Heavy Flavours and PDF's

P. Nason

INFN

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Outline

- History
- Recent results within the FONLL approach
- Issues related to the measurement of $F_{2,c}$

History

How do we treat a heavy flavour (mass $m_h \gg \Lambda_{\rm QCD}$) in QCD processes?

In standard $\overline{\rm MS}$ scheme, heavy flavour effects persist even for $Q \ll m_h$
(n_f, not $n_l = n_f - 1$, appear in the running coupling ...).
It would be cumbersome to worry about top when doing DIS at 10 GeV^2.

Use Decoupling renormalization scheme with n_l light flavours (all but h)
(Collins, Wilczek, Zee 1978;). CWZ prescription:

- the $\overline{\rm MS}$ scheme for light flavours
- a zero momentum subtraction for heavy flavour graphs

heavy flavour graphs: graphs that include heavy flavour lines, or
counterterms to heavy flavour graphs.

Advantages: $m_h \to \infty$ limit easily treated (forget the heavy quark!)
(i.e., if the scale of the process is $\ll m_h$, forget the heavy quark)

How decoupling works in CWZ

If the external momenta $p \ll m_h$:

- Convergent graphs with heavy lines are dominated by internal momenta $k \approx p$, so the heavy lines yield $k/m_h \approx p/m_h$ suppression factors.
- Divergent graphs with heavy lines are dominated by momenta $k \gg m_h$; subtracting them at zero momentum:

$$\Sigma(k, p, m_k) - \Sigma(k, 0, m_k) \approx \mathcal{O}(p/m_k)$$

Evolution of α_s and parton densities

- Decoupling (CWZ) scheme: as in $\overline{\text{MS}}$ scheme with n_l flavours
- Standard $\overline{\text{MS}}$: evolution with $n_f = n_l + 1$ flavours

In both cases m_h does not enter in the evolution.

Relations among $\overline{\rm MS}$ and decoupling schemes (suffix d for decoupling scheme)

$$\begin{aligned}
\alpha(\mu) &= \alpha_d(\mu) + c_1(\mu/m_h)\,\alpha_d^2(\mu) + c_2(\mu/m_h)\,\alpha_d^3(\mu) + \ldots \\
f_i(\mu) &= \sum_j A_{ij}(\mu/m_h) \otimes f_j^d(\mu) \\
A_{ij}(\mu/m_h) &= \delta_{ij} + A_{ij}^{(1)}(\mu/m_h)\alpha_d(\mu) + \underbrace{A_{ij}^{(2)}(\mu/m_h)\alpha_d^2(\mu)}_{\text{(Buza etal, 1996)}} + \ldots
\end{aligned}$$

In particular, the heavy flavour parton density is given in terms of the light ones in the massless scheme.

It turns out that: $c_1(1)=0$, $A_{ij}^{(1)}(1)=0$, i.e.: at NLO α_s and f_i are continuous at $\mu=m$, with $f_{h/\bar{h}}=0$ at $\mu=m$. Continuity is no longer true at NNLO.

Easy applications: μ not much larger than m_h

Use the decoupling scheme! (jargon: Massive scheme)

Accuracy: (if Born term is $\mathcal{O}(\alpha_s^b)$) an $\mathcal{O}(\alpha_s^{b+n})$ calculation has reminder of $\mathcal{O}(\alpha_s^{b+n+1})$; however, for $\mu \gg m_h$, terms of order $(\alpha_s L)^n$ (with $L = \log\frac{\mu}{m_h}$) arise at all orders, and the remainder is $\mathcal{O}(\alpha_s^{b+n+1} L^n)$ (for $\alpha_s L \approx 1$, $\mathcal{O}(\alpha_s^b)$)

In some cases (F_2) powers of L also arise in the Born term.

Easy applications: $\mu \gg m_h$

$\overline{\text{MS}}$ bar, neglecting m_h (jargon: Massless scheme)

If we do not ask explicitly for the presence or absence of h in the final state (i.e. for INCLUSIVE cross sections) we can treat all $n_f = n_l + 1$ partons as massless, throwing away effects suppressed by powers of m_h/μ.

Cross section formulae as in massless n_f flavour theory.

Accuracy: $\mathcal{O}(\alpha_s^{b+n})$ calculation has reminder of $\mathcal{O}(\alpha_s^{b+n+1})$

all terms of order $(\alpha_s \log\frac{\mu}{m_h})^k$ are resummed to all orders in k, for any n

However: powers suppressed effects (by powers of m_h/μ) are not included

Summary: (k and l stand for ANY integer from 0 to ∞)

$$\sigma = \sum_{j=1}^{n_l} f_j^{(n_l)}(x,\mu)\,\hat{\sigma}_j^{n_l}(px,\mu,m_h,....)\ (\text{Massive scheme})$$

Born	NLO	NNLO	...
$\alpha_s^b \times (\alpha_s \log \mu/\Lambda)^k$	$\alpha_s^{b+1} \times (\alpha_s \log \mu/\Lambda)^k$	$\alpha_s^{b+2} \times (\alpha_s \log \mu/\Lambda)^k$	

$$\sigma = \sum_{j=1}^{n_f} f_j^{(n_f)}(x,\mu)\,\hat{\sigma}_j^{(n_f)}(px,\mu,...)\ (\text{Massless scheme})$$

Born	NLO	NNLO	...
$\alpha_s^b \times (\alpha_s \log \mu/\Lambda)^k \times (\alpha_s \log \mu/m_h)^l + \mathcal{O}(m_h/\mu)$	$\alpha_s^{b+1} \times (\alpha_s \log \mu/\Lambda)^k \times (\alpha_s \log \mu/m_h)^l + \mathcal{O}(m_h/\mu)$	$\alpha_s^{b+2} \times (\alpha_s \log \mu/\Lambda)^k \times (\alpha_s \log \mu/m_h)^l + \mathcal{O}(m_h/\mu)$	

Phenomenological applications

The decoupling scheme has been used in all calculations of heavy flavour production processes involving incoming hadrons:

- Hadroproduction (Dawson, Ellis, P.N., 1988; Beenakker etal, 1991)
- Photoproduction (Ellis, P.N. 1989; Smith, Van Neerven, 1992)
- Electroproduction(Laenen, Riemersma, Smith, Van Neerven, 1993)

All these calculations include consistently mass effects.
The massless scheme has been used in high p_T hadro and photoproduction of charm and bottom (Cacciari and Greco, 1994)

Gluck,Reya,Vogt,(1992): straightforward application of the decoupling scheme in DIS fits. They work within a 3-flavour scheme, and compute heavy flavour effect from the $\gamma^\star g \to h\bar{h}$ process.

Charm in DIS

Charm contributes sensibly to the electromagnetic structure functions.

In DIS fits it is generally generated dynamically, most of the time using the massless scheme.

According to the Variable Flavour Number Scheme (Collins, Tung, 1986), depending upon the Q^2 value, one treats according to the decoupling scheme all flavours with $m > Q$, everything else being treated in the $\overline{\rm MS}$ scheme.

Heavy flavours are treated in the decoupling limit, i.e. neglecting Q/m corrections.

Gluck,Reya,Vogt,(1992): straightforward application of the decoupling scheme in DIS fits. They work within a 3-flavour scheme, and compute heavy flavour effect from the $\gamma^\star g \to h\bar{h}$ process. They neglect higher order $\alpha \log Q/m$ terms.

Matched calculations

Can we get the best of both worlds?
Mass effects present in the decoupling scheme,
plus log resummation present in massless scheme?

Several proposals have appeared;

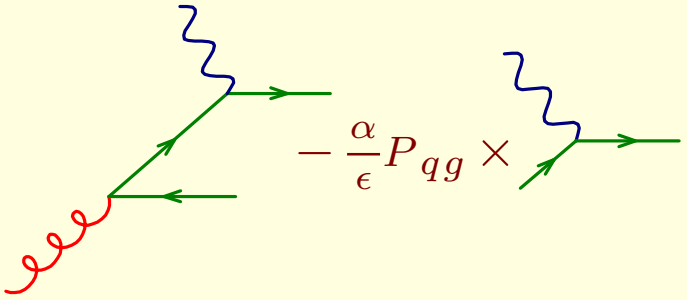
- ACOT scheme (Aivazis, Collins, Olness, Tung, 1988,1994):
  use the $\overline{\text{MS}}$ scheme above m_h without setting m_h to zero.
- (Thorne and Roberts, 1998,) Modify massless scheme coeff. function
  to achieve continuity with structure functions from massive calculation
- FONLL scheme (Cacciari, Greco, P.N., 1998):
  use the massless scheme, replace terms that are known in the massive scheme with the exact massive result.

ACOT (Aivazis, Collins, Olness, Tung, 1988,1994)

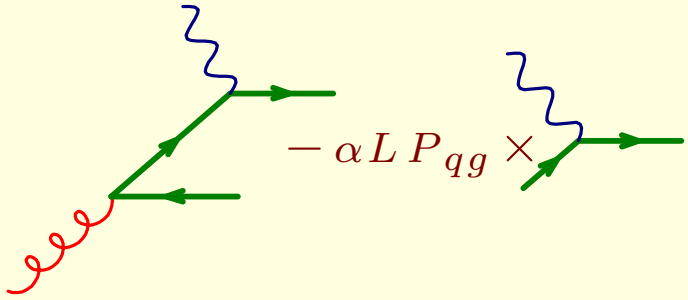
Use the $\overline{\text{MS}}$ scheme above m_h without setting m_h to zero.

If $m_h = 0$: $1/\epsilon$ poles to subtract; if $m_h > 0$, $L = \log Q/m_h$ terms to subtract

massless

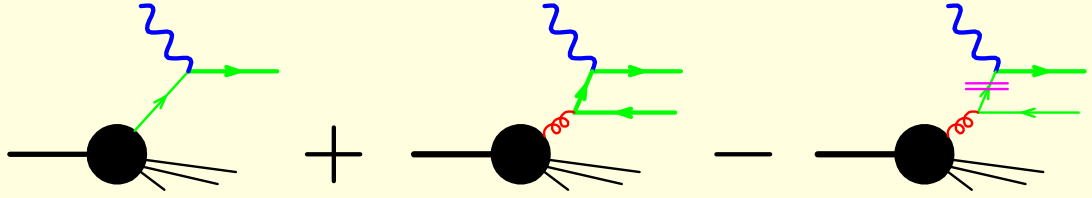


massive



Formal basis: factorization with massive quarks (Collins,1998)

ACOT At NLO: PDF subtraction (3rd graph) depends upon 1st graph.



How to include mass in the 1st graph is not fully specified ...

3rd graph takes away from 2nd graph what was already included in 1st.

In spite of ACOT formulation in 1994, up to CTEQ 6.1, the massless approximation (VFNS) has been used in the computation of DIS structure functions.

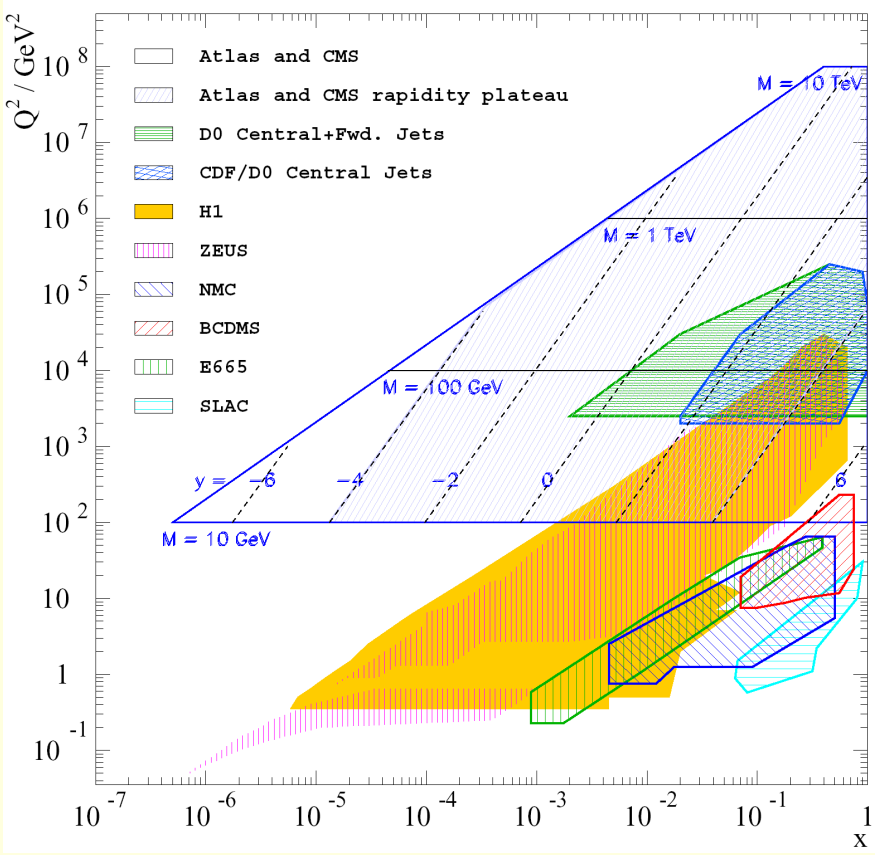
From CTEQ6.5 (end of 2006) the ACOT scheme has been implemented.

Unespected change in W cross section:

- $\Delta\sigma_W/\sigma_W = 3.5\%$ at Tevatron,
- $\Delta\sigma_W/\sigma_W = 8\%$ at LHC

Assuming that the charm mass is important up to $Q = 10\,\mathrm{GeV}$, we see that the region $x = 10^{-5} - 10^{-3}$ should be mostly affected by an incorrect treatment of charm mass effects.

Inclusion of correct mass effects has reduced the charm contribution to F_2, thus forcing the up and down quark to increase in this region of x.

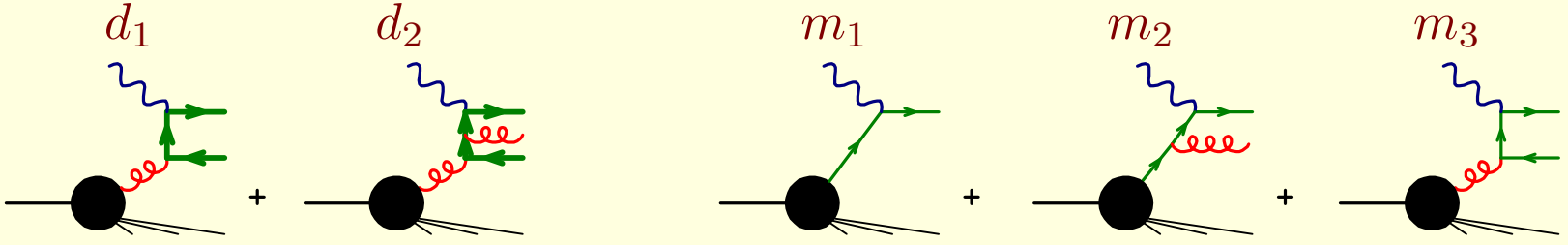


The affected region of x is relevant for W/Z production at the LHC,and so the change has not gone unnoticed.

(Tung,Lay,Belyaev,Pumplin,Stump,Yuan, 2007,Martin,Stirling,Thorne,Watt 2007)

TR SCHEME (Thorne and Roberts, 1998)

Basic idea: a structure function computed in the decoupling scheme does not match a structure function computed in the massless scheme when $Q \approx m_h$.
Correct the massless scheme so that they match.

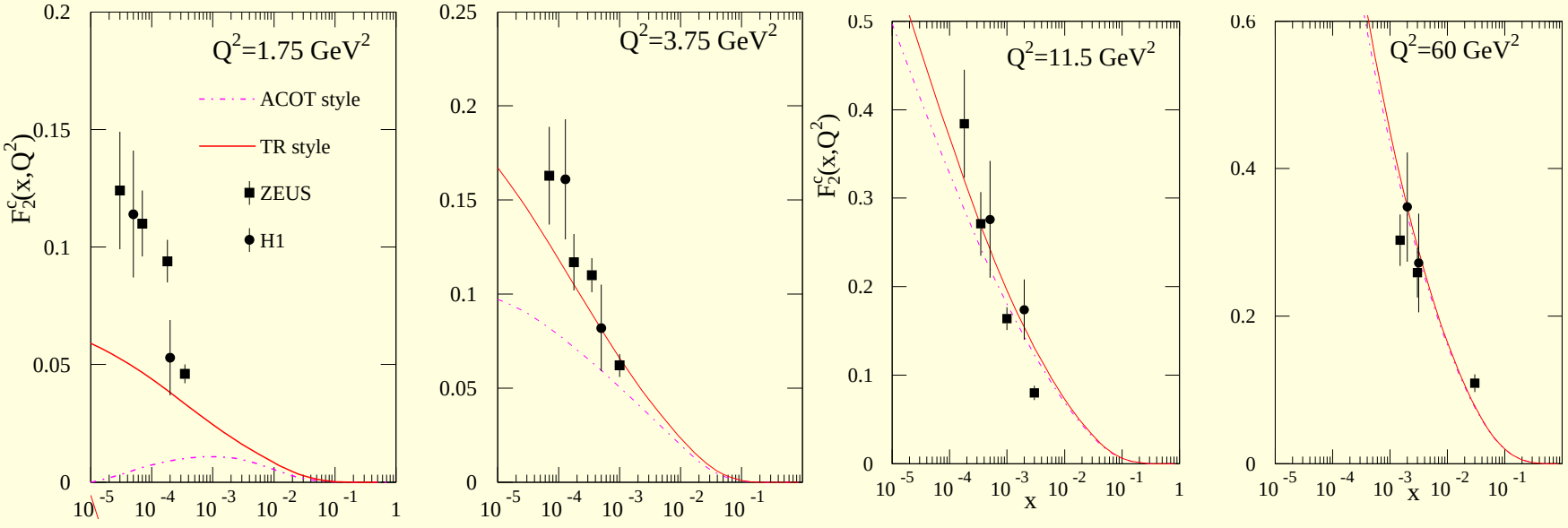


$Q \approx m_h$: $\mathcal{O}(\alpha_s)$ $\mathcal{O}(\alpha_s^2)$ $\mathcal{O}(\alpha_s)$ $\mathcal{O}(\alpha_s^2)$ $\mathcal{O}(\alpha_s)$

(When counting the order for $Q \approx m_h$, remember that $f_h \approx \alpha_s$)
Since F_2 is $\mathcal{O}(1)$, matching at NLO can be interpreted as: $\mathcal{O}(\alpha_s)$ terms only.
This approach essentially recovers ACOT

TR try to match at NLO, including $\mathcal{O}(\alpha_s^2)$ terms by imposing continuity at $Q = M$ at $\mathcal{O}(\alpha_s^2)$, up to the derivative with respect to Q.
Since m_1 and m_2 vanish at $Q = m$, this implies that d_2 is added to their result. They add $d_2(Q = m_h)$, to avoid $(\alpha_s L)^2$ terms arising for $Q \gg m_h$.

Thorne and Tung (2009) now seem to agree that this constant term summarizes the difference in their approaches (is it ALL the difference?)
It is beyond the declared accuracy of ACOT (Since it is $\mathcal{O}(\alpha_s^2)$).
Notice: it is frozen at $Q = m_h$, so $(\alpha L)^2 \approx 1$ terms do not arise at large Q^2.
Thorne (2006) shows that it is relevant at low x and Q^2.



Thorne and Tung (2009) paper has represented a considerable step forward in understanding similarities and differences between the two approaches.

From Collins (1998) factorization paper:

> Roberts and Thorne [10, 11] appear to have a scheme similar to the one in the present paper. But they do not present complete proofs, and they make a number of incorrect or misleading statements. For example, they state that "the detailed construction of the coefficient functions ... is extremely difficult if not impossible." As regards the general formalism, the construction is exactly as difficult as in the light-quark case. The only computational complication is that in a calculation of the coefficient functions, heavy quark masses must be retained. All the necessary Feynman-graph calculations for computing the coefficient functions at order α_s^2 have been done in Refs. [8], and all that remains is to organize them to form the coefficient function by use of the recursion relation Eq. (65). This recursion relation is of the same form as the one used to obtain the coefficient functions in the massless case.

so, some similarities were recognized, but differences were not fully understood.

The core of the difference:

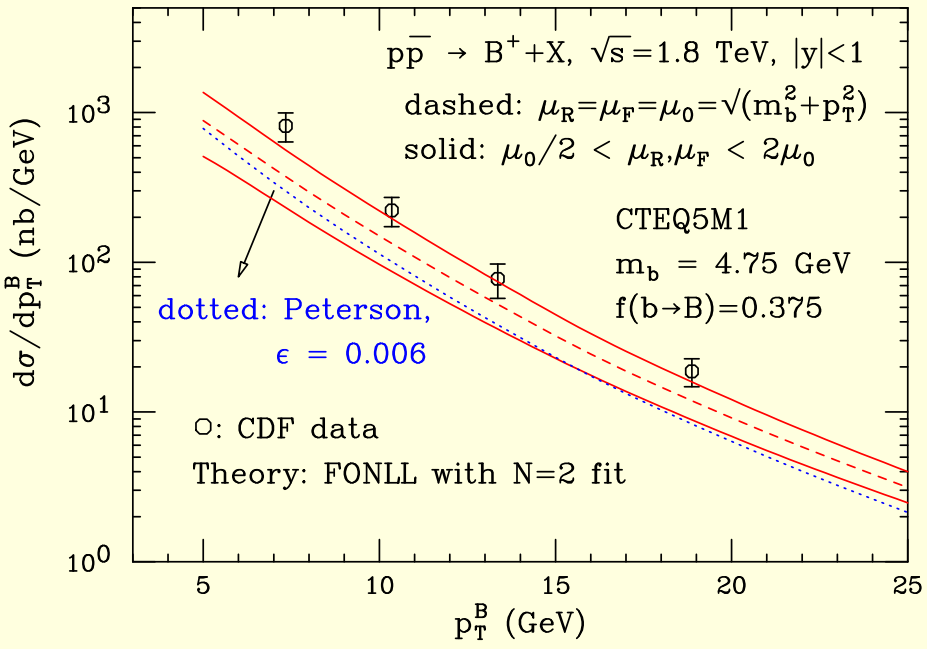
In ACOT, the massive result is included up to $\mathcal{O}(\alpha_s)$
In TR, it is included up to $\mathcal{O}(\alpha_s^2)$.

The same mismatch is present at NNLO, where an estimate of the $\mathcal{O}(\alpha_s^3)$ massive result is needed in TR (Thorne, 2006).

Within ACOT, at NLO, only the $\mathcal{O}(\alpha_s)$ massive result enters naturally within the heavy flavour factorization scheme propose by Collins.
It should be stressed, however, that the $\mathcal{O}(\alpha_s^2)$ massive result is available, it requires NLO α_s and pdf's. Why should it not be used in an NLO fit?

FONLL (Cacciari, Greco, P.N., 1998)

Totally independent approach introduced in the context of heavy flavour production at high p_T, in order to address the discrepancy between theory and Tevatron data in b production. Besides the pdf, it also deals with b fragmentation.



It was used to match the NLO heavy flavour production calculation of Ellis, Dawson and P.N. (massive scheme) with that of Cacciari and Greco (massless scheme). The method is totally general. It has been applied to heavy flavour production in $e^+ e^-$ annihilation. Its first application to DIS is illustrated in the present talk.

FONLL in few words

A cross section in the decoupling scheme can be seen as a fixed order power expansion in α_s with mass dependent coefficients. The coefficients have logarithmic behaviour at large scale.
A cross section in the massless scheme can be seen as a power series in α_s with coefficients that are polynomials in L. All large logs are resummed.

So: add them up, deleting from the second the terms of the same order in α_s present in the first.

This way, the coefficients of powers of α_s that are only approximate in the massless expression, are replaced with the coefficients that include the exact mass dependence.

Advantages: it is simple, the proof takes one page. It works at all orders (in spite of the name...). It does not need new calculations. (The heavy flavour calculation was done by putting together NDE and CG programs.)

FONLL in details

- Call σ a heavy flavour cross section in the decoupling scheme.
  σ is given in terms of $\alpha_s^{(n_l)}(\mu)$ and $f_i^{(n_l)}(\mu)$.
  Express σ in terms of $\alpha_s^{(n_f)}(\mu)$ and $f_{i\neq h}^{(n_f)}(\mu)$, using the matching equations. We call $\tilde{\sigma}$ the new expression (this procedure is elementary at NLO)
- In the limit $m \to 0$, $\tilde{\sigma} \to \tilde{\sigma}_0$, where $\tilde{\sigma}_0$ is a polynomial in α_s and L with mass independent coefficients. It is the massless limit of $\tilde{\sigma}$, in the sense $\lim_{m\to 0}(\tilde{\sigma} - \tilde{\sigma}_0) = 0$.
- The massless scheme cross section, $\sigma_{\overline{\rm MS}}$, is given in terms of $\alpha_s^{(n_f)}(\mu)$ and $f_i^{(n_f)}(\mu)$. If we express $f_h^{(n_f)}(\mu)$ as a functional of the $f_{i\neq h}^{(n_f)}(\mu)$ using the evolution equations and the matching conditions, $f_h^{(n_f)}(\mu)$ is a power series in α_s and L with mass independent coefficients. In this way $\sigma_{\overline{\rm MS}}$ can be viewed as a power series in α_s and L with mass independent coefficients.
  Let us call $\tilde{\sigma}_0^{\rm ov}$ (ov for overlap) what we get from $\tilde{\sigma}_0$ deleting all terms that are not in $\sigma_{\overline{\rm MS}}$.
- The FONLL expression is $\sigma_{\rm FONLL} = \sigma_{\overline{\rm MS}} - \tilde{\sigma}_0^{\rm ov} + \tilde{\sigma}$

FONLL in DIS

(Forte, Laenen, Piccione, Rojo, P.N.)

First: separate a structure function (F_2 or F_L) into two contributions

$$F = F_h + F_l,$$

where F_h is the contribution to F obtained by switching off all light quark charges, and F_l is all the rest.

Up to $\mathcal{O}(\alpha^2)$ in the coefficient functions, F_l corresponds to switching off the heavy quark charge. Starting at $\mathcal{O}(\alpha^3)$, terms proportional to the product of light and heavy quark charges appear in F_l.

Change of scheme

We must express the CWZ calculation of F in terms of $\alpha_s^{(n_f)}$, $f^{(n_f)}$.
We have

$$a_s^{(n_l)}(Q) = a_s(Q) \times \left(1 - a_s L \frac{2T_R}{3}\right) + \mathcal{O}(a_s^3)$$

$$f_g^{(n_l)}(x,Q) = f_g(x,Q) \times \left(1 + a_s L \frac{2T_R}{3}\right) + \mathcal{O}(a_s^2)$$

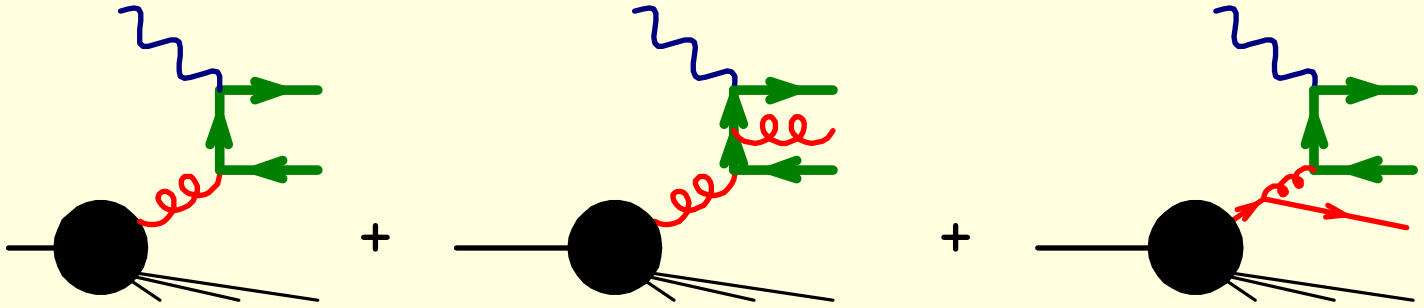
$$f_i^{(n_l)}(x,Q) = f_i(Q) - a_s^2 \int_x^1 \frac{dz}{z} K_{\rm qq}(z,L) f_j\left(\frac{x}{z}, Q\right) + \mathcal{O}(a_s^3).$$

where $L = \log Q^2/m^2$, $a_s = \alpha_s/2\pi$, and $K_{\rm qq}$ from Buza etal, 1996.
Also, we write from now on

$$a_s = a_s^{(n_f)}, \qquad \alpha_s = \alpha_s^{(n_f)}, \qquad f_i = f_i^{(n_f)}.$$

Terms of higher order than the above in the change of scheme are not needed in the computation of structure functions up to NNLO.

F_h in the CWZ scheme



$\mathcal{O}(\alpha_s)$ photon gluon fusion,
plus $\mathcal{O}(\alpha_s^2)$ photon gluon and photon light-quark fusion (Laenen etal, 1983).

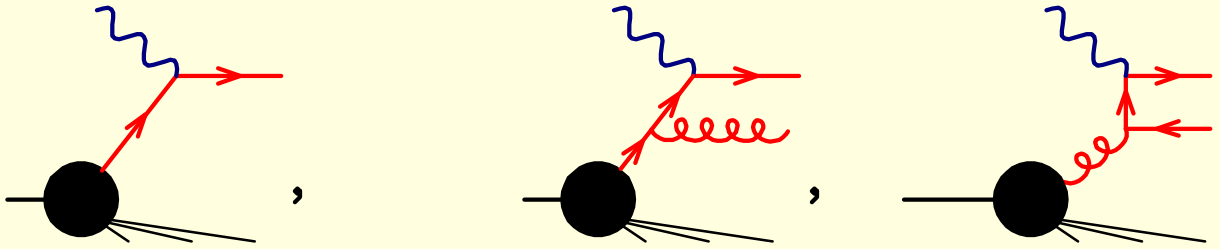
At order $\mathcal{O}(\alpha_s^2)$, the change of scheme is only relevant in the first graph;
It amounts to two factors, one for α_s and one for f_g:

$$\left(1 - a_s L \frac{2T_R}{3}\right) \times \left(1 + a_s L \frac{2T_R}{3}\right) = 1 + \mathcal{O}(a_s^2)$$

Thus: no effects at all up to $\mathcal{O}(\alpha_s^2)$ in F_h from the change of scheme!

F_l in the CWZ scheme

Here the change of scheme may affect the following contributions:



In the first graph, the $\mathcal{O}(\alpha_s^2)$ modification of the light quark density under the change of scheme, combined with a $\mathcal{O}(1)$ coefficient function, induces a contribution of order to F of $\mathcal{O}(\alpha_s^2)$.

In the second graph, of order $\mathcal{O}(\alpha_s)$, the change from $\alpha_s^{(n_l)}$ to α_s again induces a correction of order $\mathcal{O}(\alpha_s^2)$.

In the third graph, both the change of α_s and of f_g come into play, but they cancel exactly among each other.

Finally, notice that in the case of the longitudinal structure function, only the second graph contributes, since the $\mathcal{O}(1)$ coefficient function vanishes.

So, the change of scheme amounts to a change in the coefficient function of the quark

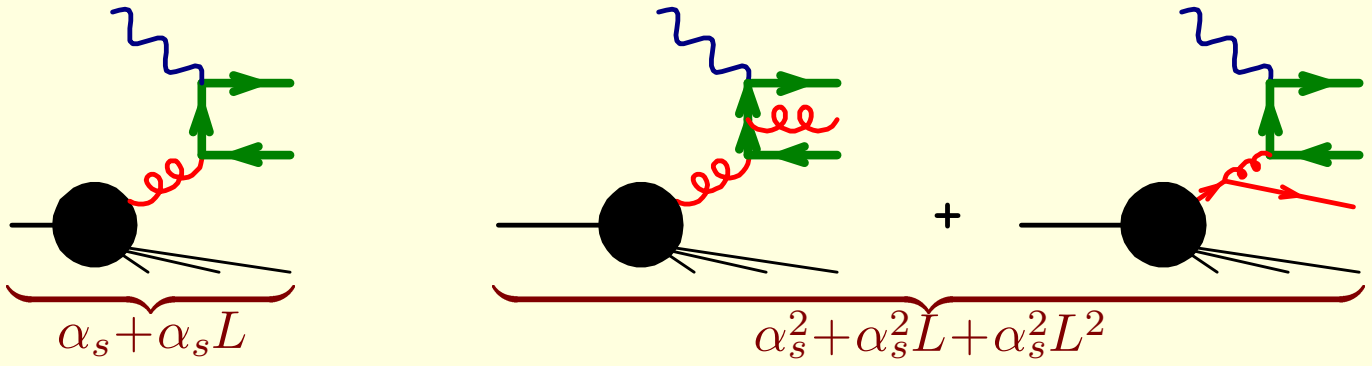
$$\begin{aligned} C_j(z) \;\Rightarrow\; & C_j(z) - C_j^{(0)} a_s^2 \int_x^1 \frac{dz}{z} K_{\rm qq}(z,L)\, f_j\Big(\frac{x}{z},Q\Big) \\ - \;\; & a_s^2 L \frac{2T_R}{3} \int_x^1 \frac{dz}{z} C_q^{(1)}(z)\, f_j\Big(\frac{x}{z},Q\Big), \end{aligned}$$

where (at NNLO)

$$C_j(z) = C_j^{(0)} \delta(1-z) + a_s C_q^{(1)}(z) + a_s^2 C_q^{(2)}(z)$$

(with $C_j^{(0)} = e_j^2$ for F_2 and 0 for F_L) is the coefficient function for the structure function under consideration.

FONLL for $F_{2,h}$



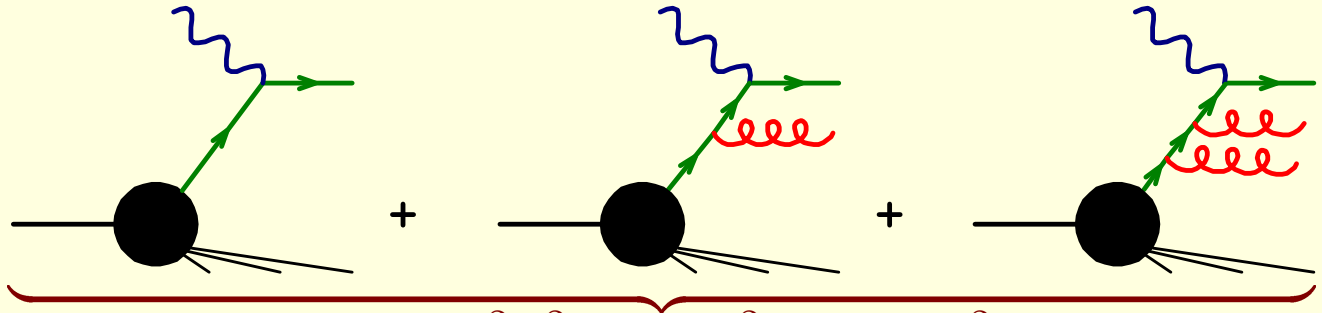
In CWZ, powers of L only arise in the coefficient functions at finite order.

In the massless calculation, powers of L only arise in $f_h(Q)$ at all orders:

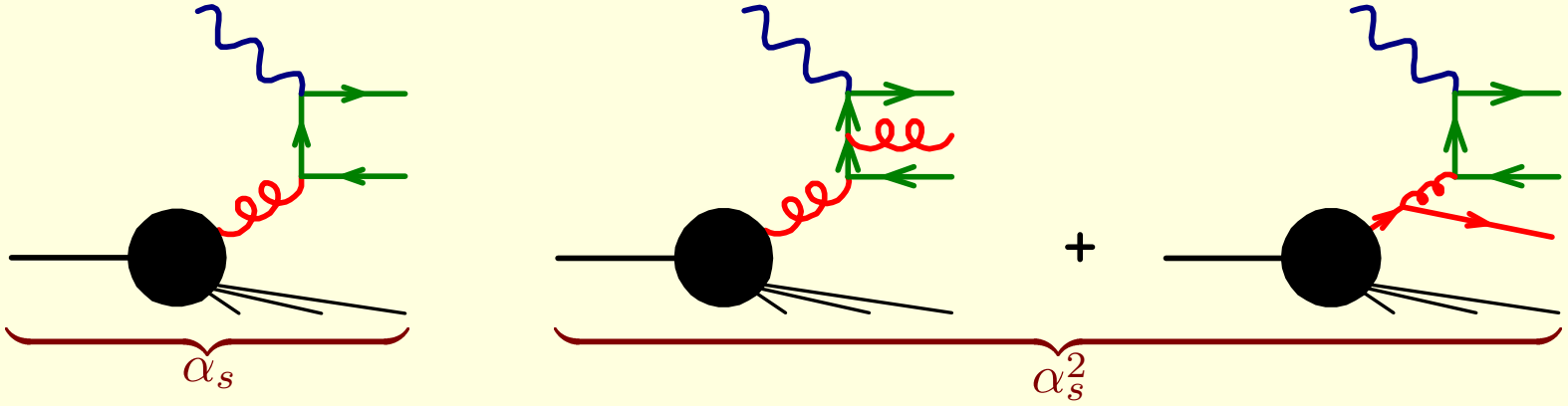
$$\begin{aligned} f_h(Q) &= \alpha_s L + \alpha_s^2 L^2 + \alpha_s^3 L^3 + \ldots (\text{Leading logs}) \\ &+ \alpha_s^2 L + \alpha_s^3 L^2 + \alpha_s^4 L^3 + \ldots (\text{Next to Leading}) \\ &+ \alpha_s^2 \ + \alpha_s^3 L \ + \alpha_s^4 L^2 + \ldots (\text{Next to Next}) \end{aligned}$$

Notice: α_s (no logs) accidentally missing from NLL, but α_s^2 present at NNLL

So, $F_{2,h}$ in the massless scheme (up to $\mathcal{O}(\alpha_s^2)$) has the log structure



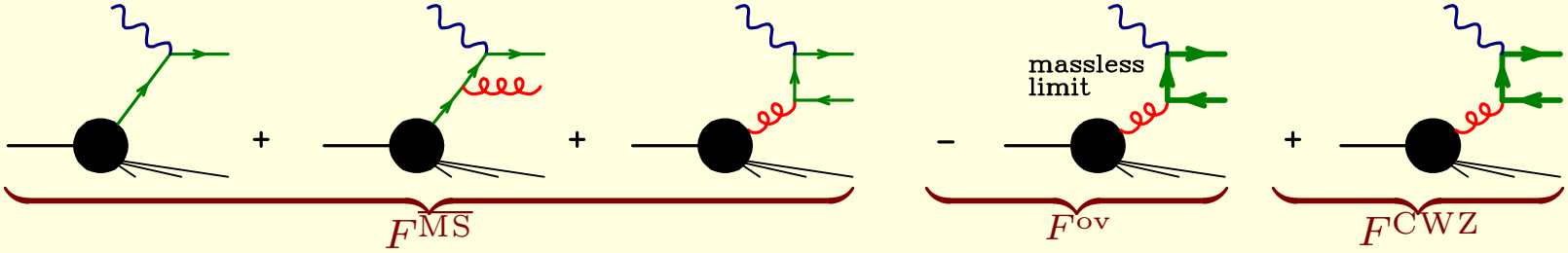
The coefficient functions have no logs; thus, further terms with no logs arise from gluon and light quark initiated processes, at order α_s and α_s^2



The FONLL prescription requires that terms that are present in the massless calculation should be replaced by the corresponding massive ones.

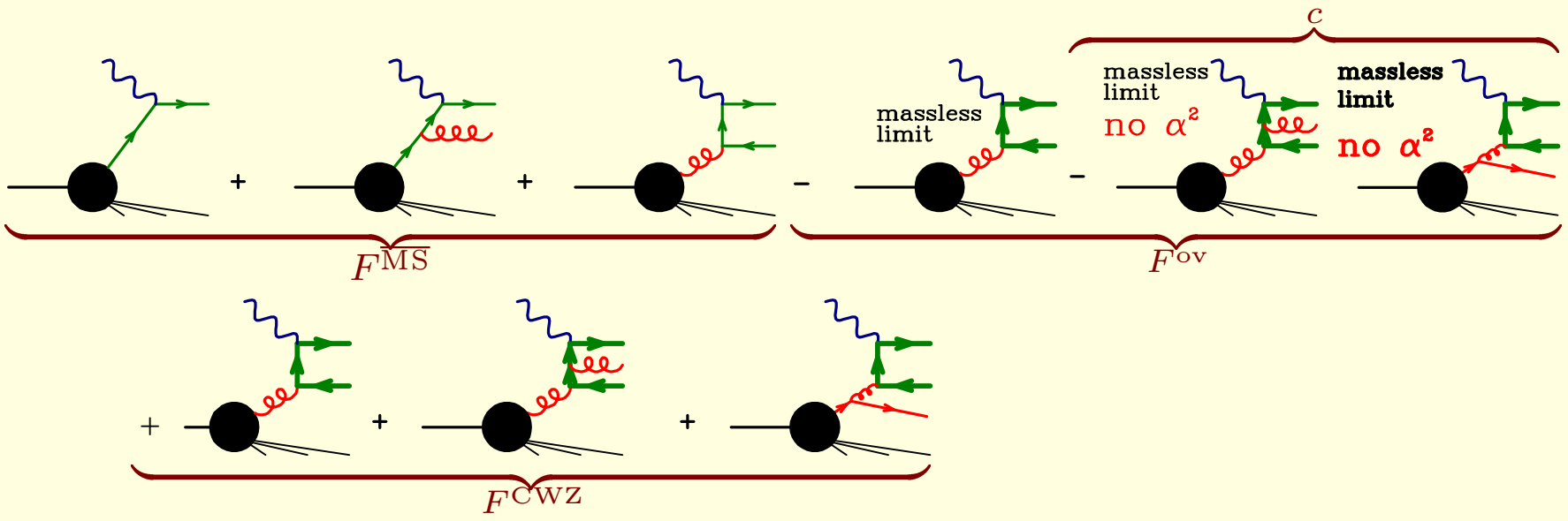
Suppose we only want to use the $\mathcal{O}(\alpha_s)$ CWZ computation, together with the NLO massless calculation. Then we get:

FONLL, scheme A:



In this case, the massless limit of the CWZ result, of order $\alpha_s + \alpha_s L$, cancels exactly the $\alpha_s L$ term in the first graph of the massless result, and the α_s term in the last graph of the massless result.

If we use NLO for massive (scheme B in FLNPR):



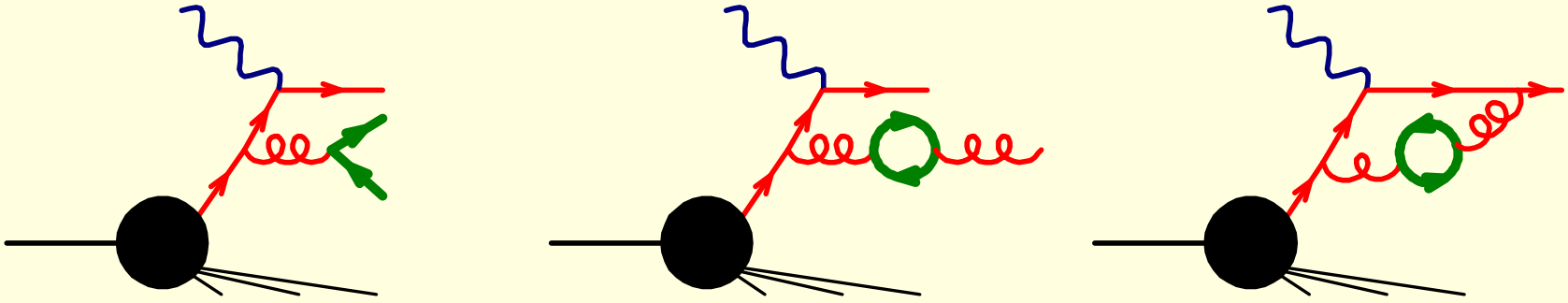
In scheme C we use NNLO for $F^{\overline{\rm MS}}$, and NLO for massive. In this case, $F^{\rm ov}$ is the full massless limit of $F^{\rm CWZ}$ (i.e. not limited to logarithmic terms).

F_l in FONLL

the computation of F_l in CWZ and in $\overline{\rm MS}$ has the following origin:

- The change of scheme, at $\mathcal{O}(\alpha_s^2)$
- The coeff. funct. for $\gamma q \to q h \bar{h}$, (γ coupled to q), plus virtual
- The coeff. funct. for $\gamma h \to h q \bar{q}$, (γ coupled to q) only present in $\overline{\rm MS}$

All these elements are of order α_s^2, thus FONLL at NLO is trivial for F_l.
The second element is summarized by the graphs:



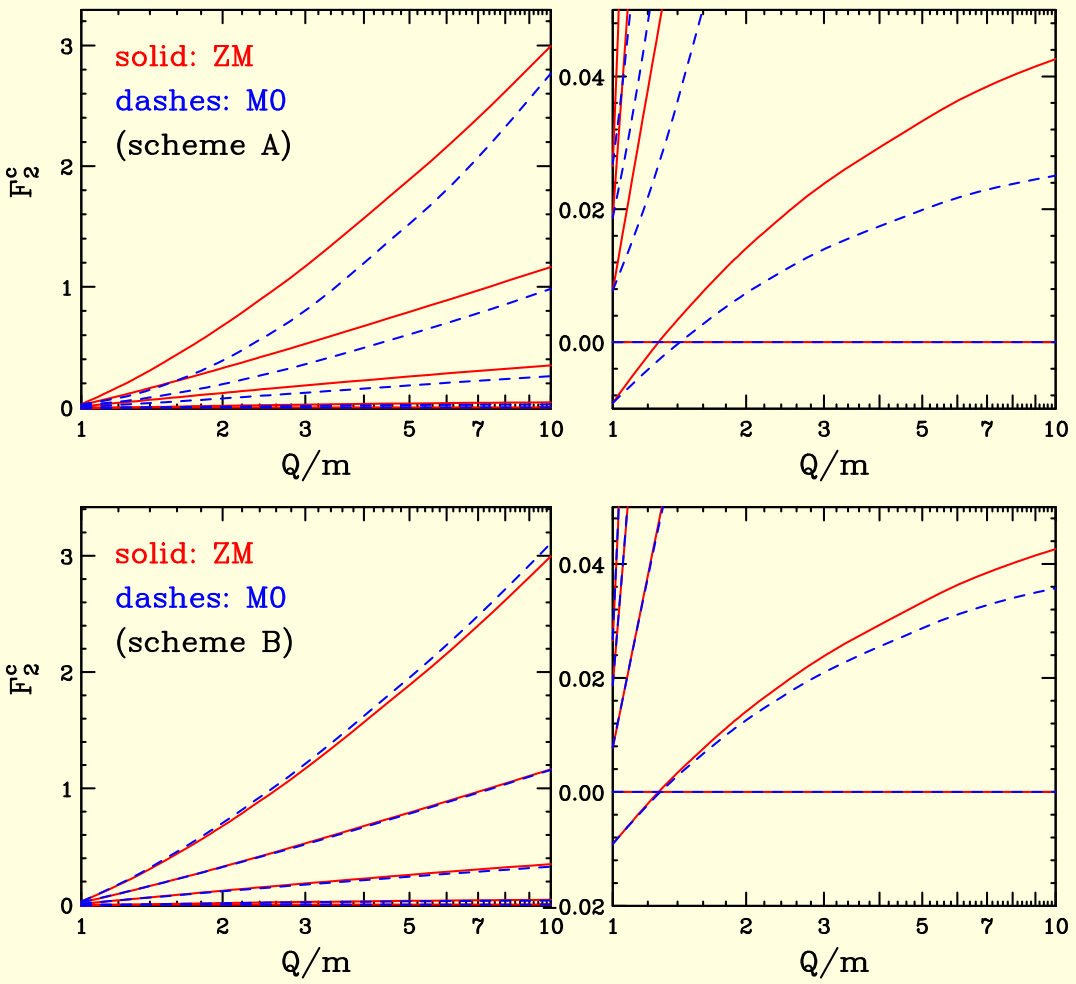
The second graph is zero both in the massless and in the CWZ scheme. The first two differ. The last one is proportional to $\delta(1-z)$.

The FONLL formula can be worked out to be

$$
\begin{aligned}
F_l^{\rm FONLL} &= F_l^{\overline{\rm MS}} - F_l^{\rm ov} + F_l^{\rm CWZ} \\
F_l^{\overline{\rm MS}} - F_l^{\rm ov} &= x \sum_{i=h,\bar{h}} \int_x^1 \frac{dy}{y} C_i\left(\frac{x}{y}, \alpha_s\right) f_i(y,Q) \\
F_l^{\rm CWZ} &= F_l^{\overline{\rm MS}} - x \sum_{i=h,\bar{h}} \int_x^1 \frac{dy}{y} C_i\left(\frac{x}{y}, \alpha_s\right) f_i(y,Q) \\
&+ x \sum_{i\neq h,\bar{h},g} \int_x^1 \frac{dy}{y} D_i\left(\frac{x}{y}, \frac{Q^2}{m^2}, \alpha_s\right) f_i(y,Q) \\
D_i\left(z, \frac{Q^2}{m^2}, \alpha_s\right) &= L_i\left(z, \frac{Q^2}{m^2}, \alpha_s\right) - \delta(1-z) \int_0^1 dz L_i\left(z, \frac{Q^2}{m^2}, \alpha_s\right) \\
&- a_s^2 \left[e_i^2 K_{qq}(z,L) + \frac{2T_R}{3} L C_i^{(1)}(z) \right] - \frac{\partial}{\partial n} C_i(z, \alpha_s)
\end{aligned}
$$

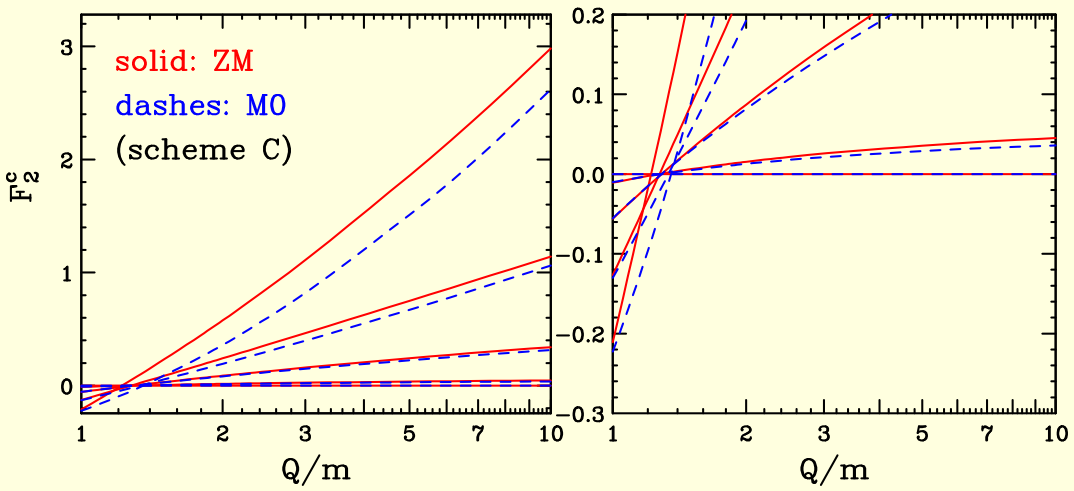
In practice, FONLL differs from the massless formula by the D term only, that vanishes for $Q^2 \gg m^2$. Its contribution is truly negligible.

Compare massless (ZM) and massless limit of CWZ (M0)

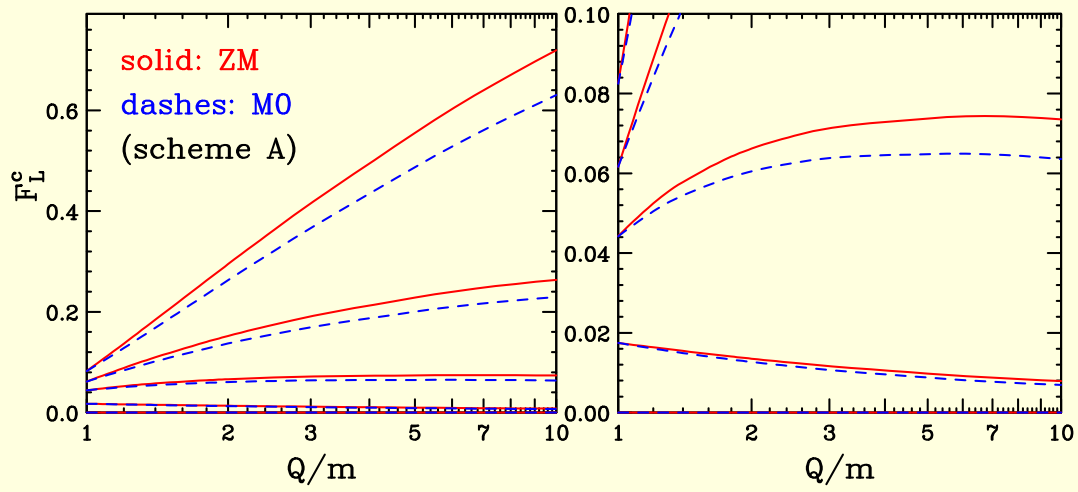


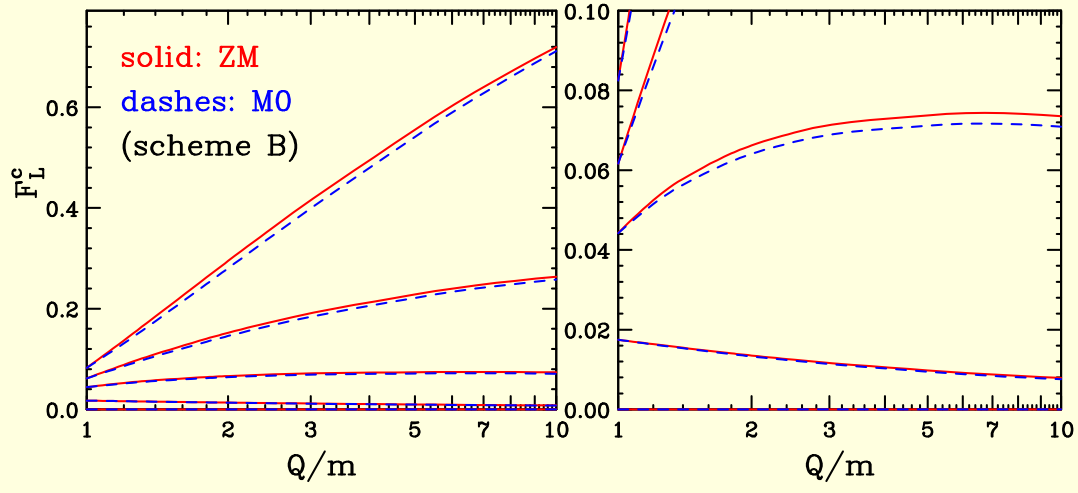
In scheme A ZM and M0 agree in value at threshold; they differ in slope by terms of order α_s^2

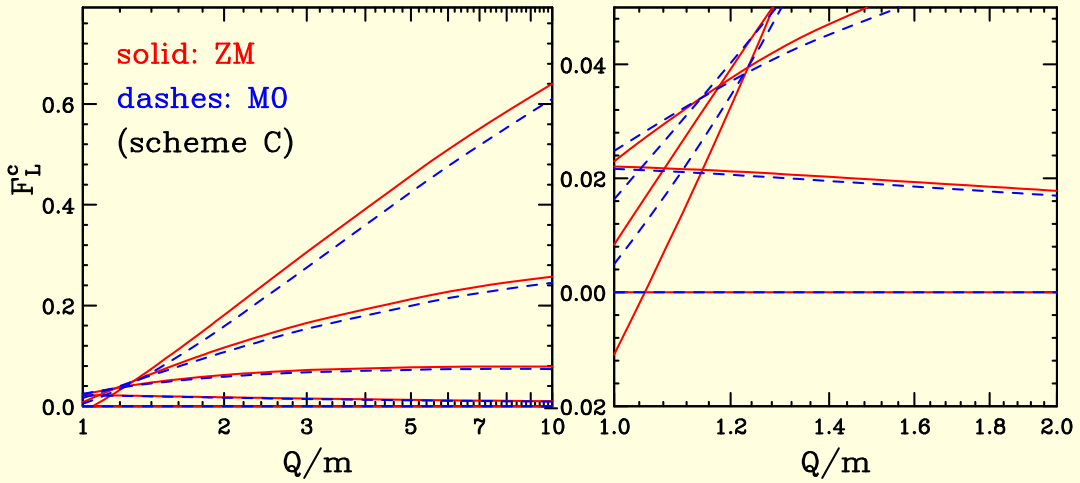
Curves for different values of x:
$x = 10^{-5}$ (top curve)
$x = 1$ (0 constant curve)
4 x value equally spaced logarithmically

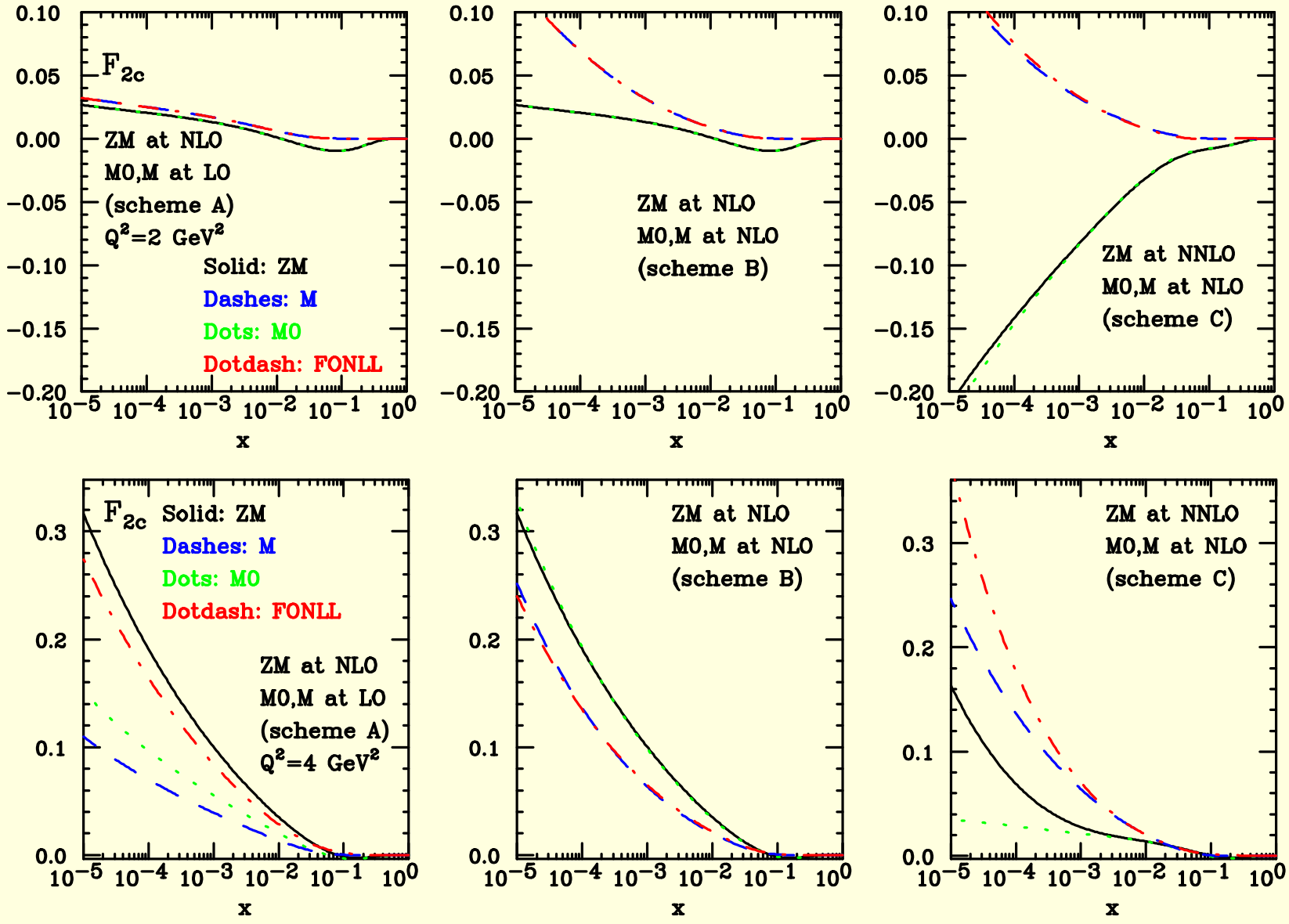


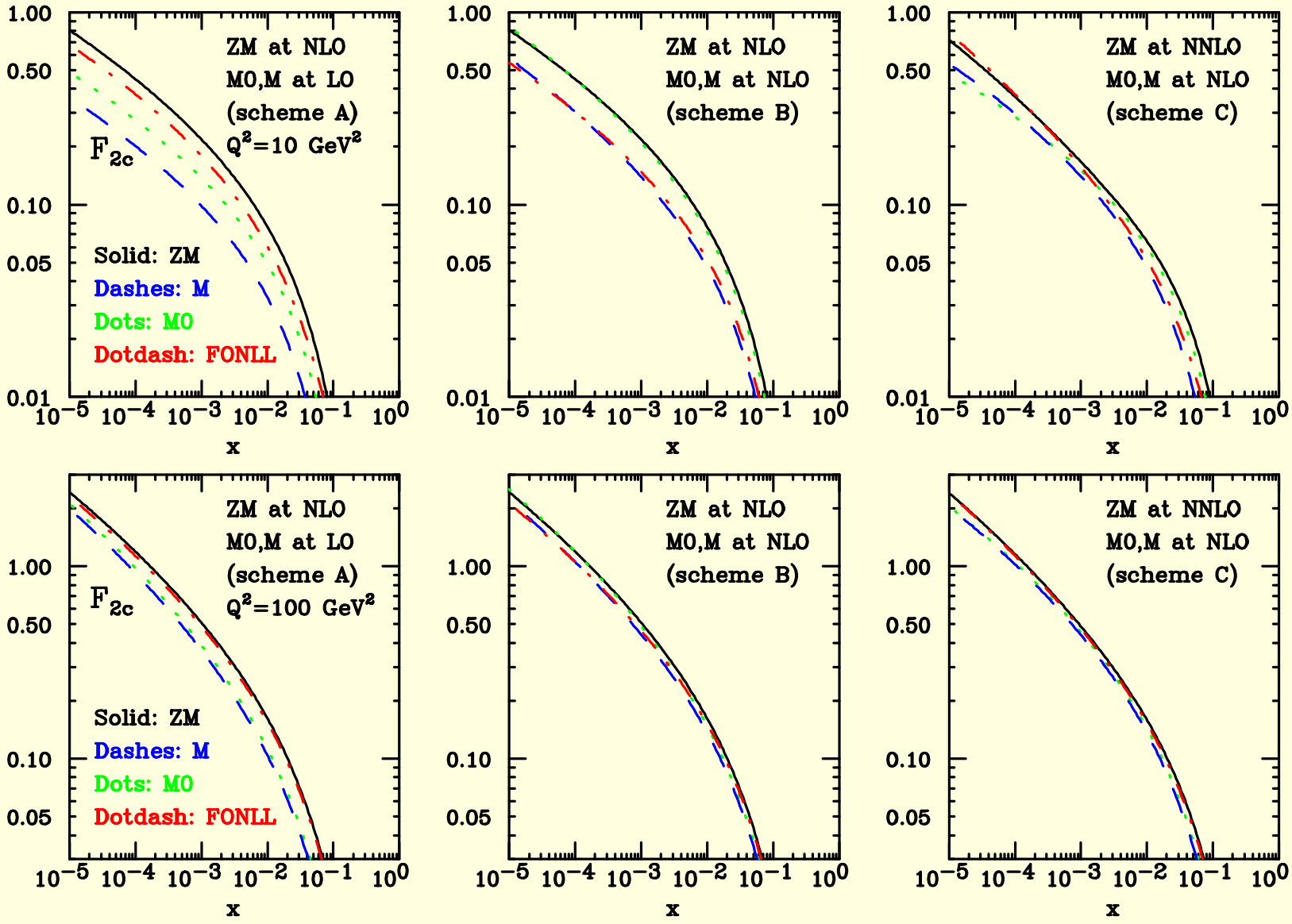
ZM and M0 differ also at $Q = m$ by terms of order α_s^3. The slopes differ at very small x becaus of small x enhanced $L\alpha_s^3$ terms.

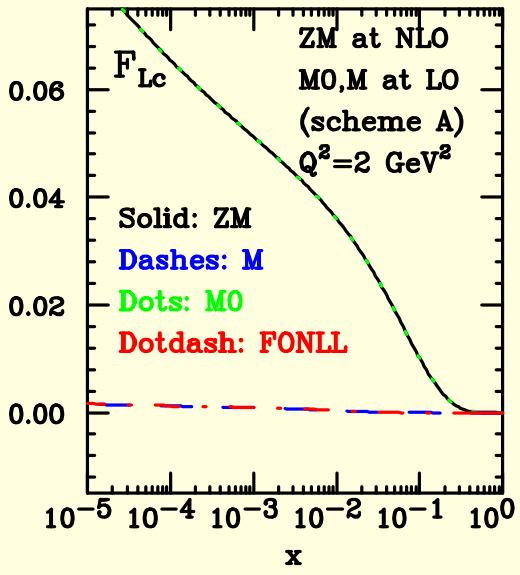

solid: ZM
dashes: M0
(scheme A)
F_L^c
Q/m
Q/m

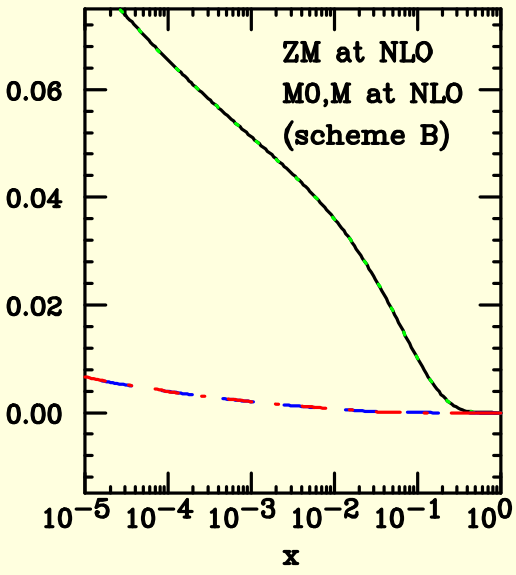

solid: ZM
dashes: M0
(scheme B)
F_L^c
Q/m
Q/m

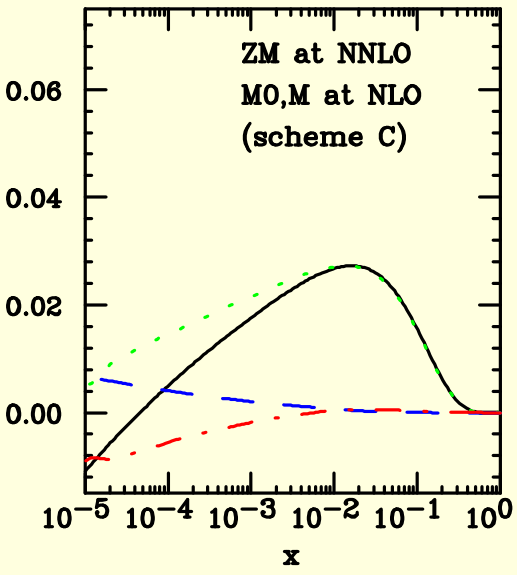

solid: ZM
dashes: M0
(scheme C)
F_L^c
0.6
0.4
0.2
0.0
1
2
3
5
7
10
Q/m
0.04
0.02
0.00
-0.02
1.0
1.2
1.4
1.6
1.8
2.0
Q/m

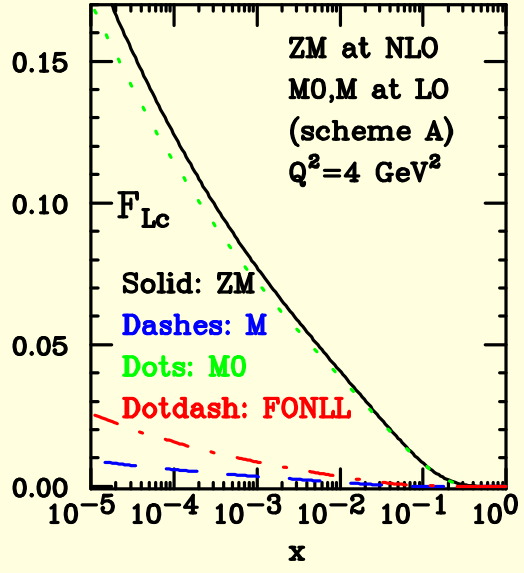

F_{2c}
ZM at NLO
M0,M at LO
(scheme A)
Q^2=2 GeV^2
Solid: ZM
Dashes: M
Dots: M0
Dotdash: FONLL
ZM at NLO
M0,M at NLO
(scheme B)
ZM at NNLO
M0,M at NLO
(scheme C)
F_{2c} Solid: ZM
Dashes: M
Dots: M0
Dotdash: FONLL
ZM at NLO
M0,M at LO
(scheme A)
Q^2=4 GeV^2
ZM at NLO
M0,M at NLO
(scheme B)
ZM at NNLO
M0,M at NLO
(scheme C)
x

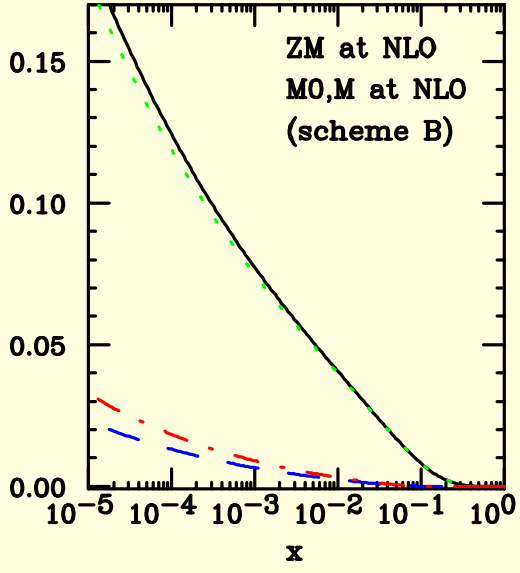

ZM at NLO
M0,M at LO
(scheme A)
Q²=10 GeV²
F2c
Solid: ZM
Dashes: M
Dots: M0
Dotdash: FONLL
ZM at NLO
M0,M at NLO
(scheme B)
ZM at NNLO
M0,M at NLO
(scheme C)
ZM at NLO
M0,M at LO
(scheme A)
Q²=100 GeV²
F2c
Solid: ZM
Dashes: M
Dots: M0
Dotdash: FONLL
ZM at NLO
M0,M at NLO
(scheme B)
ZM at NNLO
M0,M at NLO
(scheme C)
1.00
0.50
0.10
0.05
0.01
10⁻⁵ 10⁻⁴ 10⁻³ 10⁻² 10⁻¹ 10⁰
x

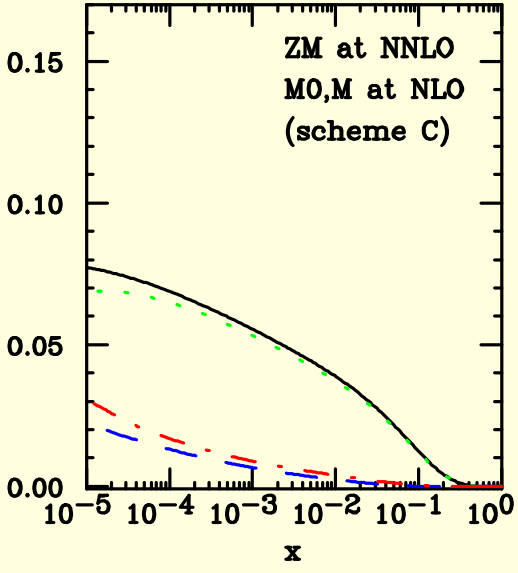

ZM at NLO
M0,M at LO
(scheme A)
Q^2=2 GeV^2
F_{Lc}
Solid: ZM
Dashes: M
Dots: M0
Dotdash: FONLL
0.06
0.04
0.02
0.00
10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0}
x

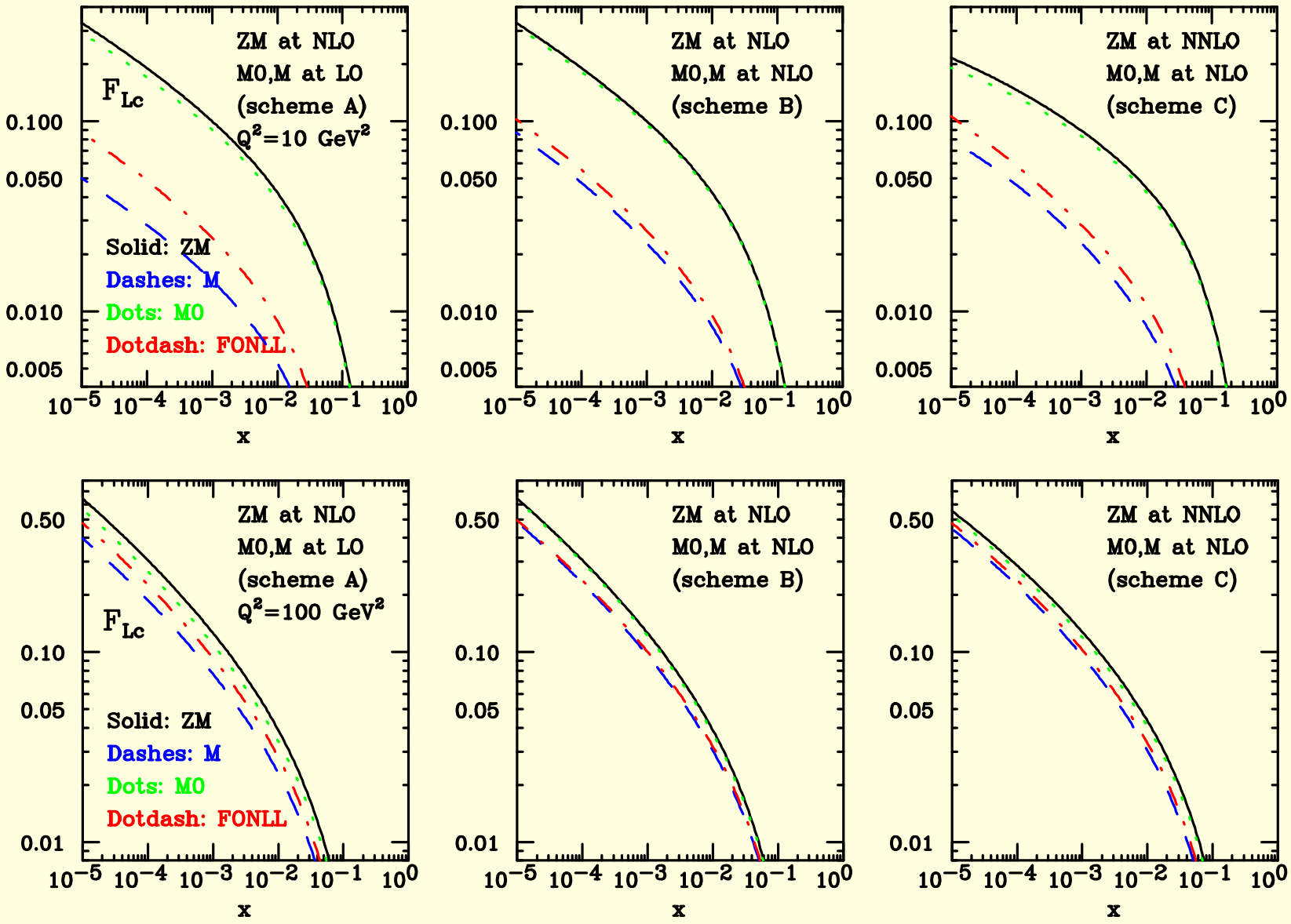

ZM at NLO
M0,M at NLO
(scheme B)
0.06
0.04
0.02
0.00
10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0}
x

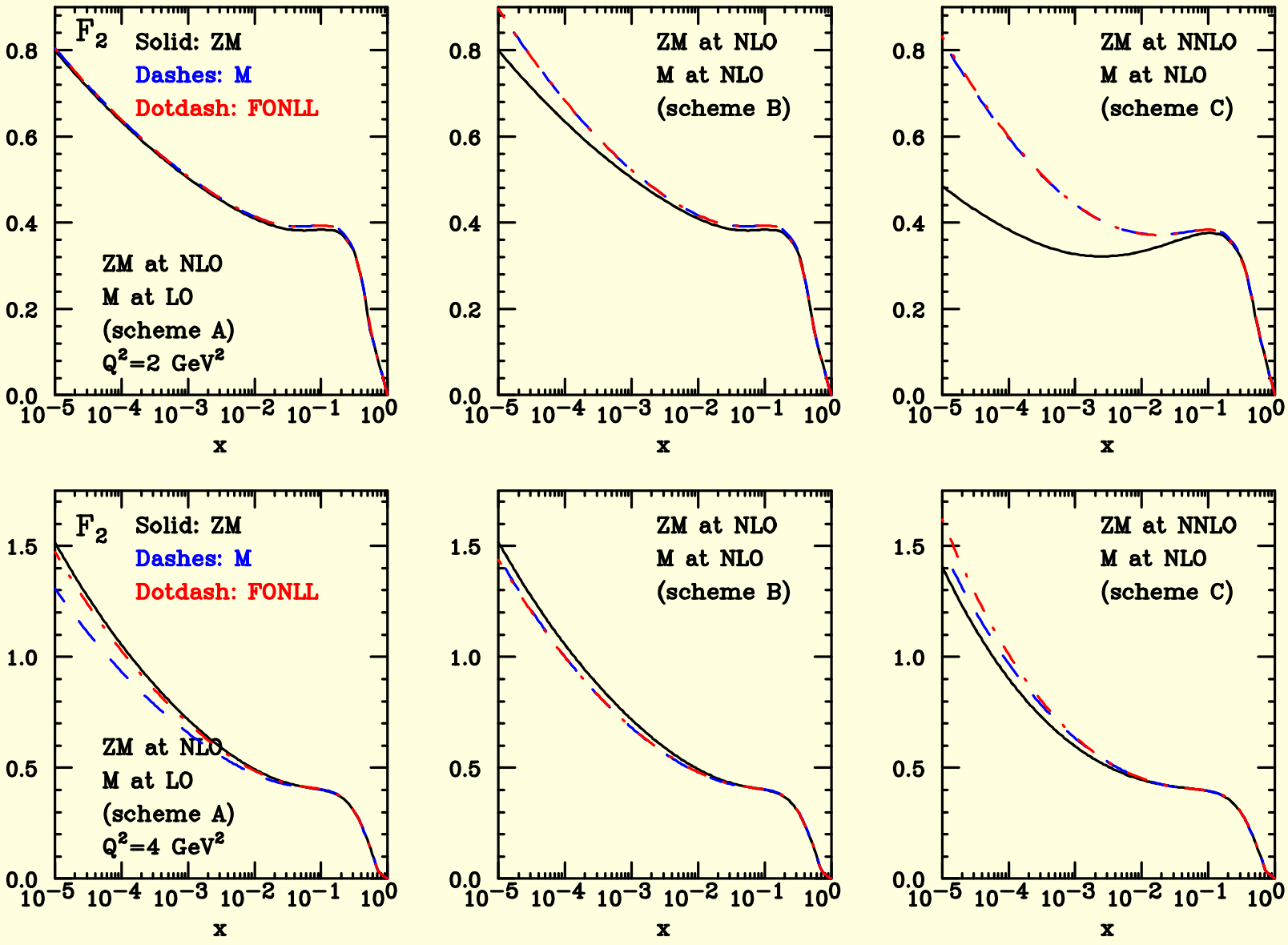

ZM at NNLO
M0,M at NLO
(scheme C)
0.06
0.04
0.02
0.00
10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0}
x

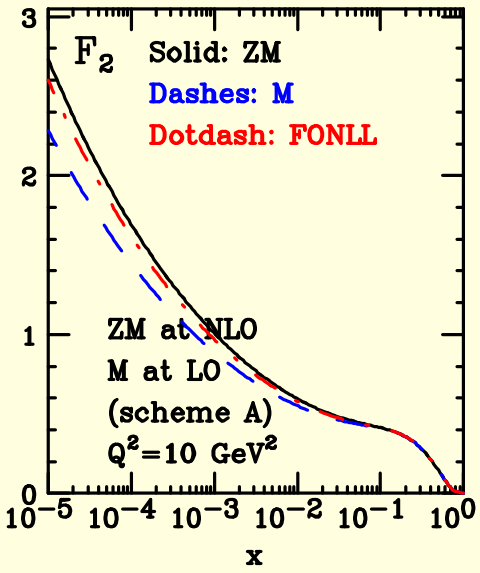

ZM at NLO
M0,M at LO
(scheme A)
Q^2=4 GeV^2
F_{Lc}
Solid: ZM
Dashes: M
Dots: M0
Dotdash: FONLL
0.15
0.10
0.05
0.00
10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0}
x

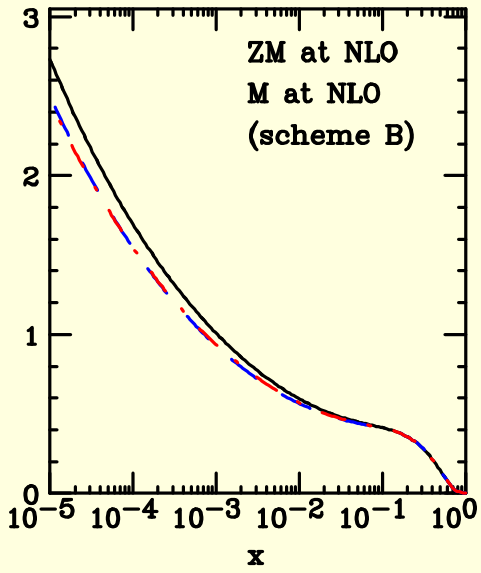

ZM at NLO
M0,M at NLO
(scheme B)
0.15
0.10
0.05
0.00
10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0}
x

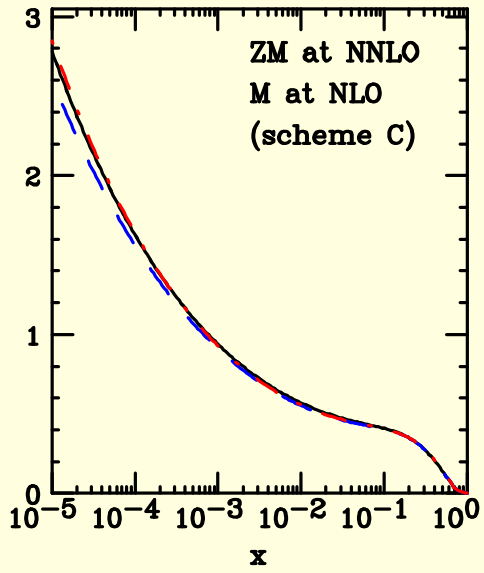

ZM at NNLO
M0,M at NLO
(scheme C)
0.15
0.10
0.05
0.00
10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0}
x

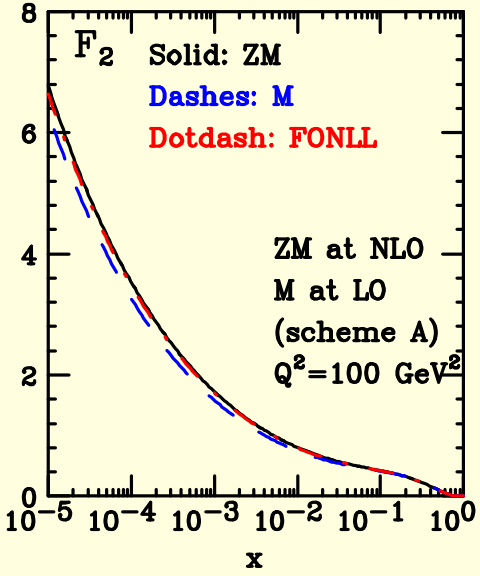

ZM at NLO
M0,M at LO
(scheme A)
Q^2=10 GeV^2
F_{Lc}
Solid: ZM
Dashes: M
Dots: M0
Dotdash: FONLL
ZM at NLO
M0,M at NLO
(scheme B)
ZM at NNLO
M0,M at NLO
(scheme C)
ZM at NLO
M0,M at LO
(scheme A)
Q^2=100 GeV^2
ZM at NLO
M0,M at NLO
(scheme B)
ZM at NNLO
M0,M at NLO
(scheme C)
x

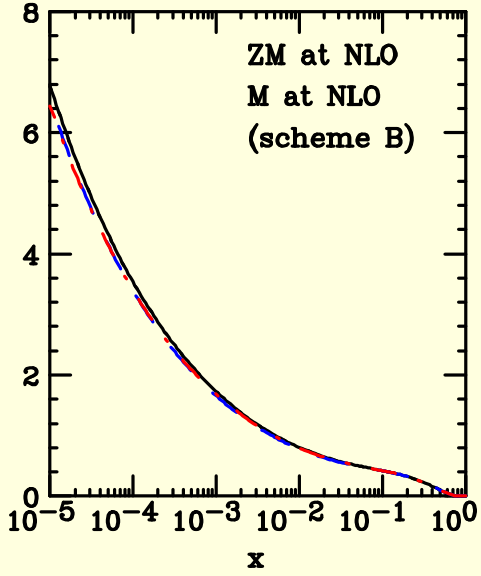

F_2
Solid: ZM
Dashes: M
Dotdash: FONLL
ZM at NLO
M at LO
(scheme A)
Q^2=2 GeV^2
ZM at NLO
M at NLO
(scheme B)
ZM at NNLO
M at NLO
(scheme C)
F_2
Solid: ZM
Dashes: M
Dotdash: FONLL
ZM at NLO
M at LO
(scheme A)
Q^2=4 GeV^2
ZM at NLO
M at NLO
(scheme B)
ZM at NNLO
M at NLO
(scheme C)
x

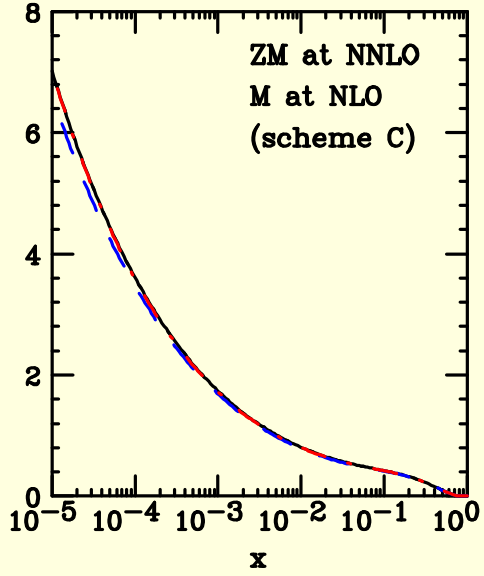

F_2
Solid: ZM
Dashes: M
Dotdash: FONLL
ZM at NLO
M at LO
(scheme A)
Q^2=10 GeV^2
x

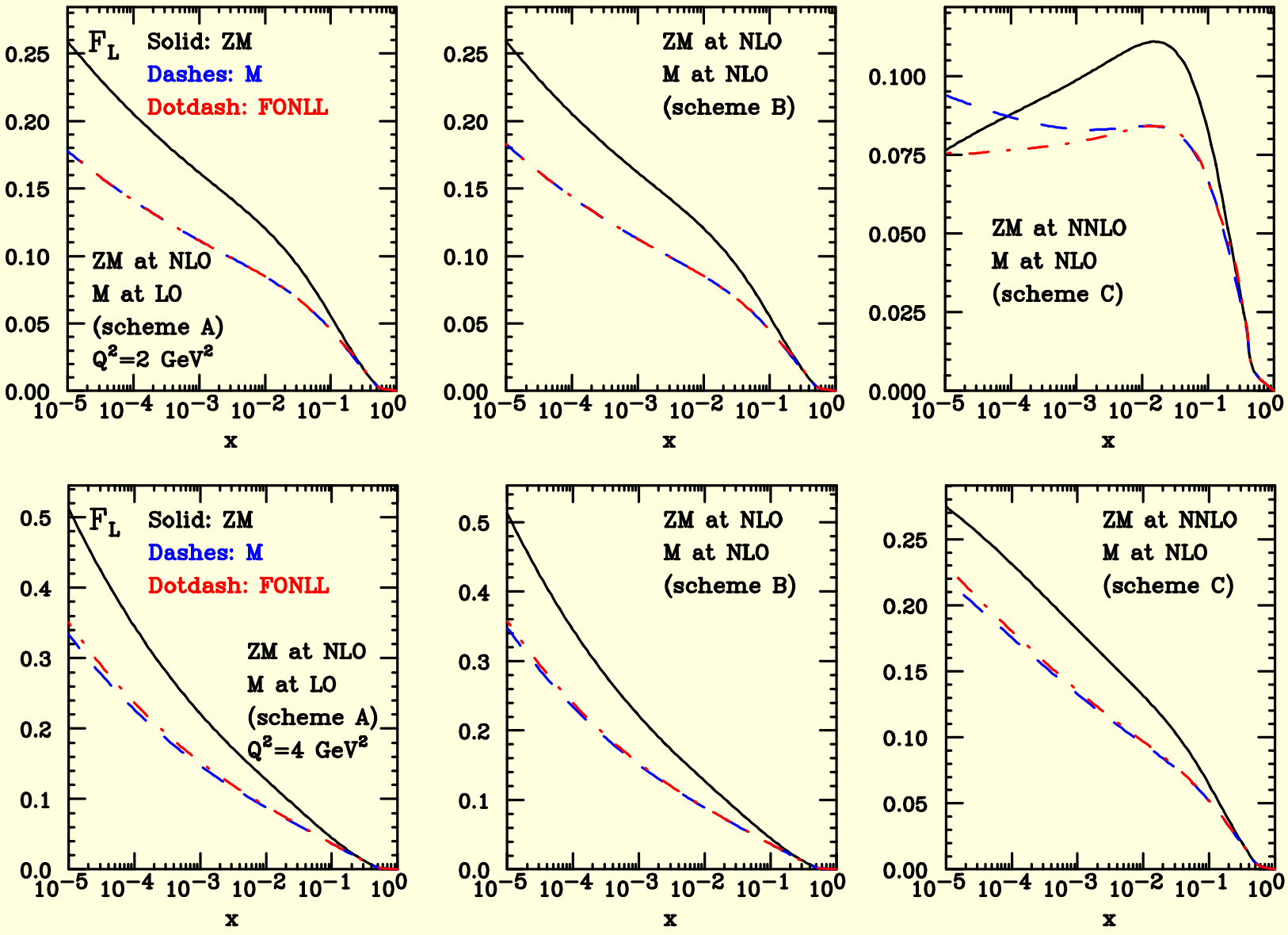

ZM at NLO
M at NLO
(scheme B)
x

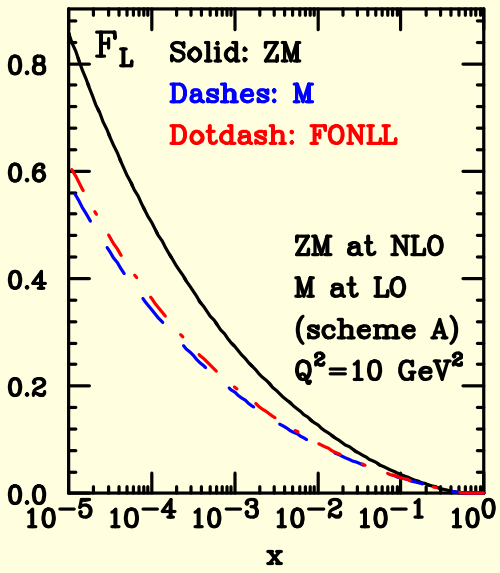

ZM at NNLO
M at NLO
(scheme C)
x

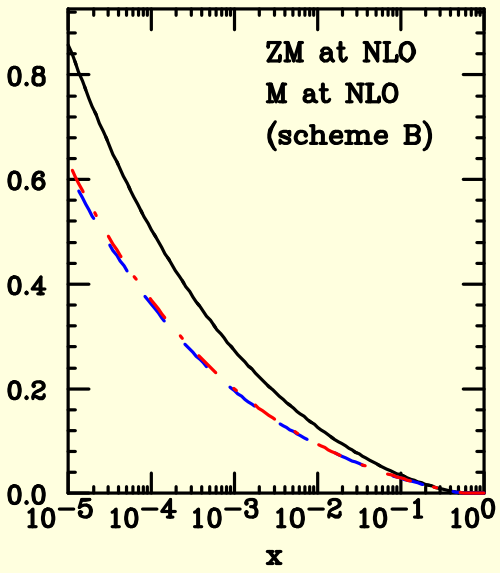

F_2
Solid: ZM
Dashes: M
Dotdash: FONLL
ZM at NLO
M at LO
(scheme A)
Q^2=100 GeV^2
x

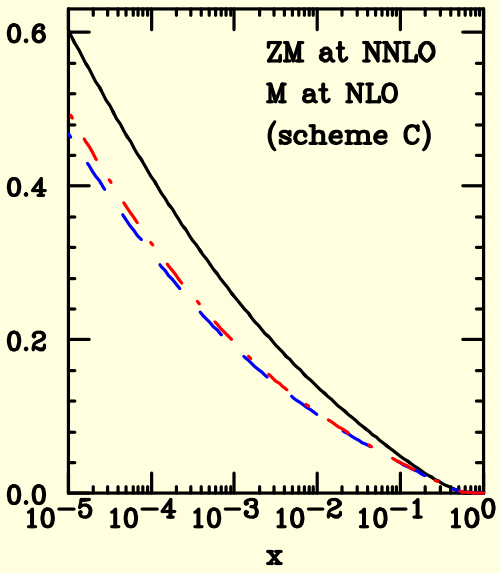

ZM at NLO
M at NLO
(scheme B)
x

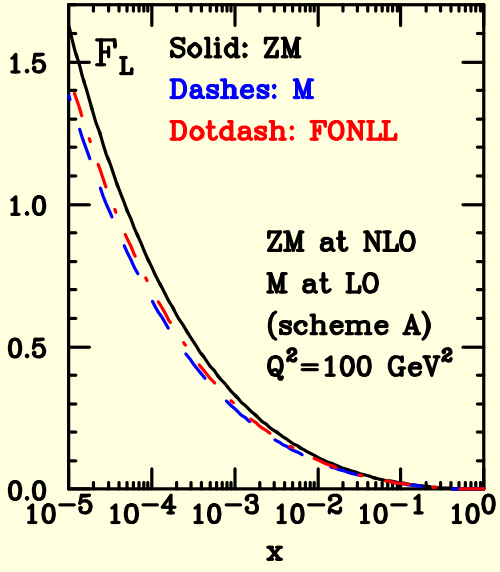

ZM at NNLO
M at NLO
(scheme C)
x

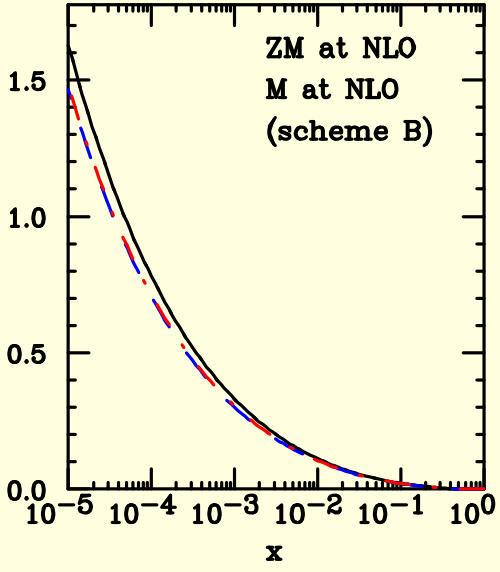

F_L
Solid: ZM
Dashes: M
Dotdash: FONLL
ZM at NLO
M at LO
(scheme A)
Q^2=2 GeV^2
ZM at NLO
M at NLO
(scheme B)
ZM at NNLO
M at NLO
(scheme C)
Q^2=4 GeV^2
x

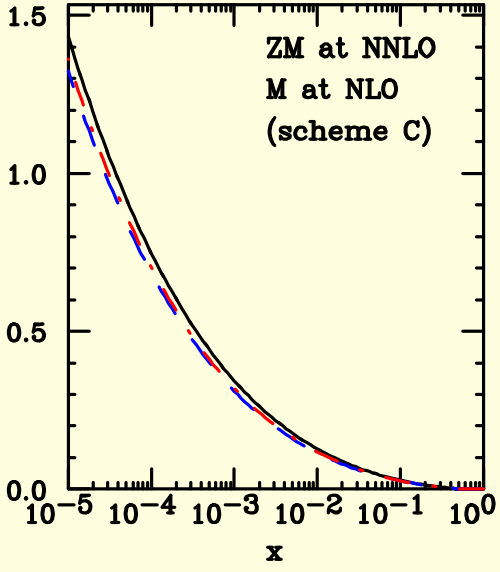

F_L
Solid: ZM
Dashes: M
Dotdash: FONLL
ZM at NLO
M at LO
(scheme A)
Q^2=10 GeV^2
x


ZM at NLO
M at NLO
(scheme B)
x


ZM at NNLO
M at NLO
(scheme C)
x


F_L
Solid: ZM
Dashes: M
Dotdash: FONLL
ZM at NLO
M at LO
(scheme A)
Q^2=100 GeV^2
x


ZM at NLO
M at NLO
(scheme B)
x


ZM at NNLO
M at NLO
(scheme C)
x

Mass ambiguities in FONLL

In $\sigma_{\rm FONLL} = (\sigma_{\overline{\rm MS}} - \tilde{\sigma}_0^o) + \tilde{\sigma}$, we always have the freedom to introduce mass dependent modifications of the round bracket, that are suppressed by powers of the mass. For example, in DIS:

$$(\sigma_{\overline{\rm MS}} - \tilde{\sigma}_0^o) + \tilde{\sigma} \Rightarrow (\sigma_{\overline{\rm MS}} - \tilde{\sigma}_0^o)\, f_{\rm thr}(Q) + \tilde{\sigma}\,,$$

as long as $f_{\rm thr}(Q) \to 1$ for $Q \gg m_h$. (For $Q \ll m$, $\sigma_{\overline{\rm MS}} - \tilde{\sigma}_0^o$ is of higher order with respect to the required accuracy. It is thus totally undetermined.)

In the case of DIS, the value of x in $F^{\overline{\rm MS}} - F^{\rm ov}$ can be rescaled:
$x \to \chi_h = x(1 + 4m_h^2/Q^2)$.

This freedom follows from the facts:

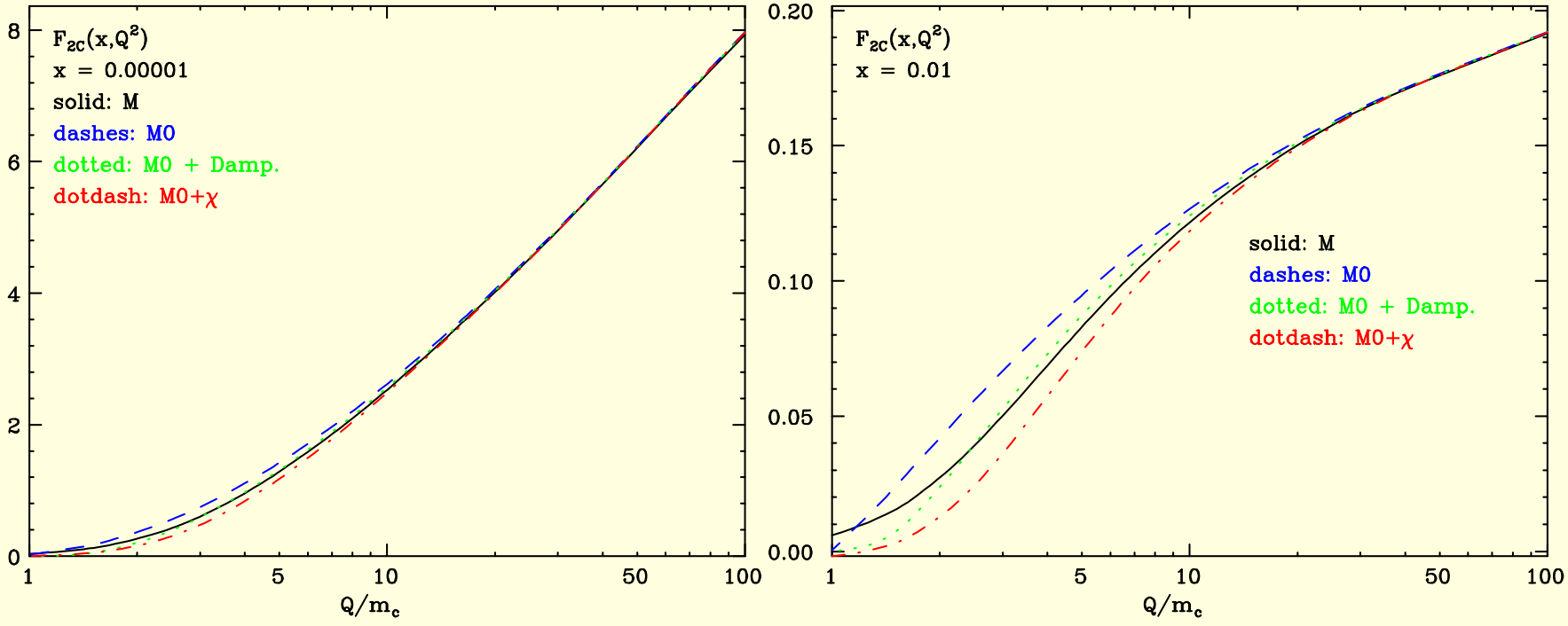
- $\sigma_{\overline{\rm MS}} - \tilde{\sigma}_0^o$ does not contain terms of the same order of those in $\tilde{\sigma}$
- $\sigma_{\overline{\rm MS}} - \tilde{\sigma}_0^o$ is valid only for $Q \gg m_h$

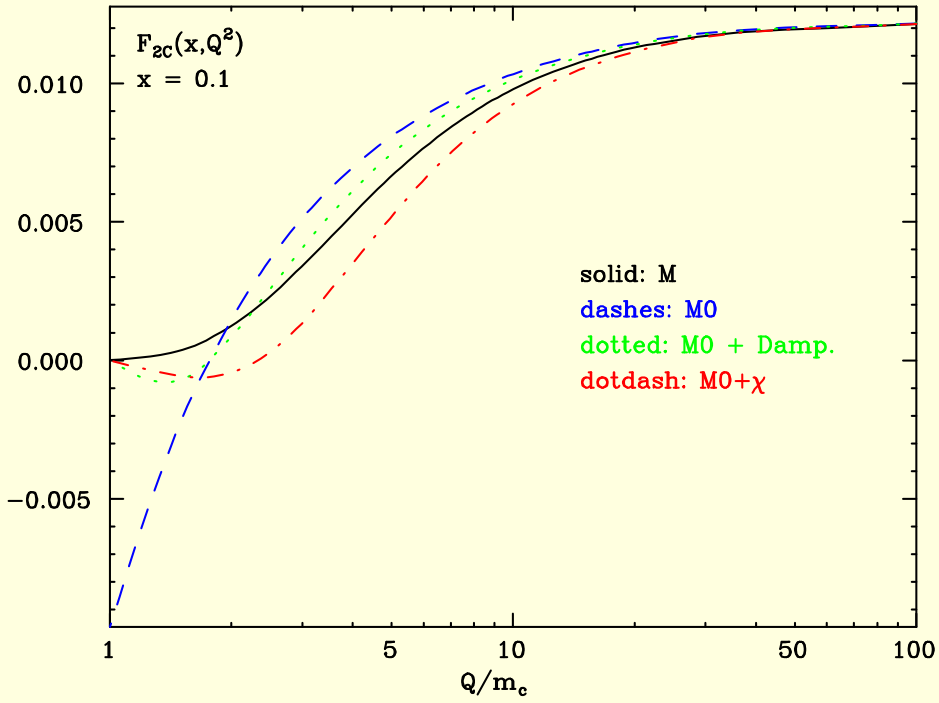
The treatment of terms computed in the massless approximation can be modified by a mass suppressed correction.

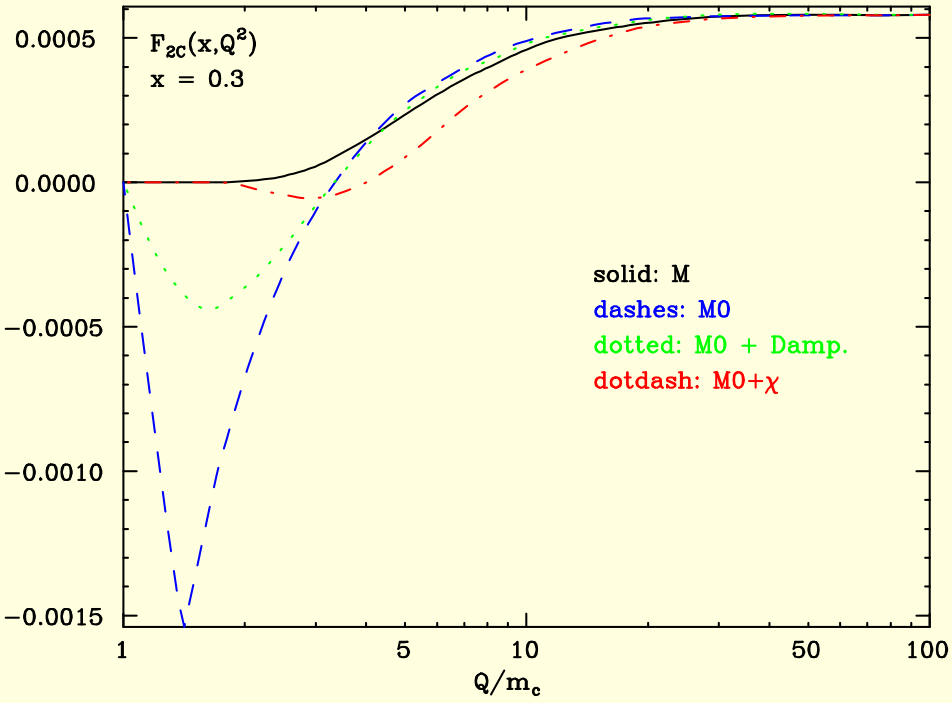
To what extent can the mass suppression mimic the real mass effect?

Compare massless limit with threshold prescription with massive result;

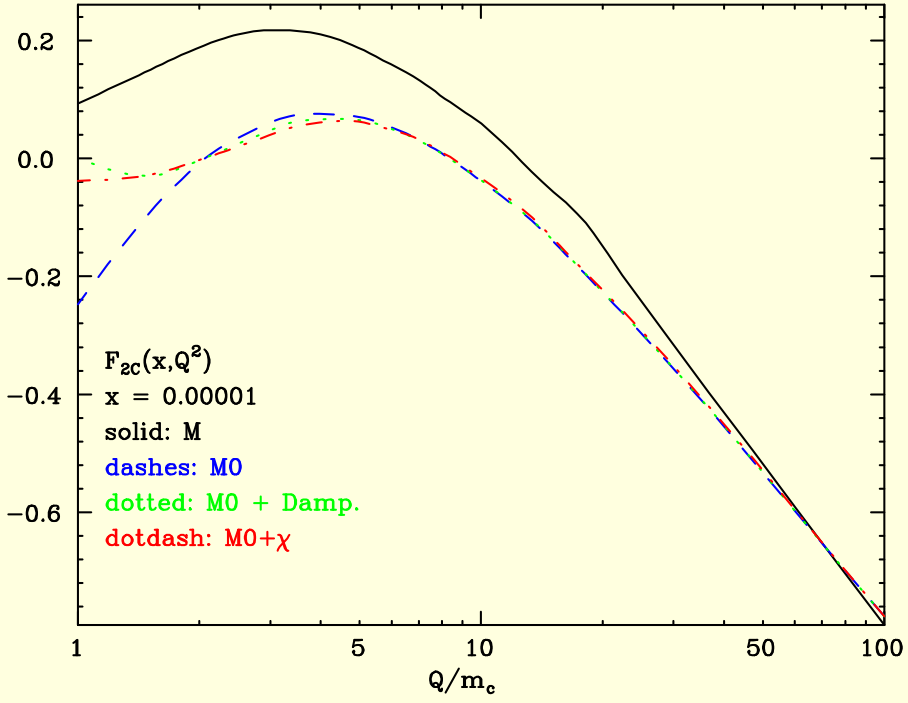
at $\mathcal{O}(\alpha_s)$:

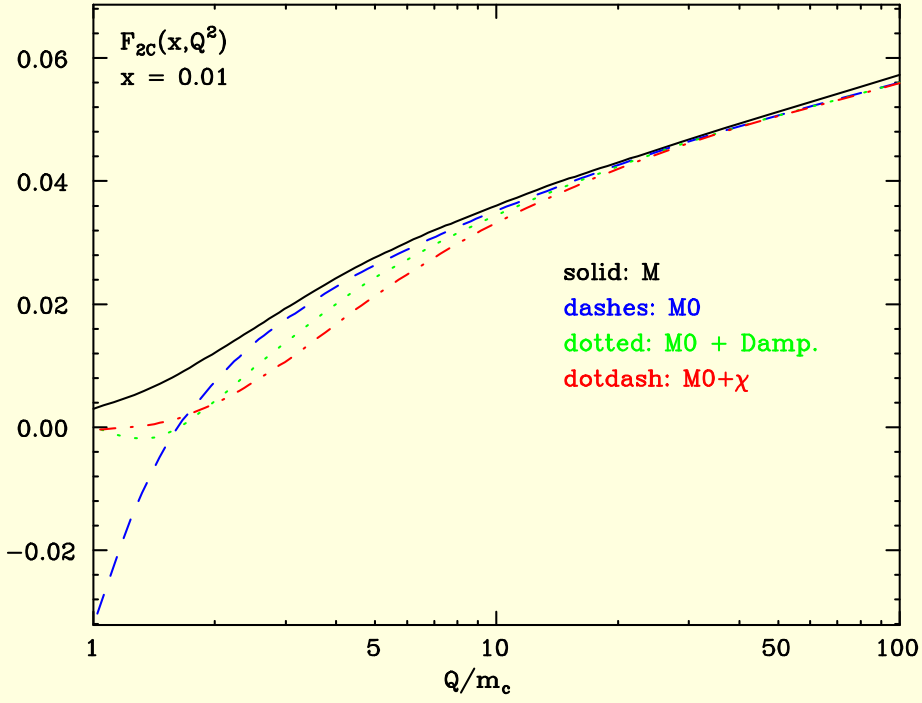


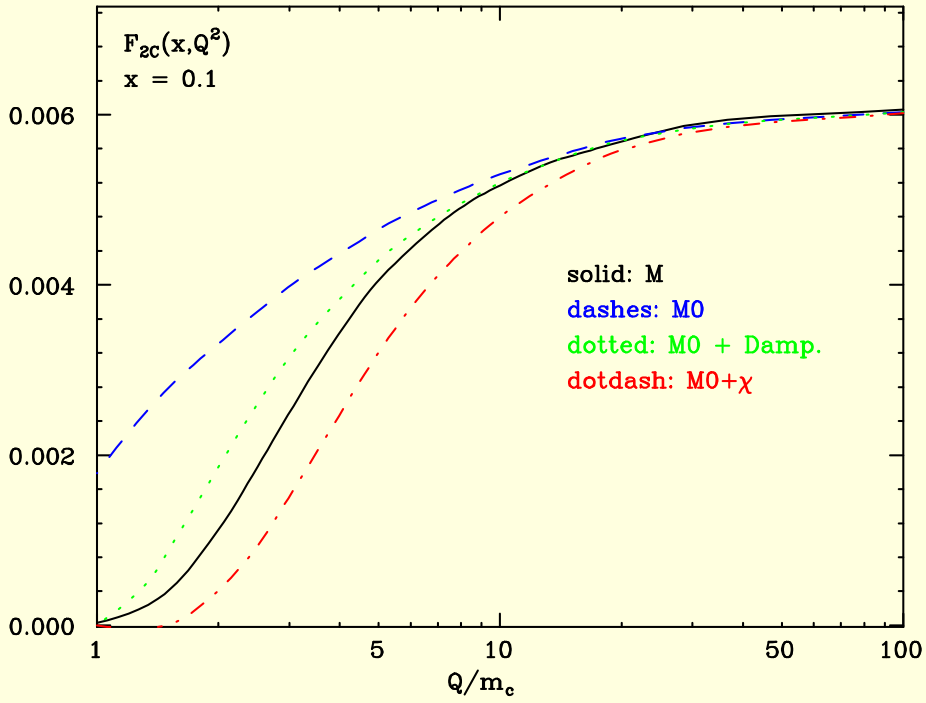

$F_{2c}(x,Q^2)$
x = 0.1
0.010
0.005
0.000
−0.005
1
5
10
50
100
Q/m$_c$
solid: M
dashes: M0
dotted: M0 + Damp.
dotdash: M0+χ

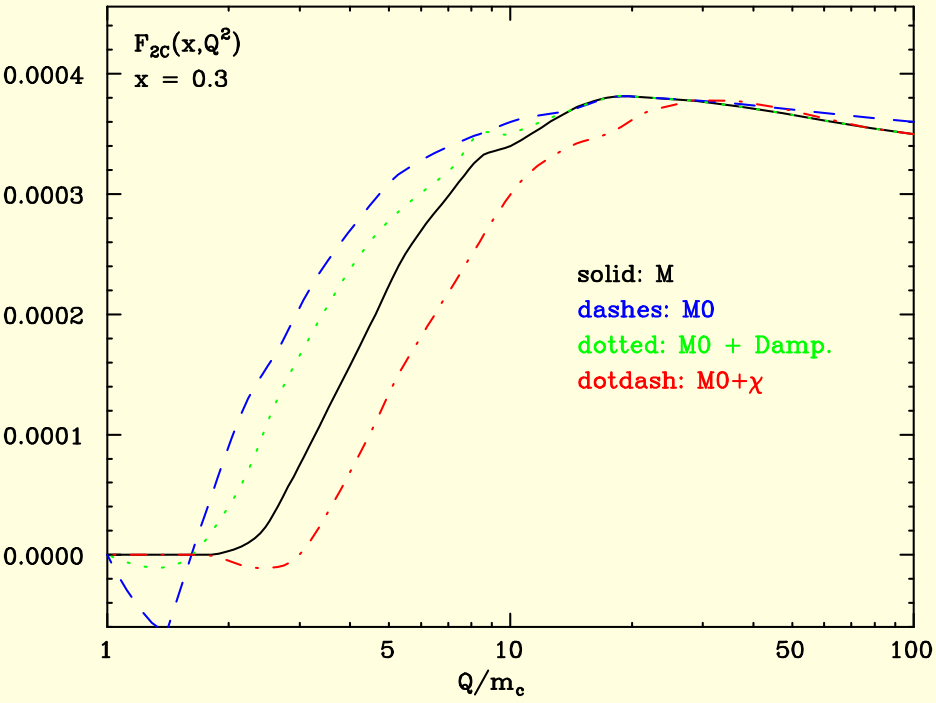

$F_{2c}(x,Q^2)$
x = 0.3
0.0005
0.0000
−0.0005
−0.0010
−0.0015
1
5
10
50
100
Q/m$_c$
solid: M
dashes: M0
dotted: M0 + Damp.
dotdash: M0+χ

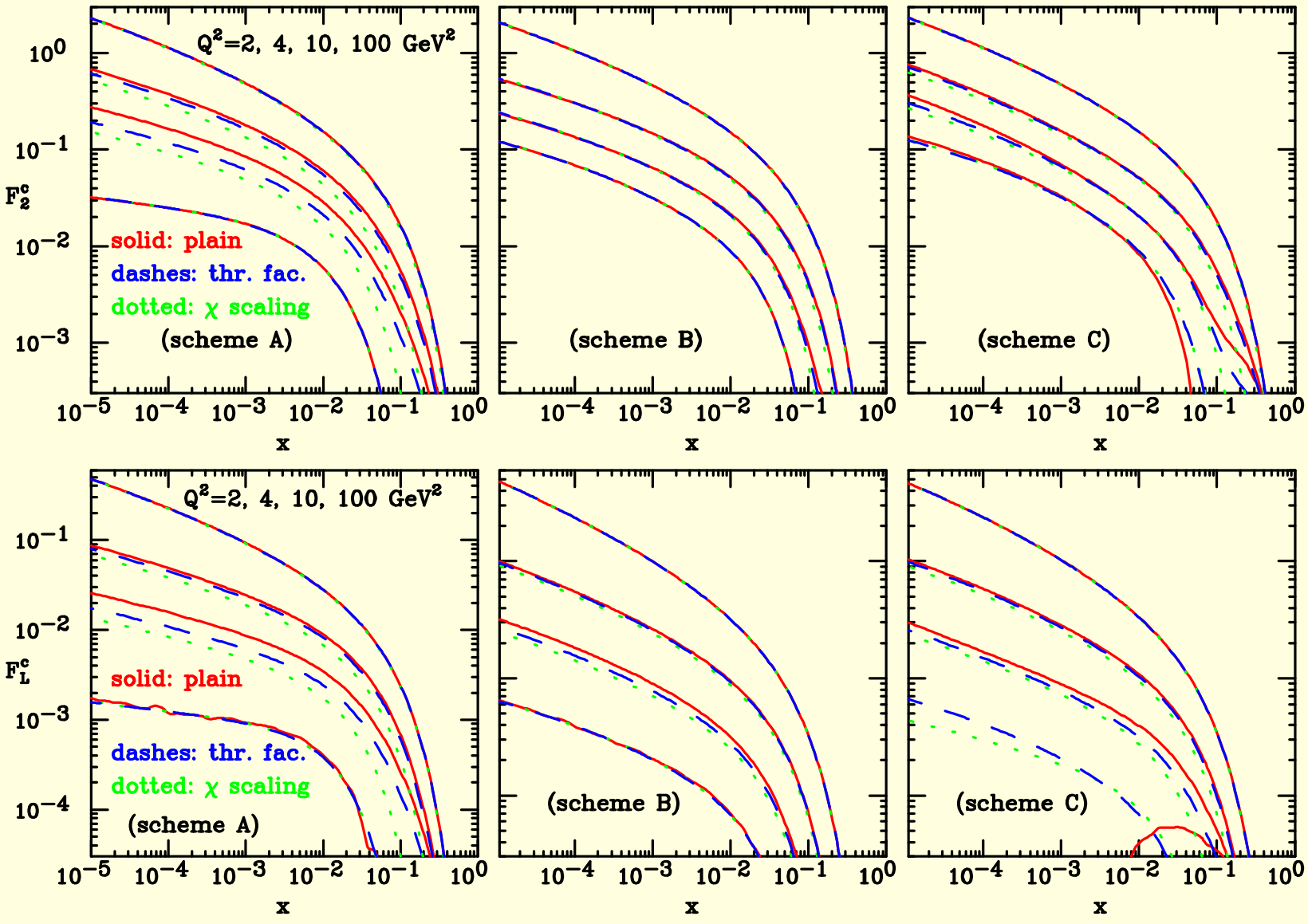
At $\mathcal{O}(\alpha_s^2)$:

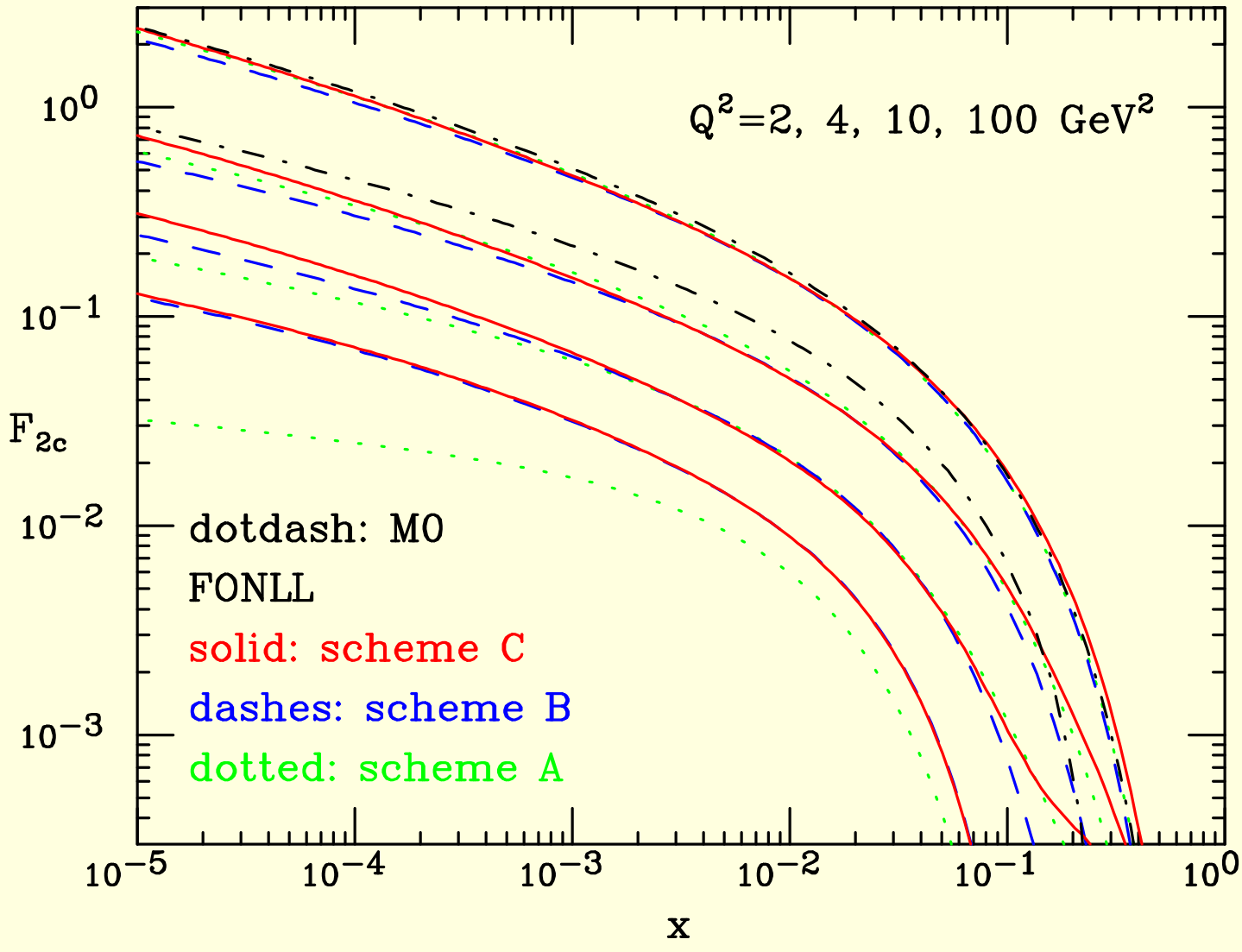


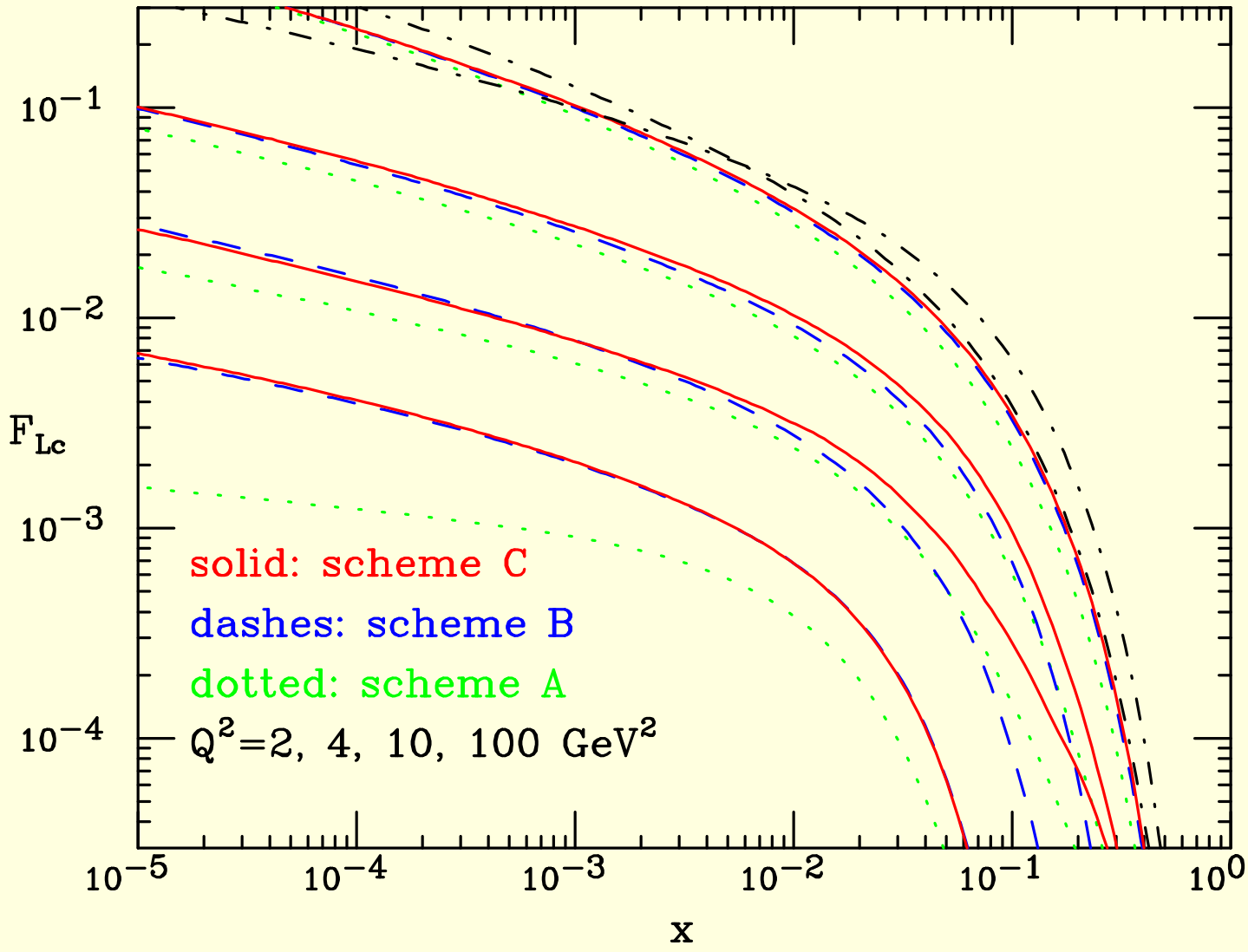



$F_{2c}(x,Q^2)$
x = 0.1
0.006
0.004
0.002
0.000
1
5
10
50
100
Q/m_c
solid: M
dashes: M0
dotted: M0 + Damp.
dotdash: M0+χ


$F_{2c}(x,Q^2)$
x = 0.3
0.0004
0.0003
0.0002
0.0001
0.0000
1
5
10
50
100
Q/m_c
solid: M
dashes: M0
dotted: M0 + Damp.
dotdash: M0+χ


Q^2=2, 4, 10, 100 GeV^2
F_2^c
solid: plain
dashes: thr. fac.
dotted: χ scaling
(scheme A)
(scheme B)
(scheme C)
x
F_L^c


Q^2=2, 4, 10, 100 GeV^2
dotdash: M0
FONLL
solid: scheme C
dashes: scheme B
dotted: scheme A
F_{2c}
x
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^0


10^{-1}
10^{-2}
10^{-3}
10^{-4}
F_{Lc}
solid: scheme C
dashes: scheme B
dotted: scheme A
Q^2=2, 4, 10, 100 GeV^2
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
x

Relation of FONLL to previous schemes

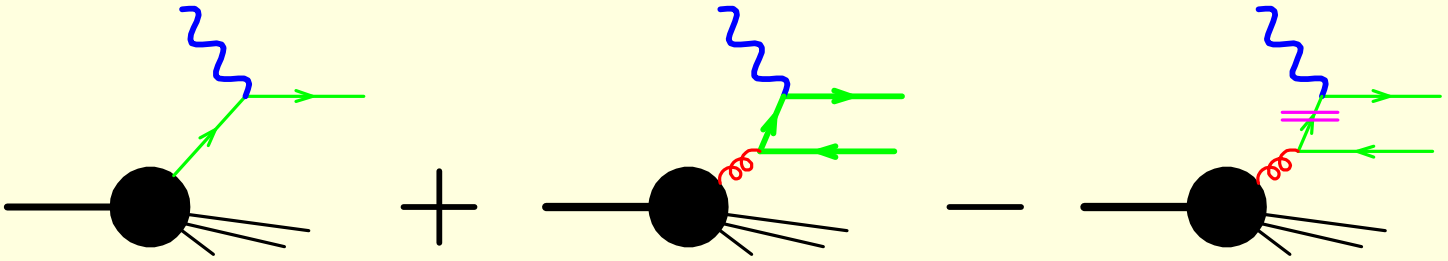
- Scheme A identical to SACOT. If the χ-scaling prescription (in the form used by the CTEQ group) is used, it is identical to SACOT-χ.

- Scheme B: no analogue in ACOT. Reminiscent of TR scheme (but not identical to it

- Scheme C: should be again equivalent to SACOT (no full discussion of ACOT at NNLO has appeared in the literature).

Note:

S-ACOT (for Simplified ACOT, Kräemer, Olness, Soper, 2000).
Variant over ACOT, exploiting freedom in the choice of mass effects.
Use mass only in heavy quark lines not coming from the hadron

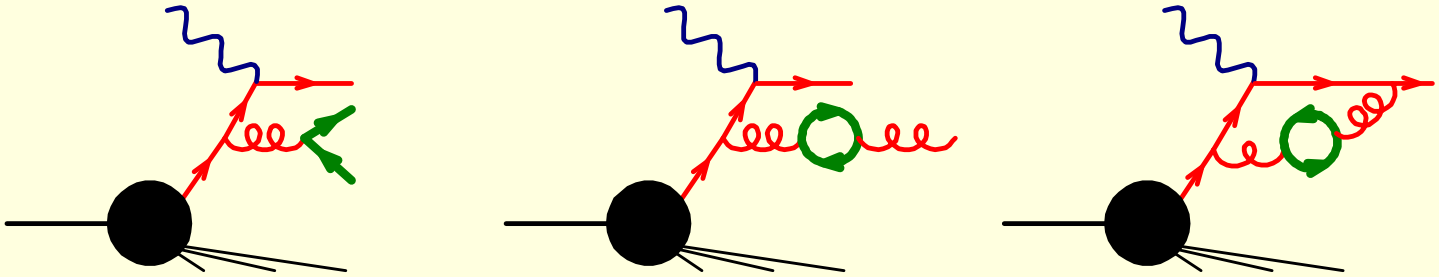


Modern CTEQ and MRST implementations use S-ACOT
(plus: χ-scaling, S-ACOT-χ).

Measured F_c versus computed F_c

The theoretical definition of F_c (i.e. the structure function will all electric charges of light quarks switched off) is not experimentally accessible.

Howeve, one can define an experimental $\tilde{F}_c$, by requiring tha charm is present in the final state. The difference between the two is due to the graphs



All of them contribute to F_c, but only the first contributes to $\tilde{F}_c$.

The first graph behave as $\log^3 Q/m$ for small m, while these large logs cancel in the total. At higher order, diagrams where the gluon emits more gluons before splitting are double-log enhanced (multiplicity logs, Mangano, P.N. 92)

These effects, although potentially enhanced at high Q^2, have been shown to be at the percent level (and thus negligible) in the HERA range (FNPR2009).

Final considerations

- Matched calculations are easy to implement; no escuse to leave out mass effects or large logs from DIS fits.
- Proposals to use kinematic procedure to fudge mass effects can be made to work (Tung, Nadolski, 2009). Can be useful, but should not replace exact methods. Also: not universal, i.e. confined to DIS.
- Much to understand on the real impact of mass effects and large logarithms in hadron collisions.
  Matched calculations can help to clarify the problem.

Topics not discussed

- Which mass (pole, $\overline{\rm MS}$, etc.)
- Logs in mass effects (i.e. higher twist evolution)
- Intrinsic charm (non perturbative effects in charm initial condition)