Status of POWHEG

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RADCOR 2009, Ascona, 27/10/2009

POWHEG

Positive Weight Hardest Emission Generator

A method for interfacing NLO calculations with Shower Monte Carlo's (SMC)

It generates the hardest emission first, with NLO accuracy, independently of the SMC (P.N. 2004), and (as the name says) with positive weights.

Motivations for NLO+SMC

Comparisons of data with NLO result require now correcting for:

- Detector effects
- Underlying event
- Hadronization

All these effects are estimated using a SMC generator;

None of this is needed if NLO+SMC is implemented.

Status of POWHEG

Up to now, the following processes have been implemented in POWHEG:

- $hh \to ZZ$ (Ridolfi, P.N., 2006)
- $e^+e^- \to \mathrm{hadrons}$, (Latunde-Dada,Gieseke,Webber, 2006), $e^+e^- \to t\bar{t}$, including top decays at NLO (Latunde-Dada,2008),
- $hh \to Q\bar{Q}$ (Frixione, Ridolfi, P.N., 2007)
- $hh \to Z/W$ (Alioli, Oleari, Re, P.N., 2008;)
  (Hamilton,Richardson,Tully, 2008;)
- $hh \to H$ (gluon fusion) (Alioli, Oleari, Re, P.N., 2008)
- $hh \to H$, $hh \to HZ/W$ (Hamilton,Richardson,Tully, 2009;)
- $hh \to t + X$ (single top) NEW (Alioli, Oleari, Re, P.N., 2009)
- $hh \to Z + \mathrm{jet}$, Preliminary (Alioli, Oleari, Re, P.N., 2009)
- VBF Higgs, Preliminary, (Oleari,P.N., 2009).
- The POWHEG BOX, Preliminary, (Alioli, Oleari, Re, P.N., 2009)

Outline

- What NLO+SMC calculations do in general
- Examples: `MC@NLO` and `POWHEG`
- Towards automation: the `POWHEG BOX`.
- Perspectives and Conclusions

NLO+SMC basics

Hardest emission in a Shower Monte Carlo

For illustration: assume there is only one radiating line.
SMC formula for hardest emission (P.N. 2004):

$$d\sigma = B(\Phi_B) d\Phi_B \left[\Delta_{t_0}^{\mathrm{MC}} + \underbrace{\Delta_t^{\mathrm{MC}} \frac{R^{\mathrm{MC}}(\Phi)}{B(\Phi_B)} d\Phi_r^{\mathrm{MC}}}_{d\,\Delta_t^{\mathrm{MC}}} \right]$$

- t is the radiation transverse momentum
- $B(\Phi_B)d\Phi_B$: Born differential cross section
- $\Delta_{t_0}^{\mathrm{MC}}$: No radiation probability down to the cutoff t_0
- Δ_t^{MC}: No radiation probability down to the scale t
- $R^{\mathrm{MC}} d\Phi_r^{\mathrm{MC}}$: SMC's real cross section, $\approx B \, \frac{1}{t} \, \frac{\alpha(t)}{2\pi} P(z) \, dz \, dt \, \frac{d\phi}{2\pi}$

$$\Delta_{t_l}^{\mathrm{MC}} = \exp\left[-\int_{t_l}^{t_h} \frac{dt'}{t'} \int dz \, \frac{\alpha_s(t')}{2\pi} \, P(z) \right] = \exp\left[-\int_{t_l} \frac{R^{\mathrm{MC}}}{B} d\Phi_r^{\mathrm{MC}} \right]$$

Hardest emission in NLO+SMC: must be NLO accurate

It has the form:

$$d\sigma = \bar{B}^s(\Phi_B)d\Phi_B\left[\Delta^s_{t_0} + \Delta^s_t \frac{R^s(\Phi)}{B(\Phi_B)}d\Phi_r\right] + \left[R(\Phi) - R^s(\Phi)\right]d\Phi$$

where $R \Rightarrow R^s$ in the soft and collinear limit,

$$\bar{B}^s(\Phi_B) = B(\Phi_B) + \underbrace{\left[\underbrace{V(\Phi_B)}_{\text{infinite}} + \underbrace{\int R^s(\Phi)\,d\Phi_r}_{\text{infinite}}\right]}_{\text{finite}}$$

Imagine that soft and collinear singularities in R^{MC} are regulated as in V.

and

$$\Delta^s_t = \exp\left[-\int_{t_l} \frac{R^s}{B} d\Phi_r \theta(t(\Phi) - t_l)\right]$$

Accuracy

Small t: $\frac{R_s(\Phi)}{B(\Phi_{_B})} d\Phi_{\rm rad} \approx \frac{\alpha_s(t)}{2\pi} P(z) \frac{dt}{t} dz \frac{d\phi}{2\pi}$,

Also: $\bar{B} \approx B \times (1 + \mathcal{O}(\alpha_s))$

Thus: all features of SMC's are preserved at small t.

Large t: $\Delta \to 1, \quad d\sigma = \bar{B} \times \frac{R_s}{B} d\Phi + (R - R_s) d\Phi \approx R\, d\Phi$,
so: large t accuracy is preserved.

NLO accuracy: since $\Delta_{t_0} + \int \ \Delta_t \frac{R_s(\Phi)}{B(\Phi_{_B})} d\Phi_r = 1$, integrating in $d\Phi_r$ at fixed $\Phi_{_B}$

$$\int \delta(\Phi_{_B} - \bar{\Phi}_{_B})\, d\sigma = \left[\bar{B} + \int \ (R - R_s)\, d\Phi_r\right]_{\Phi_{_B} = \bar{\Phi}_{_B}} = \left[B + V + \int R d\Phi_r\right]_{\Phi_{_B} = \bar{\Phi}_{_B}}$$

So: NLO accuracy is preserved for inclusive quantities.

In MC@NLO: $R^s d\Phi_r = R^{\rm MC} d\Phi_r^{\rm MC}$

Furthermore: the phase space parametrization $\Phi_B, \Phi_r \Rightarrow \Phi$ is the one of the Shower Monte Carlo. We have:

$$\underbrace{\bar{B}^s(\Phi_B) d\Phi_B}_{\substack{\text{provided by MCatNLO} \\ \mathcal{S}\,\text{event}}} \left[\underbrace{\Delta^s_{t_0} + \Delta^s_t \frac{R^s(\Phi)}{B(\Phi_B)} d\Phi_r}_{\text{generated by HERWIG}} \right] + \underbrace{\left[R(\Phi) - R^s(\Phi)\right] d\Phi}_{\substack{\text{provided by MCatNLO} \\ \mathcal{H}\,\text{event}}}$$

Recipe for MC@NLO

- Compute total cross section for $\mathcal{S}$ and $\mathcal{H}$ events:

$$\sigma_S = \int |\bar{B}^{\rm MC}(\Phi_B)|\, d\Phi_B, \;\; \sigma_H = \int |R - R^{\rm MC}|\, d\Phi$$

- Chose an $\mathcal{S}$ or $\mathcal{H}$ event with probability proportional to σ_S, σ_H
- For an $\mathcal{S}$ event:
  - generate Born kinematics with probability

$$|\bar{B}^{\rm MC}(\Phi_B)| = \left| B(\Phi_B) + \left[V(\Phi_B) + \int R^{\rm MC}(\Phi)\, d\Phi_r^{\rm MC} \right] \right|$$

  - Feed the Born kinematics to the MC for subsequent shower with weight ± 1, same sign as $\bar{B}^{\rm MC}(\Phi_B)$ (mostly $+1$).
- For an H event:
  - generate Radiation kinematics with probability $|R - R^{\rm MC}|$.
  - Feed to the MC (with weight ± 1, same sign as $R - R^{\rm MC}$)

Issues:

- Must use of the MC kinematic mapping $(\Phi_B, \Phi_r^{\mathrm{MC}}) \Rightarrow \Phi$.
- $R - R^{\mathrm{MC}}$ must be non singular: the MC must reproduce exactly the soft and collinear singularities of the radiation matrix element. (Many MC's are not fully accurate in the soft limit)
- Negative weights in the output (not like standard MC's).

In POWHEG: $R^s d\Phi_r = R F(\Phi)$

where $0 \leqslant F(\Phi) \leqslant 1$, and $F(\Phi) \Rightarrow 1$ in the soft or collinear limit.
$F(\Phi) = 1$ is also possible, and often adopted.
The parametrization $\Phi_B, \Phi_r \Rightarrow \Phi$ is within POWHEG, and there is complete freedom in its choice.

$$\underbrace{\bar{B}^s(\Phi_B) d\Phi_B}_{\texttt{POWHEG}} \left[\underbrace{\Delta^s_{t_0} + \Delta^s_t \frac{R^s(\Phi)}{B(\Phi_B)} d\Phi_r}_{\texttt{POWHEG}} \right] + \underbrace{\left[R(\Phi) - R^s(\Phi)\right] d\Phi}_{\texttt{POWHEG}}$$

All the elements of the hardest radiation are generated within POWHEG

Recipe

- POWHEG generates an event, with $t = t_{\texttt{powheg}}$
- The event is passed to a SMC, imposing no radiation with $t > t_{\texttt{powheg}}$.

Improvements over MC@NLO:

- Positive weighted events: $R - R_s = R(F-1) \geqslant 0$!
- Independence on the Shower MC: The hardest emission is generated by POWHEG; less hard emissions are generated by the shower.
- No issues with SMC inaccuracies

MC@NLO and POWHEG yield the exact total NLO cross section;

However, differential distributions are affected by induced higher order terms:

$$d\sigma = d\Phi_{B}\bar{B}\left[\Delta_{t_0} + \Delta_t \frac{R_s}{B} d\Phi_r\right] + (R - R_s)\, d\Phi, \qquad \bar{B} = B + \left[V + \int R_s\, d\Phi_r\right]$$

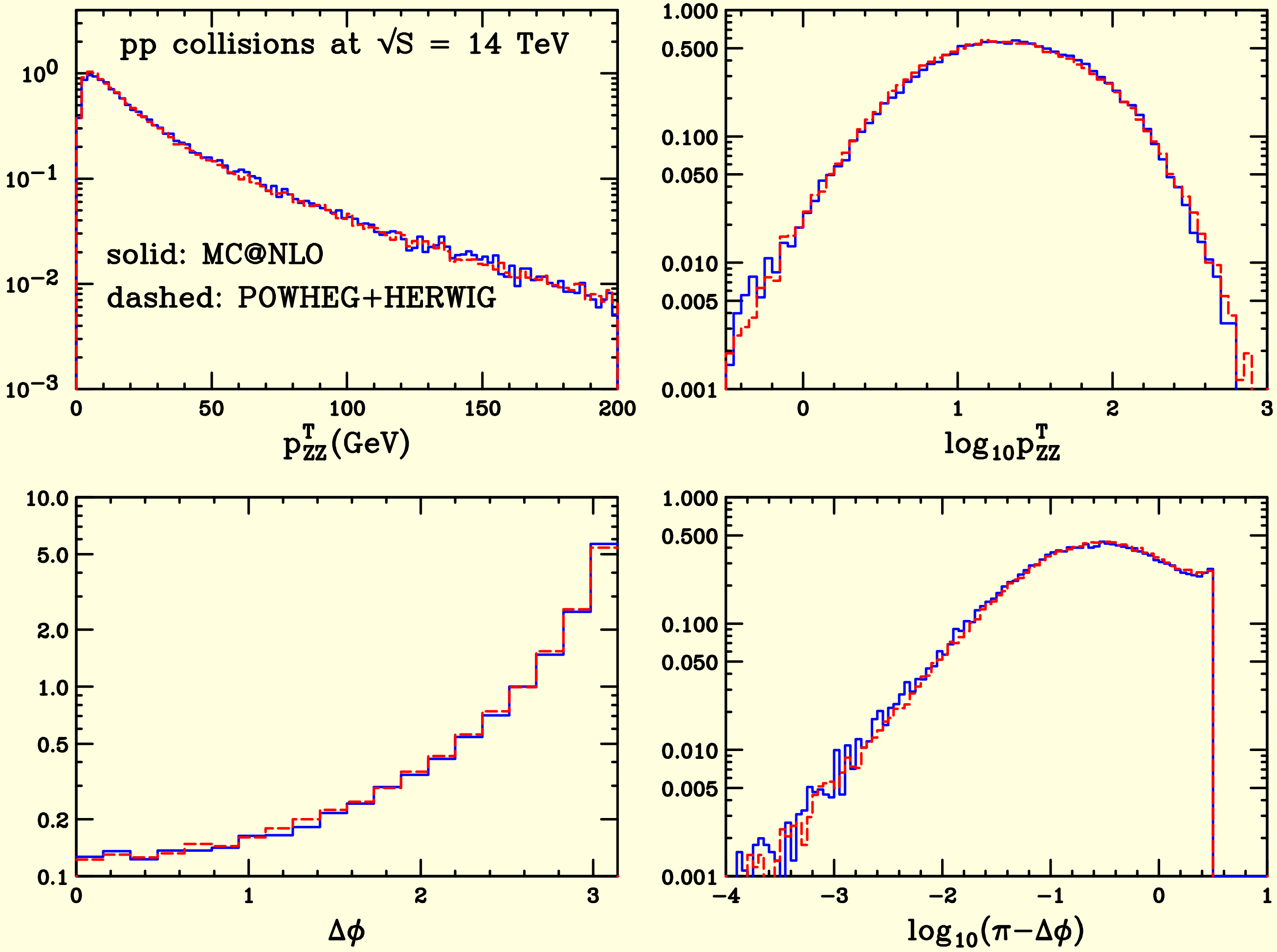
- The expression for $\Delta_{t_1,t} = \exp\left[-\int \frac{R}{B}\, d\Phi_r\, \theta(k_T - t)\right]$ generates terms of all orders, and suppresses the distributions at small p_T.
- Most important: the square bracket term in $\bar{B}$, multiplied by R_s/B, generates NNLO terms. For large t:

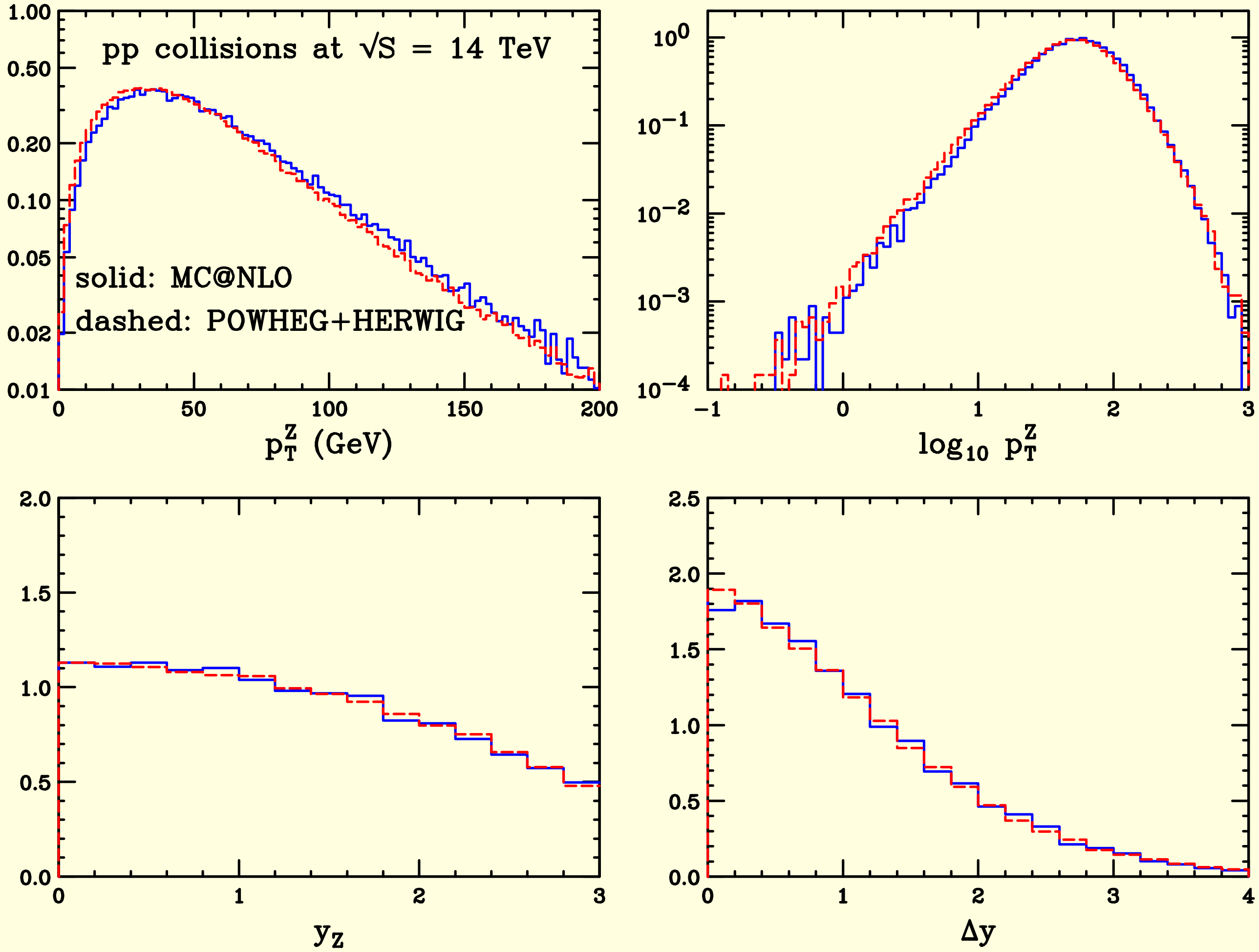
$$d\sigma = d\Phi_{B}\bar{B}\left[\Delta_{t_0} + \Delta_t \frac{R_s}{B} d\Phi_r\right] + (R - R_s)\, d\Phi \Rightarrow \left[\underbrace{\left(\frac{\bar{B}}{B} - 1\right) R_s}_{\mathrm{NNLO}} + R\right] d\Phi$$

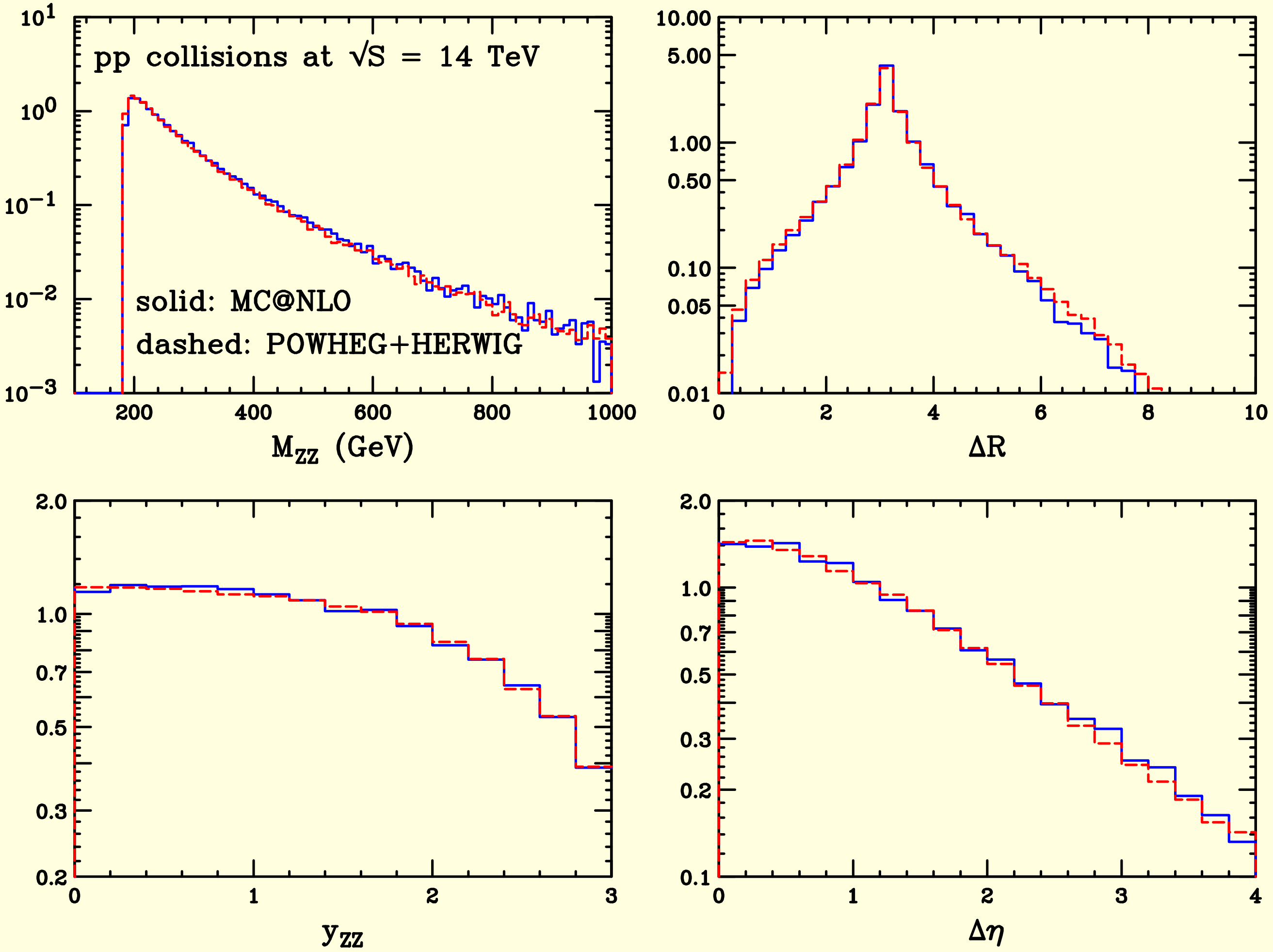
(in NLO corrections are positive, it typically enhances the distributions).

Comparisons of POWHEG+HERWIG vs. MC@NLO

Z pair production

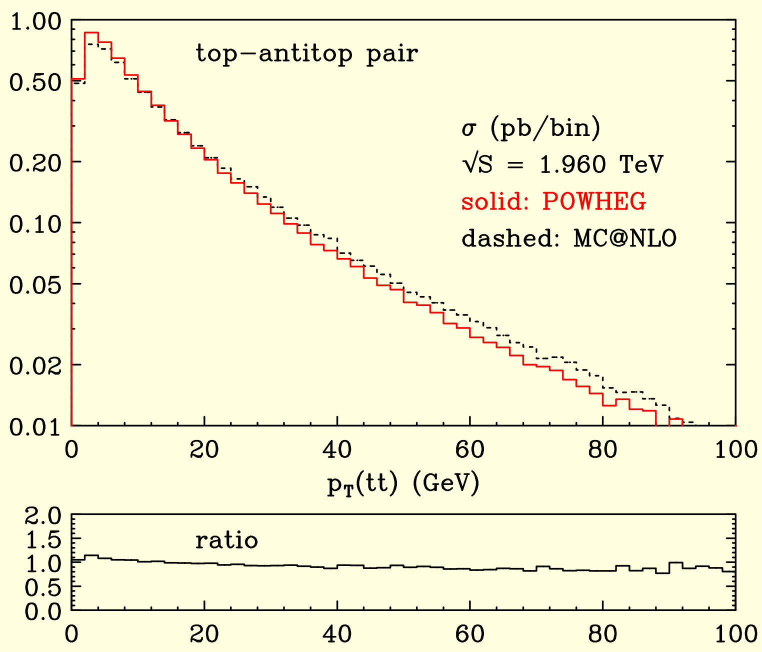


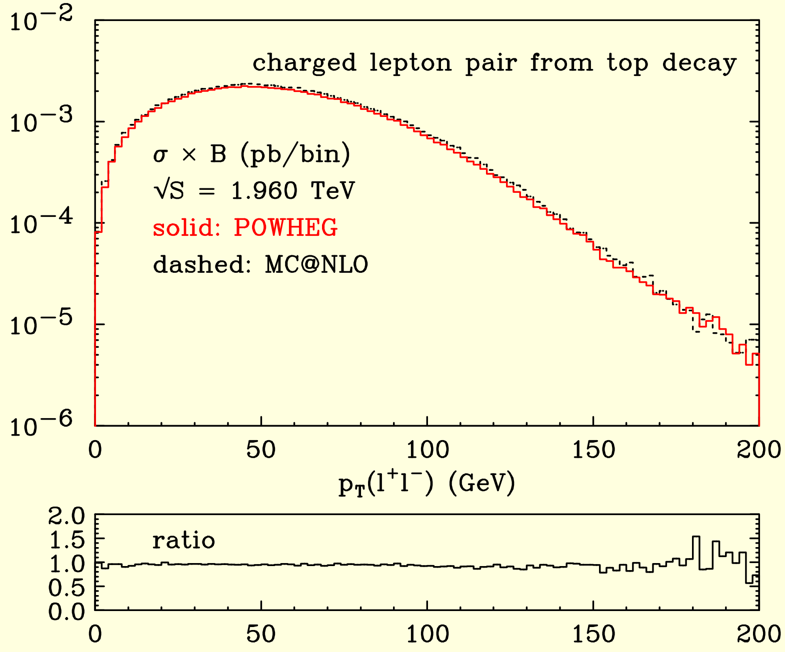

pp collisions at √S = 14 TeV
solid: MC@NLO
dashed: POWHEG+HERWIG
p_T^Z (GeV)
$\log_{10} p_T^Z$
y_Z
Δy



Remarkable agreement for most quantities;

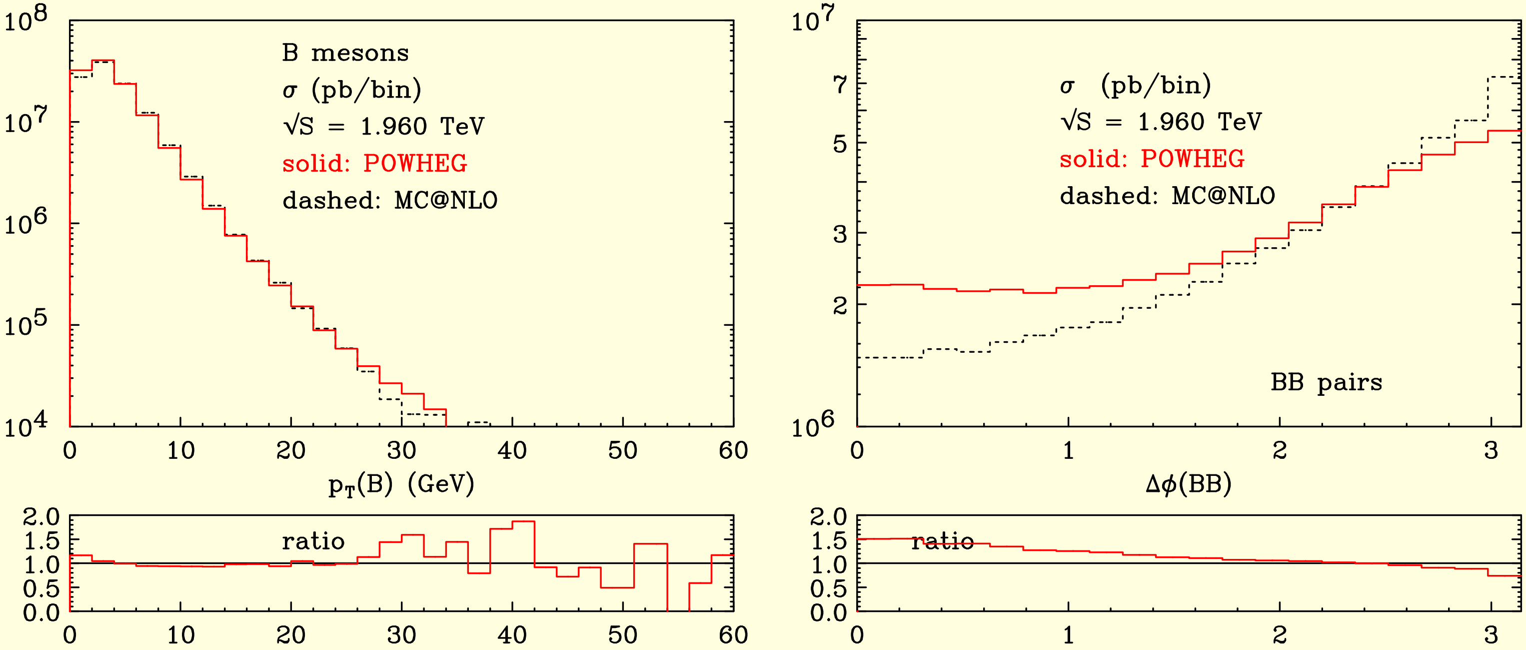
POWHEG and MC@NLO comparison:
Top pair production





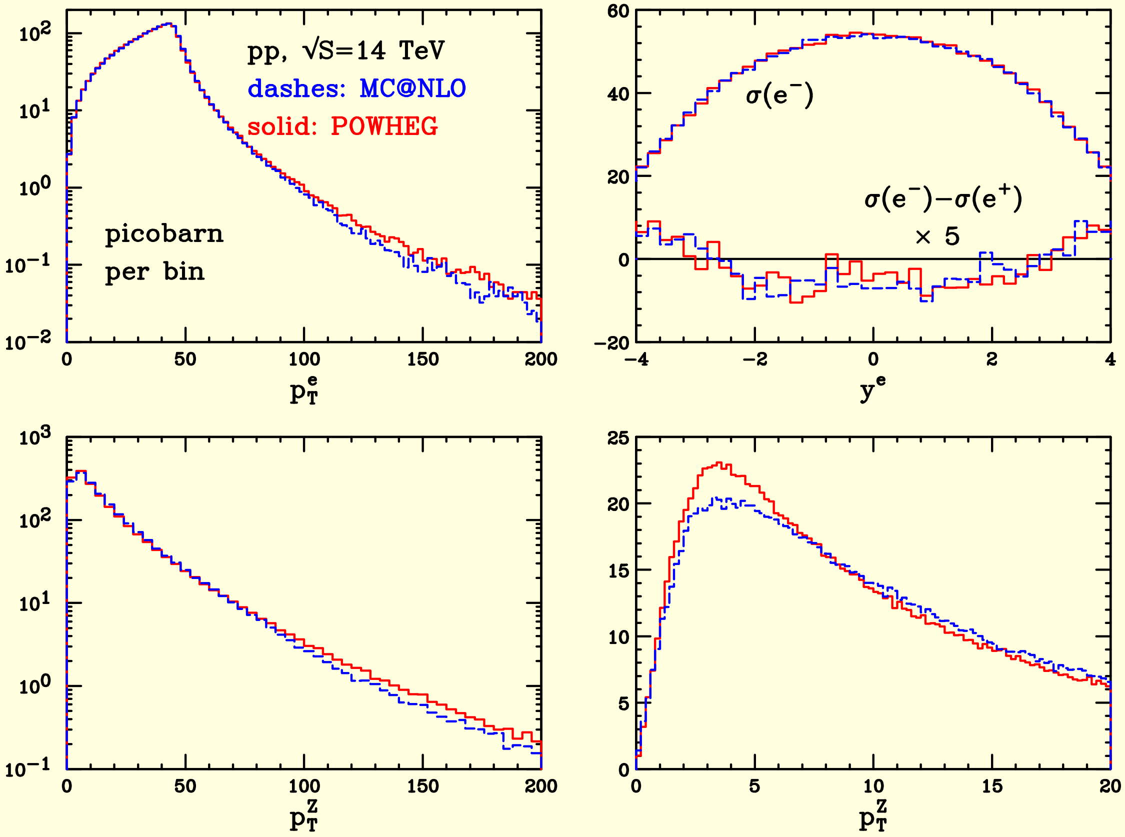
Good agreement for most observables considered
(differences can be ascribed to different treatment of higher order terms)

Bottom pair production



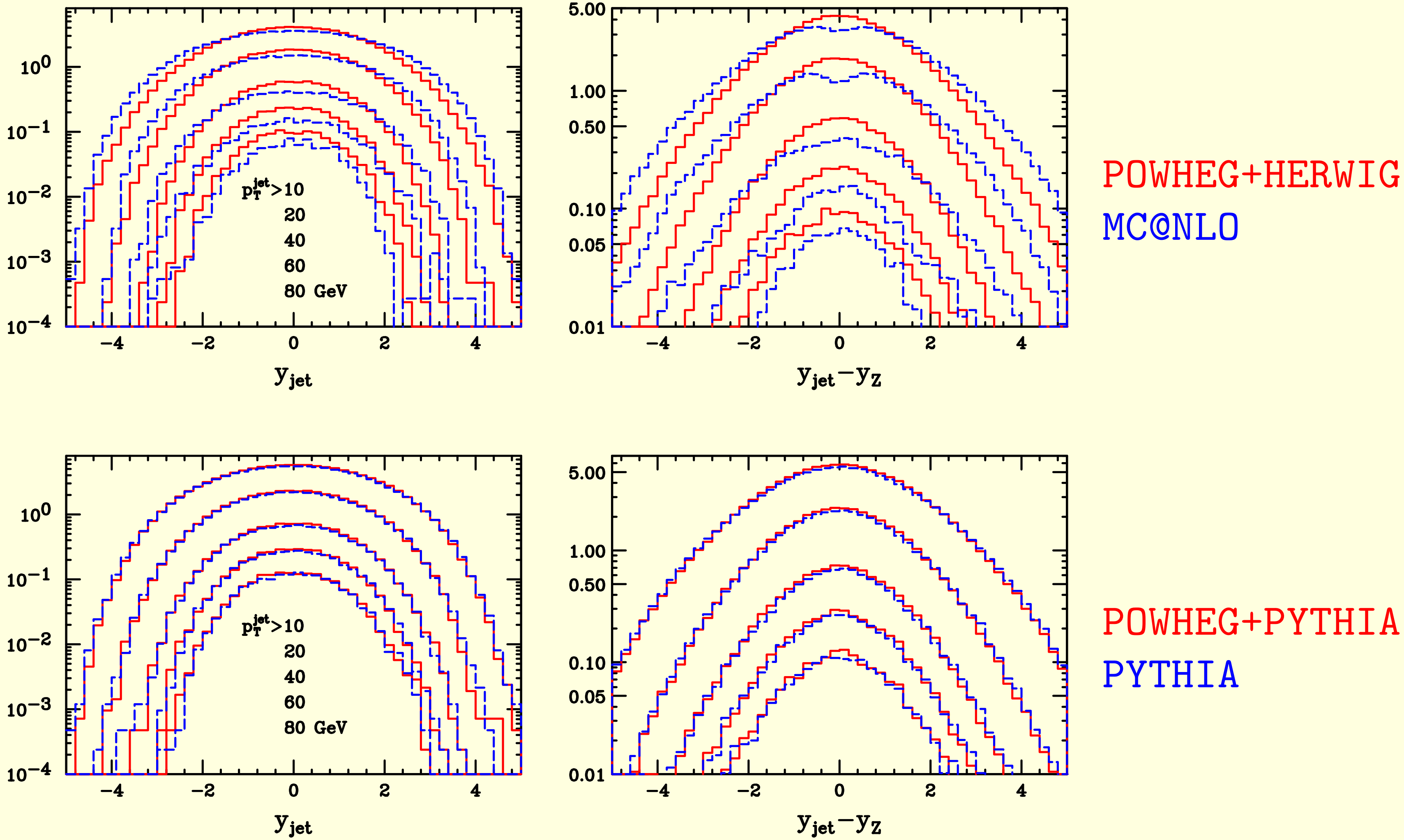
- Very good agreement For large scales (ZZ, $t\bar{t}$ production)
- Differences at small scales ($b\bar{b}$ at the Tevatron)
- `POWHEG` more reliable in extreme cases like $b\bar{b}, c\bar{c}$ at LHC
  (yields positive results, `MC@NLO` has problems with negative weights)

Z production: POWHEG+HERWIG vs. MC@NLO

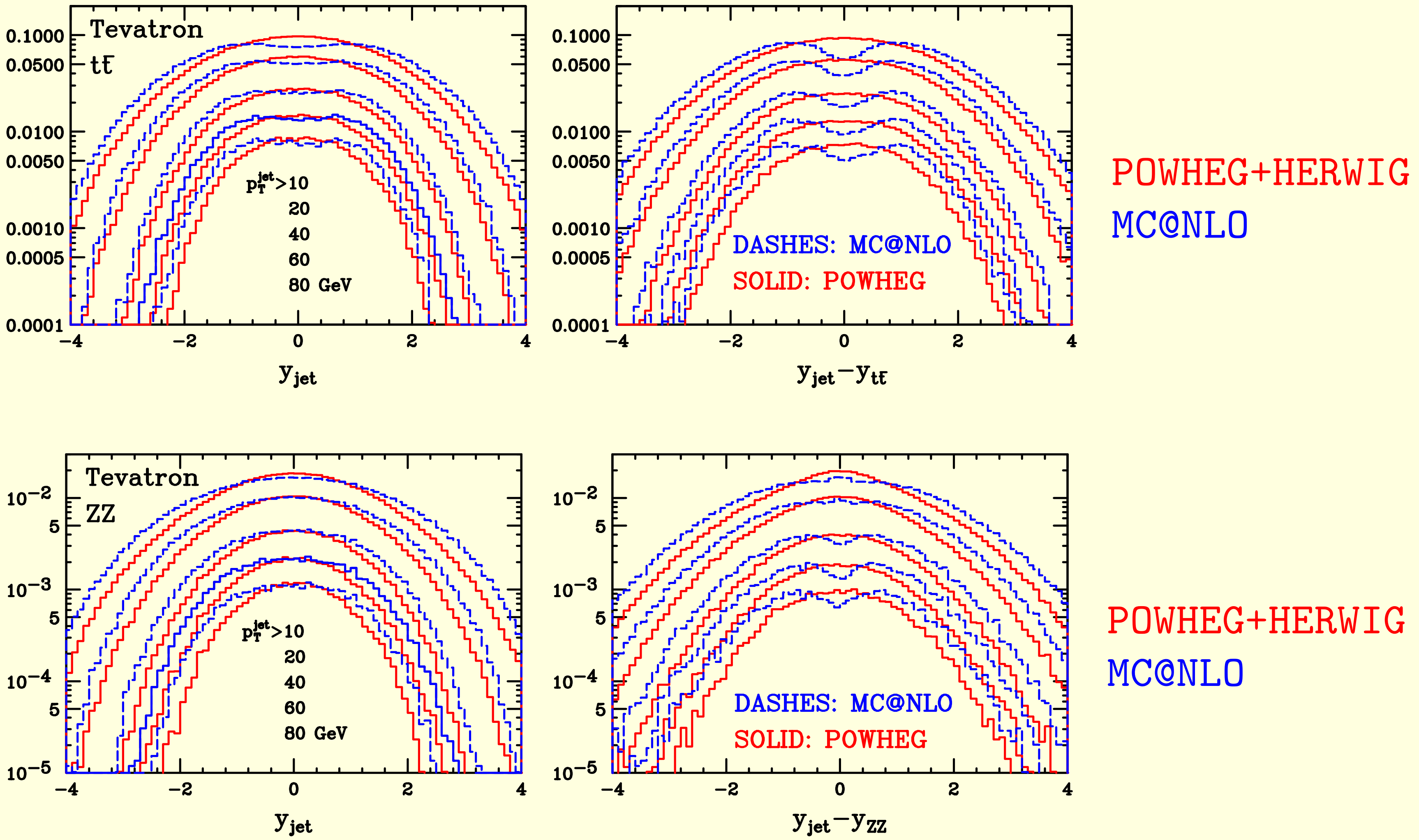


Small differences in high and low p_T region

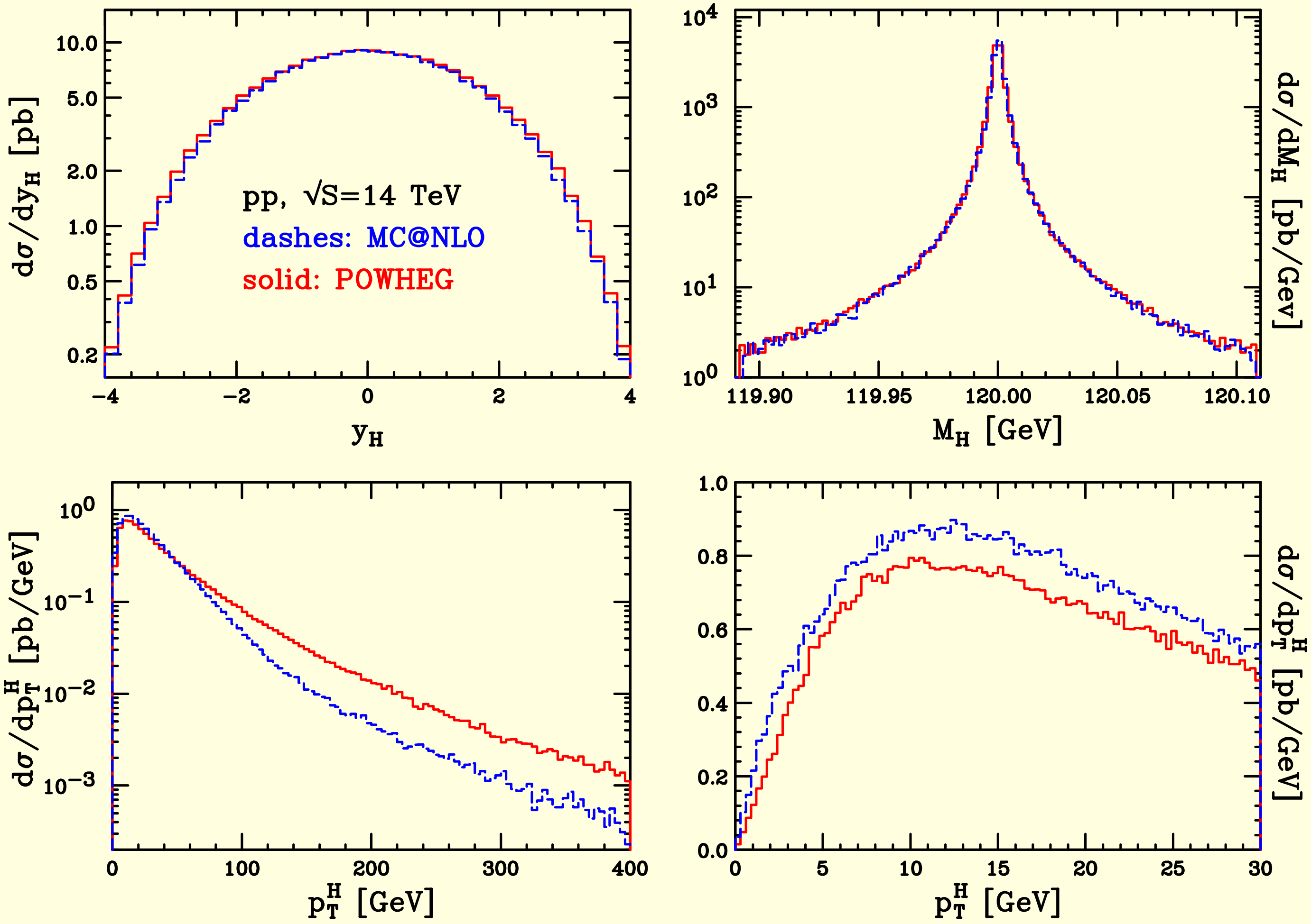
Z production: rapidity of hardest jet (TEVATRON)



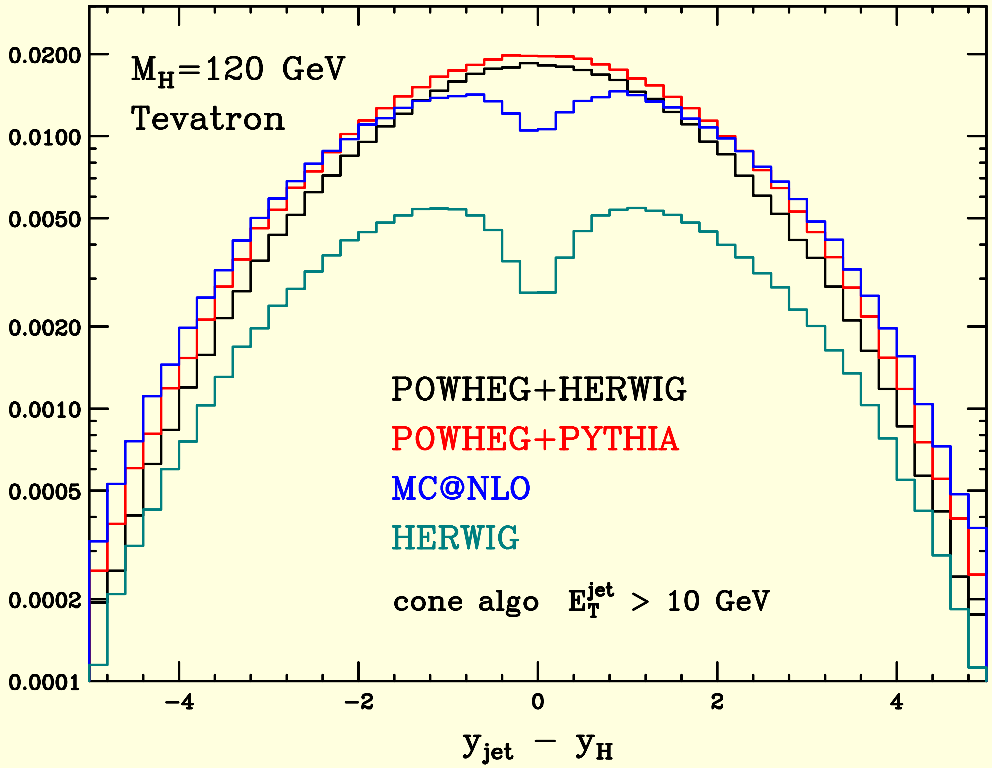
Dip in central region in MC@NLO also in $t\bar{t}$ and ZZ



Higgs boson via gluon fusion at LHC



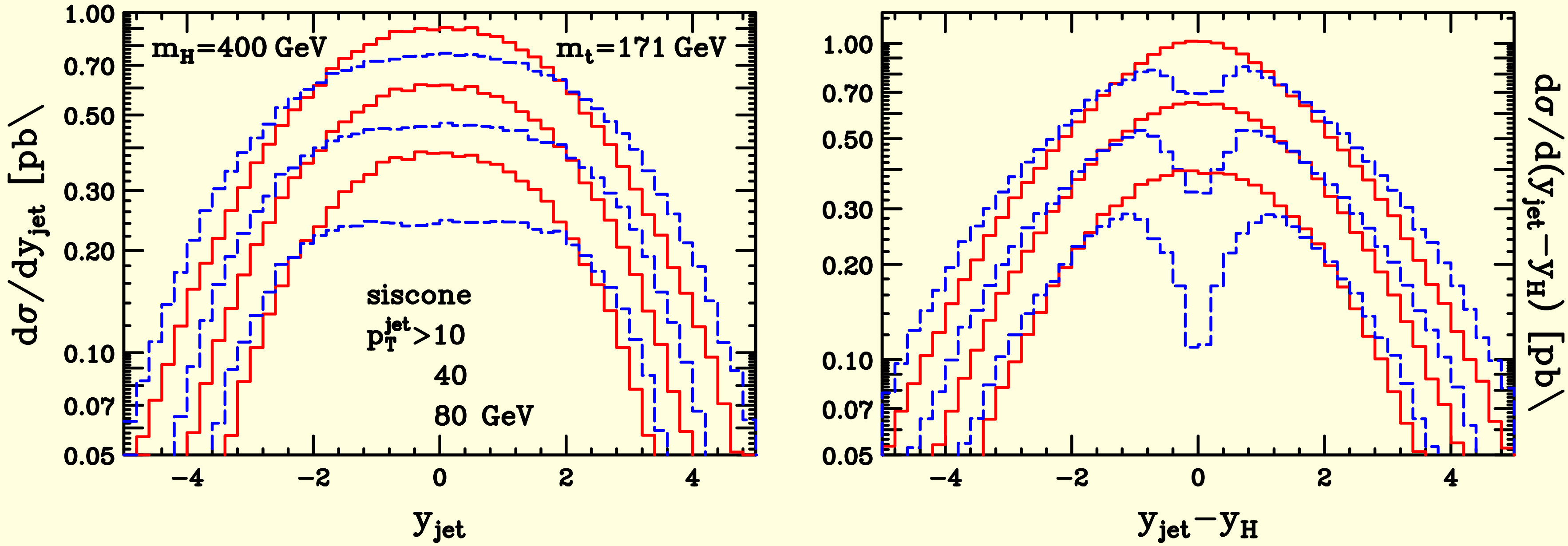
Jet rapidity in h production



Dip in MC@NLO inerithed from even deeper dip in HERWIG

(MC@NLO tries to fill dead regions in HERWIG, a mismatch remains).

Gets worse for larger E_T cuts:

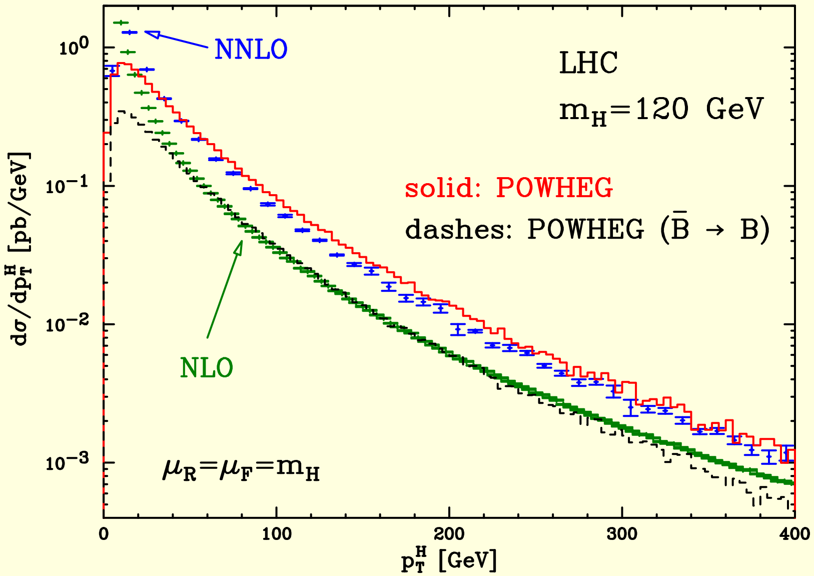


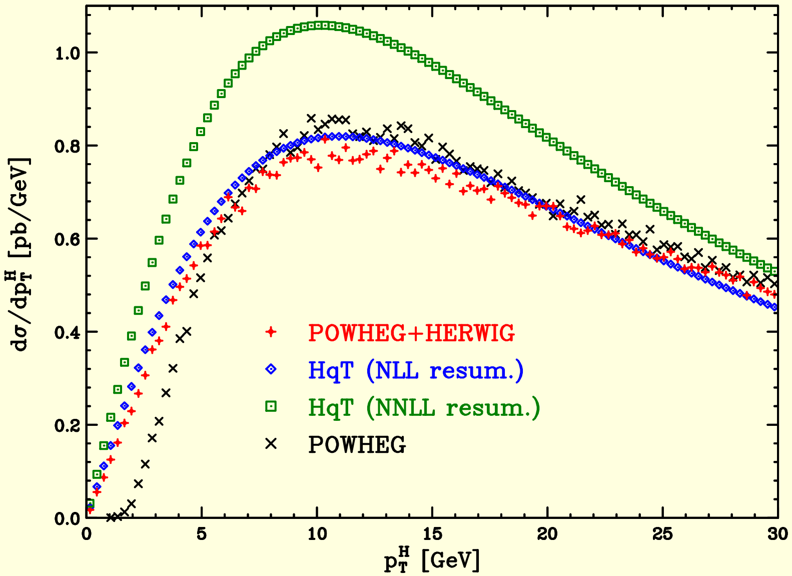
Questions:

Why MC@NLO has a dip in the hardest jet rapidity?

Why POWHEG has no dip?
(Some have (wrongly) argued that the dip is filled by the hardest p_T spectrum)

Hard p_T spectrum: POWHEG vs. NNLO vs. NNLL





$$d\sigma = \bar{B}\, d\Phi_B \left\{ \Delta_{t^0} + \Delta_t \frac{R}{B} d\Phi_r \right\} \approx \frac{\bar{B}}{B} R\, d\Phi_B\, d\Phi_r = \underbrace{\{1 + \mathcal{O}(\alpha_s)\}}_{\approx 2 \text{ for here!}} R\, d\Phi$$

Large enhancement because of the large K factor in Higgs production.

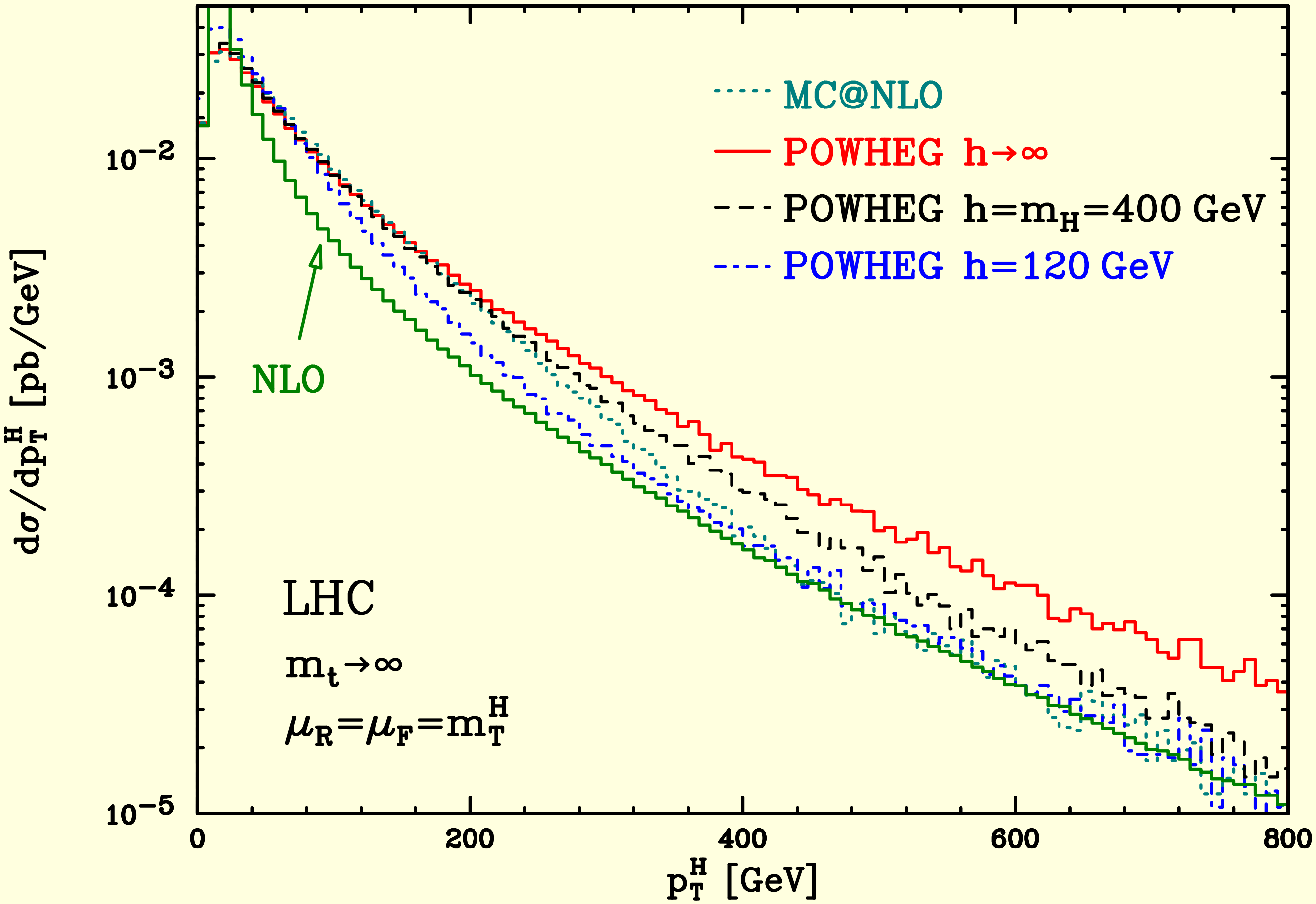
Higher p_T spectrum because of the choice $R_s = R$.
(Better agreement with NNLO this way)

Use the flexibility in POWHEG to choose $R_s \neq R$

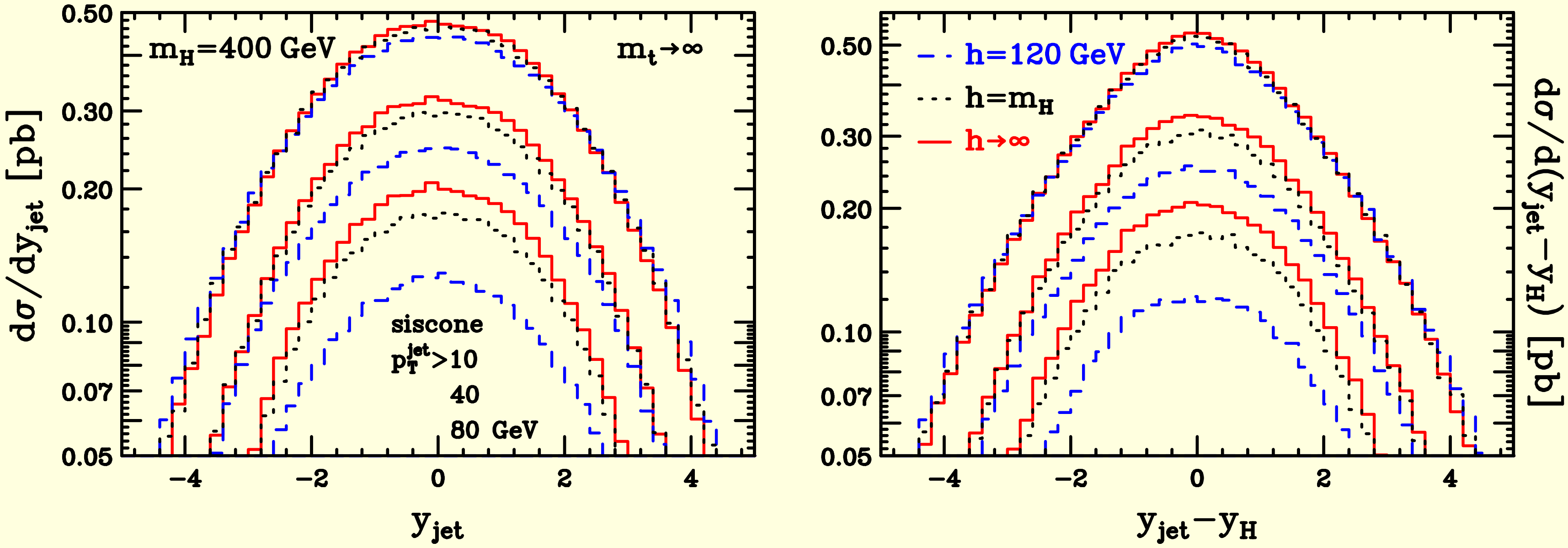
$$R_s = R\frac{h^2}{k_T^2 + h^2}$$

Agrees with NLO at high p_T.

However ...



No dips arise in the jet rapidity distributions:



So: extra radiation at high k_T and dips are unrelated issues in POWHEG.

Why is there a dip in MC@NLO?

For large k_T:

$$d\sigma = \frac{\bar{B}^{\mathrm{MC}}}{B} R^{\mathrm{MC}}\, d\Phi_B d\Phi_r^{\mathrm{MC}} + [R - R^{\mathrm{MC}}] d\Phi$$

$$= \underbrace{R d\Phi}_{\text{no dip}} + \underbrace{\left(\frac{\bar{B}^{\mathrm{MC}}}{B} - 1\right)}_{\substack{\mathcal{O}(\alpha_s),\\ \text{large for Higgs!}}} \times \underbrace{R^{\mathrm{MC}}}_{\text{Herwig dip}} d\Phi$$

So: a contribution with a dip is added to the exact NLO result;

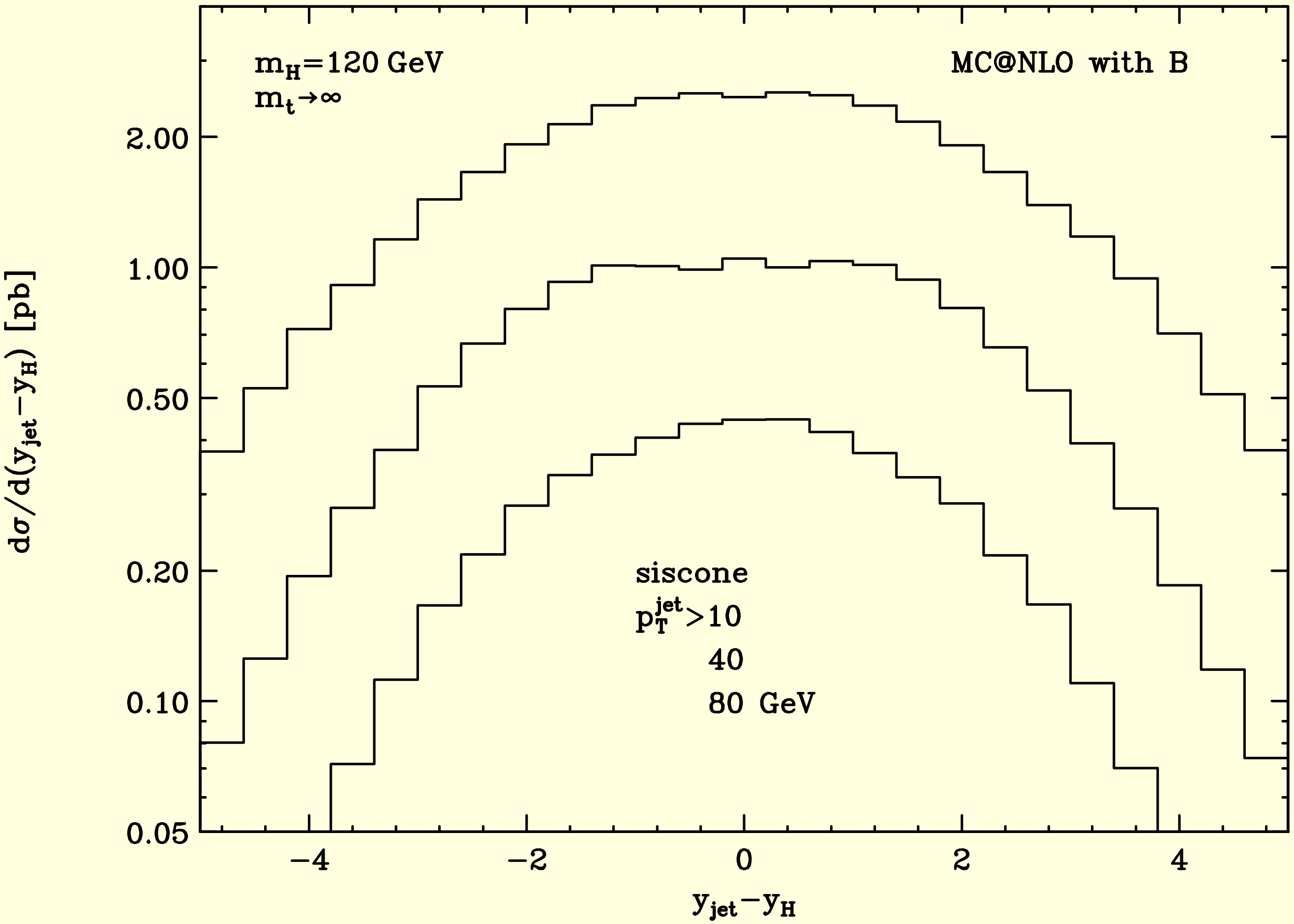
The contribution is $\mathcal{O}(\alpha_s R)$, i.e. NNLO!

but is large in processes with large K-factors.

Can we test this hypothesis? Replace $\bar{B}^{\mathrm{MC}}(\Phi_n) \Rightarrow B(\Phi_n)$ in MC@NLO!

the dip should disappear ...

MC@NLO with $\bar{B}^{\rm MC}$ replaced by B



No visible dip is present! (see also Hamilton,Richardson,Tully, 2009)

Summary of MC@NLO and POWHEG comparisons

- Fairly good agreement on most distributions
- Areas of disagreement can be tracked back to NNLO terms, arising mostly because of the use of an NLO inclusive cross section (the $\bar{B}$ function) to shower out the hardest radiation.
- In POWEG, since the hardest radiation is generated by POWHEG itself, one has the flexibility of tuning the magnitude of these NNLO terms.
- For MC@NLO, these NNLO terms can generate unphysical behaviour in physical distributions, reflecting the dead zones structure of the underlying shower Monte Carlo.

Towards automation: the POWHEG BOX

The MIB (Milano-Bicocca) group (Alioli, Oleari, Re, P.N.) is working on an automatic implementation of POWHEG for generic NLO processes.

The framework has been tested in processes already implemented, like single vector boson production and single top production

The new processes $hh \to Z + 1\text{jet}$, and the VBF higgs production, have been implemented in this framework.

The POWHEG BOX

A computer code framework, such that, given the Born cross section, the finite part of the virtual corrections, and the real graph cross section, one builds immediately a POWHEG generator. More precisely, the user must supply:

- The Born phase space
- The lists of Born and Real processes (i.e. $u\,\bar{s} \rightarrow W^{+} c\,\bar{c}$, etc.)
- The Born squared amplitudes $\mathcal{B} = |\mathcal{M}|^2$, $\mathcal{B}_{ij}$, $\mathcal{B}_{j,\mu_j,\mu'_j}$, for all relevant partonic processes; $\mathcal{B}_{ij}$ is the colour ordered Born amplitude squared, $\mathcal{B}_{j,\mu\nu}$ is the spin correlated amplitude, where j runs over all external gluons in the amplitude. All these amplitudes are common ingredient of an NLO calculation.
- The Real squared amplitude, for all relevant partonic processes.
- The finite part of the virtual amplitude contribution, for all relevant partonic processes.

Strategy

Use the FKS framework according to the general formulation of `POWHEG` given in FNO2007 (Frixione, Oleari, P.N. 2007). FKS details hidden in the BOX: we use FKS, but the user does not need to understand it.
(Attempts to use the Catani-Seymour method did not work ...)
It includes:

- Combinatorics
- The phase space for ISR and FSR, according to FNO2007.
- The calculation of soft and coll. limits of the real cross section (to implement accurately the subtraction method)
- The calculation of the soft and collinear remnants
- The calculation of $\bar{B}$ (spinoff: general NLO implementation using the FKS method)
- The generation of radiation
- Writing the event to the Les Houches interface for user processes

Combinatorics

The different singular regions of R must be treated separately.
The `BOX`, given the Born and Real flavour processes, generates all regions.

Slice of combinatorics output for VBF Higgs production:

1 flavour structure yielding 3 regions
Round bracket surrounds emitter

```
  c~  s~   ==>   H (s~) c~  g      mult= 1
  c~  s~   ==>   H  s~  c~         <=== uborn
4 6  5 6  0 6
  c~  s~   ==>   H  s~ (c~) g      mult= 1
  c~  s~   ==>   H  s~  c~         <=== uborn
4 6  5 6  0 6
 (c~  s~)  ==>   H  s~  c~  g      mult= 1
  c~  s~   ==>   H  s~  c~         <=== uborn
4 6  5 6  0 6
```

For example, the second contribution equals:

$$R_2 = R\,\frac{\frac{1}{d_{56}}}{\frac{1}{d_{46}}+\frac{1}{d_{56}}+\frac{1}{d_{06}}}\,,\qquad d_{ij}\to 0 \;\;\text{when}\;\; i\parallel j;\qquad d_{oj}\to 0 \;\;\text{when}\;\; p_T^{(j)}\to 0$$

and is singular only when 6 (the gluon) is collinear to 5 (anti-charm)

Phase space mappings

R is separated into terms (with definite flavour structure) that are singular in a single region: R_{α_r}. For each R_{α_r} the full phase space Φ is parametrized in terms of an underlying Born phase space Φ_B and a radiation phase space Φ_r. It is required that in the singular limit Φ_B coincides with the phase space of the underlying Born process.

2 kinds of mappings: FSR and ISR, given in FNO2007 paper.
ISR: FKS phase space mapping (introduced in Mele, Ridolfi, PN 1991 for ZZ)
FSR: variant of FKS, different kinematics, same remnants.

The $\bar{B}$ calculation

For each underlying Born flavour configuration f_B there is a single $\bar{B}_{f_B}$:

$$\bar{B}^{f_b}(\Phi_B) = [B(\Phi_B) + V(\Phi_B)]_{f_b} + \sum_{\alpha_r \in \{\alpha_r | f_b\}} \int [d\Phi_r R(\Phi)]_{\alpha_r}$$

The R_{α_r} appearing here have singularities regulated by $+$ prescriptions in the FKS framework (collinear remnants are not shown here.)

- $\{\alpha_r | f_b\}$ is the set of all singular regions having the underlying Born configuration with flavour structure f_b.
- $[\ldots]_{\alpha_r}$ means that everything inside is relative to the α_r singular term: R is R_{α_r}, and the parametrization (Φ_B, Φ_r) is the one appropriate to the α_r singular region

Radiation

Sudakov FF also carries an f_b index:

$$\Delta^{f_b}(\Phi_n, p_T) = \exp\left\{ - \sum_{\alpha_r \in \{\alpha_r | f_b\}} \int \frac{\left[d\Phi_r\, R(\Phi_n, \Phi_r)\theta(k_T - p_T)\right]_{\alpha_r}}{B^{f_b}(\Phi_n)} \right\}$$

or

$$\Delta^{f_b}(\Phi_n, p_T) = \prod_{\alpha_r \in \{\alpha_r | f_b\}} \exp\left\{ - \sum \int \frac{\left[d\Phi_r\, R(\Phi_n, \Phi_r)\theta(k_T - p_T)\right]_{\alpha_r}}{B^{f_b}(\Phi_n)} \right\}$$

The Sudakov form factor is a product of elementary Sudakov form factors associated with each radiation region. Technically, one generates radiation by generating a k_T with each elementary form factor, and choosing the one with the largest k_T at the end.

POWHEG BOX prospects

The aim of the project is to provide a framework to build implementations of NLO corrections in POWHEG.

Programming details, in a well written code, are understood by reading the code. Program organization and algorithms must instead be documented.

We are writing up a description of the POWHEG BOX, that, together with the source code, should be sufficient for a user to learn how to use, and even modify it.

Conclusions

- NLO accuracy with Shower MC has become a reality in recent years.
- The POWHEG method is progressing, with new processes being included
- Progress in understanding agreement and differences between MC@NLO and POWHEG
- A path to full automation of POWHEG implementations of arbitrary NLO calculations is open
- Many interesting problems remain to be addressed: interfacing POWHEG to CKKW style showers; CKKW at NLO, etc.

Backup slides

Flavour and singularities separation

There are several allowed flavour assignments in the n body process.
B and V contributions are labelled by the flavour structure index f_b.

There are several allowed flavour structures in the $n+1$ body process.
Thus R is labelled by a flavour structure index f_r.
Each component R_{f_r} has several singularity regions. We thus write

$$R = \sum_{\alpha_r} R^{\alpha_r}$$

where each R^{α_r} has a specific flavour structure, and is singular in only one singular region. In FKS one writes

$$R^{\alpha_r} = R_{f_r} \times \mathcal{S}_{\alpha_r}, \qquad \sum_{\alpha_r} S_{\alpha_r} = 1$$

The S factors in the FKS formalism are defined as

$$\mathcal{S}_i = \frac{1}{N d_i}, \ \mathcal{S}_{ij} = \frac{1}{N d_{ij}} h\left(\frac{E_i}{E_i + E_j}\right),$$

where N is define so that $\sum_{\alpha_r} S_{\alpha_r} = 1$,

$$d_i = \left(\sqrt{s} E_i/2\right)^a (1 - \cos^2\theta_i)^b, \ \ d_{ij} = (E_i E_j)^a (1 - \cos\theta_{ij})^b,$$

$$\lim_{z \to 0} h(z) = 1, \ \ \lim_{z \to 1} h(z) = 0, \ h(z) + h(1-z) = 1.$$

For example:

$$h(z) = \frac{(1-z)^c}{z^c + (1-z)^c}$$

So, the $\mathcal{S}_i$ factors single out the region where parton i is collinear to either initial state line, or is soft, while S_{ij} single out the region where parton i is collinear to parton j or is soft.

The underlying Born

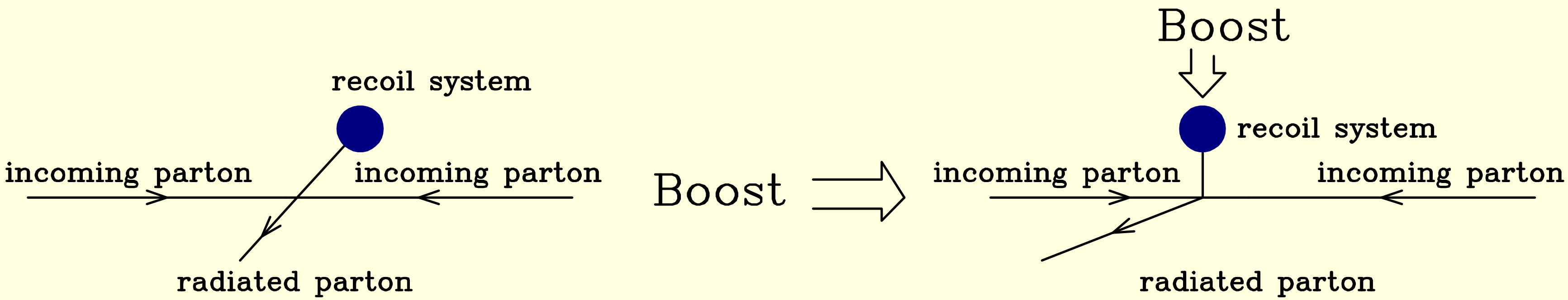
This is a basic concept in the POWHEG formalism;
To each region α_r we associate an underlying Born flavour configuration f_b, obtained as follows:

- If the singular region is associated to a parton becoming soft, then the parton must be a gluon, and it is simply removed to get the underlying Born configuration
- If the region is associated to two parton becoming collinear, then, in order for the region to be singular, the two partons must come from the splitting of another parton. The two partons are removed, and are replaced by the single parent parton with the appropriate flavour

Notice that in a shower Monte Carlo one first generates the Born process (i.e. the underlying Born configuration) and then lets one initial or final line undergo collinear splitting. Here we look at each singular region of the real matrix element, and ask from which underlyng Born process it could have been produced via a shower.

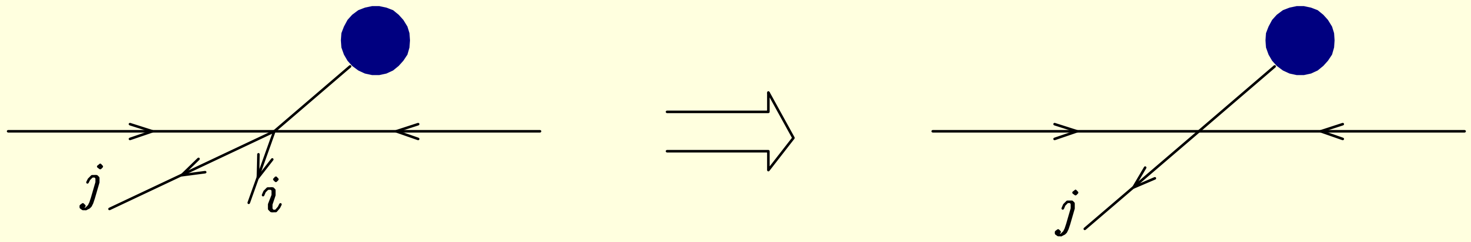
The underlying Born kinematics

To each kinematic configuration for the full radiation phase space Φ, one associates an underlying Born kinematics Φ_B and a set of radiation variables $\Phi_r = (y, z, \phi)$. For initial state radiation Φ_B is obtained by going with a longitudinal boost to the frame where the system recoiling against radiation has zero longitudinal momentum. In this frame one boosts the recoil system in the transverse direction, so that its transverse momentum becomes zero



The radiation variables are $y = \cos\theta$, θ being the angle between the radiated parton and the positive rapidity incoming parton, $\xi = 2E/\sqrt{s}$, where E is the energy of the radiated parton, and ϕ is its azimuth.

For final state radiation, the splitting partons are merged by summing their 3-momenta in the partonic CM frame. The 3-momentum is scaled, and the recoil system is boosted so that momentum and energy are conserved.



The radiation variables are $y = \cos\theta$, θ being the angle between the radiated partons, $\xi = 2E_i/\sqrt{s}$, ϕ is the azimuth of the ij plane relative to $\vec{k}_i + \vec{k}_j$. (This differs from FKS kinematics , where ϕ is relative to $\vec{k}_j$).