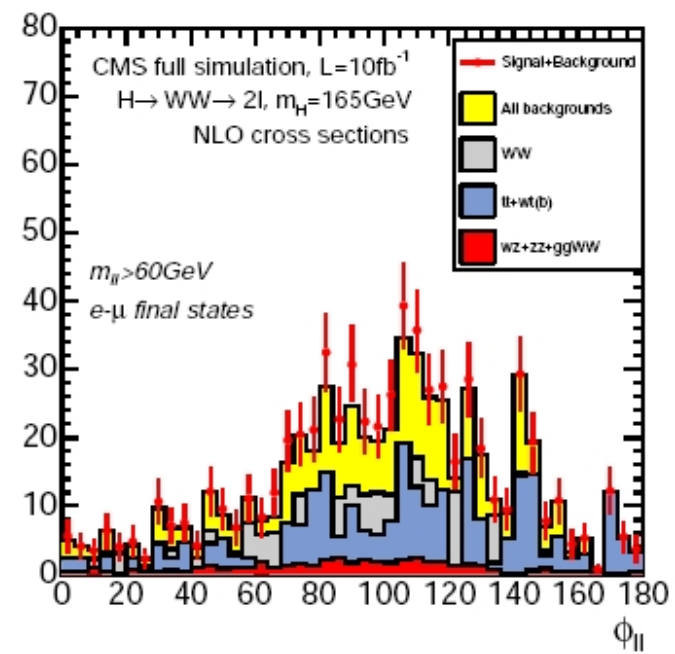
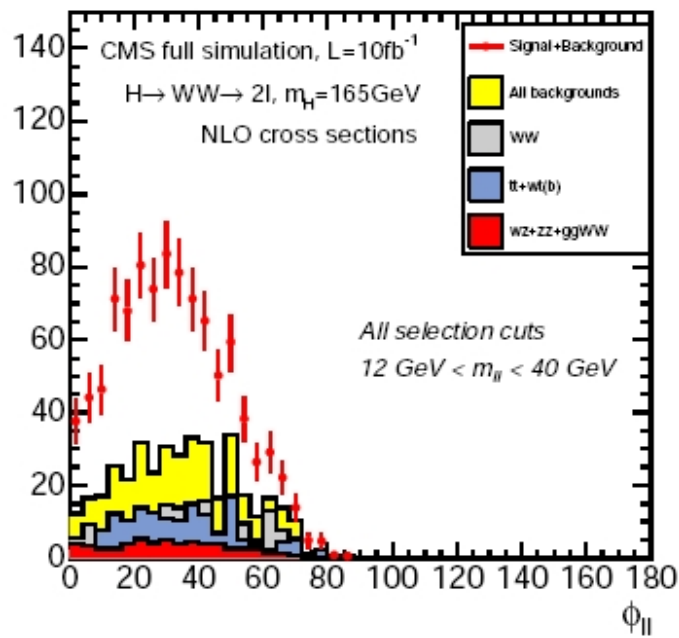
- ✓ Exploit  $\ell^+ \ell^-$  angular correlations
- ✗ Background and signal have similar shape  $\implies$  must know the background normalization precisely

ATLAS TDR



$m_H = 170 \text{ GeV}$

integrated luminosity =  $20 \text{ fb}^{-1}$



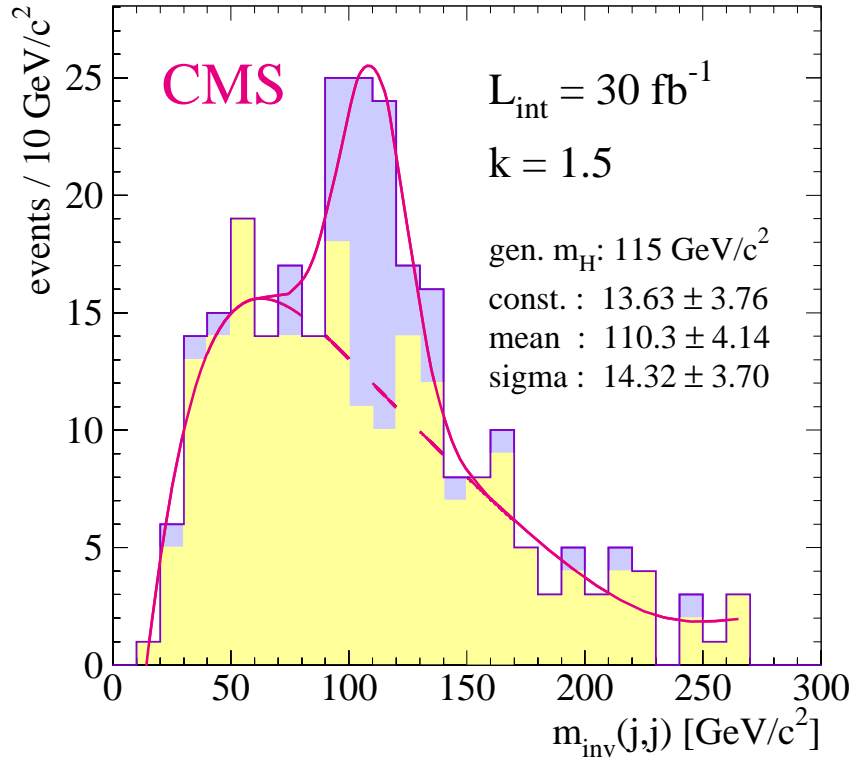
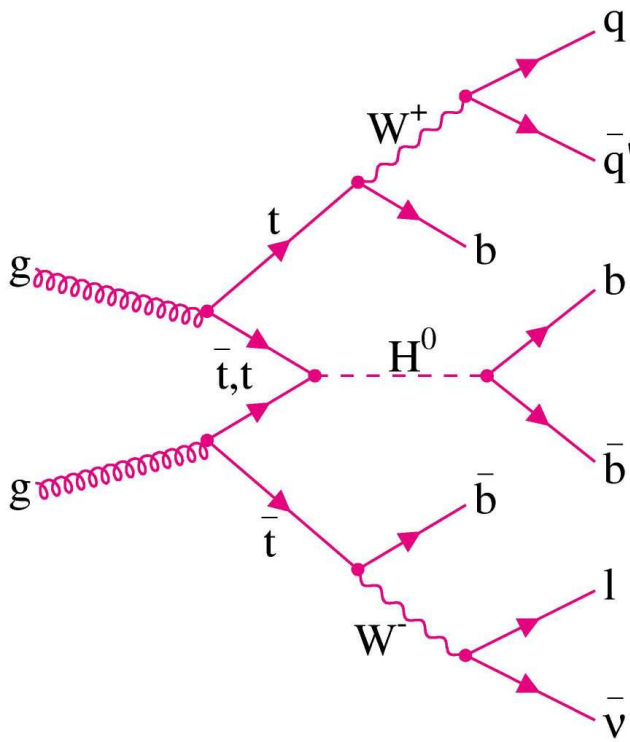
- ✓ best channel for  $m_H \sim 160 - 170$  GeV
- ✓ systematic uncertainty 10 - 20%
- ✓  $m_H$  can be determined to 2 - 2.5 GeV
- ✓ production cross section known at  $\sim 10\%$

## Associated production search channels

- $pp \rightarrow t\bar{t}H \rightarrow t\bar{t}b\bar{b}$   
for  $m_H < 120-130$  GeV
- $qq \rightarrow Hqq$   
in vector-boson fusion (VBF)

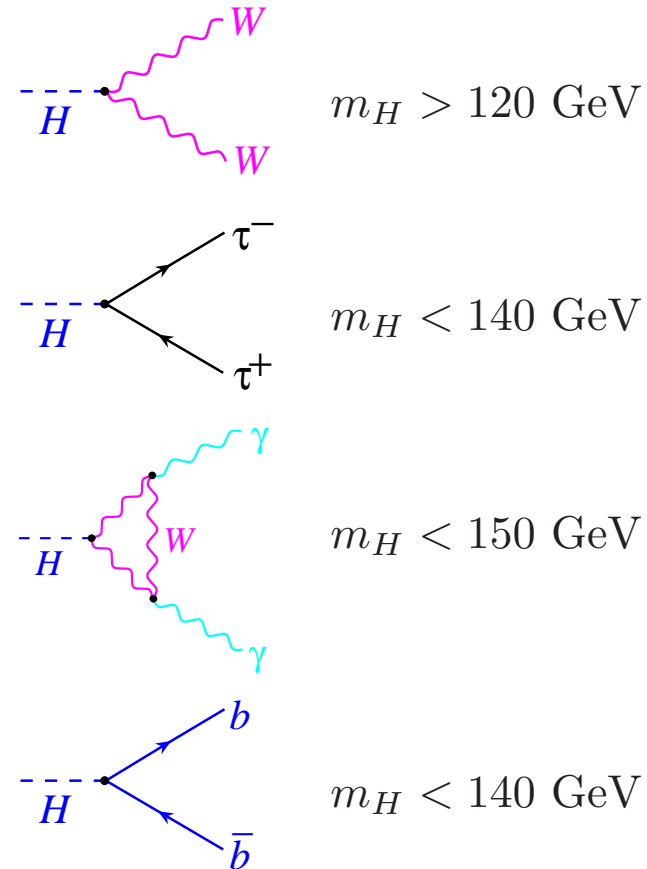
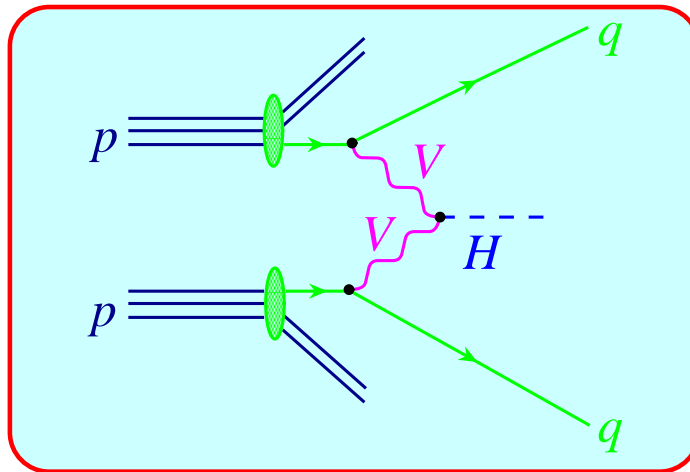
The particles produced in [association](#) with the Higgs boson are the [trigger](#) of the event.

$t\bar{t}H \rightarrow t\bar{t}b\bar{b}$



- ✓  $h_t = t\bar{t}H$  Yukawa coupling  $\implies$  measure  $h_t^2$   $BR(H \rightarrow b\bar{b})$
- ✗ must know the background normalization precisely
- ✗ it has been shown recently that this channel is **no longer feasible**

## Weak Boson Fusion

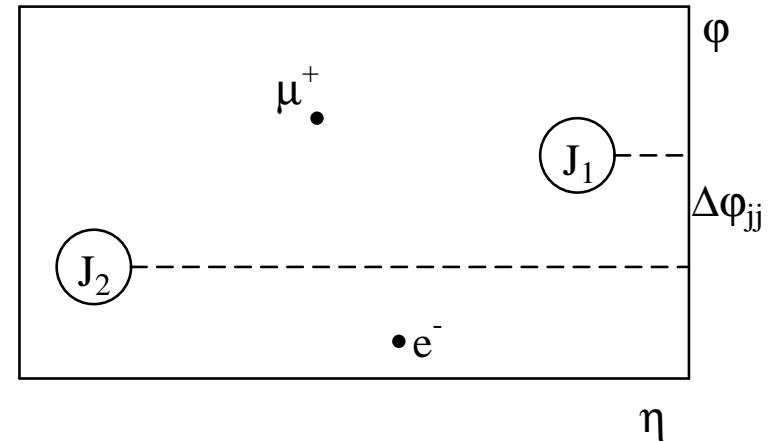
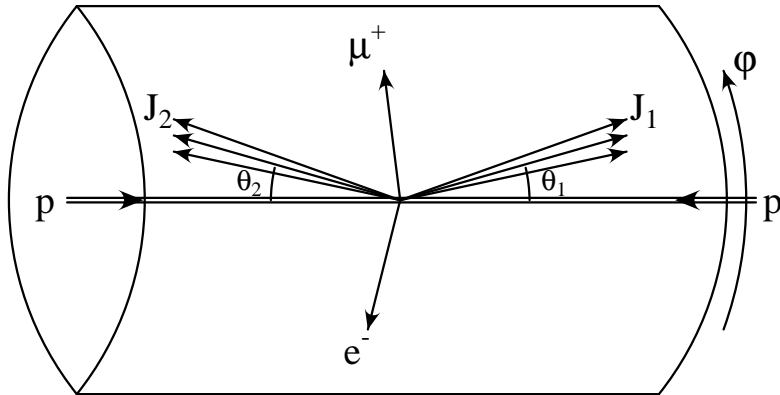


[Eboli, Hagiwara, Kauer, Plehn, Rainwater, Zeppenfeld ...] [Mangano, Moretti, Piccinini, Pittau, Polosa ('03)]

These measurements can be performed at the LHC with **statistical accuracies** on the measured cross sections times decay branching ratios,  $\sigma \times \text{BR}$ , of **order 10%** (sometimes even better).



## VBF signature

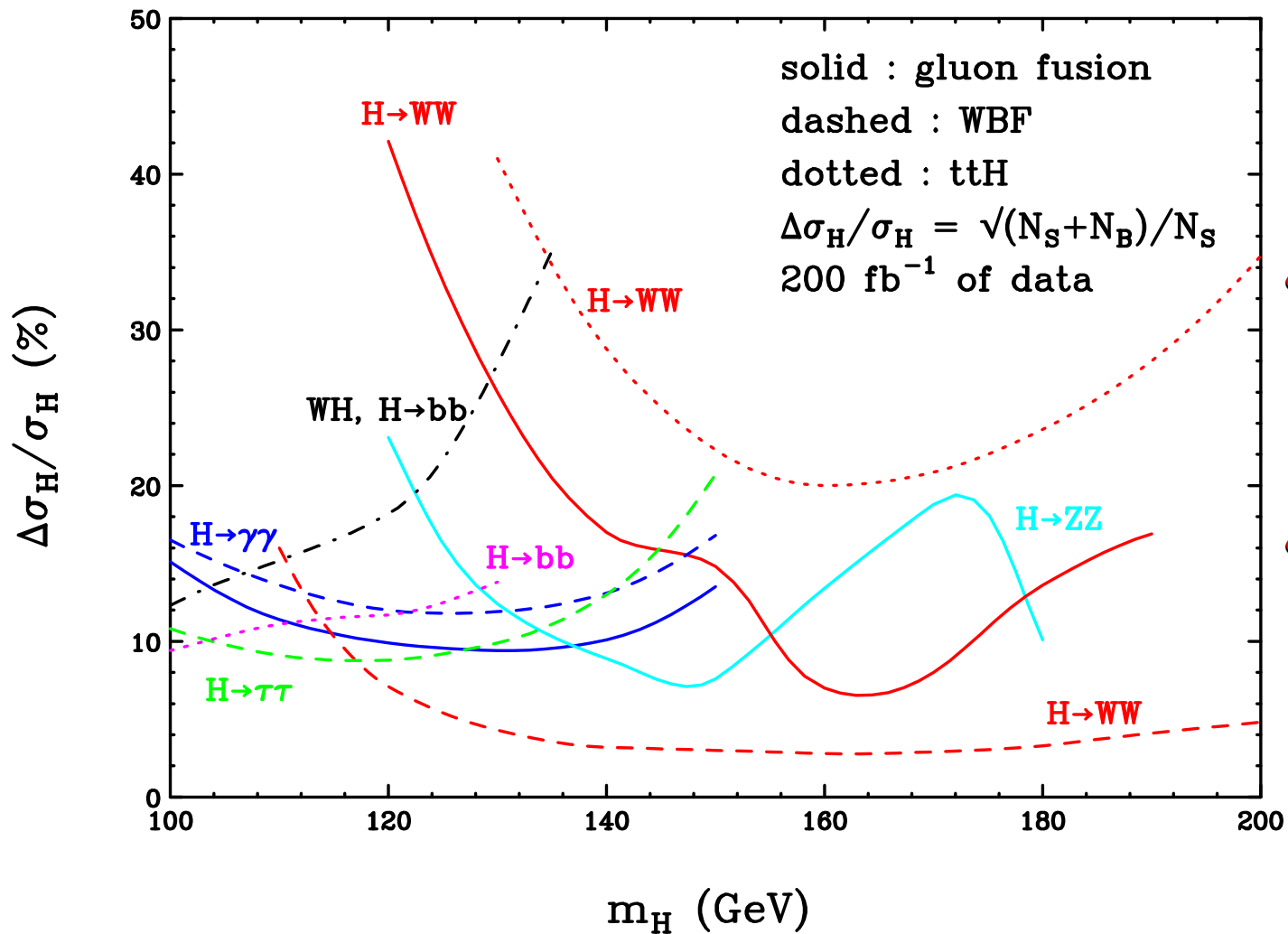


$$\eta = \frac{1}{2} \log \frac{1 + \cos \theta}{1 - \cos \theta}$$

### Characteristics:

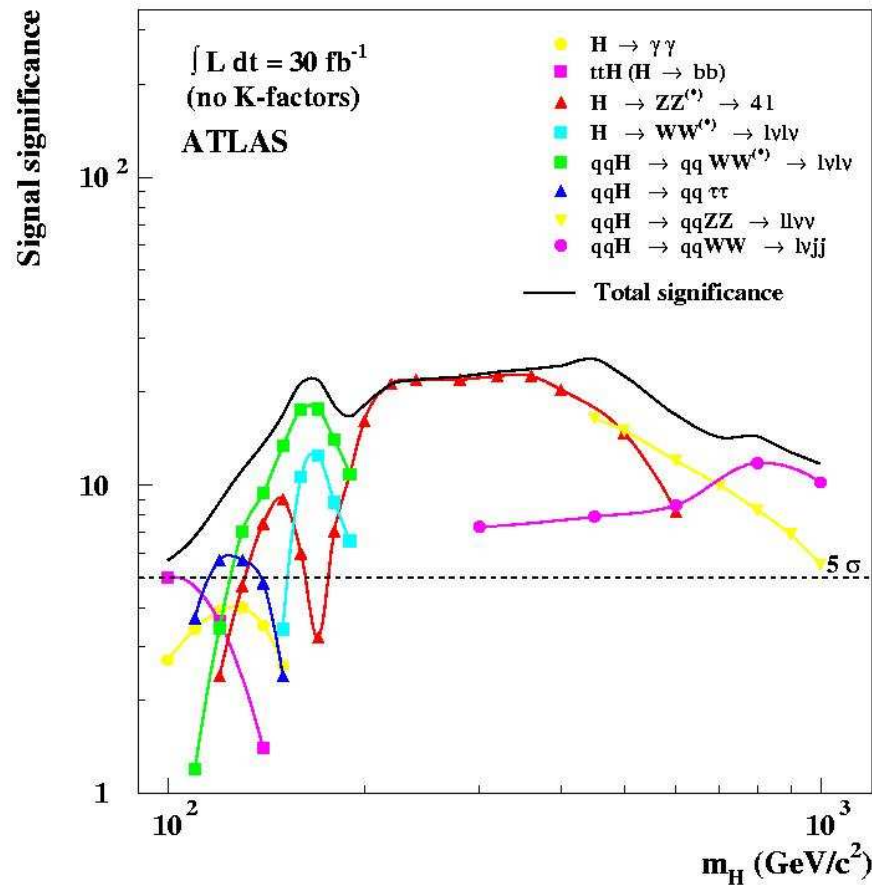
- energetic jets in the **forward** and **backward** directions ( $p_T > 20$  GeV)
- **large rapidity separation** and large **invariant mass** of the two tagging jets
- **Higgs decay products between** tagging jets
- Little gluon radiation in the central-rapidity region, due to **colorless**  $W/Z$  exchange  
(**central jet veto**: no extra jets with  $p_T > 20$  GeV and  $|\eta| < 2.5$ )

# Statistical and systematic errors at LHC

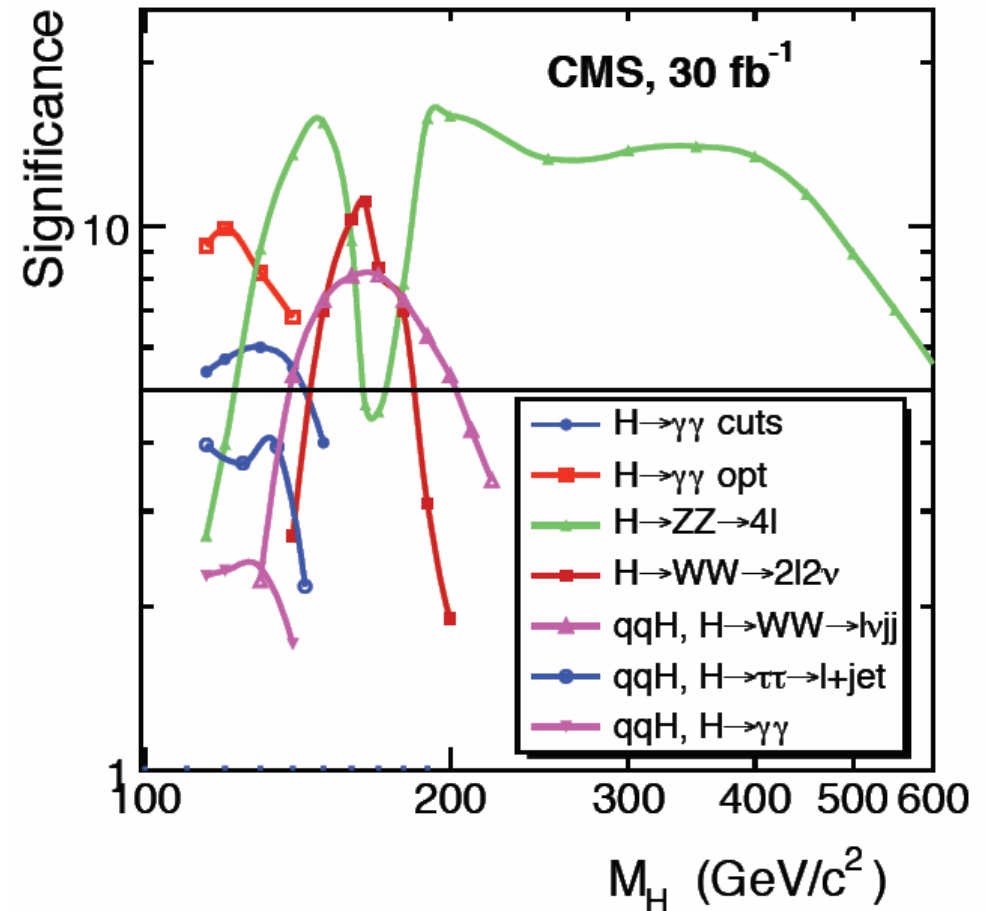


- QCD/PDF uncertainties
  - ±5% for Weak Boson Fusion
  - ±20% for gluon fusion
- luminosity/acceptance uncertainties
  - ±5%

# Higgs discovery potential with $30 \text{ fb}^{-1}$



**2003**      no K-factors

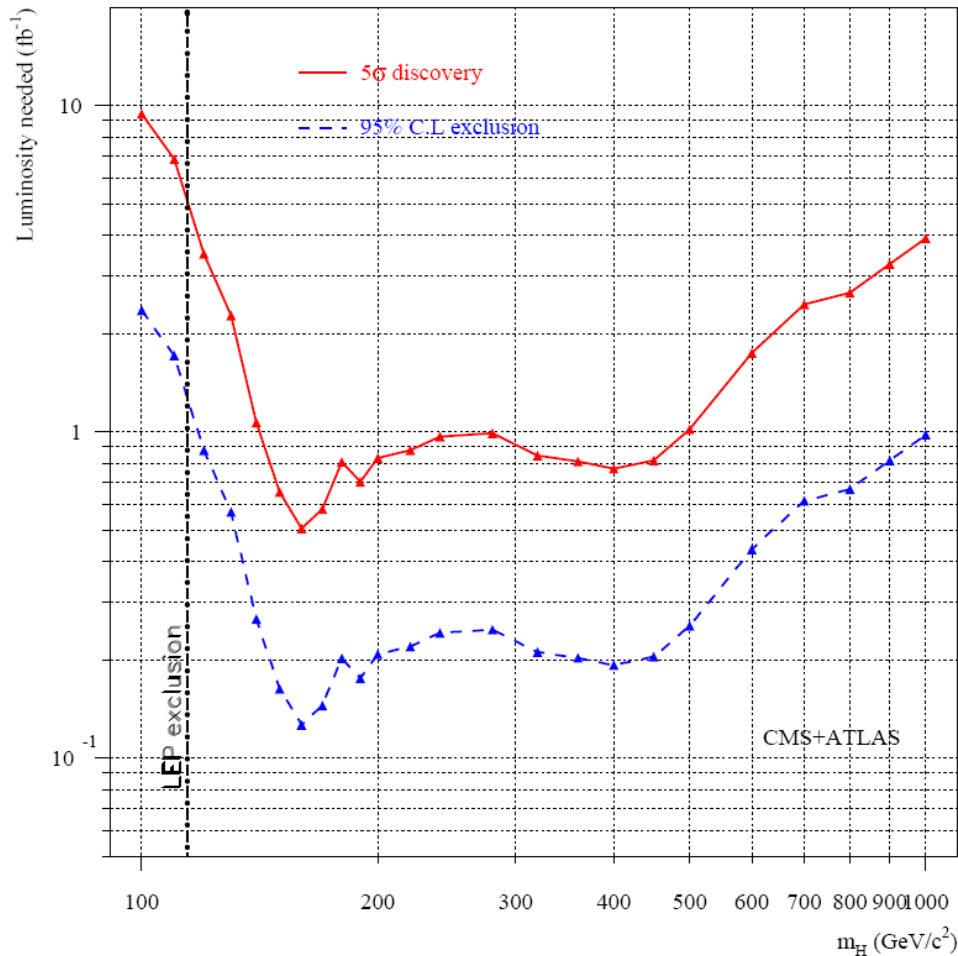


**2006**      K-factors included

Full mass range can already be covered after a few years at low luminosity.

Vector-boson fusion channels play an important role at low mass!

# ATLAS and CMS combined



Luminosity required for a 5  $\sigma$  discovery or a 95% CL exclusion

- $\sim 5 \text{ fb}^{-1}$  needed to achieve a 5  $\sigma$  discovery (well-understood and calibrated detector)
- $< 1 \text{ fb}^{-1}$  needed to set a 95% CL limit

## Conclusions

More can be said about:

- Higgs boson **couplings** to **bosons** and **fermions**
- Higgs boson **spin** measurement from decay products and jet-angular correlations in VBF and gluon fusion
- **CP** properties
- Higgs boson **self couplings**
- **SUSY** Higgs bosons
- ...