Heavy Flavours and PDF's

P. Nason

INFN
Milano-Bicocca

QCD at the LHC, ECT*, Trento, 28 Sept. 2010

History

How do we treat a heavy flavour (mass $m_h \gg \Lambda_{\mathrm{QCD}}$) in QCD processes?

In standard $\overline{\mathrm{MS}}$ scheme, heavy flavour effects persist even for $Q \ll m_h$
(n_f, not $n_l = n_f - 1$, appear in the running coupling ...).
It would be cumbersome to worry about top when doing DIS at 10 GeV^2.

Use Decoupling renormalization scheme with n_l light flavours (all but h)
(Collins, Wilczek, Zee 1978;). CWZ prescription:

    —   the $\overline{\mathrm{MS}}$ scheme for light flavours

    —   a zero momentum subtraction for heavy flavour graphs

heavy flavour graphs: graphs that include heavy flavour line, or
counterterms to heavy flavour graphs.

Advantages: $m_h \to \infty$ limit easy: forget the heavy quark!
(i.e., if the scale of the process is $\ll m_h$, forget the heavy quark)

Evolution of α_s and parton densities

- Decoupling scheme: as in $\overline{\text{MS}}$ scheme with n_l flavours

- Standard $\overline{\text{MS}}$: evolution with $n_f = n_l + 1$ flavours

In both cases m_h does not enter in the evolution.

Relations among the schemes (suffix d for decoupling scheme)

$$\alpha(\mu) = \alpha_d(\mu) + c_1(\mu/m_h)\,\alpha_d^2(\mu) + c_2(\mu/m_h)\,\alpha_d^3(\mu) + ...$$

$$f_i(\mu) = \sum_j A_{ij}(\mu/m_h) \otimes f_j^d(\mu)$$

$$A_{ij}(\mu/m_h) = \delta_{ij} + A_{ij}^{(1)}(\mu/m_h)\alpha_d(\mu) + \underbrace{A_{ij}^{(2)}(\mu/m_h)\alpha_d^2(\mu)}_{\text{(Buza etal, 1996)}} + ...$$

VFNS (Collins, Tung, 1986):
  Use CWZ scheme treating as heavy all quarks heavier than μ.
  VFNS has variable flavour number depending upon the scale.

Easy applications: μ not much larger than m_h

Use the decoupling scheme! (jargon: Massive scheme)

Accuracy: (if Born term is $\mathcal{O}(\alpha_s^b)$) an $\mathcal{O}(\alpha_s^{b+n})$ calculation has reminder of $\mathcal{O}(\alpha_s^{b+n+1})$; however, for $\mu \gg m_h$, terms of order $(\alpha_s L)^n$ (with $L = \log\frac{\mu}{m_h}$) arise at all orders, and the remainder is $\mathcal{O}(\alpha_s^{b+n+1} L^n)$ (for $\alpha_s L \approx 1$, $\mathcal{O}(\alpha_s^b)$)

In some cases (F_2) powers of L also arise in the Born term.

Easy applications: $\mu \gg m_h$

$\overline{\text{MS}}$, neglecting m_h (jargon: Massless scheme)

If we do not ask explicitly for the presence or absence of h in the final state (i.e. for INCLUSIVE cross sections) we can treat all $n_f = n_l + 1$ partons as massless, throwing away effects suppressed by powers of m_h/μ.

Cross section formulae as in massless n_f flavour theory.

Accuracy: $\mathcal{O}(\alpha_s^{b+n})$ calculation has reminder of $\mathcal{O}(\alpha_s^{b+n+1})$

all terms of order $(\alpha_s \log\frac{\mu}{m_h})^k$ are resummed to all orders in k, for any n

However: powers suppressed effects (by powers of m_h/μ) are not included

Accuracy: (k and l stand for ANY integer from 0 to ∞;)

$$\sigma = \sum_{j=1}^{n_l} f_j^{(n_l)}(x,\mu)\,\hat{\sigma}_j^{n_l}(px,\mu,m_h,....)\ (\text{Massive scheme})$$

Born	NLO	NNLO	...
$\alpha_s^b \times \left(\alpha_s \log \mu/\Lambda\right)^k$	$\alpha_s^{b+1} \times \left(\alpha_s \log \mu/\Lambda\right)^k$	$\alpha_s^{b+2} \times \left(\alpha_s \log \mu/\Lambda\right)^k$	

$$\sigma = \sum_{j=1}^{n_f} f_j^{(n_f)}(x,\mu)\,\hat{\sigma}_j^{(n_f)}(px,\mu,...)\ (\text{Massless scheme})$$

Born	NLO	NNLO	...
$\alpha_s^b \times \left(\alpha_s \log \mu/\Lambda\right)^k$ $\times \left(\alpha_s \log \mu/m_h\right)^l$ $+\, \mathcal{O}(m_h/\mu)$	$\alpha_s^{b+1} \times \left(\alpha_s \log \mu/\Lambda\right)^k$ $\times \left(\alpha_s \log \mu/m_h\right)^l$ $+\, \mathcal{O}(m_h/\mu)$	$\alpha_s^{b+2} \times \left(\alpha_s \log \mu/\Lambda\right)^k$ $\times \left(\alpha_s \log \mu/m_h\right)^l$ $+\, \mathcal{O}(m_h/\mu)$	

So: two ways to count the order: in both $\alpha_s \log \mu/\Lambda \approx 1$;
In massless scheme also $\alpha_s \log \mu/m_h \approx 1$

Phenomenological applications

The decoupling scheme has been used in all calculations of heavy flavour production processes involving incoming hadrons:

- Hadroproduction (Dawson, Ellis, P.N., 1988; Beenakker etal, 1991)

- Photoproduction (Ellis, P.N. 1989; Smith, Van Neerven, 1992)

- Electroproduction(Laenen, Riemersma, Smith, Van Neerven, 1993)

All these calculations include consistently mass effects.
The massless scheme has been used in high p_T hadro and photoproduction of charm and bottom (Cacciari and Greco, 1994)

Gluck,Reya,Vogt,1992; Delgado,Reya,2008:
straightforward application of the decoupling scheme in DIS fits.
They work within a 3-flavour scheme, and compute heavy flavour effect from the $\gamma^\star g(q) \to h\bar{h}$ process.

Matched calculations

Can we get the best of both worlds?
Mass effects present in the decoupling scheme,
plus log resummation present in massless scheme?

Several proposals have appeared; focus upon:

- ACOT scheme (Aivazis, Collins, Olness, Tung, 1988,1994):
  use the $\overline{\text{MS}}$ scheme above m_h without setting m_h to zero.

- (Thorne and Roberts, 1998,) Modify massless scheme coeff. function
  to achieve continuity with structure functions from massive calculation

- FONLL scheme (Cacciari, Greco, P.N., 1998):
  use the massless scheme, replace terms that are known in the massive
  scheme with the exact massive result.

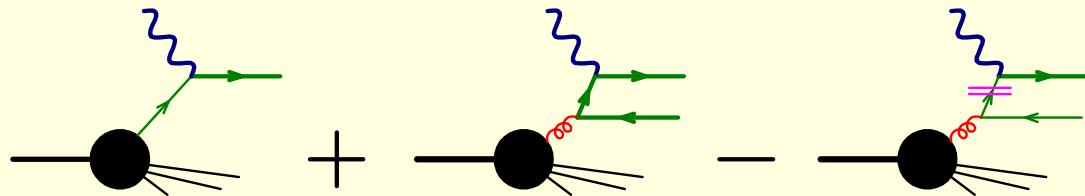# ACOT (Aivazis, Collins, Olness, Tung, 1988,1994)

Use the $\overline{\text{MS}}$ scheme above m_h without setting m_h to zero.

If $m_h = 0$: $1/\epsilon$ poles to subtract; if $m_h > 0$, $L = \log Q/m_h$ terms to subtract

massless

$$-\frac{\alpha}{\epsilon}P_{qg} \times$$

massive

$$-\alpha L\, P_{qg} \times$$
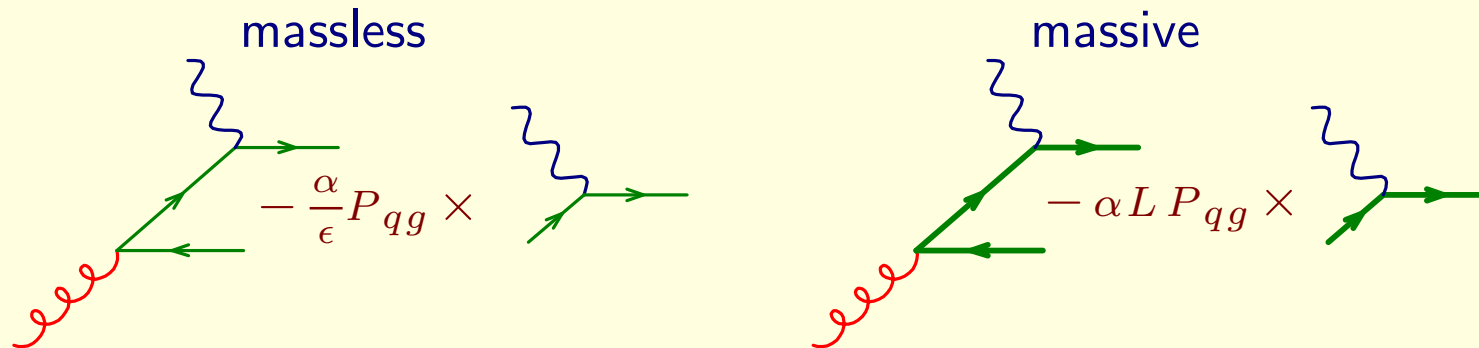
Formal basis: factorization with massive quarks (Collins,1998)

ACOT At NLO: PDF subtraction (3rd graph) depends upon 1st graph.

How to include mass in the 1st graph is not fully specified ...

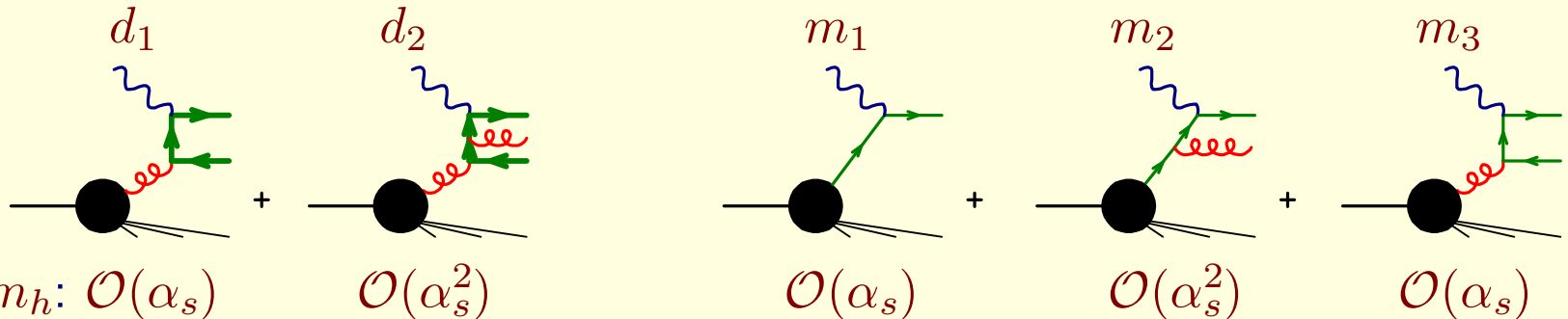3rd graph takes away from 2nd graph what was already included in 1st.

In spite of ACOT formulation in 1994, up to CTEQ 6.1, the massless approximation has been used in the computation of DIS structure functions.

From CTEQ6.5 (end of 2006) the ACOT scheme has been implemented.

TR SCHEME (Thorne and Roberts, 1998)

Basic idea: a structure function computed in the decoupling scheme does not match a structure function computed in the massless scheme when $Q \approx m_h$. Correct the massless scheme so that they match.

$Q \approx m_h$: $\mathcal{O}(\alpha_s)$ $\qquad$ $\mathcal{O}(\alpha_s^2)$ $\qquad\qquad$ $\mathcal{O}(\alpha_s)$ $\qquad\qquad$ $\mathcal{O}(\alpha_s^2)$ $\qquad\qquad$ $\mathcal{O}(\alpha_s)$

(When counting the order for $Q \approx m_h$, remember that $f_h \approx \alpha_s$)

Since F_2 is $\mathcal{O}(1)$, matching at NLO can be interpreted as: $\mathcal{O}(\alpha_s)$ terms only. This approach essentially recovers ACOT

TR try to match at NLO, including $\mathcal{O}(\alpha_s^2)$ terms by imposing continuity at $Q = M$ at $\mathcal{O}(\alpha_s^2)$, up to the derivative with respect to Q.
Since m_1 and m_2 vanish at $Q = m$, this implies that d_2 is added to their result. They add $d_2(Q = m_h)$, to avoid $(\alpha_s L)^2$ terms arising for $Q \gg m_h$.

Thorne and Tung (2009) now agree that this constant term
summarizes the difference in their approaches.
It is beyond the declared accuracy of ACOT (Since it is $\mathcal{O}(\alpha_s^2)$).
Notice: it is frozen at $Q = m_h$, so $(\alpha L)^2 \approx 1$ terms do not arise at large Q^2.
Thorne (2006) shows that it is relevant at low x and Q^2.
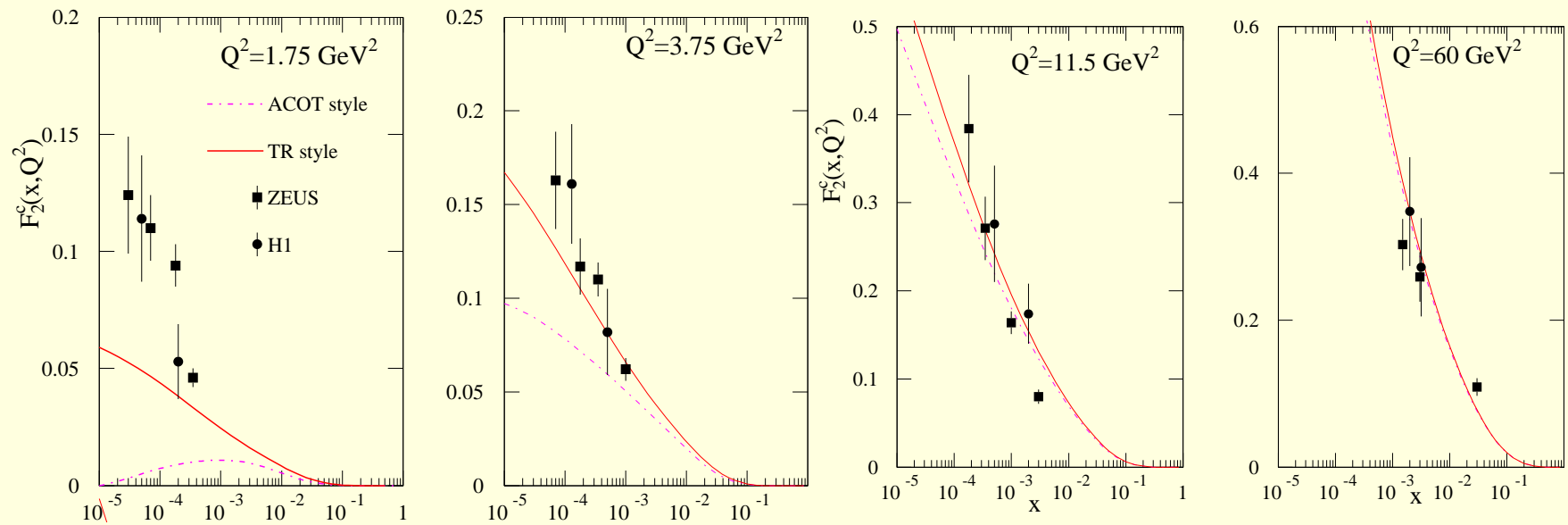
The core of the difference:

In ACOT, the massive result is included up to $\mathcal{O}(\alpha_s)$
In TR, it is attempted to included it up to $\mathcal{O}(\alpha_s^2)$.

The same mismatch is present at NNLO, where an estimate of the $\mathcal{O}(\alpha_s^3)$ massive result is needed in TR (Thorne, 2006).

Within ACOT, at NLO, only the $\mathcal{O}(\alpha_s)$ massive result enters naturally within the heavy flavour factorization scheme propose by Collins.
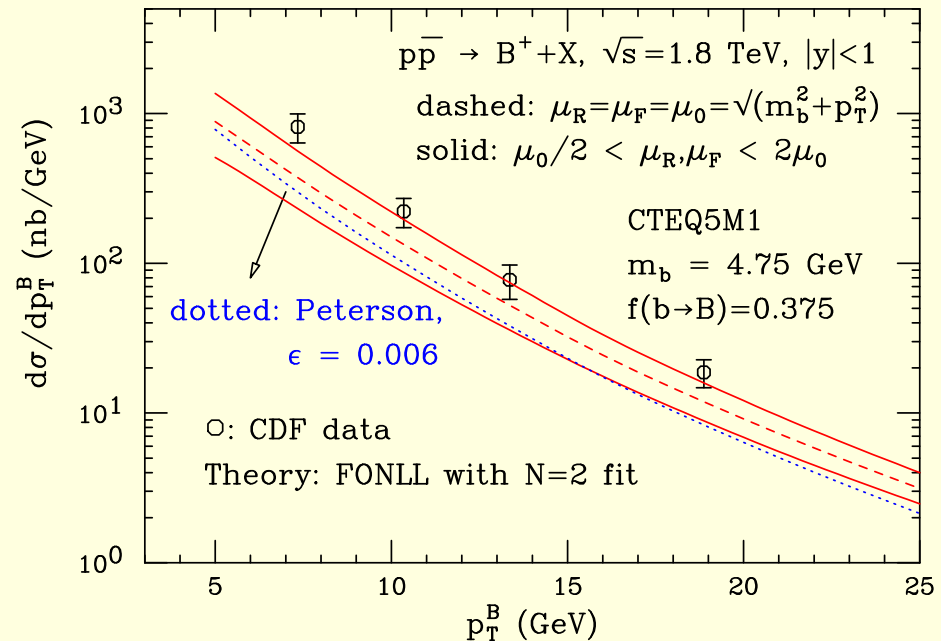
The question however remains:
The $\mathcal{O}(\alpha_s^2)$ massive result is available, it requires NLO α_s and pdf's.
Why should it not be used in an NLO fit?

FONLL (Cacciari, Greco, P.N., 1998)

Totally independent approach introduced in the context of heavy flavour production at high p_T, in order to address the discrepancy between theory and Tevatron data in b production. Besides the pdf, it also deals with b fragmentation.

$p\bar{p} \to B^+ + X,\ \sqrt{s} = 1.8$ TeV, $|y| < 1$

dashed: $\mu_R = \mu_F = \mu_0 = \sqrt{(m_b^2 + p_T^2)}$

solid: $\mu_0/2 < \mu_R, \mu_F < 2\mu_0$

CTEQ5M1

$m_b = 4.75$ GeV

$f(b \to B) = 0.375$

dotted: Peterson, $\epsilon = 0.006$

$\circ$: CDF data

Theory: FONLL with N=2 fit

It was used to match the NLO heavy flavour production calculation of Ellis, Dawson and P.N. (massive scheme) with that of Cacciari and Greco (massless scheme). The method is totally general. It has been applied to heavy flavour production in $e^+ e^-$ annihilation, but never to DIS.

FONLL in few words

A cross section in the decoupling scheme can be seen as a fixed order power expansion in α_s with mass dependent coefficients. The coefficients have logarithmic behaviour at large scale.
A cross section in the massless scheme can be seen as a power series in α_s with coefficients that are polynomials in L. All large logs are resummed.

So: add them up, deleting from the second the terms of the same order in α_s present in the first.

This way, the coefficients of powers of α_s that are only approximate in the massless expression, are replaced with the coefficients that include the exact mass dependence.

Advantages: it is simple, the proof takes one page. It works at all orders (in spite of the name...). It does not need new calculations. The heavy flavour calculation was done by putting together NDE and CG programs.

FONLL

Assuming that the massive result is known at order not higher than the massless one, the FONLL formula can be written as

$$\sigma^{\mathrm{FONLL}} = \sigma + \sigma^d - \sigma_0^d$$

σ=massless scheme, σ^d=massive (decoupling) scheme,
σ_0^d is the massless limit of the massive cross section, in the following sense:
it is a polynomial in L with mass independent coefficients, such that

$$\lim_{m \to 0} \left(\sigma^d - \sigma_0^d \right) = 0.$$

Obviously:

$$\sigma^{\mathrm{FONLL}} = \sigma + \mathcal{O}(m^2/Q^2).$$

We also need $\sigma - \sigma_0^d$ to be of order higher than σ^d in α_s (counting $L\alpha \approx \alpha$).
This is easily proven to all orders, provided σ^d and σ_0^d are given in terms of the same α_S and pdf's used in σ (i.e. the n_f rather than the n_l $\overline{\mathrm{MS}}$ scheme).
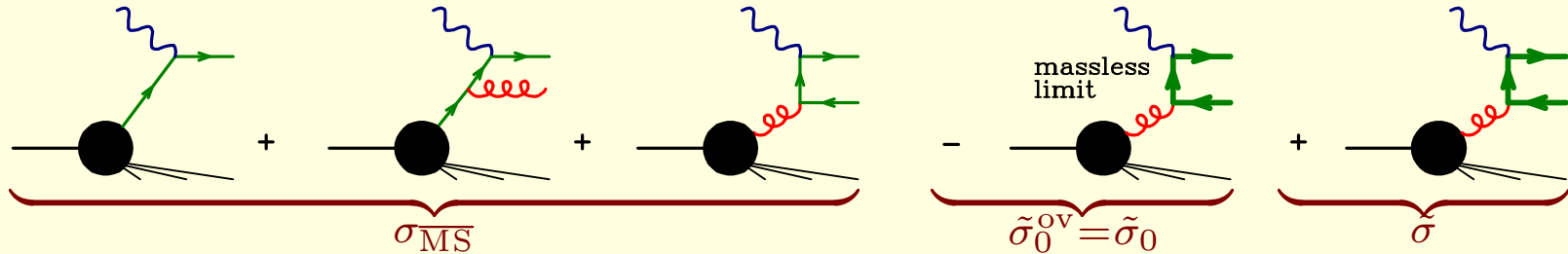
FONLL in details

- Call σ^d a heavy flavour cross section in the decoupling scheme. σ^d is given in terms of $\alpha_s^{(n_l)}(\mu)$ and $f_i^{(n_l)}(\mu)$.

  Express σ^d in terms of $\alpha_s^{(n_f)}(\mu)$ and $f_{i\neq h}^{(n_f)}(\mu)$, using the matching equations. We call $\tilde{\sigma}$ the new expression.

- In the limit $m \to 0$, $\tilde{\sigma} \to \tilde{\sigma}_0$, where $\tilde{\sigma}_0$ is a polynomial in α_s and L with mass independent coefficients. It is the massless limit of $\tilde{\sigma}$, in the sense $\lim_{m\to 0}(\tilde{\sigma} - \tilde{\sigma}_0) = 0$.

- The massless scheme cross section, $\sigma_{\overline{MS}}$, is given in terms of $\alpha_s^{(n_f)}(\mu)$ and $f_i^{(n_f)}(\mu)$. If we express $f_h^{(n_f)}(\mu)$ as a functional of the $f_{i\neq h}^{(n_f)}(\mu)$ using the evolution equations and the matching conditions, $f_h^{(n_f)}(\mu)$ is a power series in α_s and L with mass independent coefficients, and so is $\sigma_{\overline{MS}}$'.

  Let us call $\tilde{\sigma}_0^{\text{ov}}$ (ov for overlap) what we get from $\tilde{\sigma}_0$ deleting all terms that are not in $\sigma_{\overline{MS}}$.

- The FONLL expression is $\sigma_{\text{FONLL}} = \sigma_{\overline{MS}} - \tilde{\sigma}_0^{\text{ov}} + \tilde{\sigma}$
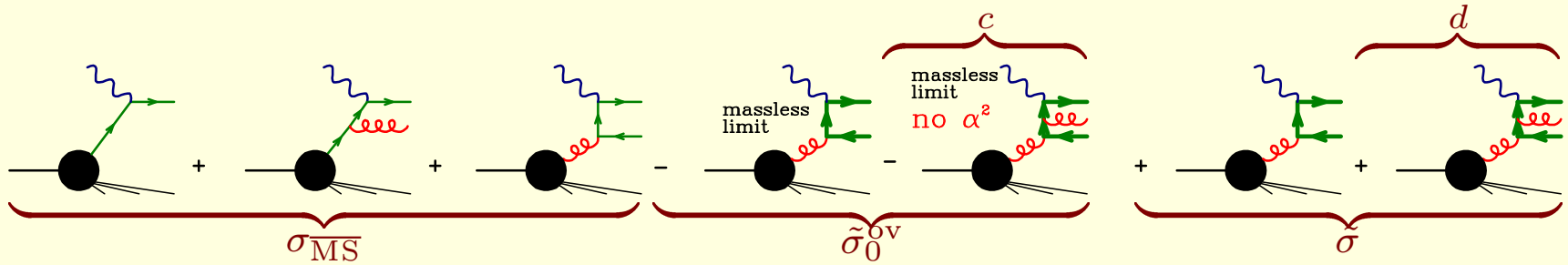
FONLL at NLO

$$\alpha_s + \alpha_s L \qquad \alpha_s^2 + \alpha_s^2 L + \alpha_s^2 L^2 \qquad \alpha_s L + (\alpha_s^2 L + ...) \quad \alpha_s^2 L + (\alpha_s^2 L^2 + ...) \qquad \alpha_s$$
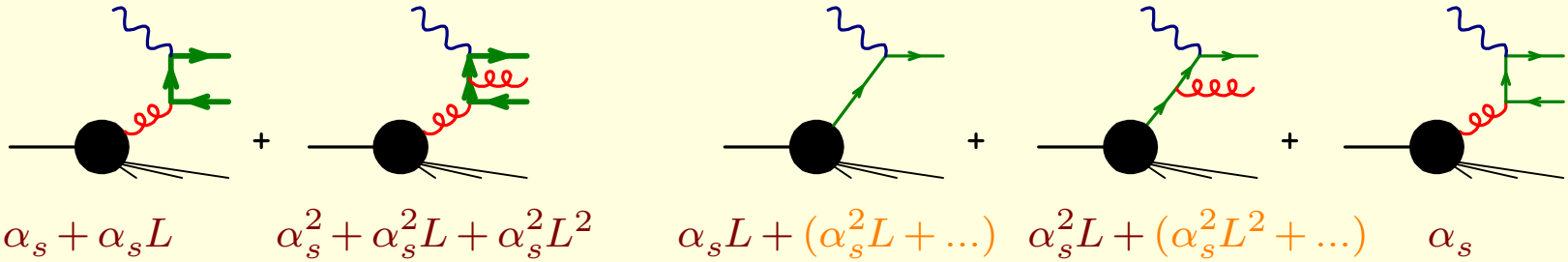
If we uso LO for massive: (Same as S-ACOT!)

$$\underbrace{\qquad}_{\sigma_{\overline{\text{MS}}}} \qquad \underbrace{\qquad}_{\tilde{\sigma}_0^{\text{ov}} = \tilde{\sigma}_0} \qquad \underbrace{\qquad}_{\tilde{\sigma}}$$

If we use NLO for massive: (reminiscent of TR!) $d - c \approx \mathcal{O}(\alpha_s^2), \text{no } L \text{ powers}$

$$\underbrace{\qquad}_{\sigma_{\overline{\text{MS}}}} \qquad \underbrace{\qquad}_{\tilde{\sigma}_0^{\text{ov}}} \qquad \underbrace{\qquad}_{\tilde{\sigma}}$$

Mass ambiguities in FONLL

In $\sigma_{\mathrm{FONLL}} = (\sigma_{\overline{\mathrm{MS}}} - \tilde{\sigma}_0^o) + \tilde{\sigma}$, we always have the freedom to introduce mass dependent modifications of the round bracket, that are suppressed by powers of the mass. For example, in DIS:

$$(\sigma_{\overline{\mathrm{MS}}} - \tilde{\sigma}_0^o) + \tilde{\sigma} \Rightarrow (\sigma_{\overline{\mathrm{MS}}} - \tilde{\sigma}_0^o) f_{\mathrm{thr}}(Q) + \tilde{\sigma},$$

as long as $f_{\mathrm{thr}}(Q) \to 1$ for $Q \gg m_h$. Or the value of x in $\sigma_{\overline{\mathrm{MS}}} - \tilde{\sigma}_0^o$ can be rescaled: $x \to \chi_h = x(1 + 4m_h^2/Q^2)$. This freedom follows from the facts:

- $\sigma_{\overline{\mathrm{MS}}} - \tilde{\sigma}_0^o$ does not contain terms of the same order of those in $\tilde{\sigma}$

- $\sigma_{\overline{\mathrm{MS}}} - \tilde{\sigma}_0^o$ is valid only for $Q \gg m_h$

Terms computed in the massless approximation can be modified by a mass suppressed correction.

FONLL has being applied to DIS (Forte, Laenen, Rojo, P.N. 2010)

Advantages:

- Extreme simplicity (does not rely upon factorization with masses, etc.)

- No new calculation needed (σ_0 easily derived numerically from σ; also available in DIS from Buza etal, 1996)

- More general: it allows inclusion of α_s^2 term in NLO implementation

Without the α_s^2 term it is identical to SACOT at NLO; if χ-scaling is included, identical to ACOT-χ (FONLL scheme A)

With α_s^2 term: as in TR, but more natural (FONLL B)

NNLO (FONLL C): implemented in a straightforward way; should be identical to SACOT (but NNLO SACOT not yet there)

In principle a FONLL D scheme could be realized, including an estimate of the $\mathcal{O}(\alpha_s^3)$ massive result, as done by Thorne (2006).

Note:

S-ACOT (for Simplified ACOT, Kräemer, Olness, Soper, 2000,
        N.B.: AFTER FONLL).

Variant over ACOT, exploiting freedom in the choice of mass effects.
Use mass only in heavy quark lines not coming from the hadron
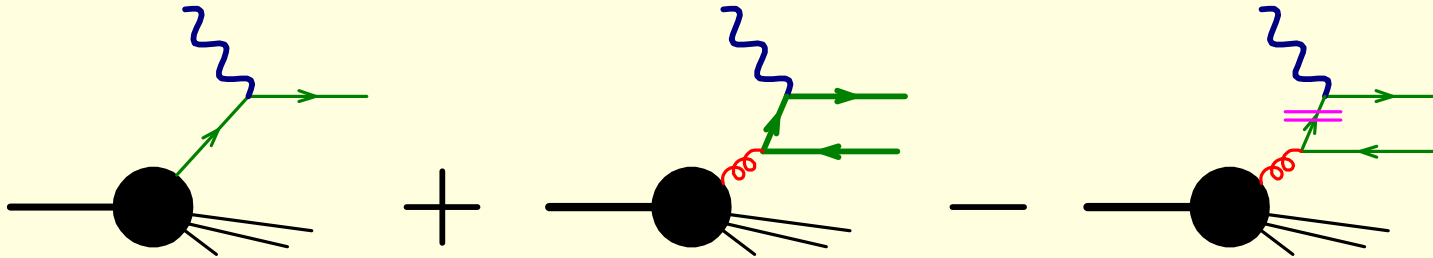
The S-ACOT variant makes the ACOT scheme practically useful without
having to deal with cumbersome calculations, involving for example massive
fermions with massless incoming momenta.
At the end, it is equivalent to FONLL.

Modern CTEQ and MRST implementations use S-ACOT
(plus: χ-scaling, S-ACOT-χ).

The BMSN scheme (Buza,Matiounine,Smith,van Neerven,96 and 97)

propose (v1)

$$F^{\mathrm{BMSN}} = F^{\mathrm{VFNS}} + F^d - F_0^d$$

$$F^{\mathrm{VFNS}}(Q^2) = C(n_f, \alpha_s, Q^2/\mu^2) \otimes f(n_f, \mu^2), \quad f(n_f, \mu^2) = A(\alpha_s) \otimes f(n_l, \mu^2)$$

where A and C are evaluated at $\mathcal{O}(\alpha_s^2)$ in the massless limit,
or alternatively (v2)

$$F^{\mathrm{BMSN}} = F^{\mathrm{PDF}} + F^d - F_0^d, \quad F^{\mathrm{PDF}} = C \otimes f^{\mathrm{PDF}}(n_f),$$

f^{PDF} from standard n_f flavours pdf.

They notice that $F^d - F_0^d \approx \mathcal{O}(m^2/Q^2)$.
They notice that $F^{\mathrm{VFNS}} - F_0^d$ is zero when $Q \approx m$, if only $\mathcal{O}(\alpha_s)$ is kept.

However, they conclude that this cannot be generalized to higher orders
for several reasons (see BMSN 96 and 97):

1. Mass factorization does not apply to power suppressed terms

2. Mismatch in number of flavours in the two expressions

3. in $F^{\mathrm{VFNS}} - F_0^d$ terms of $\mathcal{O}(\alpha_s^3)$ are included, not present in F_0^d; in $F^{\mathrm{PDF}} - F_0^d$ even terms to all order in $L\alpha_s$ are also included, that make the mismatch worse.

4. in the charm contribution to F_2, at $\mathcal{O}(\alpha_s^2)$ a term $L^3 \alpha_s^2$ arises that is not present in either F^{VFNS} or F^{PDF}.

In 1996 they find a large numerical mismatch in F^{VFNS} vs. F_0^d, and use it to dismiss the method.

In 1997, using the v2 version, they find a small numerical mismatch, and use this to justify their procedure.

In FONLL, F_0^d in the n_f scheme is instead demonstrated to match the massless result at any desired order in perturbation theory.

Problem 4: their $F^{\mathrm{VFNS/PDF}}$ and their F^d/F^d_0 refer to different observables:

$F^{\mathrm{VFNS/PDF}}$: F^c_2 defined as F_2 with the γ coupled only the heavy flavour;
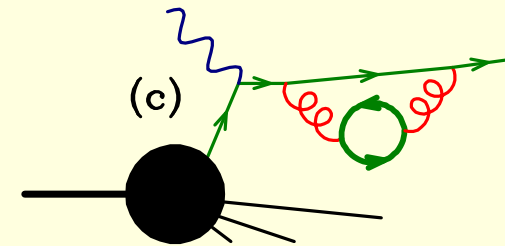
F^d/F^2_0: F^c_2 defined as F_2 with charm in the final state.
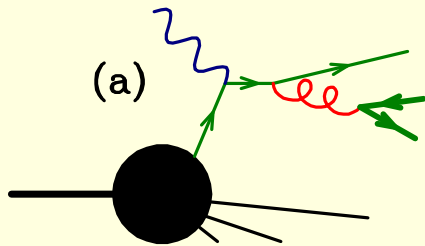
(a): 2 collinear and one soft log;   (b,c): Same with opposite sign
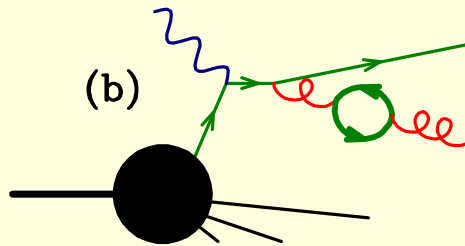
That's why they find a $\alpha_s^2 L^3$ mismatch.

In Forte,Rojo,P.N.2010 we define F^c_2 as the contribution to F_2 where only the heavy quark interacts with the photon. The above contributions enter then together in $F^l = F_2 - F^c_2$, and the $L^3 \alpha_s^2$ terms cancel also there.
We estimate the term of graph (a) using a partonic Monte Carlo generator.

Note: ABKM (Alekhin, Blümlein, Klein, Moch, 2009)

propose: $\quad F^{\mathrm{BMSN}} = F^{\mathrm{VFNS}} + F^d - F^d_0,$

$$F^{\mathrm{VFNS}}(Q^2) = C(n_f, \alpha_s, Q^2/\mu^2) \otimes f(n_f, \mu^2), \quad f(n_f, \mu^2) = A(\alpha_s) \otimes f(n_l, \mu^2),$$

(as in BMSN v1) except that they expand it and truncate it to $\mathcal{O}(\alpha_s^2)$.

They argue that the $L^3 \alpha_s^2$ mismatch is small and can be neglected

They find good numerical agreement of F^{VFNS} with F^d_0 at low Q^2.

In ABKM, F^{VFNS} includes heavy flavour contributions where a light quark is struck by a photon. However, also the virtual heavy flavour loops are included, while they are not included in F^d. Thus a $L^2 \alpha_s^3$ mismatch is still present.

So, with respecto to BMSN v1 more higher order log terms are neglected.

Notice: going one step further in expanding and truncating:

If we express $\alpha_s(n_f)$ in terms of $\alpha_s(n_l)$, expand and truncate to $\mathcal{O}(\alpha_s^2)$ F^{VFNS} becomes identically equal to F_0^d, and $F^{\mathrm{BMSN}} = F^d$

There is no clear motivation on why this last step should not be performed.

So: ABKM keep some (but not all) log enhanced terms: no formal improvement over massive scheme; incorrect logarithmic structure for large L.

However: once the $\mathcal{O}(\alpha_s^2)$ terms are all included higher order logs have a very modest effect; so, the method becomes closer to the GRV method.

Comparisons

Detail comparisons of the ACOT, TR and FONLL methods have been carried out for the "Les Houches benchmarks for GM-VFN schemes" (2010), with common pdf's, α and m_c.
Comparisons with ABKM are in progress.

Aim: to see the differences only due to the schemes
Thus: adopt a common set of pdf, and common parameters.
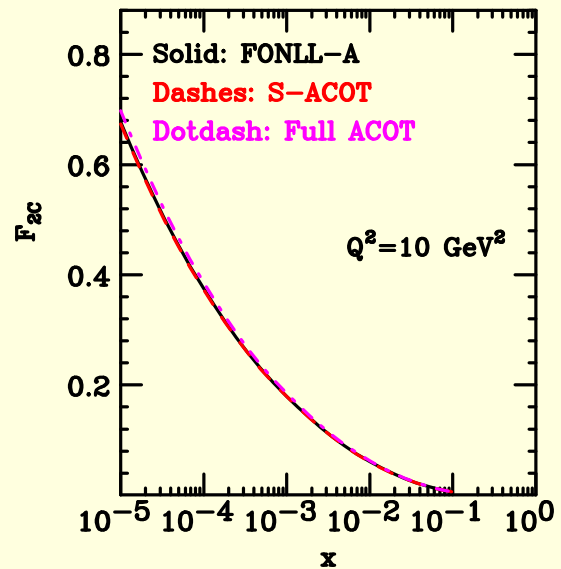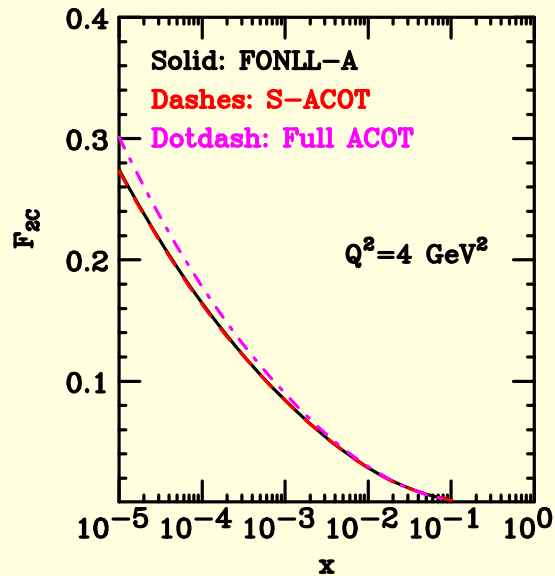   (all methods but ABKM use n_f flavours pdf's).

Mass suppression methods: it was pointed out in Forte,Laenen,Rojo,P.N. 2010 that two different interpretations of χ-scaling were adopted in the literature:

1. $F_{2c}^{(\chi)}(x, Q^2) = \frac{x}{\chi} F_{2c}(\chi, Q^2)$

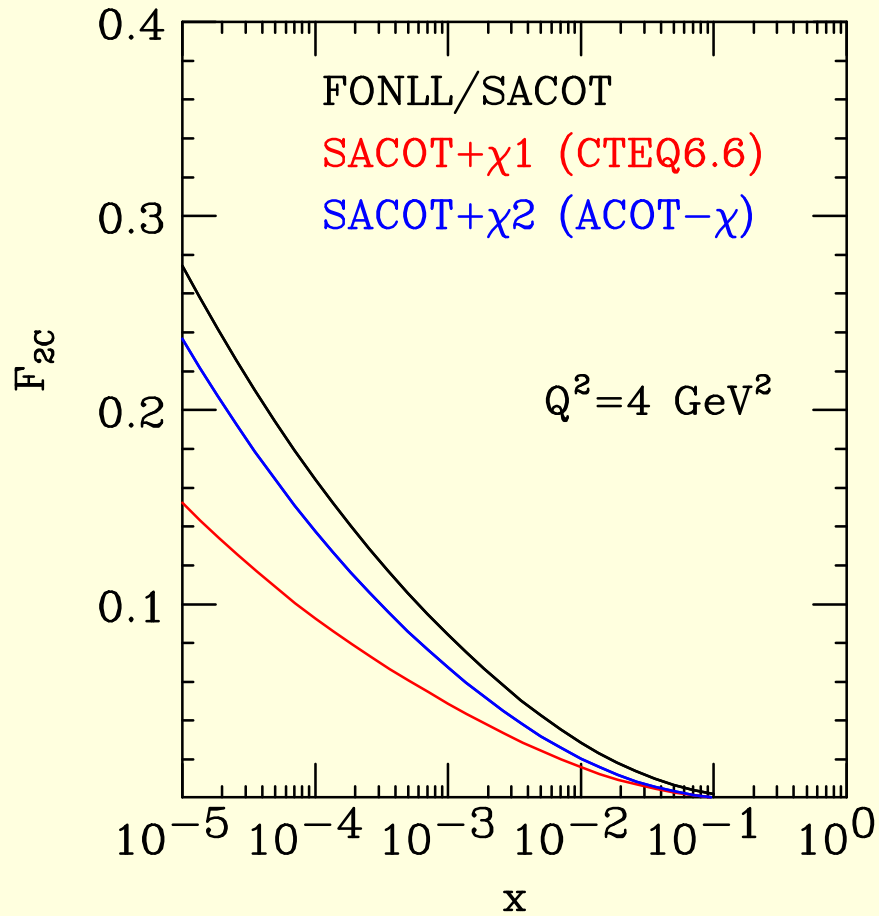2. $F_{2c}^{(\chi)}(x, Q^2) = F_{2c}(\chi, Q^2)$

(1) adopted in FONLL and in CTEQ6.5/6.6, and (2) adopted in TR'.

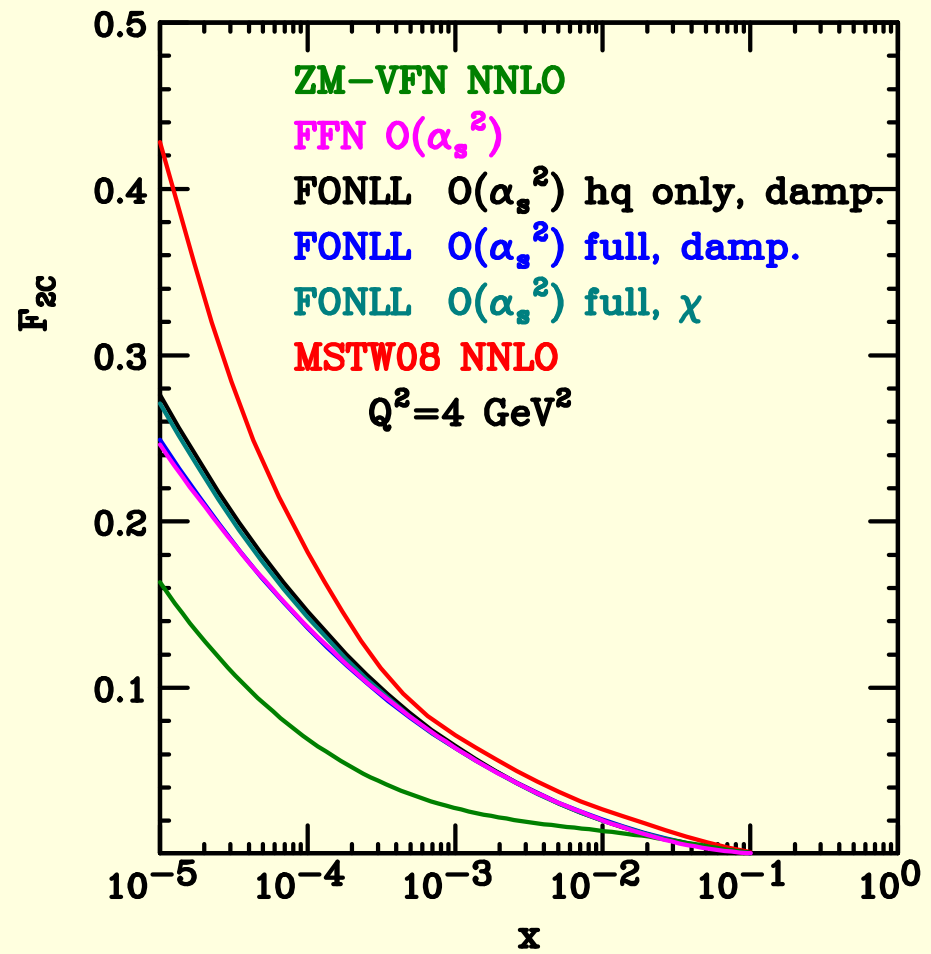# FONLL $\mathcal{O}(\alpha_s)$ vs. ACOT vs. SACOT:
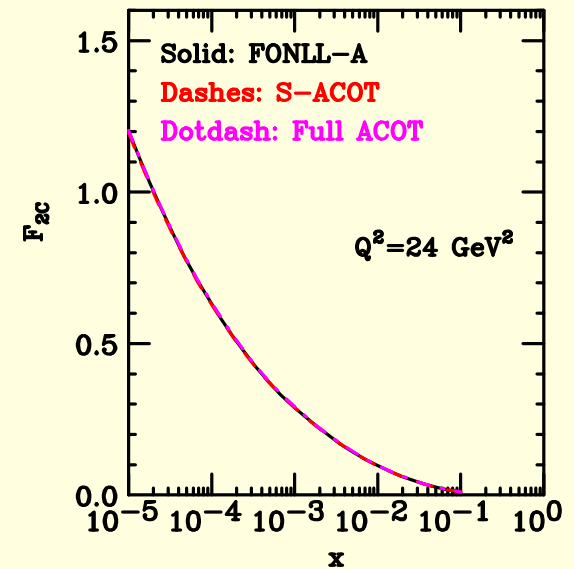
FONLL exactly equivalent to S-ACOT at NLO.
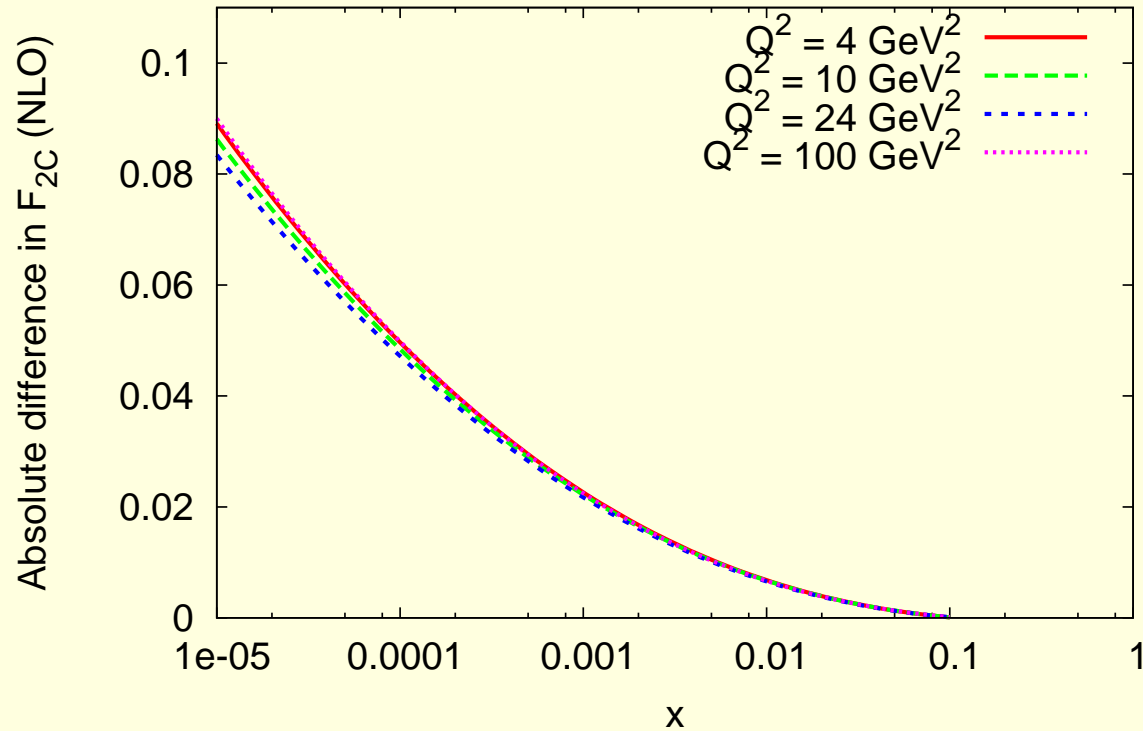Small power suppressed difference between ACOT and S-ACOT.
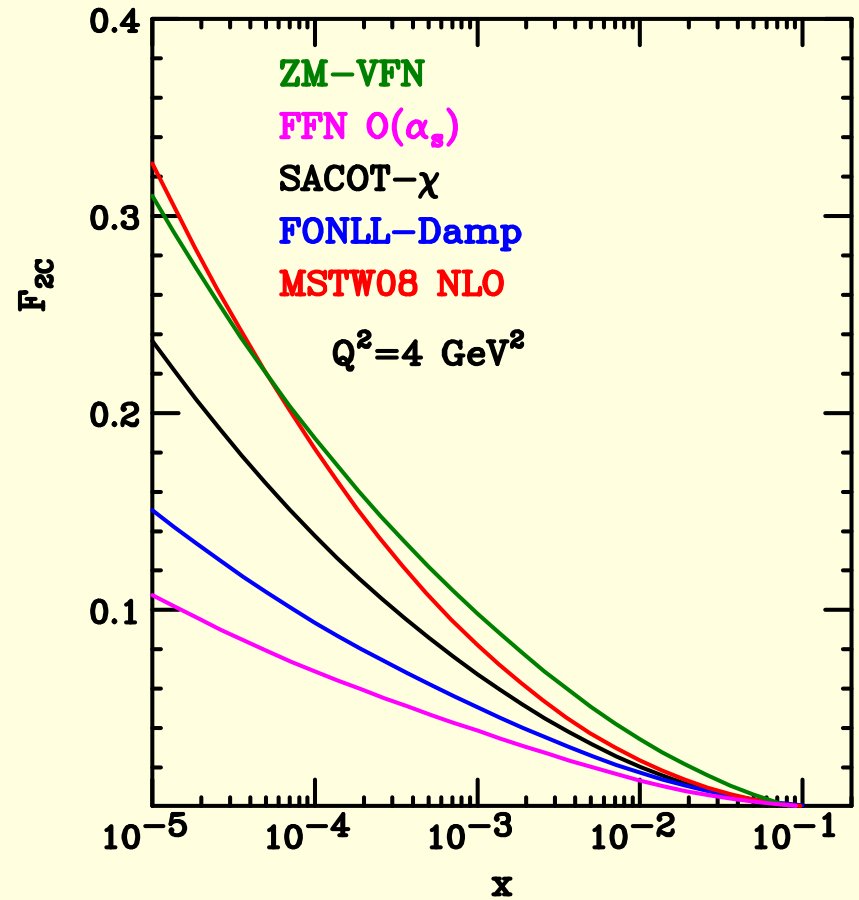
At NLO: threshold prescriptions non negligible (relative to spread ZM - FFN)

At NNLO the impact of threshold suppression is strongly reduced.
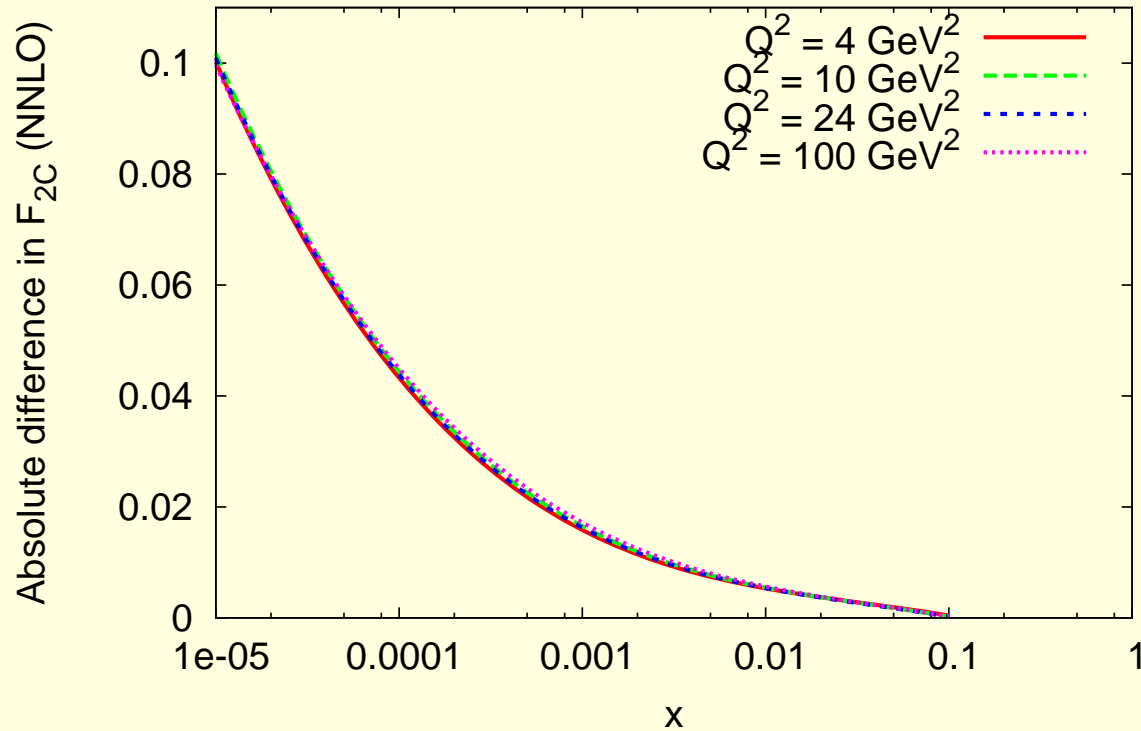
ZM−VFN NNLO
FFN $O(\alpha_s^2)$
FONLL $O(\alpha_s^2)$ hq only, damp.
FONLL $O(\alpha_s^2)$ full, damp.
FONLL $O(\alpha_s^2)$ full, χ
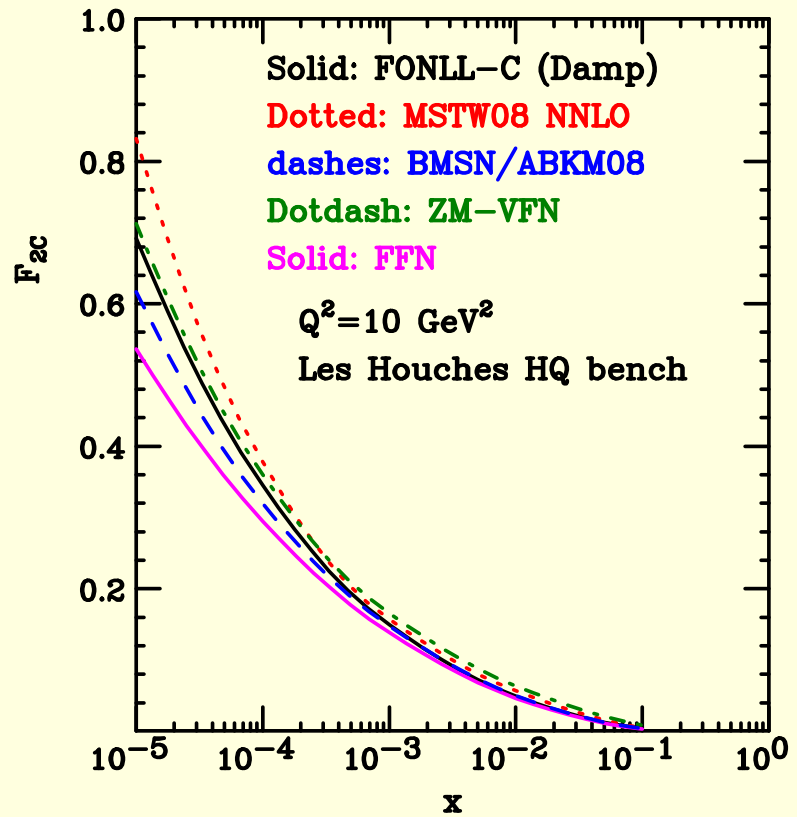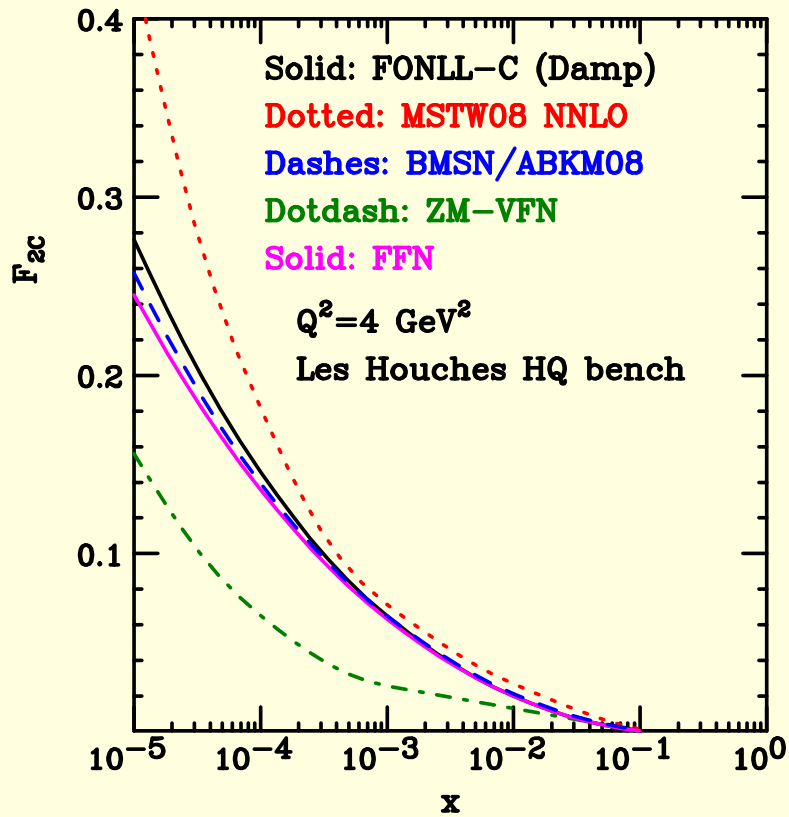MSTW08 NNLO
$Q^2 = 4$ GeV^2

Absolute difference between MSTW08 and plain FONLL/SACOT is verified to be Q^2 independent (frozen α_s^2 correction in TR scheme)
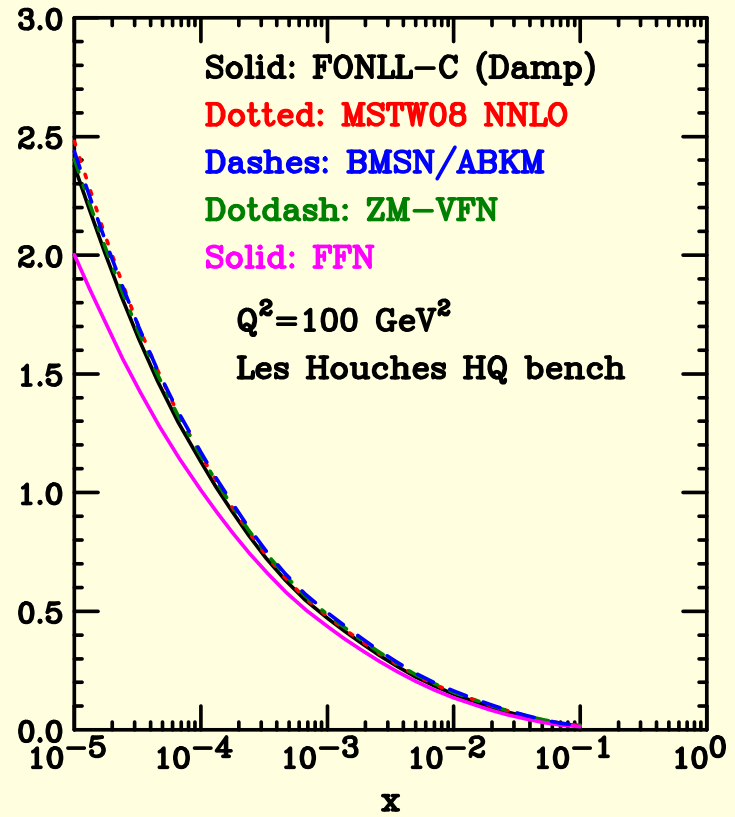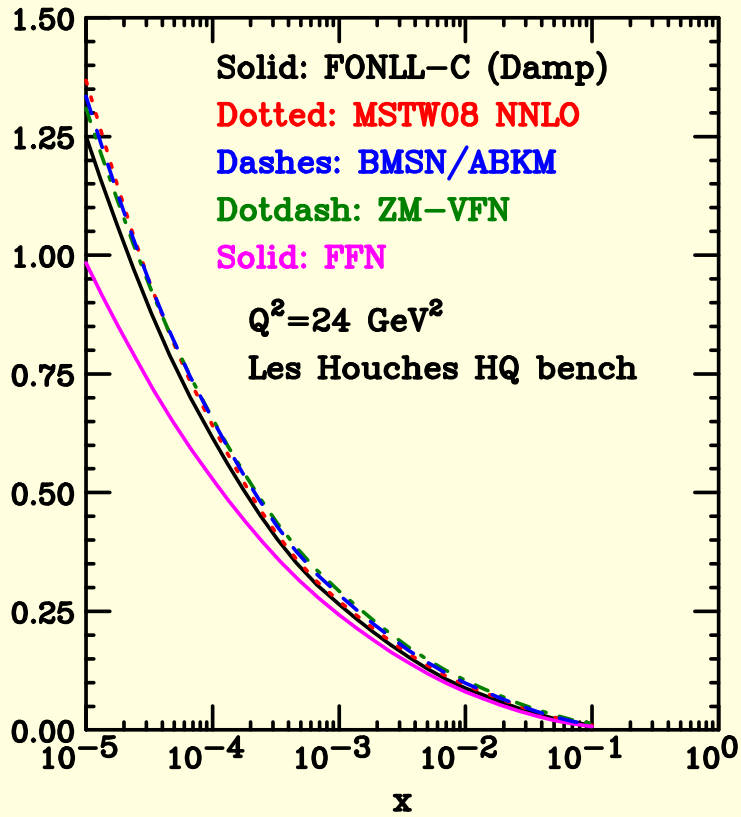
MSTW08 NNLO (plain) - FONLL scheme C (plain)

The absolute difference between MSTW08 NNLO and plain FONLL-C is verified to be Q^2 independent (frozen α_s^3 correction in TR scheme)

NNLO comparisons: (BMSN/ABKM08 very preliminary)

Left panel:
- F_{2c} vs x
- Solid: FONLL-C (Damp)
- Dotted: MSTW08 NNLO
- Dashes: BMSN/ABKM08
- Dotdash: ZM-VFN
- Solid: FFN
- $Q^2=4$ GeV^2
- Les Houches HQ bench

Right panel:
- F_{2c} vs x
- Solid: FONLL-C (Damp)
- Dotted: MSTW08 NNLO
- dashes: BMSN/ABKM08
- Dotdash: ZM-VFN
- Solid: FFN
- $Q^2=10$ GeV^2
- Les Houches HQ bench

Left panel:

Solid: FONLL-C (Damp)
Dotted: MSTW08 NNLO
Dashes: BMSN/ABKM
Dotdash: ZM-VFN
Solid: FFN

$Q^2 = 24$ GeV^2
Les Houches HQ bench

x

Right panel:

Solid: FONLL-C (Damp)
Dotted: MSTW08 NNLO
Dashes: BMSN/ABKM
Dotdash: ZM-VFN
Solid: FFN

$Q^2 = 100$ GeV^2
Les Houches HQ bench

x

MRST and FONLL scheme B

FONLL-B coincides with FFN NLO at these scales; Seems to go toward MSTW08 at low scales, but it remains rather different from it; on the other hand, FONLL succedes in matching the FFN NLO result very closely

Conclusions

- Two methods (ACOT, FONLL) for the inclusion of mass effects and large logs in a consistent way, up to any order in PT.

- FONLL simple and practical: old applications in heavy flavour hadroproduction, recent application to DIS up to NNLO

- FFN fits to DIS seem to work, once the NNLO level is reached (α_s^3 coefficient function for heavy flavour production for high Q^2/m^2 may help further in this direction (Bierenbaum,Blümlein,Klein,2009).

- Detailed work in comparing different schemes has proven very useful to clarify differences among various scheme (Les Houches Benchmarks). Still some space for improvement.