Towards event simulation at Next-to-Next-to leading accuracy

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Some History

1978: Large Perturbative Corrections to the Drell-Yan Process in QCD,
(Altarelli, Ellis, Martinelli)
The message: NLO corrections are BIG.

Extended effort in the computation of NLO corrections to strong processes, culminating in the present avaliability of automated methods for the computation of NLO corrections in complex processes

1980: First shower Monte Carlo algorithms
(Fox and Wolfram, Odorico, Sjöstrand)

- Backward evolution (Sjöstrand, 1985)
- Coherence in soft radiation (Marchesini and Webber, 1987)

Before LEP ended:

Shower Monte Carlo: event simulation, analysis planning, etc.
NLO calculations: mostly QCD tests and refinement of "new physics" predictions (W and Z, $t\bar{t}$)

After LEP:

emphasis shifting from QCD tests to event modeling;

- CKKW paper (Catani, Krauss, Marchesini, Webber, 2001)
  Method for interfacing fixed order, multi-jet production matrix elements with parton shower generators
- `MC@NLO` (Frixione and Webber, 2002), `POWHEG` (P.N. 2004)

Fixed order calculation and parton showers are merged, in an attempt to achieve higher accuracy in fully exclusive event simulation.

Developments in fixed order calculations

A series of theoretical developments (among the most relevant ones: Ossola, Papadopoulos, Pittau, 2007) for simplifying NLO calculations and extend numerical feasibility;
automated NLO calculation for complex processes (`HELAC`, `AMC@NLO`, `GoSam`)

Outstanding developments in NNLO calculations:

- $e^+e^- \to 3\,\mathrm{jets}$, (G-De Ridder, Gehrmann, Glover, Heinrich, 2007)
- Fully differential $gg \Rightarrow H$ at NNLO (Anastasiou, Melniko, Petriello, 2005)
- General method for fully differential production of a heavy colourless system (Catani, Grazzini, 2007),
  HW at NNLO (Ferrera,Grazzini,Tramontano 2011)
- $t\bar{t}$ production at NNLO (Czakon, Mitov 2012; +Fiedler, 2013)
- gluon dijet at NNLO (G-De Ridder, Gehrmann, Glover, Pires, 2013)
- $H + j$ at NNLO (Boughezal,Caola,Melnikov,Petriello, Schulze, 2013)

Developments in event simulation

ME-PS:
AlpGen (Mangano,Moretti,Piccinini,Pittau,Polosa)
Sherpa (Gleisberg, Höche,Krauss,Schonherr,Schumann,Siegert,Winter)
MadGraph (Alwall,Herquet,Maltoni,Mattelaer,Stelzer)

Full simulation, accurate at tree level in jet observables for widely separated jets, LL (partial NLL) accuracy for small angle jets

`NLO-PS`:
`(a)MC@NLO` (Frixione,Webber,Frederix,Hirschi,Maltoni,Pittau,Torielli ...)
`POWHEG-BOX` (Alioli,Oleari,Re,Hamilton,Zanderighi,P.N. + ...)
`Sherpa` (`POWHEG` and `MC@NLO` variants, Höche,Krauss,Schonherr,Siegert)
Herwig++ (`POWHEG` and `MC@NLO` variants, Platzer and Gieseke)
New proposal: `VINCIA` (Giele et al, 2013), `GENEVA` (Alioli et al,),
CKKW-L extensions (Lönnblad, Prestel, 2013)

NLO-PS: Full simulation, accurate at NLO level for observables that are non-trivial at the Born level, and involve widely separated jets. LO and LL accuracy for observables that are non trivial for real emissions.

At variance with ME-PS, NLO accuracy is not maintained when jets are added.

Example: (Higgs)

	CKKW	NLO-PS H	NLO-PS HJ	NLO-PS HJJ
Inclusive	$\mathcal{O}(\alpha_s^2)$ (LO)	$\mathcal{O}(\alpha_s^2 + \alpha_s^3)$ (NLO)	NO	NO
1 jet	$\mathcal{O}(\alpha_s^3)$ (LO)	$\mathcal{O}(\alpha_s^3)$ (LO)	$\mathcal{O}(\alpha_s^3 + \alpha_s^4)$ (NLO)	NO
2 jet	$\mathcal{O}(\alpha_s^4)$ (LO)	approximate $\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^4)$ (i.e. LO)	$\mathcal{O}(\alpha_s^4 + \alpha_s^5)$ (NLO)
3 jet	$\mathcal{O}(\alpha_s^5)$ (LO)		approximate $\mathcal{O}(\alpha_s^5)$	$\mathcal{O}(\alpha_s^5)$ (i.e. LO)
4 jet	$\mathcal{O}(\alpha_s^6)$ (LO)			approximate $\mathcal{O}(\alpha_s^6)$

What current research is aiming to: NLO+PS merging, NNLO+PS

	`H`	`H+HJ merging` (1)	`H+HJ+HJJ merging` (2)	NNLO-PS H
Incl.	$\mathcal{O}(\alpha_s^2 + \alpha_s^3)$ (NLO)	$\mathcal{O}(\alpha_s^2 + \alpha_s^3)$ (NLO)	$\mathcal{O}(\alpha_s^2 + \alpha_s^3)$ (NLO)	$\mathcal{O}(\alpha_s^2 + \alpha_s^3 + \alpha_s^4)$ (NNLO)
1 jet	$\mathcal{O}(\alpha_s^3)$ (LO)	$\mathcal{O}(\alpha_s^3 + \alpha_s^4)$ (NLO)	$\mathcal{O}(\alpha_s^3 + \alpha_s^4)$ (NLO)	$\mathcal{O}(\alpha_s^3 + \alpha_s^4)$ (NLO)
2 jet	approximate $\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^4)$ (LO)	$\mathcal{O}(\alpha_s^4 + \alpha_s^5)$ (NLO)	$\mathcal{O}(\alpha_s^4)$ (LO)
3 jet		approximate $\mathcal{O}(\alpha_s^5)$	$\mathcal{O}(\alpha_s^5)$ (LO)	approximate $\mathcal{O}(\alpha_s^5)$
4 jet			approximate $\mathcal{O}(\alpha_s^5)$	

Notice: 1st-level NLO-PS merging approaches NNLO-PS

Main claim

(Zanderighi, Hamilton, Oleari, P.N. 2013) From 1st-level NLO+PS merging, NNLO accuracy can be reached by reweighting.

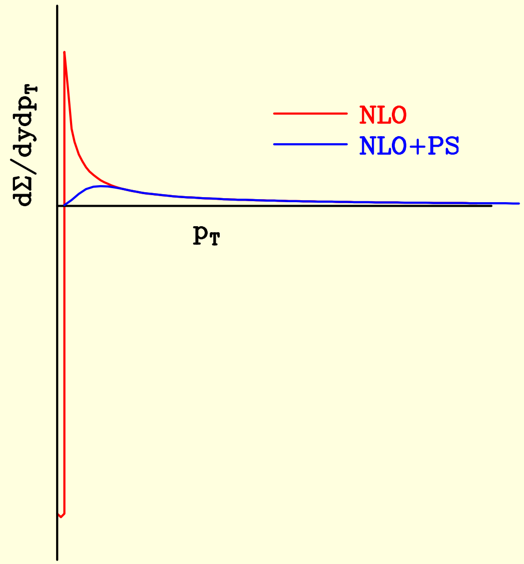
Here we prove this in the example of Higgs production (proof easely extended to the general case.

Begin with the following (trivial) theorem:
A parton level Higgs boson production generator that is accurate at $\mathcal{O}(\alpha_s^4)$ *for all IR safe observables that vanish with the maximum transverse momenta of all light partons, and that also reaches accuracy for the* $\mathcal{O}(\alpha_s^4)$ *inclusive Higgs rapidity distribution, achieves the same level of precision for all IR safe observables, i.e. it is fully NNLO accurate.*
Proof: $F(\Phi)$ is an IR safe observables; $F(y_H)$ is its "Born level" value

$$\langle F \rangle = \int \mathrm{d}\Phi \frac{\mathrm{d}\sigma}{\mathrm{d}\Phi} F(\Phi) = \underbrace{\int \mathrm{d}\Phi \frac{\mathrm{d}\sigma}{\mathrm{d}\Phi} \left(F(\Phi) - F(y_\Phi)\right)}_{\text{accurate at } \mathcal{O}(\alpha_s^4) \text{ by 1st hypothesis}} + \underbrace{\int \mathrm{d}y \frac{\mathrm{d}\sigma}{\mathrm{d}y} F(y)}_{\mathcal{O}(\alpha_s^4) \text{ by 2nd hypothesis}}$$

Does the 1st hypothesis apply also to an NLO-PS generator?

The difference with respect to a parton level generator is that the soft and collinear singularities are spread out over the Sudakov regions. For the Higgs:



NLO result: divergent distribution at low p_{T};
Negative divergent spike at $p_{\mathrm{T}}=0$, so that

$$\int \frac{d\sigma^{\mathrm{NLO}}}{dy\,dp_{\mathrm{T}}}\,\mathrm{d}p_{\mathrm{T}}=\frac{d\sigma^{\mathrm{NLO}}}{dy}$$

NLO+PS result: smooth Sudakov shape at small p_{T}, all positive, with

$$\int \frac{d\sigma^{\mathrm{NLO+PS}}}{dy\,dp_{\mathrm{T}}}\,\mathrm{d}p_{\mathrm{T}}=\frac{d\sigma^{\mathrm{NLO}}}{dy}$$

(The proof of the 1st hypothesis for a NLO+PS generator can be carried out by expanding the Sudakov form factors in terms of a normalized "+" distribution plus higher order terms)

As for the 2nd hypothesis, a NLO+PS merged generator has only $\mathcal{O}(\alpha_s^3)$ accuracy in the $\int \mathrm{d}y \frac{\mathrm{d}\sigma}{\mathrm{d}y} F(y)$ term.

However, the reweighted cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi} \frac{\left(\frac{\mathrm{d}\sigma^{\mathrm{NNLO}}}{\mathrm{d}y}\right)_{y_\Phi}}{\left(\frac{\mathrm{d}\sigma}{\mathrm{d}y}\right)_{y_\Phi}} = \frac{\mathrm{d}\sigma}{\mathrm{d}\Phi} \times (1 + C(y)\, \alpha_s^2)$$

is NNLO accurate. In fact, in our generic observable:

$$\langle F \rangle = \int \mathrm{d}\Phi \frac{\mathrm{d}\sigma}{\mathrm{d}\Phi} F(\Phi) = \underbrace{\int \mathrm{d}\Phi \frac{\mathrm{d}\sigma}{\mathrm{d}\Phi} (F(\Phi) - F(y_\Phi))}_{\text{accurate at } \mathcal{O}(\alpha_s^4) \text{ by 1st hypothesis}} + \underbrace{\int \mathrm{d}y \frac{\mathrm{d}\sigma}{\mathrm{d}y} F(y)}_{\mathcal{O}(\alpha_s^4) \text{ by 2nd hypothesis}}$$

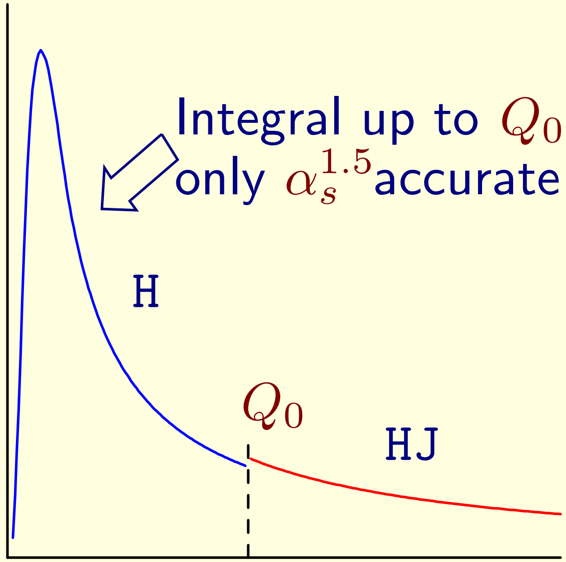
the first term, $\int \mathrm{d}\Phi \frac{\mathrm{d}\sigma}{\mathrm{d}\Phi} (F(\Phi) - F(y_\Phi))$, is of order $\alpha_s^3 + \alpha_s^4$ (the Born term does not contribute), and the reweighting factor only introduces terms of order α_s^5.

Also: $\int \mathrm{d}y \frac{\mathrm{d}\sigma}{\mathrm{d}y} F(y)$ becomes $\int \mathrm{d}y \frac{\mathrm{d}\sigma^{\mathrm{NNLO}}}{\mathrm{d}y} F(y)$.

The real challenge is to set up a 2nd level NLO+PS merged generator.

Why is it difficult? Naive approach:

- Start with the H and HJ generator
- Introduce a separation scale: $\Lambda_{\rm QCD} \ll Q_0 \ll M_H$
- Use H for $p_T^H < Q_0$
- Use HJ for $p_T^H > Q_0$



The Sudakov peak is at $\alpha_s L^2 \approx 1$ ($L = \log M_H/p_T^H$). Even if the Sudakov form factor is accurate at the NLL level in the H generator, missing NNLL terms of order $\alpha_s^2 L \approx \alpha_s^{1.5}$ spoil NLO accuracy, that requires α_s^2 neglected terms.

The only way out seems to require $\alpha_s^2 L \approx \alpha_s^2$, i.e. $Q_0 \approx M_H$!!

Remark on counting orders

- When $\alpha L^2 \lesssim 1$, it is sensible to count $L \sim \alpha^{-\frac{1}{2}}$
- The region $\alpha L^2 > 1$ is an important one, but can only be accessed with the counting $L \sim \alpha^{-1}$. Shower MC go down to $L \sim \alpha^{-1}$.
- When integrating over the Sudakov region, it can be demonstrated that $\int \frac{\mathrm{d}q_T^2}{q_T^2} L^m \alpha_s^n \exp \mathcal{S}(Q, q_T) = \int \mathrm{d}L\, L^m \alpha_s^n \exp \mathcal{S}(Q, q_T) \approx [\alpha_s(Q^2)]^{n-\frac{m+1}{2}}$ (the Sudakov peak is at $\alpha L^2 \sim 1$, so this makes sense).

In NLO+PS, NLO accuracy of the TOTAL integral of the Sudakov peak is granted by some tricks (i.e. unitarity constraints). When breaking the integral, NLO accuracy is lost, unless the Sudakov reaches NNLL accuracy.

Current merging approaches:

- SHERPA, [Hoeche, Krauss, Schonherr, Siegert, arXiv:1207.5030], traditional merging with matching scales.

- aMCNLO, [Frederix, Frixione, arXiv:1209.6215], traditional merging with matching scales; scales kept high to avoid above problems.

- [Platzer, arXiv:1211.5467], [Lönnblad, Prestel, arXiv:1211.7278], force unitarity by subtracting appropriate terms.

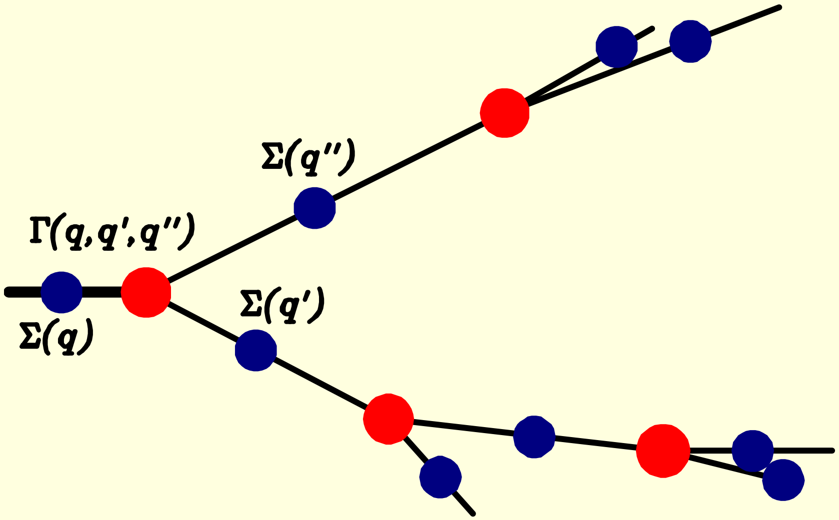
- GENEVA, [Alioli, Bauer, Berggren, Hornig, Tackmann, Vermilion, Walsh, Zuberi, arXiv:1211.7049], increase precision in LL resummation to reach accurate matching

Approach presented here

(Sequel of papers in collaboration with Hamilton, Oleari, Re and Zanderighi)

- The `MiNLO` method: a method for assigning scales and Sudakov form factors in NLO calculations that captures large renormalization, factorization and Sudakov logarithms (Hamilton, Zanderighi, P.N., June 2012).
- The `MiNLO` method applied to processes like `HJ` makes it integrable over the whole p_T^H range (i.e., no cut needed), and it yields a cross section that is accurate at order $\mathcal{O}(\alpha_s^3)$, neglecting terms of order $\mathcal{O}(\alpha_s^{3.5})$.
  By a suitable modification of the Sudakov form factor, neglected terms can be made of order $\mathcal{O}(\alpha_s^{3.5})$, thus making possible an NNLO+PS generator (Hamilton, Oleari, Zanderighi, P.N., December 2012)
- Construction of an NNLO+PS generator for Higgs production (Hamilton, Re, Zanderighi, P.N., August 2013)

Shower basics



Matrix elements computed in collinear approximation (Leading Log)
Splitting vertices computed using the Altarelli-Parisi approximation
Leading Log virtual corrections also included by inserting RG improved vertices and self-energy corrections.

At leading log: $\Gamma(q, q', q'') \approx \Gamma(q = \max(q, q', q''))$ (the largest scale), and

$$\Gamma(q)\left(\sqrt{\Sigma(q)}\right)^3 = \lambda(q)$$

So, virtual corrections are included by inserting in the squared amplitude:

- $\lambda^2(q)$ at each vertex (instead of λ^2), where q is the incoming virtuality
- $\Sigma(q')/\Sigma(q)$ in each intermediate line beginning at a scale q and ending at a scale q'. $\Sigma(q) = \Delta(q)$, "Sudakov" form factor.

CKKW basics:

- Use LO matrix elements, rather than LL approximation;
- Consider only configurations with the smallest relative transverse momentum $> Q_0$.
- Reconstruct a branching history from the kinematic of the event, using a clustering algorithm (for example, by recursively merging the pair of partons with smallest relative transverse momentum).
- Assign running couplings and Sudakov form factors as in the Parton Shower approximation to include LL virtual corrections.
- Feed the kinematics of the event to a parton shower to generate splittings with transverse momenta below Q_0.

What we learn: scale choice in the couplings is intertwined with the presence of the Sudakov form factors, that take care of large scale mismatch in nearby vertices. Scale assignment in multiscale processes must be complemented with the inclusion of Sudakov form factors.

The MiNLO approach

In order to deal with multi-scale processes, use the following strategy:

- Compute the Born term according to the CKKW prescription
- Include virtual and real corrections in such a way that
  - NLO accuracy is preserved
  - CKKW LL (NLL?) virtues are not spoiled

Problems to solve:

1. In CKKW, several renormalization scales are present in the Born term, in the argument of the coupling constant associated to each node of the branching history. The virtual correction coefficient is generally computed for a single renormalization scale.
2. CKKW requires a Q_0 scale. What do we choose in our case?
3. The Sudakov form factor in the Born term already contain terms of NLO order.
4. How do we chose the scales of the α_s's in the NLO term?

How to set μ_R

From:

$$R = B\alpha_s^N(\mu_R) + \left[N b_0 \log\frac{\mu_R^2}{Q^2} B + C \right] \alpha_s^{N+1}(\mu_R) =$$

So, if the scales in each α_s power in the Born term is different, we use

$$B\alpha_s(\mu_1)\ldots\alpha_s(\mu_n) + \left[\sum_{i=1}^{N} b_0 \log\frac{\mu_i^2}{Q^2} B + C \right] \alpha_s^{N+1}(\mu_R) =$$

$$B\alpha_s(\mu_1)\ldots\alpha_s(\mu_n) + \left[N b_0 \log\frac{\bar{\mu}}{Q^2} B + C \right] \alpha_s^{N+1}(\mu_R)$$

where $\bar{\mu}$ is the geometric average of $\mu_1 \ldots \mu_N$. So: set μ_R to $\bar{\mu}$.

How to pick Q_0

Born term: pick the scale of the first clustering.
(Usual choice in CKKW for the highest multiplicity sample,
where we want that the parton shower generates all softer jets.)

In our case, a Born kinematic configuration with N partons will be associated with a virtual term with N partons, and with a real emission term with $N+1$ partons, where the softest clustering will yield N pseudopartons with the same kinematic configuration as the Born term. The integration of the softest cluster plays the role here of the further shower in ME+PS matching, i.e. it represent inclusive radiation that does not spoil the N jet structure. Q_0 is taken as the clustering scale for going from N to $N-1$ jets, i.e. the first clustering scale in the Born event, and the second clustering scale in the real event.

Remaining issues

The extra NLO terms present in the Sudakov form factor multiplying the Born term must be subtracted. We then have to decide what to use for the scales appearing in the powers of the coupling constant multiplying the NLO correction, and whether to include Sudakov form factors also in the NLO terms.

Guideline: treat the NLO term as much as possible as the Born term

By doing this we avoid spoiling the good LL features of the CKKW approach.

Notice that, in doing so, the real term has to be clustered once, and some (somewhat arbitrary) prescription has to be given for the choice of scale in the α_s^{N+1} power of α_s in the NLO terms.

Summary of the `MiNLO` prescription

We deal with a process of associated jet production, of order $N = m + n$, where n is the number of associated jets. For example, in Higgs plus 2 jets, $m = 2$ and $n = 2$.

1. Perform the k_T clustering of the partons in the event, determine the nodal scales $q_1 \ldots q_n$, the scale of the primary process Q, and eventualy the very first merging scale q_0 for the real process. Set $Q_0 = q_1$.

2. n powers of the coupling constant will be evaluated at the scales $\mu_1 \ldots . \mu_n$, with $\mu_i = K_R q_i$. K_R is the renormalization scale factor, that will be varied between $1/2$ and 2 to study scale uncertainties. The remaining m powers of the strong coupling are evaluated at the primary process scale $K_R Q$.

3. The (explicit) renormalization scale in the virtual term is set to

$$\mu_R = ((\mu_Q)^m \times \mu_1 \ldots \mu_n)^{\frac{1}{n+m}}.$$

   The factorization scale is taken equal to $K_F Q_0$, where K_F is the renormalization scale factor.

4. The Sudakov form factor are applied to all internal and external line of the branching skeleton. For real events, the branching skeleton after the first clustering is considered. External lines leaving a node at the scale q_i have a Sudakov form factor $\Delta_{f_i}(Q_0, q_i)$. Internal lines joining nodes i and j have the Sudakov form factor

$$\Delta_{f_{ij}}(Q_0, q_i)/\Delta_{f_{ij}}(Q_0, q_j),\; q_i > q_j$$

Note that the line leaving the node q_1 has no Sudakov:

$$\Delta(Q_0, q_1) = \Delta(Q_0, Q_0) = 1.$$

5. The subtraction of the NLO contribution already included in the Born term via the Sudakov form factor amounts to the replacement

$$B \Rightarrow B\left(1 - \sum_{ij}\left[\Delta^{(1)}_{f_{ij}}(Q_0, q_i) - \Delta^{(1)}_{f_{ij}}(Q_0, q_j)\right] - \sum_{l}\Delta^{(1)}_{f_l}(Q_0, q_i)\right)$$

the first sum runs over all pairs of nodes connected by a line, and l runs over external lines. $\Delta^{(1)}$ is the first order term in the expansion of Δ.

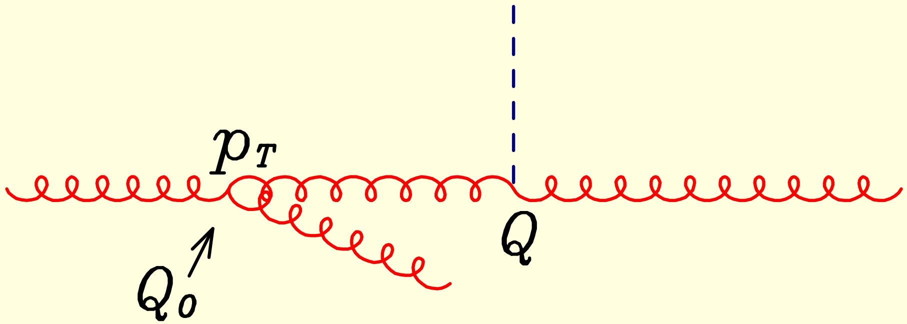
6. The $(N+1)^{\rm th}$ power of α_s multiplying the virtual, real and Sudakov subtraction term is taken equal to the average of the first N powers:

$$\alpha_s^{(N+1)} = \frac{1}{N}\left(\sum_{i=1}^{n} \alpha_s(\mu_i) + m\alpha_s(\mu_Q)\right).$$

HJ example

The CKKW factors amount to: $Q = M_{\mathrm{H}}$, $Q_0 = p_{\mathrm{T}}$, $\mu_F = Q_0$

$$F = \frac{\alpha_s(Q_0)}{\alpha_s(Q^2)} \left\{ \exp\left[-\frac{C_A}{\pi b_0} \left(\log \frac{\log \frac{Q^2}{\Lambda^2}}{\log \frac{Q_0^2}{\Lambda^2}} \left(\frac{1}{2} \log \frac{Q^2}{\Lambda^2} - \frac{\pi b_0}{C_A} \right) - \frac{1}{2} \log \frac{Q^2}{Q_0^2} \right) \right] \right\}^2$$



No Sudakov for external lines meeting at Q_0, and two identical Sudakov for the remaining gluon lines.

$$B \Rightarrow B \times \left(1 + \alpha_s \left(4\pi b_0 \log \frac{Q^2}{Q_0^2} + \frac{2C_A}{\pi} \left[\frac{1}{4} \log^2 \frac{Q^2}{Q_0^2} - \frac{\pi b_0}{C_A} \log \frac{Q^2}{Q_0^2} \right] \right) \right) \times F$$

$$(V, R) \Rightarrow (V, R) \times F \times \frac{\alpha_s(\mu_f)}{\alpha_s(Q)}$$

Very similar to Higgs p_T resummation

(Hamilton,Oleari,Zanderighi,P.N. 2012) Focus upon $H/W/Z+1\,\text{jet}$.The NNLL resummed the transverse momentum distribution of the boson is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y\mathrm{d}q_T^2} = \sigma_0 \frac{\mathrm{d}}{\mathrm{d}q_T^2}\{[C\otimes f_A](x_A, q_T)\times[C\otimes f_B](x_B, q_T)\times\exp\mathcal{S}(Q, q_T)\} + R_f$$

$$\mathcal{S}(Q, q_T) = -\int_{q_T^2}^{Q^2}\frac{\mathrm{d}q^2}{q^2}\left[A(\alpha_s(q^2))\log\frac{Q^2}{q^2} + B(\alpha_s(q^2))\right],$$

$$A(\alpha_s) = \sum_{i=1}^{\infty} A_i\alpha_s^i, \qquad B(\alpha_s) = \sum_{i=1}^{\infty} B_i\alpha_s^i, \qquad R_f = \text{finite terms}$$

This formula yields the correct NLO $\mathrm{d}\sigma/\mathrm{d}y$ when integrated over $\mathrm{d}q_T^2$.
To make contact with the MiNLO result, take explicitly $\mathrm{d}/\mathrm{d}q_T^2$, and get:

$$\sim\sigma_0\frac{1}{q_T^2}[\alpha_S, \alpha_S^2, \alpha_S^3, \alpha_S^3, \alpha_S^4, \alpha_S L, \alpha_S^2 L, \alpha_S^3 L, \alpha_S^4 L]\times\exp\mathcal{S}(Q, q_T) + R_f,$$

$(L = \log Q^2/q_T^2)$. We have: $\int\ \frac{\mathrm{d}q_T^2}{q_T^2} L^m \alpha_s^n \exp\mathcal{S}(Q, q_T) \approx [\alpha_s(Q^2)]^{n-\frac{m+1}{2}}$

Thus, if we drop all α^3 and higher terms, NLO accuracy upon integration is still preserved (worse term: $\alpha^3 L \to \alpha_s^2(Q)$, NNLO).

Same accuracy as in MiNLO! Only difference: B_2 term missing in MiNLO $\mathcal{S}$!

Correction:

$$\sigma_0 \frac{1}{q_T^2}[\alpha_S, \alpha_S^2, \alpha_S^3, \alpha_S^3, \alpha_S^4, \alpha_S L, \alpha_S^2 L] \times \exp \mathcal{S} \times \left\{ \exp\left[-\int_{q_T^2}^{Q^2} \frac{\mathrm{d}q^2}{q^2} B_2 \alpha_S^2 \right] - 1 \right\}$$

$$\Longrightarrow \frac{1}{q_T^2}[\alpha_S^3 L^2] \times \exp \mathcal{S} \Longrightarrow [\alpha_s(Q^2)]^{3-\frac{2+1}{2}} = [\alpha_s(Q^2)]^{1.5}$$

So, MiNLO yields an accuracy that is more than LO, but less than NLO, in the inclusive cross section, the neglected term having a power of α_S greater than 1 but less than 2.

In case of $H/W/Z + 1\,\mathrm{jet}$, it is in fact possible to modify the MiNLO Sudakov form factor by carefully including the B_2 term in such a way that integrating over the radiated jet we achieve NLO accuracy for inclusive $H/W/Z$ distributions. (Hamilton,Oleari,Zanderighi,P.N. 2012)

NNLO generator for Higgs production

(Hamilton,Re,Zanderighi,P.N. 2013)
Variant reweighting schemes

$$\begin{aligned} \mathrm{d}\sigma &= \mathrm{d}\sigma_A + \mathrm{d}\sigma_B \\ \mathrm{d}\sigma_A &= \mathrm{d}\sigma \times h(p_T) \\ \mathrm{d}\sigma_B &= \mathrm{d}\sigma \times (1 - h(p_T)) \end{aligned}$$

with

$$h(p_T) = \frac{(\beta m_{\mathrm{H}})^\gamma}{(\beta m_{\mathrm{H}})^\gamma + p_T^\gamma},$$

and reweight by

$$W(y, p_T) = h(p_T) \times \frac{\int \mathrm{d}\sigma_A^{\mathrm{NNLO}} \delta(y - y(\Phi))}{\int \mathrm{d}\sigma_A^{\texttt{MiNLO}} \delta(y - y(\Phi))} + (1 - h(p_T)),$$

that yields

$$\int \mathrm{d}\sigma^{\texttt{MiNLO}} \delta(y - y(\Phi))\, W(y, p_T) = \left(\frac{\mathrm{d}\sigma_A}{\mathrm{d}y}\right)^{\mathrm{NNLO}} + \left(\frac{\mathrm{d}\sigma_B}{\mathrm{d}y}\right)^{\texttt{MiNLO}}$$

We have adopted a further variant that has the advantage of yielding exactly the NNLO rapidity distribution:

$$W(y,p_T)=h(p_T)\times\frac{\int(\mathrm{d}\sigma^{\mathrm{NNLO}}-\mathrm{d}\sigma_B^{\mathtt{MiNLO}})\delta(y-y(\Phi))}{\int\mathrm{d}\sigma_A^{\mathtt{MiNLO}}\delta(y-y(\Phi))}+(1-h(p_T)).$$

Numerically one needs to compute and store the (one-parameter) functions of y that appear in the fraction. After that one generates events normally, and reweights them by the W factor.

The NNLO cross section is computed with HNNLO (Grazzini, 2008)

Uncertainties

Uncertainties are estimated by the 7 point scale variation

$$(K_R, K_F) = (0.5, 0.5), (1, 0.5), (0.5, 1), (1, 1), (2, 1), (1, 2), (2, 2)$$

that is performed independently in the NNLO calculation and in the `MiNLO` one.

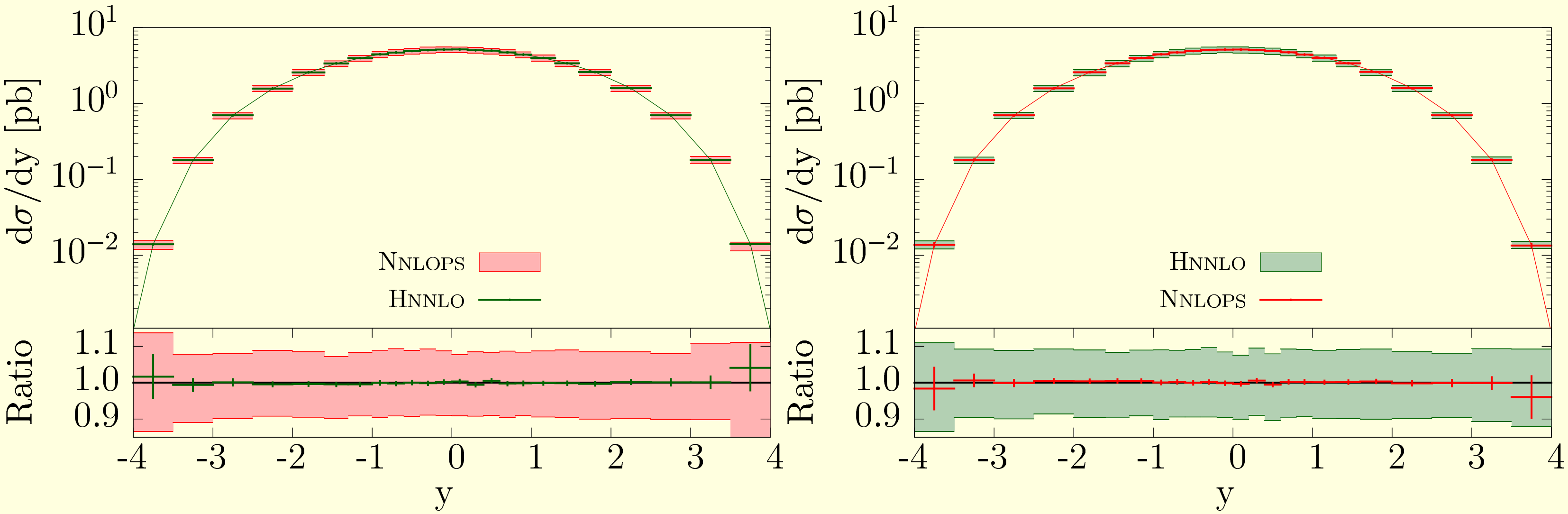
In other words, we assume conservatively that scale uncertainties in the NNLO and in the `MiNLO` results are uncorrelated.

The value of p_T in the h function has been taken as the transverse momentum of the hardest jet.

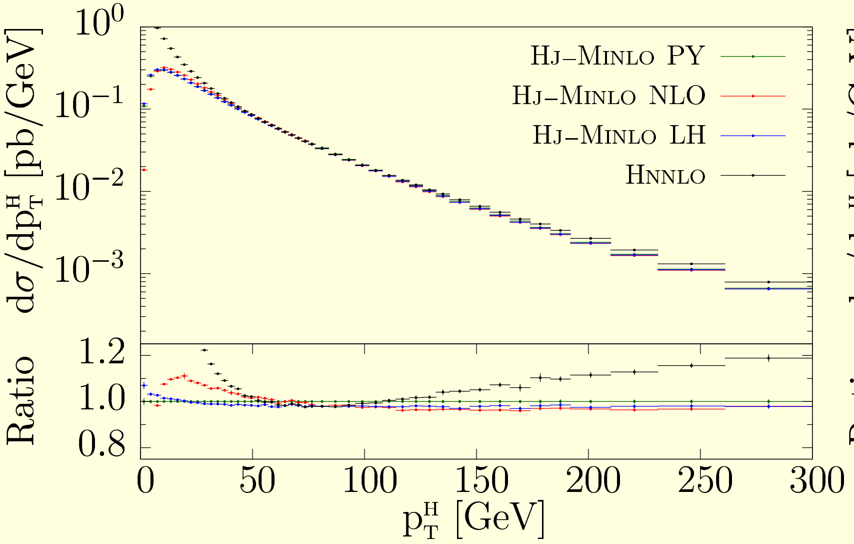
Central scale for HNNLO: $m_H/2$, in slight tension with the `MiNLO` choice.

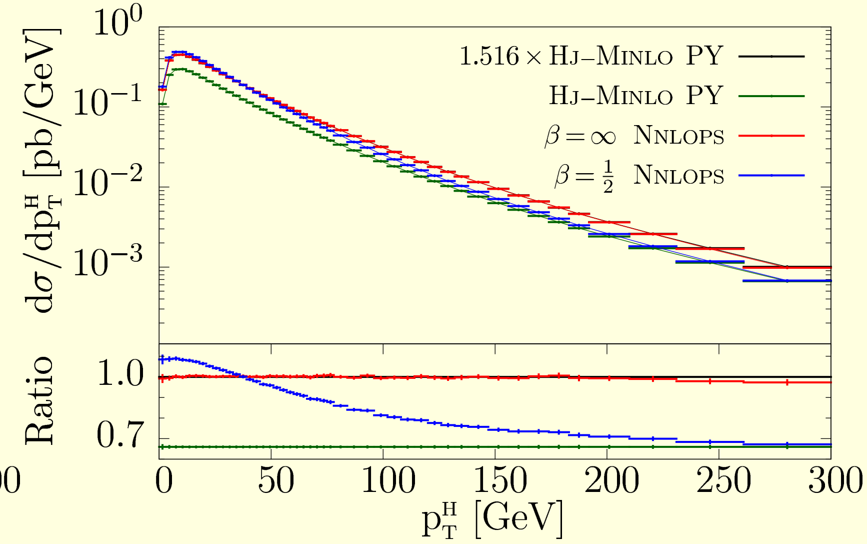
We use $\gamma = 2$ in the h function, and consider the range $0.5 < \beta < \infty$.

Results



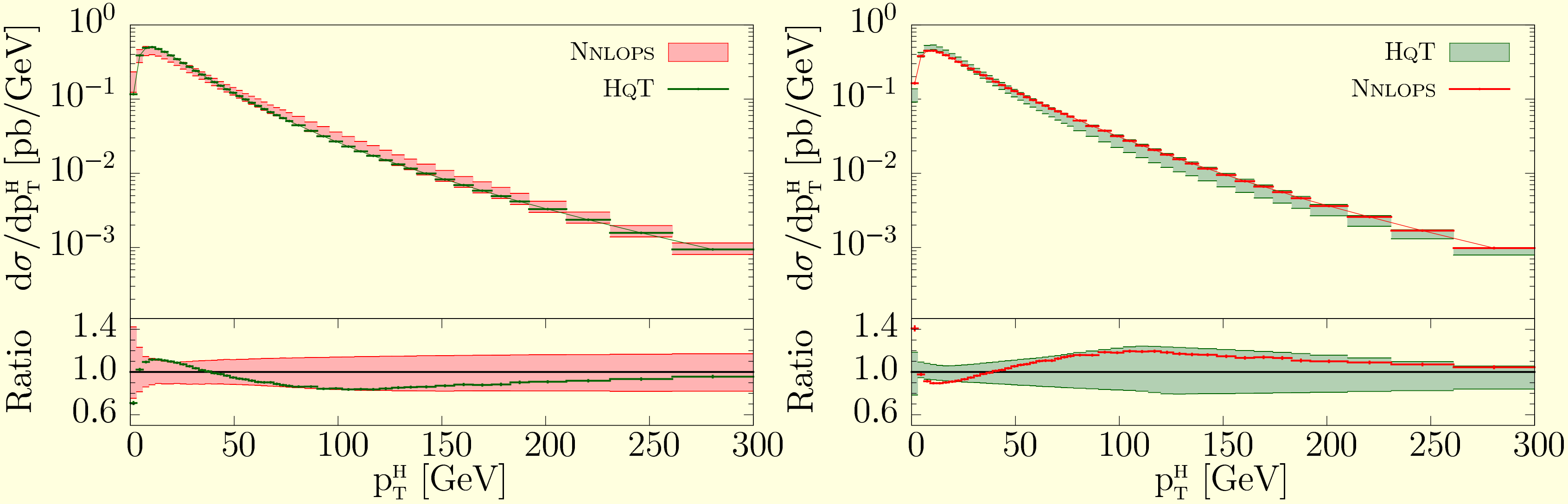
By construction, the rapidity distribution is exactly the same in NNLO-PS and in fixed order



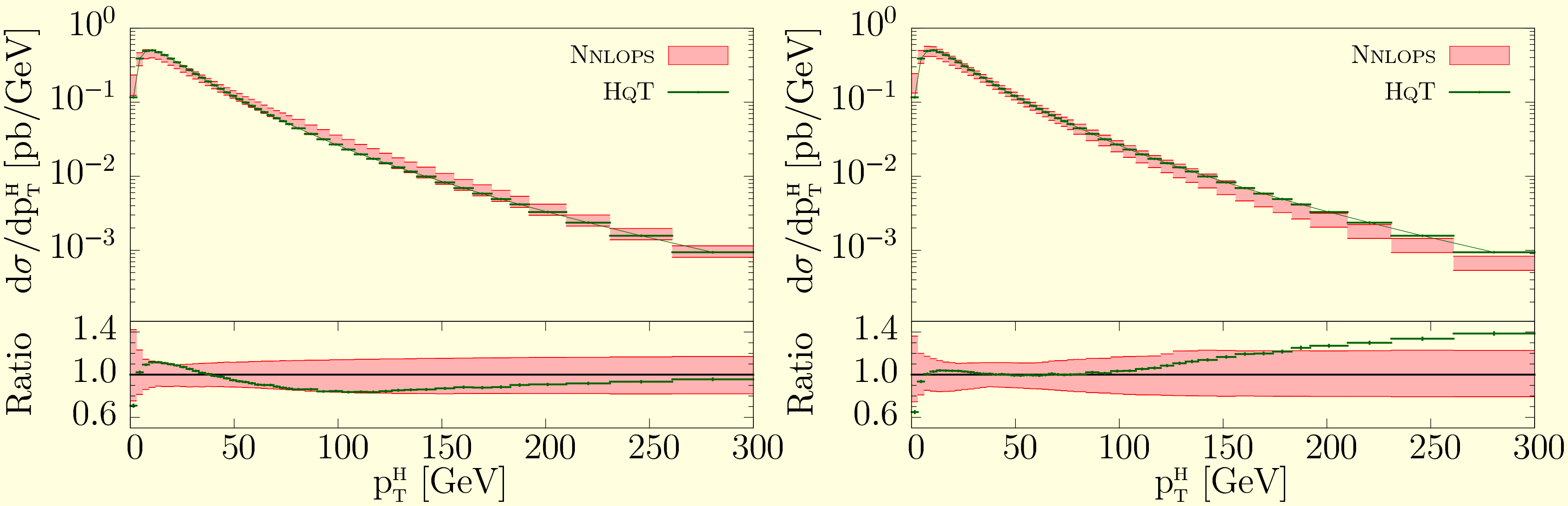


Left:
Comparison of the high p_T distribution with HNNLO, using M_H as scales

Right:
Effect of β variation

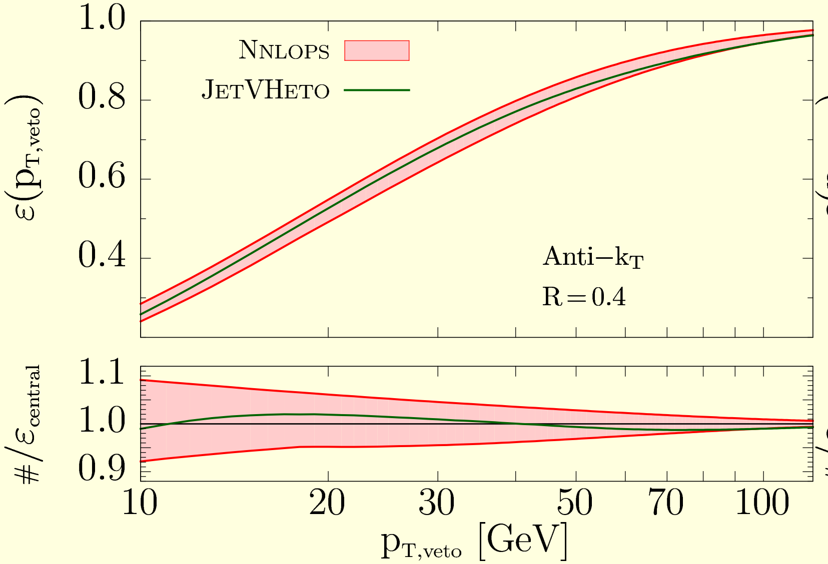


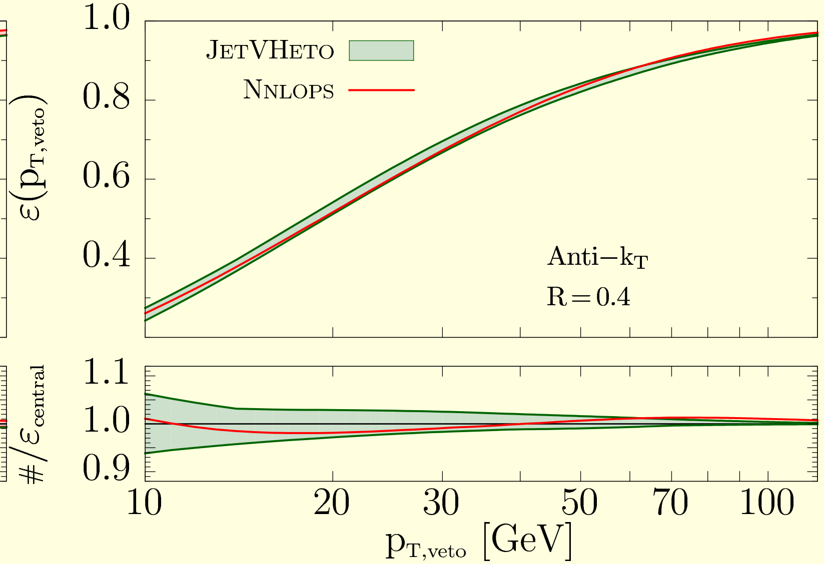
HqT and NNLO-PS error bands comparable



p_T spectrum with error bands, $\beta = \infty$ (left), $\beta = 1/2$ (right)

Choice of β analogous to the choice of the resummation scale in `HqT`.
$\beta = 1/2$ corresponds to $Q_{\rm res} = M_H/2$.





- JetVHeto: NNLL resummed, $\mu_R = \mu_F = m_H/2$, 7pts band (Banfi,Monni,Salam,Zanderighi, 2012)
- Fair agreement

Other work on NNLO+PS

- Lönnblad,Prestel,2013 hint at the possibility to build NNLO+PS generators using their method
- Alioli,Bauer,Berggren,Tackmann,Walsh,Zuberi,2013 present a general discussion on how to construct NNLO+PS generator

Discussion on the meaning/requirements of NNLO+PS generators has just started.

Conclusions

- First implementation of NNLO-PS for a simple process
- Can be immediately extended to W/Z and HW HZ production
- Extensions to more complex processes relies upon extension of the (improved) MiNLO procedure.
- Investigation in the NNLO+PS direction just starting ...