Monte Carlo at NLO

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Higgs Hunting, July 30$^{\mathrm{th}}$, Orsay, France

Plan of the talk

- Overview: PS (parton showers), ME+PS merging, NLO+PS merging.
- NLO+PS for $gg \to H$: comparison of `POWHEG` and `MC@NLO`
- VBF higgs production in `POWHEG`
- Available NLO+PS results relevant for Higgs production
- Recent results on merging NLO+PS and ME+PS

How events are simulated today

- Traditional PS (Parton Shower generators)
- ME+PS generators
- NLO+PS generators

Traditional generators

"Traditional" PS's: PYTHIA, HERWIG, HERWIG++; give a fair description of the bulk of the production process, where "fair" means LO

- They use LO matrix elements for the partonic production process ($\mathcal{O}(\alpha_s^2 h_t^2)$ for $gg \to H$)
- They generate QCD radiation using the collinear approximation, and, to a limited extent, the soft approximation.
  For example, in H production, jets at small angle with respect to the collision axis, and to a minor extent soft jets, are well described. In short: low p_T jets.
- They may or may not include spin correlations in decay.
- They include more or less sofisticated models for hadron formation and for the underlying event, including multiparton collisions.

ME+PS

Combine exact, tree-level matrix element calculations with Parton Showers.
ME+PS can achieve LO accuracy for the prorduction of a fairly large number of associated jets.

In the $gg \to H$ example, they achieve the accuracy:
$H\colon \alpha_s^2$, $H + \mathrm{jet}\colon \alpha_s^3$, $H + 2\,\mathrm{jets}\colon \alpha_s^4$, etc.

NLO+PS

NLO+PS generators are able to describe the emission of the hardest jet with LO accuracy (α_s^3 for $gg \to H$, same as ME+PS generator), but are also capable to achieve NLO accuracy (i.e. $\alpha_s^2 + \alpha_s^3$ for $gg \to H$ production) for inclusive observables.

Several proposed methods:

(Giele,Kosower,Skands 2007; Lavesson,Lonnblad,2008; Nagy,Soper, 2005, etc.)

Available generators at present:

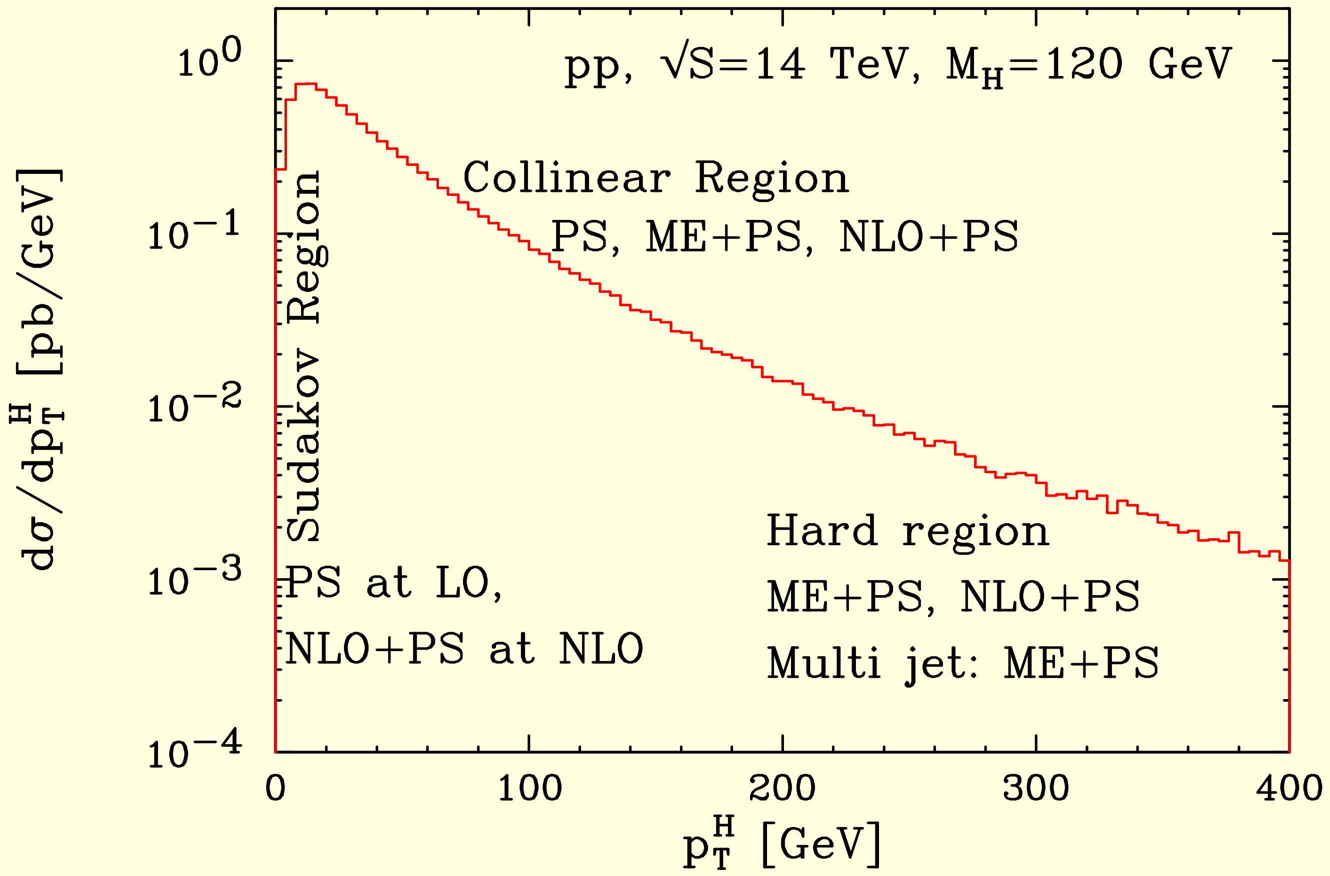
`MC@NLO` (Frixione, Webber 2002) `POWHEG` (P.N. 2004)

They use a traditional PS for radiation bejond the hardest jet, and for hadronization and event completion.

Thus, in the example of $gg \to H$, only the hardest jet is described with tree level accuracy. Further jets are generated by the shower in the collinear or soft approximation.

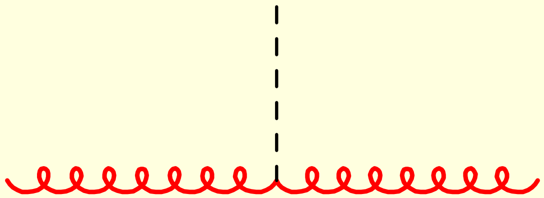
Domain of PS, ME+PS, NLO+PS

Regions: Sudakov: $p_T \lesssim \sqrt{m_H \Lambda_{\rm QCD}}$; collinear: $p_T \ll m_H$; hard: $p_T \gtrsim m_H$

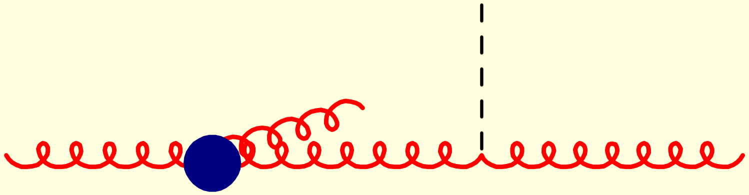


Parton Shower basic concepts

Born cross section: partonic cross section convoluted with parton densities $B(\Phi_B)d\Phi_B$, where Φ_B is the Born phase space.



The splitting algorithm is applied to each external coloured line, recursively, according to a splitting probability $P(\Phi_r)$ ($\Phi_r = \theta, z, \phi$, radiation variables)



So: from Φ_B, Φ_r we recover Φ, the full kinematics of the first radiation;

The other way around, $\Phi \Rightarrow (\Phi_r, \Phi_B)$, where Φ_B is the underlying Born of Φ.

- $P(\Phi_r)$ is such that, for $p_T \ll m_h$, (but $p_T \gg \sqrt{m_h \Lambda}$) we have

$$P(\Phi_r) \times B(\Phi_B) \approx R(\Phi)$$

- For $p_T \lesssim \sqrt{m_h \Lambda}$, $P(\Phi_r)$ is damped by a Sudakov Form Factor $\Delta(\Phi_r)$, arising from dominant virtual corrections.

- $P(\Phi_r)$ is such that (unitarity of the shower)

$$\int P(\Phi_r)\, d\Phi_r + P_0 = 1$$

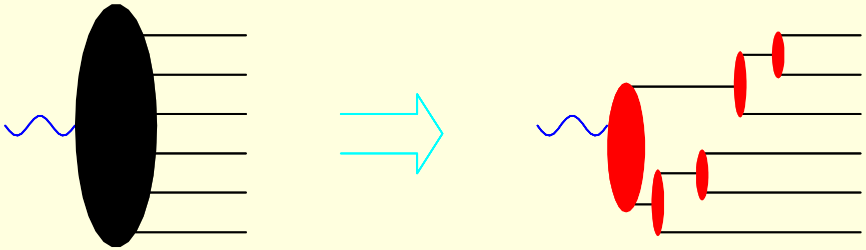
ME+PS

Historical approach: CKKW

Catani, Krauss, Küen, Webber (2001), (in e^+e^- annihilation).

In a nut-shell:

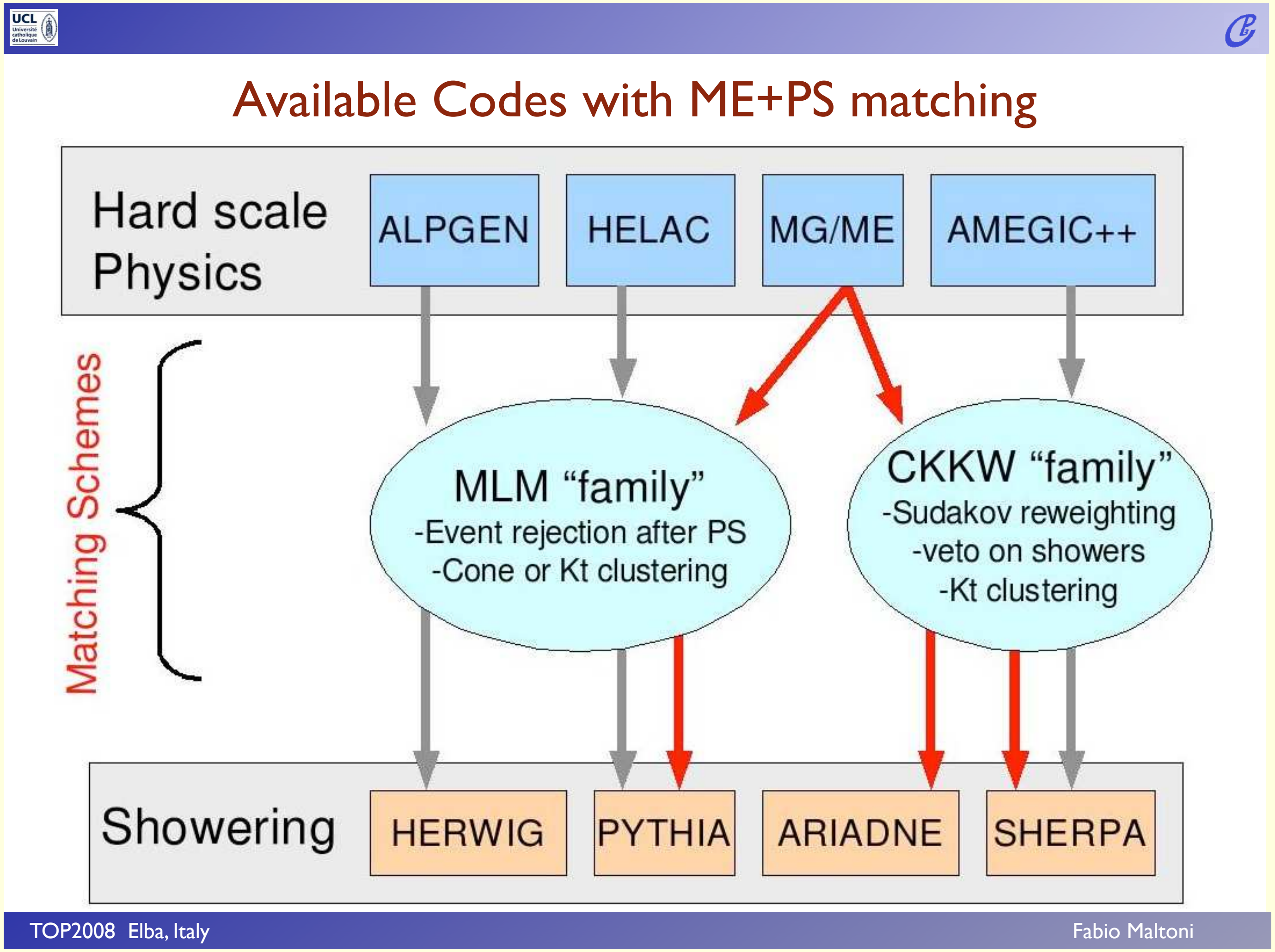
- Use exact tree level ME to compute the multiparton cross section. Clusterize ME partons to reconstruct a shower skeleton (by pairing up particles that yield smallest t recursively)



Red blobs have decreasing t values

- Correct exact tree level ME calculations with Sudakov form factor so that they reproduce the Shower results in the small k_T limit.
- Let the Shower take care of radiation with $k_T < M_{\rm cut}$, where $M_{\rm cut}$ is a cutoff on the jet separation

Alternative methods: MLM matching (no proofs, but it seems to work).
Others: CKKW-L (Lonnblad).



NLO+PS

Hardest radiation: as in PS, but corrected up to NLO:

$$d\sigma = \overbrace{\bar{B}^s(\Phi_B)}^{\text{NLO!}} d\Phi_B \left[\overbrace{\Delta^s_{t_0}}^{P_0} + \overbrace{\Delta^s_t \frac{R^s(\Phi)}{B(\Phi_B)} d\Phi_r}^{P(\Phi_r)} \right] + \overbrace{[R(\Phi) - R^s(\Phi)]}^{\text{ME correction}} d\Phi$$

where $R \Rightarrow R^s$ in the soft and collinear limit,

$$\bar{B}^s(\Phi_B) = B(\Phi_B) + \underbrace{\left[\underbrace{V(\Phi_B)}_{\text{infinite}} + \underbrace{\int R^s(\Phi)\, d\Phi_r}_{\text{infinite}} \right]}_{\text{finite}}$$

The Born cross section is replaced by the inclusive cross section at fixed underlying Born

and

$$\Delta^s_t = \exp\left[-\int_{t_l} \frac{R^s}{B} d\Phi_r \theta(t(\Phi) - t_l) \right]$$

so that

$$\Delta^s_{t_0} + \int \Delta^s_t \frac{R^s(\Phi)}{B(\Phi_B)} d\Phi_r = 1 \; (\text{Unitarity})$$

In MC@NLO: $R^s d\Phi_r = R^{\rm MC} d\Phi_r^{\rm MC}$

Furthermore:
in MC@NLO the phase space parametrization $\Phi_B,\ \Phi_r \Rightarrow \Phi$ is the one of the Shower Monte Carlo. We have:

$$\underbrace{\bar{B}^s(\Phi_B) d\Phi_B}_{\substack{\text{provided by MCatNLO} \\ \mathcal{S}\ \text{event}}} \left[\underbrace{\Delta^s_{t_0} + \Delta^s_t \frac{R^s(\Phi)}{B(\Phi_B)} d\Phi_r}_{\text{generated by HERWIG}} \right] + \underbrace{\left[R(\Phi) - R^s(\Phi)\right] d\Phi}_{\substack{\text{provided by MCatNLO} \\ \mathcal{H}\ \text{event}}}$$

More synthetically

$$\texttt{MCatNLO}\ \mathcal{S} = \frac{\bar{B}^s(\Phi_B)}{B(\Phi_B)} \times \texttt{HERWIG}\ \text{basic process}$$

$$\texttt{MCatNLO}\ \mathcal{H} = R(\Phi) - R^s(\Phi)\ \text{fed through}\ \texttt{HERWIG}$$

Issues:

- Must use the MC kinematic mapping $(\Phi_B, \Phi_r^{\mathrm{MC}}) \Rightarrow \Phi$.
- For $R - R^{\mathrm{MC}}$ to be non singular, the MC should reproduce exactly the soft and collinear singularities of the radiation matrix element.
  No existing PS can do that. For example, the azimuthal dependence of collinear singularities is neglected in the MC's.
  In `MC@NLO` this difference is essentially damped, by smoothly matching R^{MC} to R in the collinear and soft limit.
- $R - R^{\mathrm{MC}}$ can be negative: negative weights in the output.

In POWHEG: $R^s d\Phi_r = R F(\Phi)$

where $0 \leqslant F(\Phi) \leqslant 1$, and $F(\Phi) \Rightarrow 1$ in the soft or collinear limit.
$F(\Phi) = 1$ is also possible, and often adopted.
The parametrization $\Phi_B, \Phi_r \Rightarrow \Phi$ is within POWHEG, and there is complete freedom in its choice.

$$\underbrace{\bar{B}^s(\Phi_B) d\Phi_B}_{\texttt{POWHEG}} \left[\underbrace{\Delta^s_{t_0} + \Delta^s_t \frac{R^s(\Phi)}{B(\Phi_B)} d\Phi_r}_{\texttt{POWHEG}} \right] + \underbrace{\left[R(\Phi) - R^s(\Phi) \right] d\Phi}_{\texttt{POWHEG}}$$

All the elements of the hardest radiation are generated within POWHEG

Recipe

- POWHEG generates an event, with $t = t_{\texttt{powheg}}$
- The event is passed to a SMC, imposing no radiation with $t > t_{\texttt{powheg}}$.

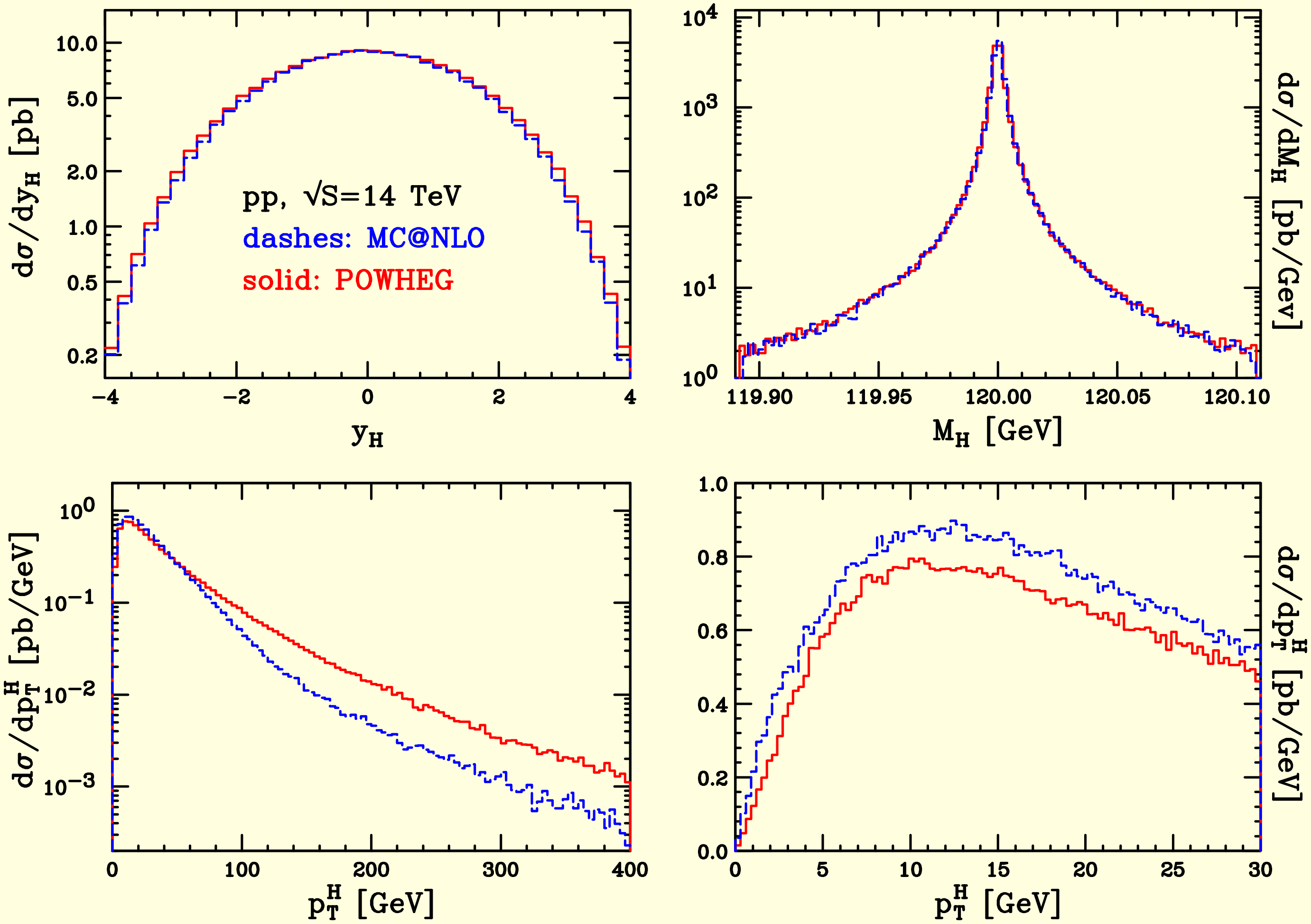
Improvements over MC@NLO:

- Positive weighted events: $R - R_s = R(F-1) \geqslant 0$!
- Independence on the Shower MC: The hardest emission is generated by POWHEG; less hard emissions are generated by the shower.
  Can switch Shower models: very valuable for theoretical studies
- No issues with improper cancellation of PS singularities

Do we expect differences btween POWHEG and MC@NLO at NLO?

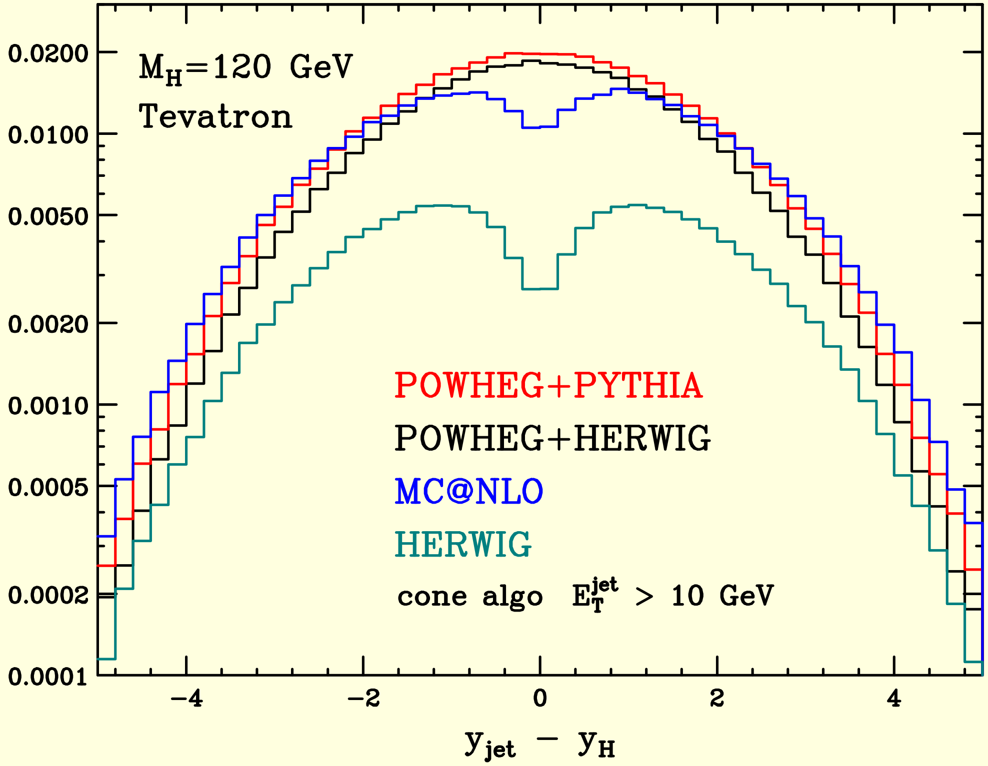
In MC@NLO: $R - R^{\rm MC}$ difference in $\mathcal{H}$ events is explicitly suppressed in the collinear and soft region. This may cause inaccuracies of NLO order when describing relatively soft jets. For example: azimuthal correlations in $R^{\rm MC}$ are not implemented in HERWIG.
Preliminary studies show differences with POWHEG in azimuthal correlations of very soft jets. These effects, however, are not yet fully understood.

POWHEG and MC@NLO in $gg \to H$



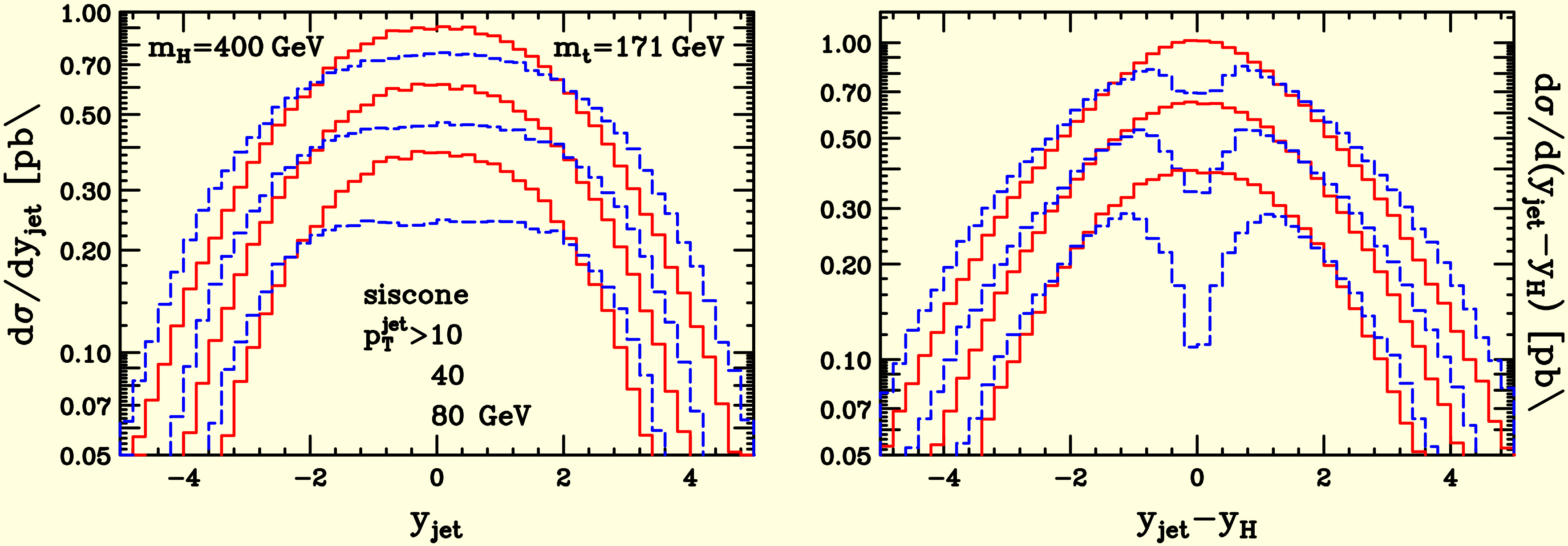
Large differences in the high pt tail, POWHEG being much harder

Jet rapidity in h production

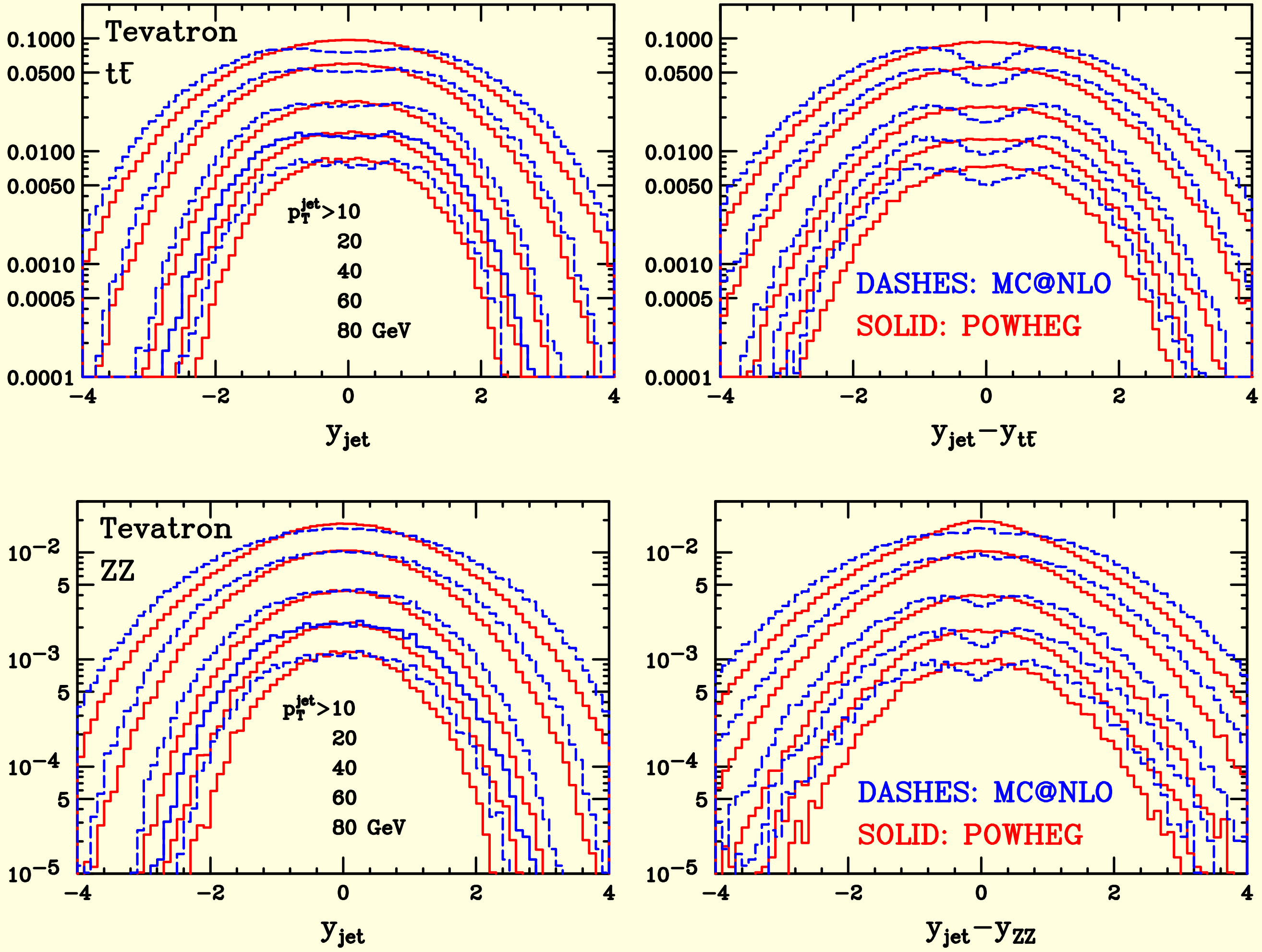


Dip in MC@NLO inerithed from even deeper dip in HERWIG (MC@NLO tries to fill dead regions in HERWIG, a mismatch remains).

Gets worse for larger E_T cuts:



Also present in several other processes:



POWHEG+HERWIG
MC@NLO

POWHEG+HERWIG
MC@NLO

Why is there a dip in MC@NLO?

The dip is already present in HERWIG alone.
How it propagates to MC@NLO has been clarified in several publications:
(Hamilton,Richardson,Tully, 2009; Alioli, Oleari, Re, P.N. 2009; P.N. 2010)

In short: in MC@NLO $\mathcal{S}$-events carry a K factor; $\mathcal{H}$-events do not

$$d\sigma = K(\Phi_B) \times \texttt{HERWIG} + [R(\Phi) - R^s(\Phi)]\, d\Phi$$

$$K(\Phi_B) = \frac{\bar{B}(\Phi_B)}{B(\Phi_B)}$$

for large k_T:

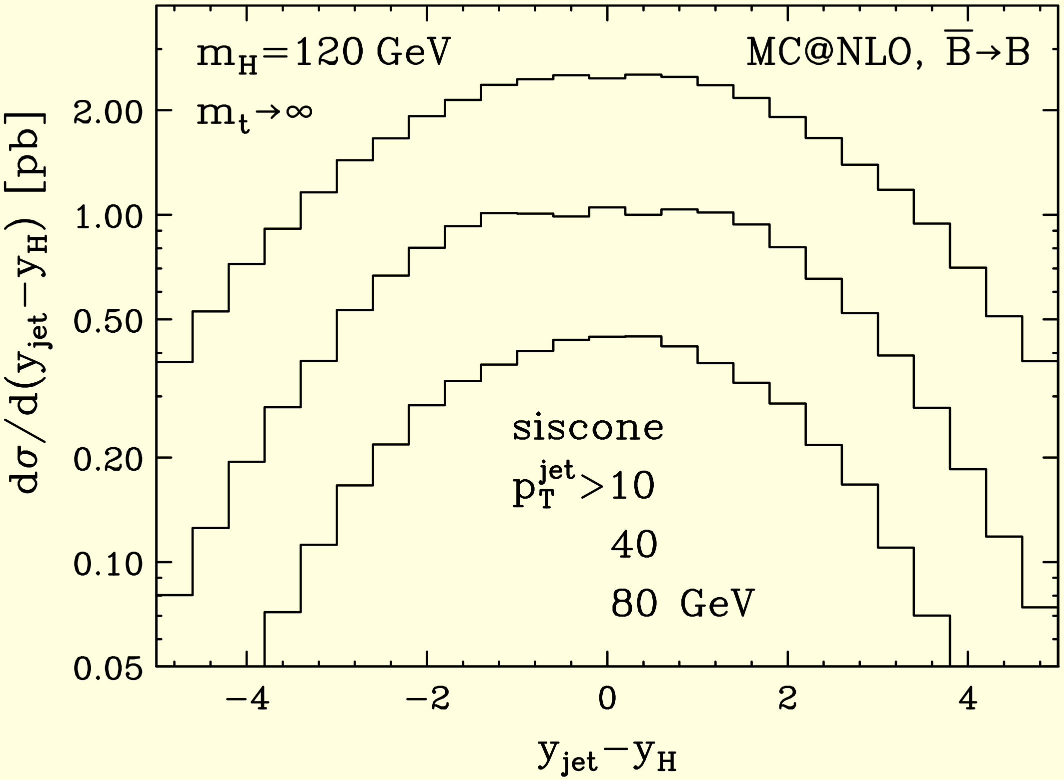
$$\frac{d\sigma}{d\Phi} = \underbrace{K\, R^s(\Phi)}_{\mathcal{S}} + \underbrace{[R(\Phi) - R^s(\Phi)]}_{\mathcal{H}}$$

The $\mathcal{H}$ contribution should cancel the dip in R^s, but, if K is large, there is a leftover. Since $K = 1 + \mathcal{O}(\alpha_s)$, this is an NNLO effect.

Can we test this hypothesis? Replace $\bar{B}^{\rm MC}(\Phi_n) \Rightarrow B(\Phi_n)$ in MC@NLO!

the dip should disappear ...

MC@NLO with $\bar{B}^{\rm MC}$ replaced by B



No visible dip is present! (see also Hamilton,Richardson,Tully, 2009)

Harder p_T spectrum in POWHEG; why?

In POWHEG we have chosen $R_s = R$; so

$$\bar{B}^s(\Phi_B) d\Phi_B \left[\Delta^s_{t_0} + \Delta^s_t \frac{R(\Phi)}{B(\Phi_B)} d\Phi_r \right] + \overbrace{[R(\Phi) - R^s(\Phi)]}^{=0} d\Phi$$

At large transverse momentum this is like

$$d\sigma = \bar{B}^s(\Phi_B) \frac{R(\Phi)}{B(\Phi_B)} d\Phi_B d\Phi_r = K(\Phi_B) R(\Phi), \qquad K(\Phi_B) = \frac{R(\Phi)}{B(\Phi_B)}.$$

In other words, $d\sigma$ at large p_T has full NLO K factor (≈ 2) in front of it.

Further reason: at large p_T, $R(\Phi) \approx \alpha_s^3$. Scale choice has large impact:

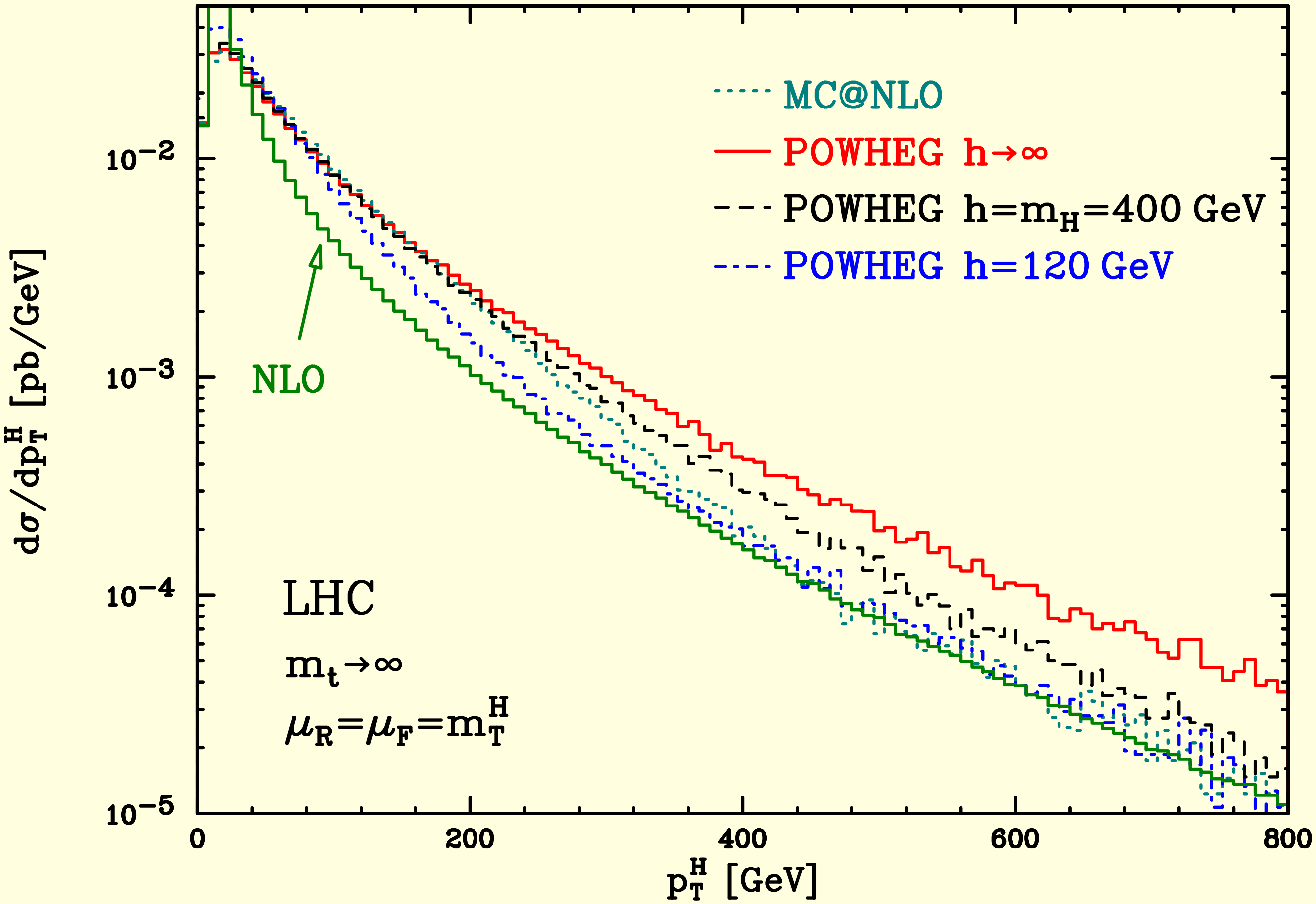
MC@NLO: $\mu^2 = M_H^2 + p_T^2$;

POWHEG: $\mu^2 = M_H^2$ for two powers of α_s, $\mu^2 = p_T^2$ for one power.

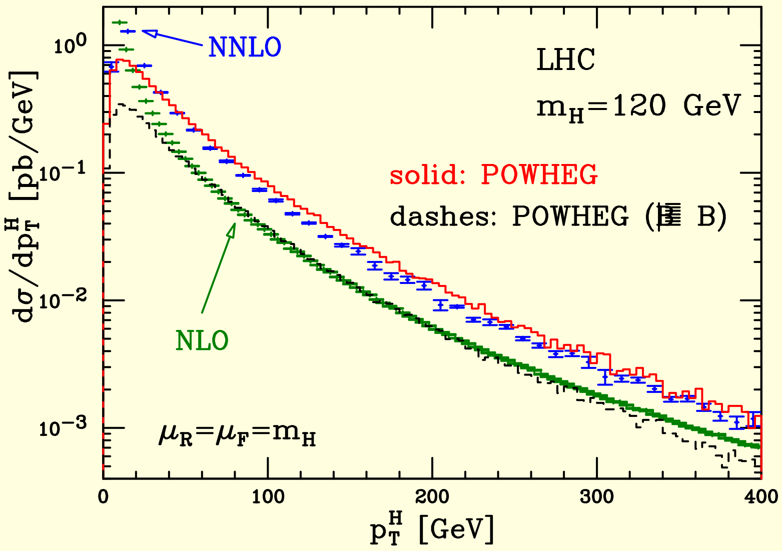
One may use the flexibility in POWHEG to choose $R_s \neq R$

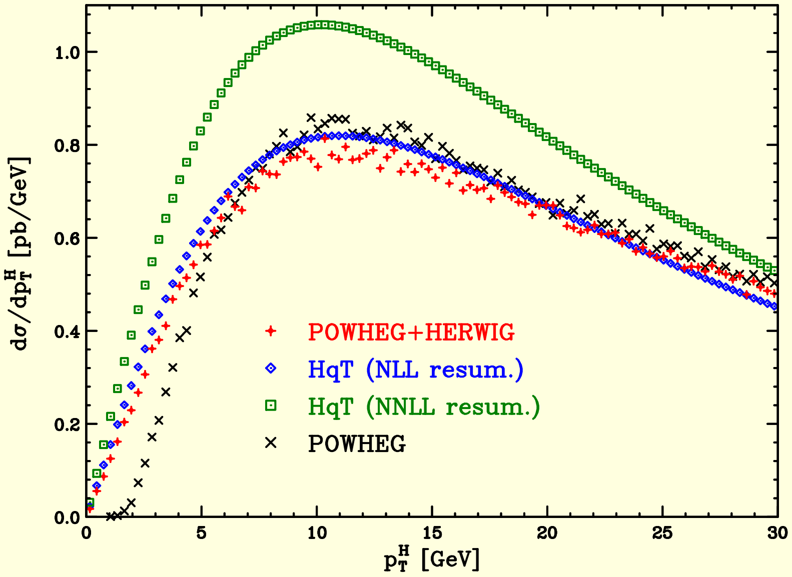
$$R_s = R\frac{h^2}{k_T^2 + h^2}$$

Agrees with NLO at high p_T.



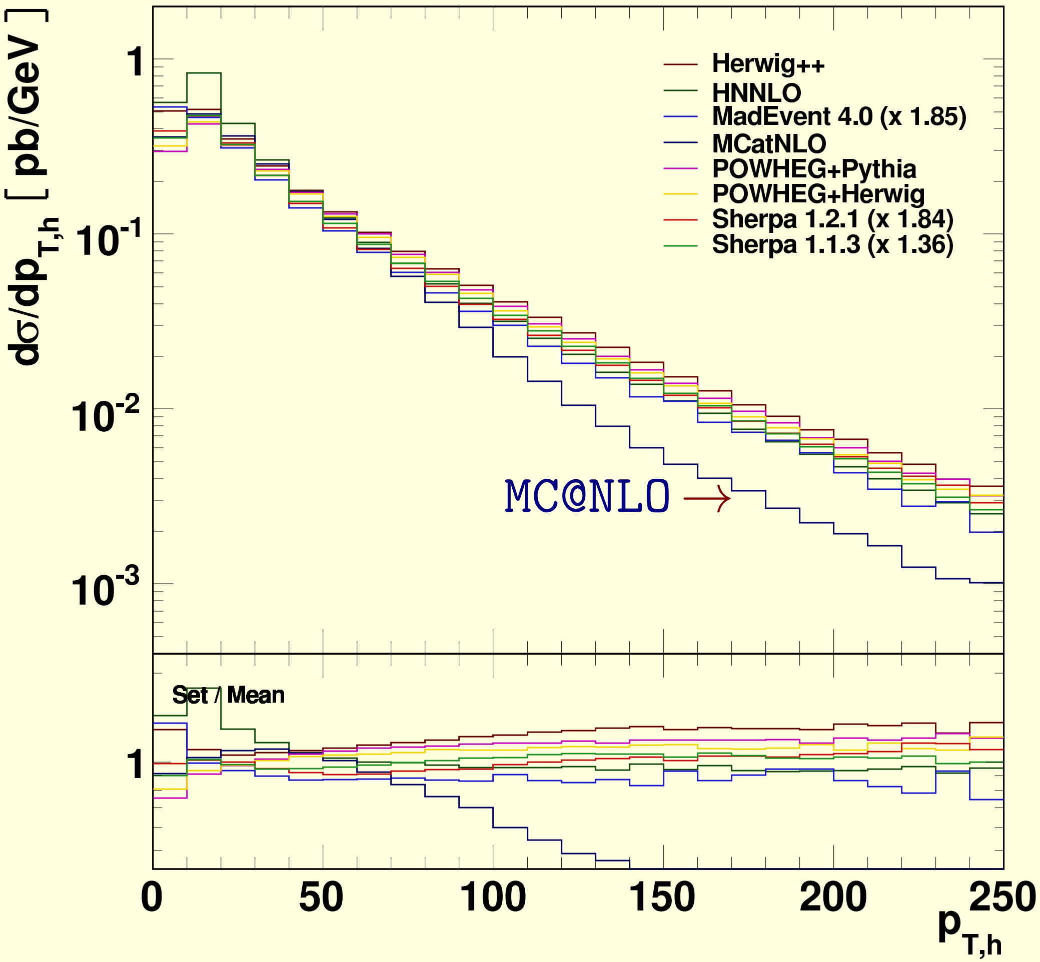
However: better agreement of POWHEG vs. NNLO





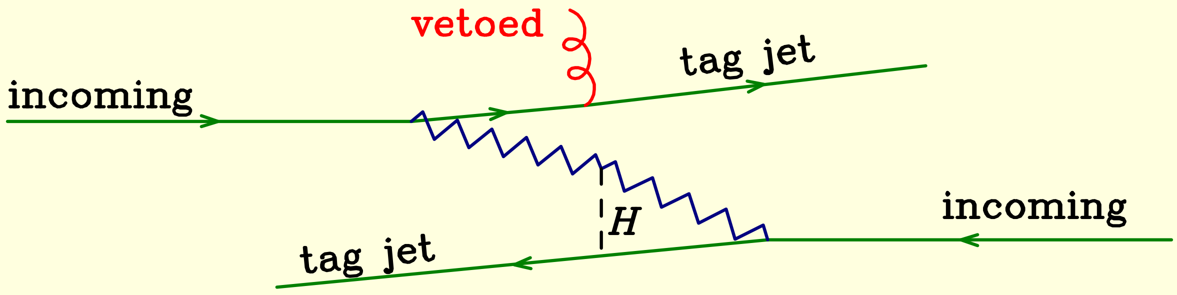
By maintaining the choice $R = R_s$, POWHEG agrees better with NNLO at high transverse momentum.

(Les Houches 2010)
Comparison of several ME generators and of POWHEG and MC@NLO for the Higgs p_T in $gg \to H$.
MC@NLO has softer tail.
All ME generators use a constant K factor.
POWHEG behaves similarly.



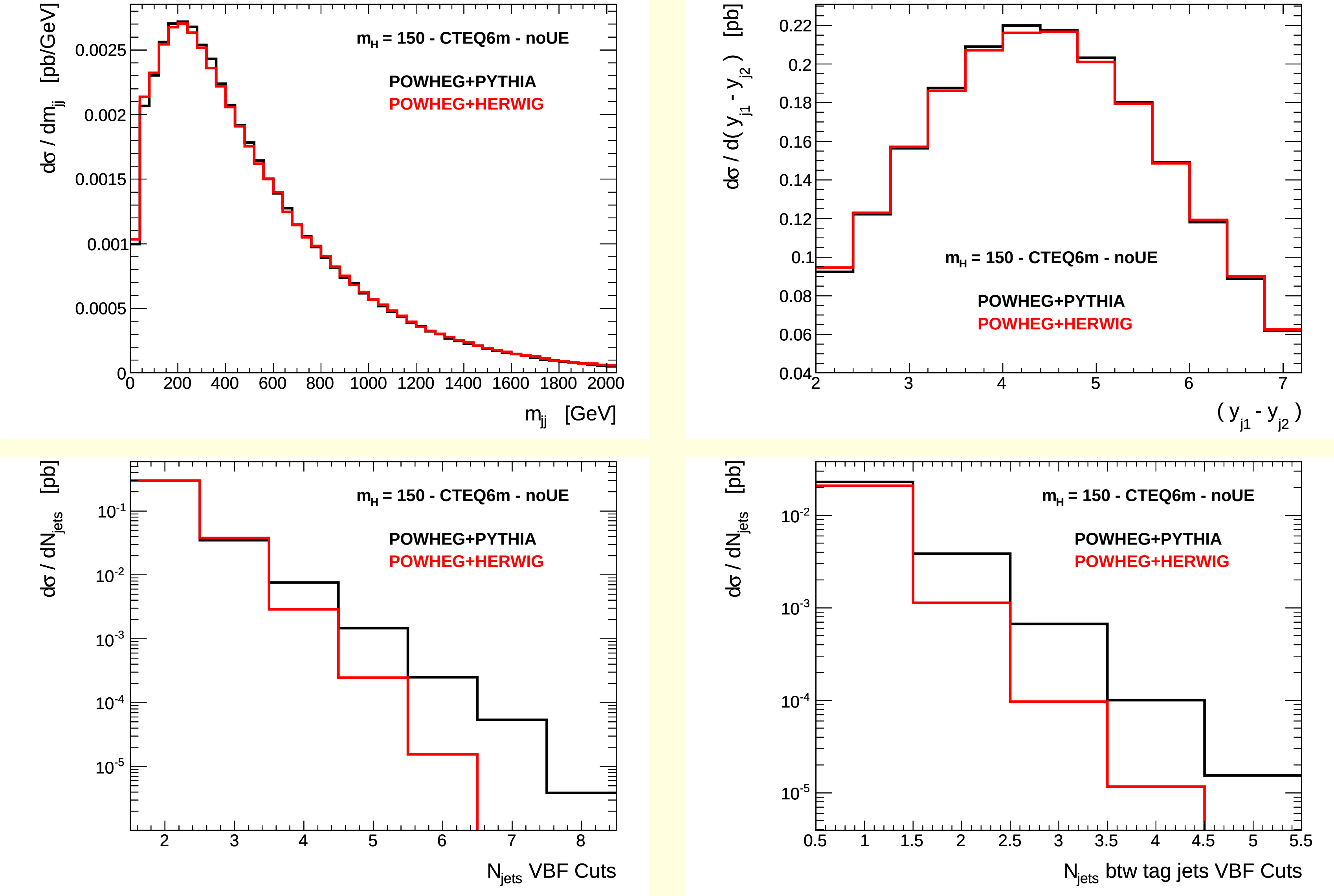
Higgs production by Vector Boson Fusion

Available in POWHEG (Oleari, P.N. 2009).
Study performed for Joint ATLAS/CMS/Theory efforts on Higgs cross sections presented by C. Oleari for the VBF subgroup (Freiburg, April 2010)

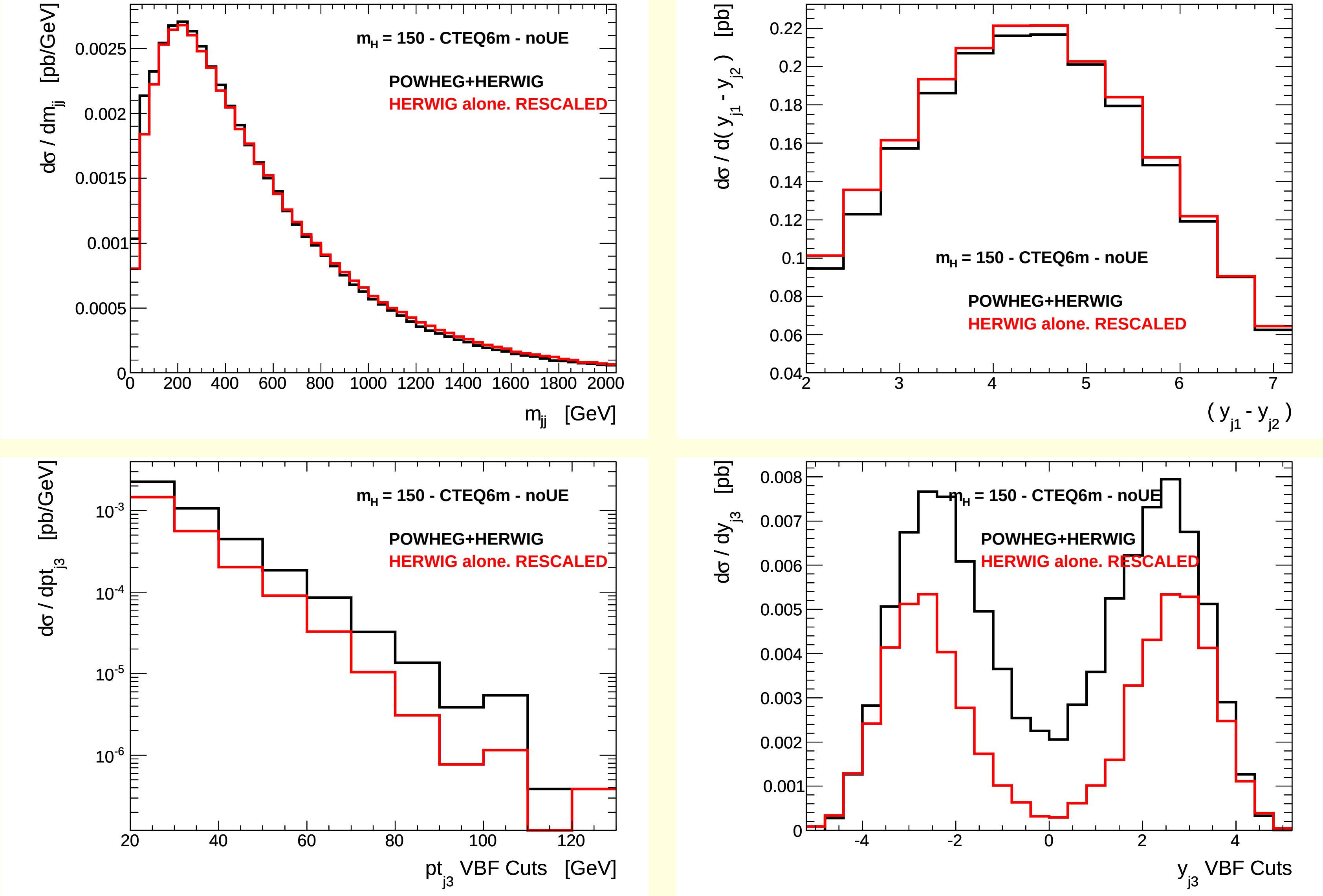


VBF cuts:

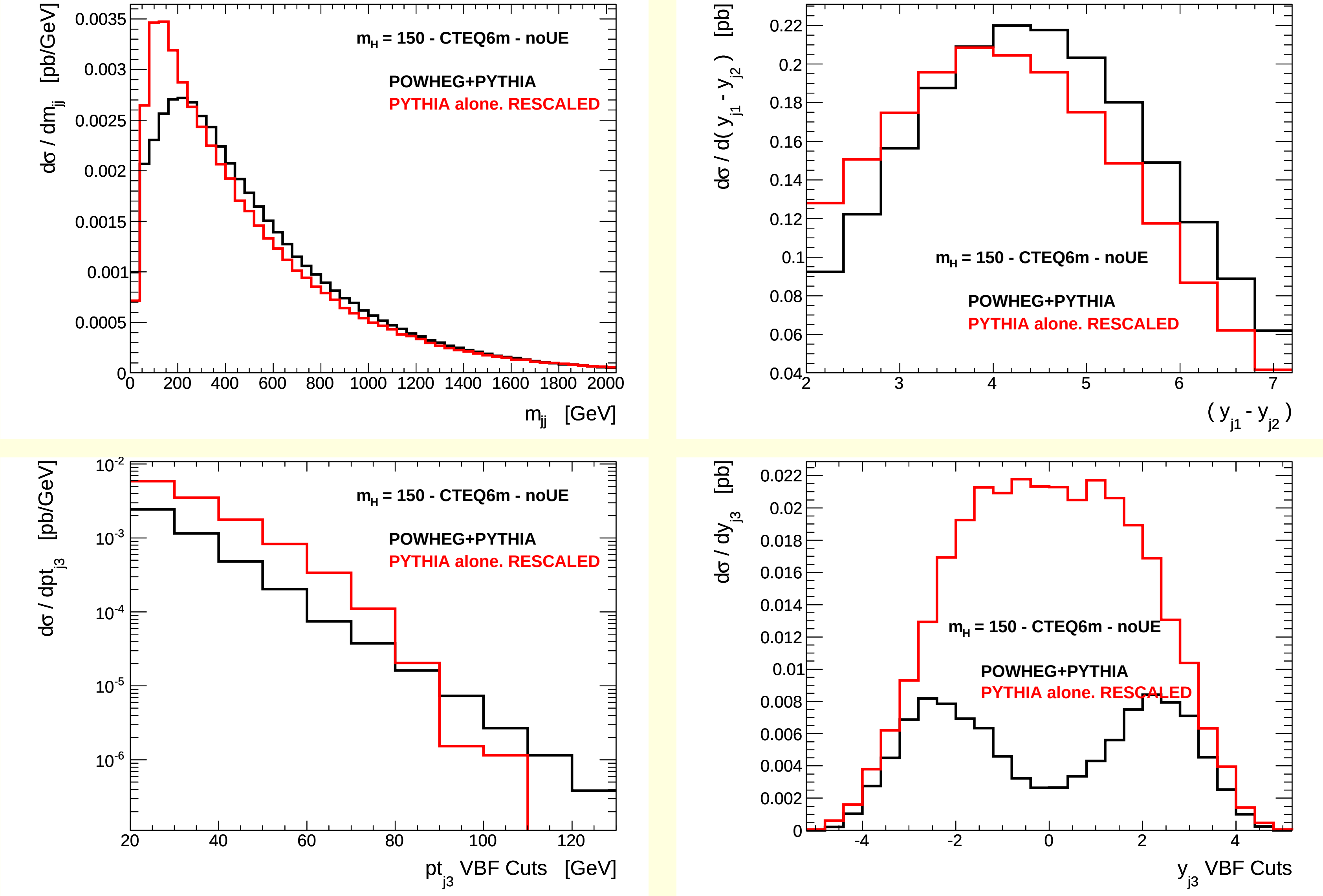
- Jets: $p_T > 20$, $|y| < 5$
- Tagging jets: $p_T^{\rm tag} > 30$, $y_1 < 0, y_2 > 0, y_2 - y_1 > 4.2$, $m_{12} > 600\,{\rm GeV}$
- veto jet: $y_1 < y_j^{\rm veto} < y_2$



Differences for $\geqslant 4$ jets, generated by the Shower.



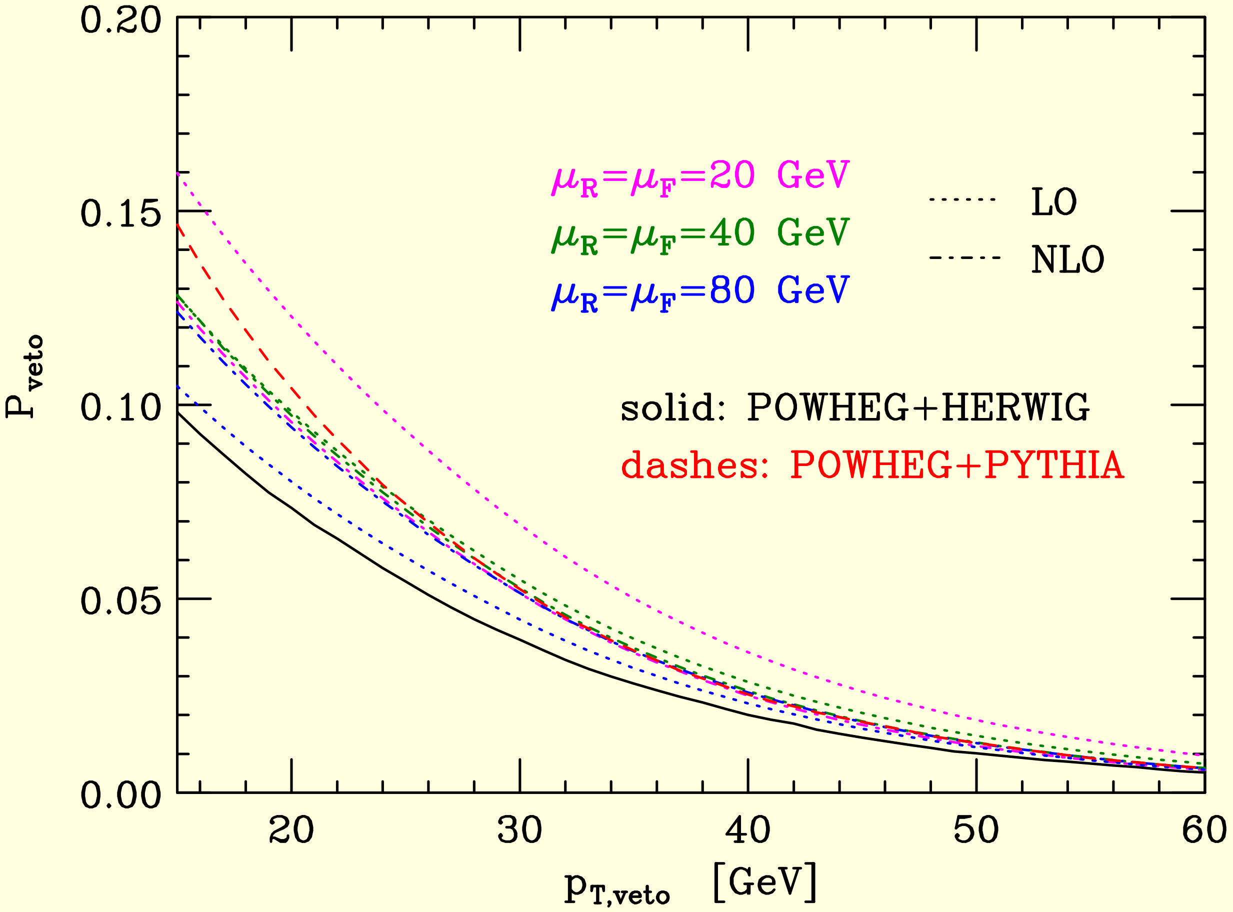
Third jet wrong in Herwig alone



Several problems in Pythia alone

Remember: third jet generated by the Shower in PYTHIA and HERWIG alone. Accurate only in the collinear regions (parallel to the incoming or outgoing quarks). In the central region, as needed for the pt veto, it cannot be trusted.

Probability to find a veto jet in a VBF sample (Oleari, P.N., 2009) compared to VBF H+1jet, LO,NLO (Figy, Hankele, Zeppenfeld, 2008)



As the p_T veto gets smaller, the subsequent shower and hadronization makes more difference than NLO scale variation.

Available programs for Higgs

Within the Joint ATLAS/CMS/Theory efforts on Higgs cross sections, there is an ongoing effort of the NLO MC subgroup to collect available result, and promote the inclusion of new processes.

Teams:

MC@NLO in HERWIG (Frixione, Webber, + others)

POWHEG at MiB (Alioli, Re, Oleari, Hamilton, P.N.)

POWHEG at HERWIG++ (Hamilton etal)

There are plans for POWHEG in SHERPA (Siegert and Krauss talks at ICHEP)

At MiB there is an effort to keep all implementations in the same uniform framework (the POWHEG BOX)

$gg \to H$, MC@NLO (Frixione, Webber) , POWHEG

(Alioli, Oleari, Re, P.N. 2009; Hamilton,Richardson,Tully, 2009)

$gg \to HV$, MC@NLO, POWHEG (Hamilton,Richardson,Tully, 2009)

VBF Higgs, POWHEG (Oleari, P.N. 2009)

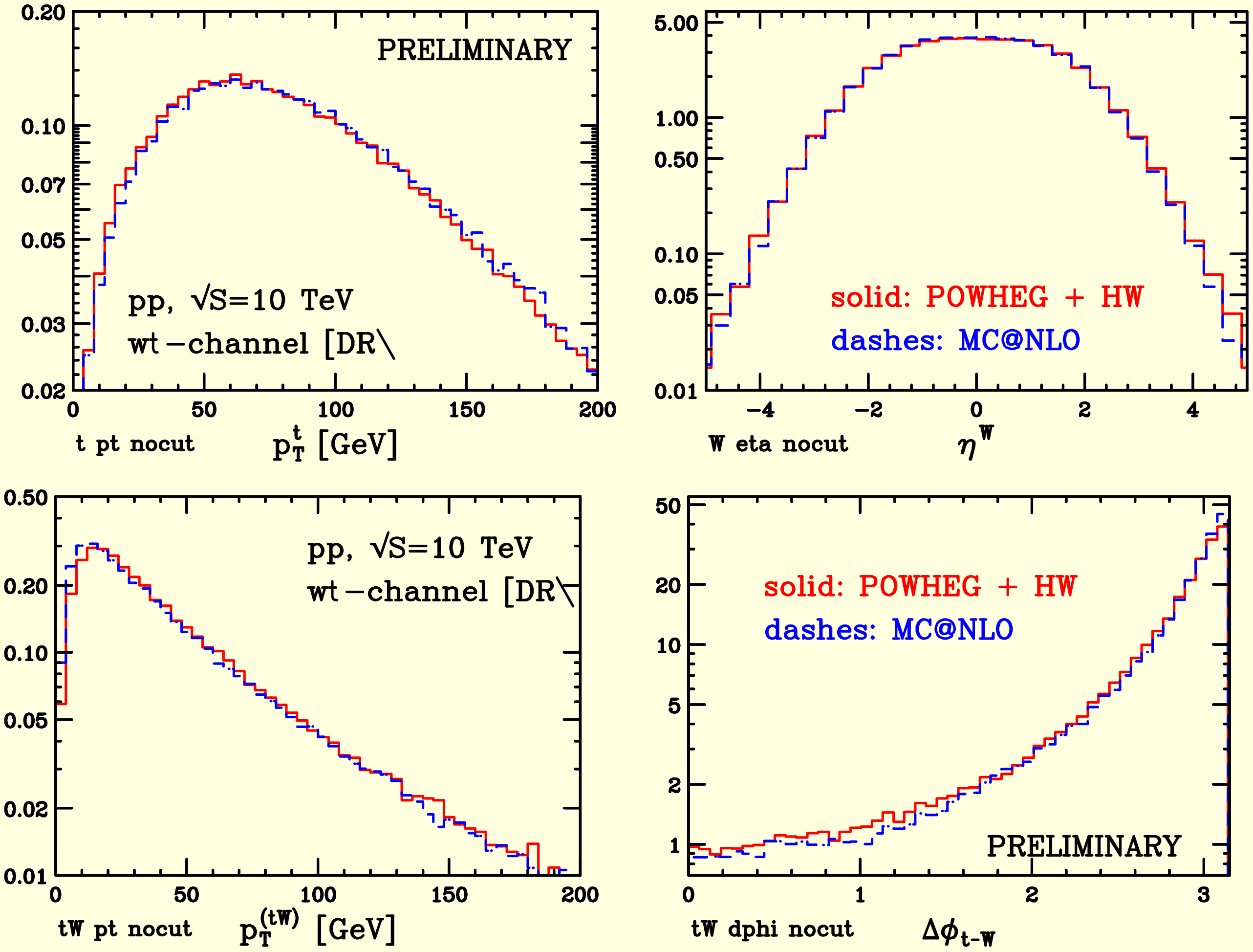
$t H^{\pm}$, MC@NLO (Weydert etal, 2009), POWHEG (Weydert etal, in preparation)

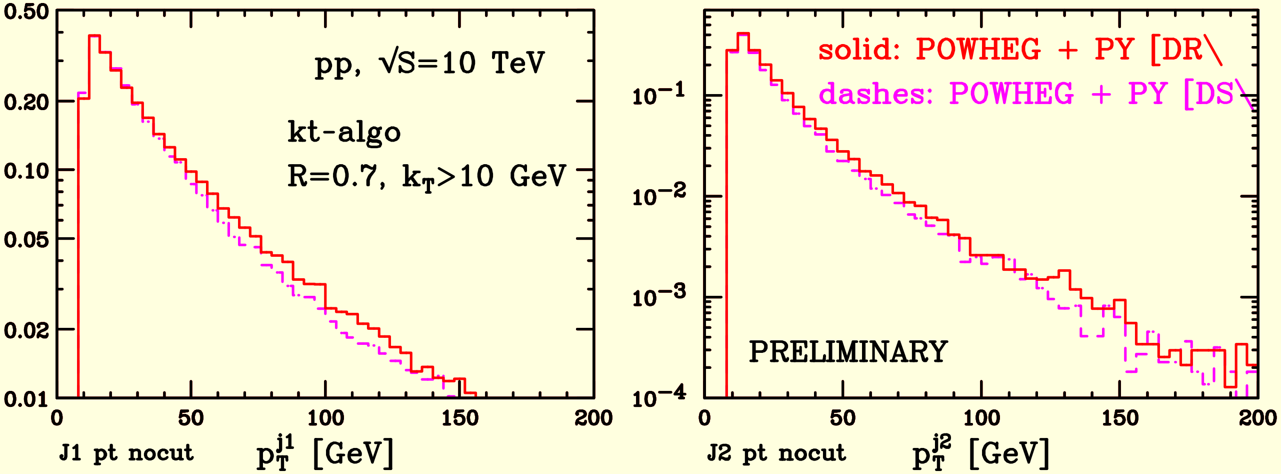
Notice: for $H \to WW$, relevant backgrounds ($t\bar{t}$, WW, tW) are all available in MC@NLO (Frixione,Webber,P.N.2003;Frixione,Webber 2002;Frixione etal, 2008).

$t\bar{t}$ is available in POWHEG (Frixione, Ridolfi, P.N. 2007), WW (Hamilton) in preparation, and tW (Re) to be relesed.

The whole analysis may be performed using NLO+PS tools.

Comparison of POWHEG and MC@NLO for Wt production (E. Re)




0.50
0.20
0.10
0.05
0.02
0.01
0
50
100
150
200
pp, √S=10 TeV
kt-algo
R=0.7, k_T>10 GeV
J1 pt nocut
p_T^j1 [GeV]
10^-1
10^-2
10^-3
10^-4
solid: POWHEG + PY [DR\
dashes: POWHEG + PY [DS\
PRELIMINARY
J2 pt nocut
p_T^j2 [GeV]

Perspective

In `POWHEG`: the `POWHEG BOX`, (Alioli, Oleari, Re, P.N. 2009)

a framework for implementing generic NLO processes has been released,

based upon previous theoretical work (Frixione, Oleari, P.N. 2007).

It has been used to implement two fairly complex processes:

VBF Higgs production, (Oleari, P.N. 2009)

$Z + \mathrm{jet}$ production, (Alioli, Oleari, Re, P.N.)

It can be applied to the new NLO results in t production.

Processes like $t\bar{t}H$ and $b\bar{b}H$ should be easy to implement in this framework

Merging NLO+PS and ME+PS

Given the fact that NLO+PS and ME+PS cover complementary aspects of the production process, the natural question arises: can they be merged?

DIFFICULT problem; proposals:

Giele, Kosower, Skands, 2008, `VINCIA` proposal

Bauer, Tackmann, Thaler, 2009 `GenEva` (e^+e^-)

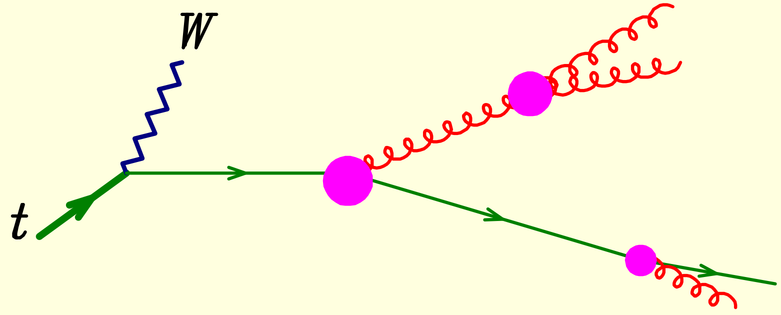
Lavesson, Lonnblad, 2009, (e^+e^-)

First attempt in hadronic collision processes: W and $t\bar{t}$ (Hamilton, P.N. 2010)

Look at ME+PS sample; what does it lack to be NLO accurate?

Simple example: t decay (just one jet!)

In the ME+PS, clusterising final state particles in order of increasing relative k_T, the configuration of hardest emission is the one just before the last clustering. From this configuration, one can also assign an underlying Born configuration to the event.



It can be demonstrated (Hamilton, P.N. 2010) that: in order to achieve NLO accuracy:

the ME+PS result should be reweighted with a $K(\Phi_B)$ factor.

$K(\Phi_B)$ hard to compute numerically (requires further studies).

merge POWHEG and ME+PS samples: MENLOPS

Alternative (approximate) method: build a sample according to the equation

$$d\sigma = d\sigma_{\rm PW}(0) + \frac{\sigma_{\rm ME}(1)}{\sigma_{\rm ME}(\geqslant 1)} \frac{\sigma_{\rm PW}(\geqslant 1)}{\sigma_{\rm PW}(1)} d\sigma_{\rm PW}(1) + \frac{\sigma_{\rm PW}(\geqslant 1)}{\sigma_{\rm ME}(\geqslant 1)} d\sigma_{\rm ME}(\geqslant 2),$$

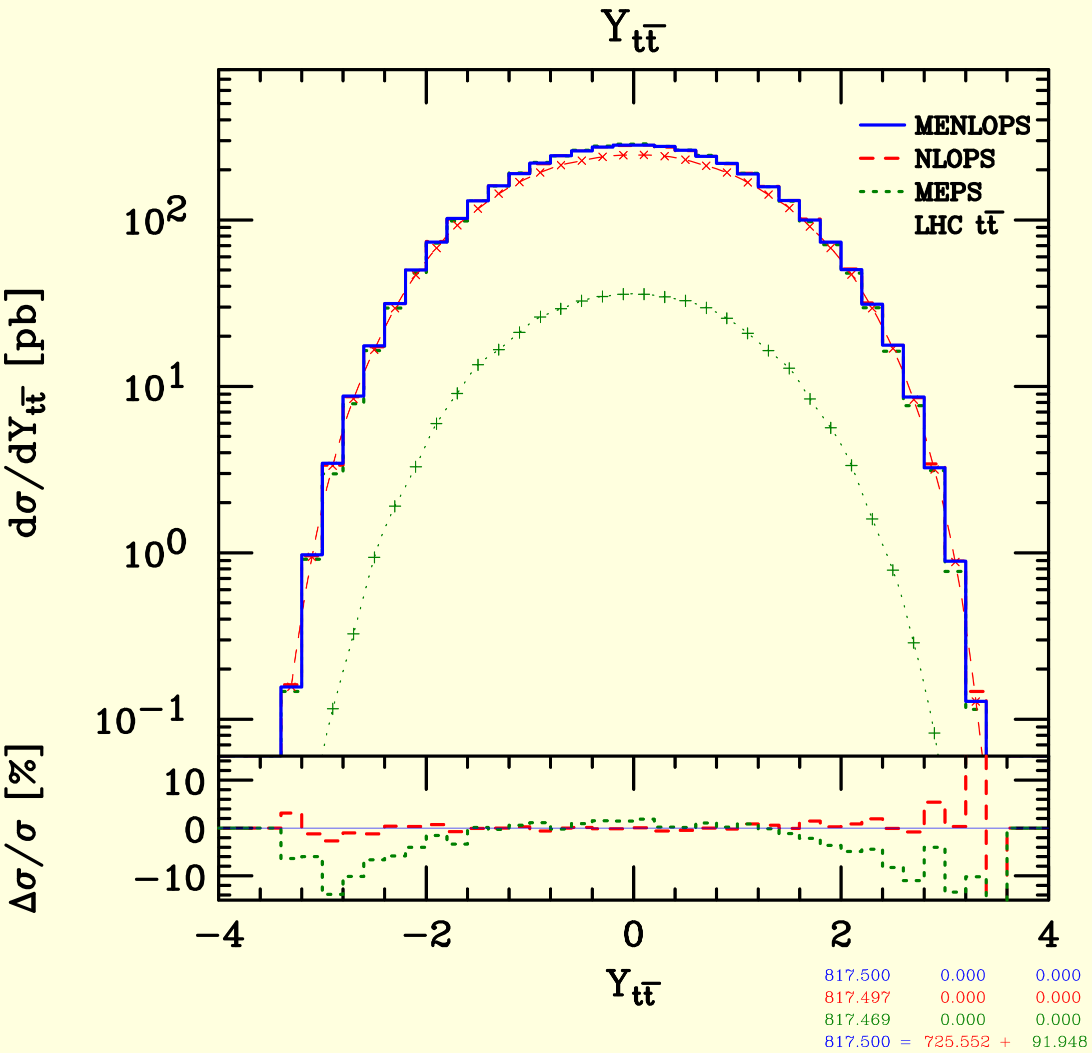
where $\sigma(j)$ is the cross section for j extra jets ($\sigma(\geqslant j)$: j or more). So:

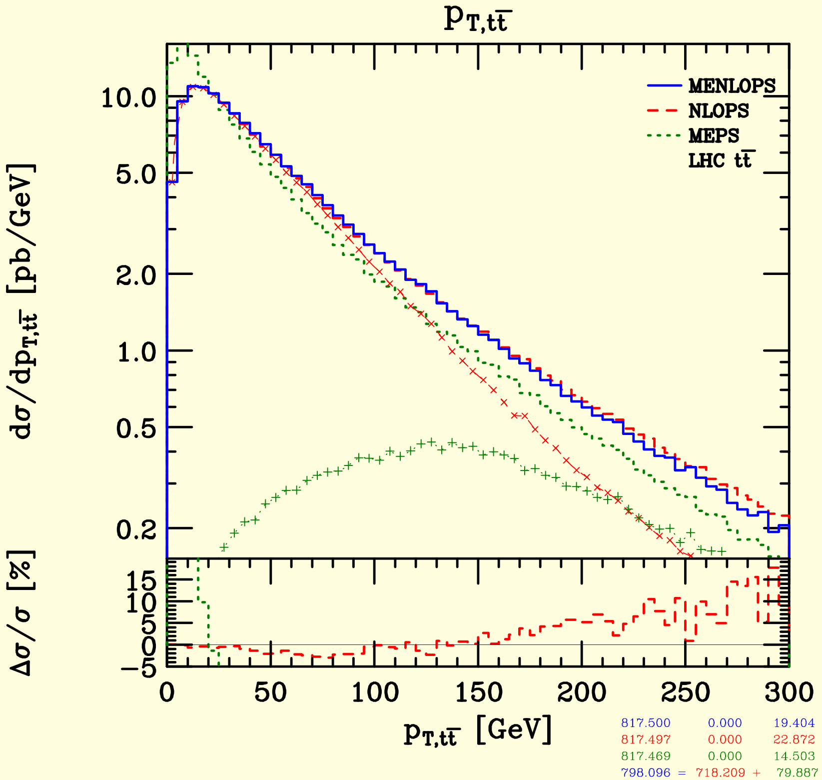
i. Events with no extra jets are always generated by POWHEG
ii. Events with one jet are also generated by POWHEG
iii. Events with more than one jet are generated by the ME+PS
iv. events ii and iii are reweighted, so that:
    the ii to iii ratio is as given by the ME+PS generator
    the total equals the POWHEG total

For our $t\bar{t}$ study:

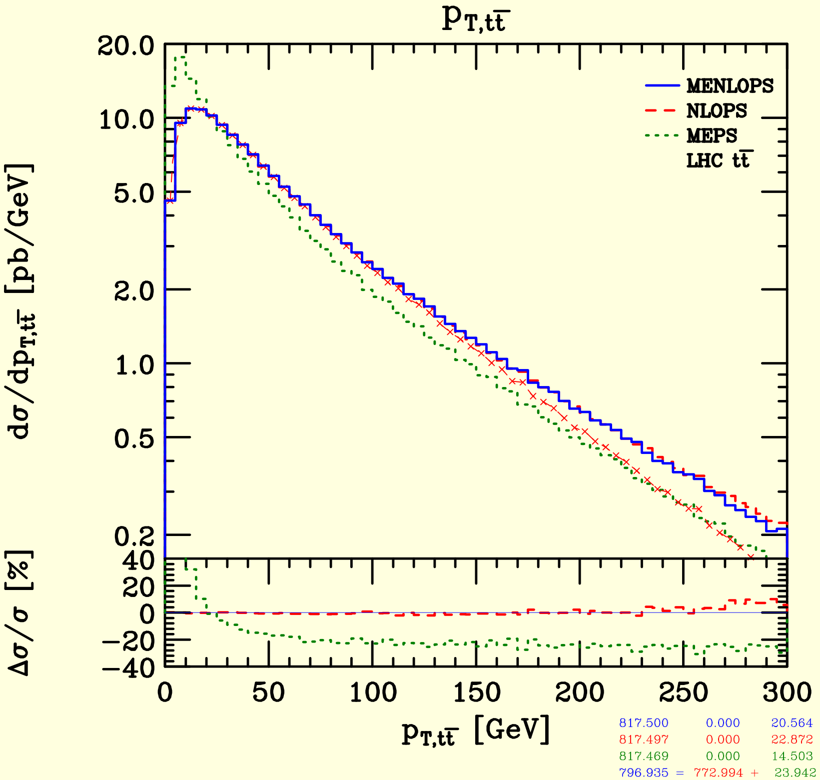
- NLO+PS sample generated using POWHEG
- ME+PS sample from Madgraph (using MLM matching, 20 GeV gen. cut, 30 GeV merging scale, virtuality ordered)
- The MENLOPS mergins scale was chosen equal to 60 GeV.

MEPS slightly too central.
NLOPS recovers NLO accuracy for this distribution.



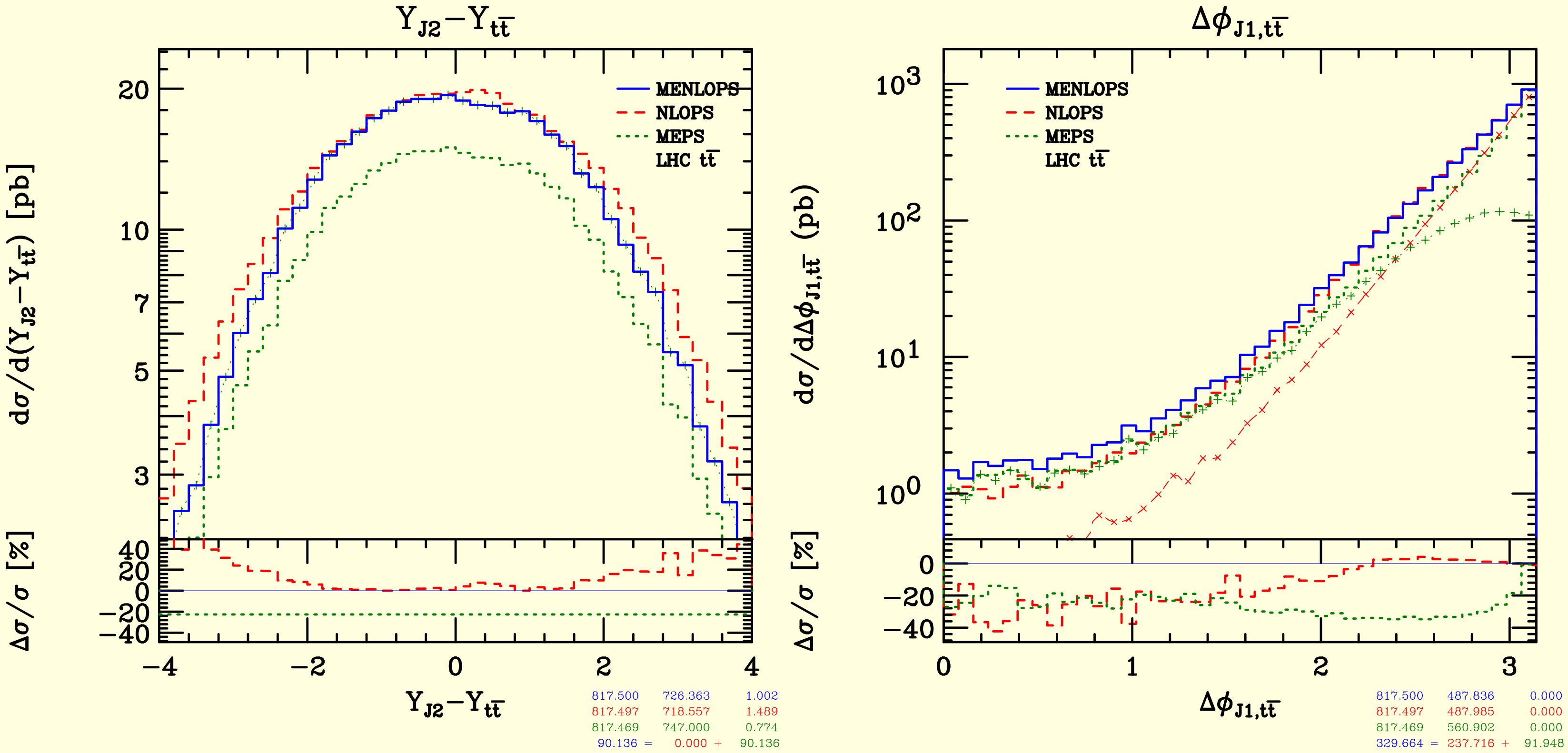


60 GeV MENLOPS scale



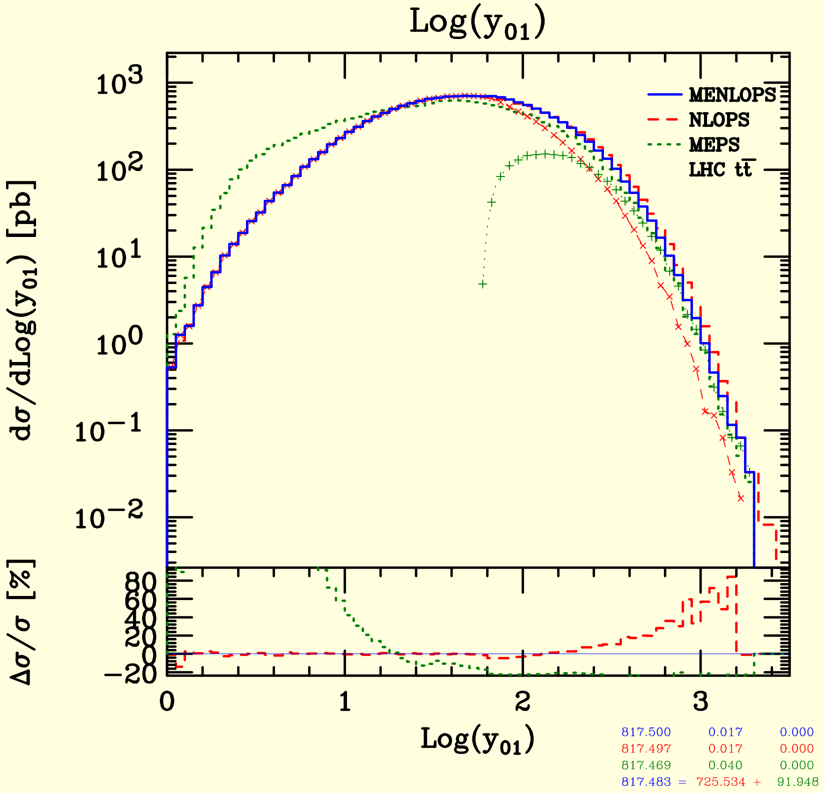
100 GeV MENLOPS scale

MENLOPS result stable with respect to variation of the merging scale

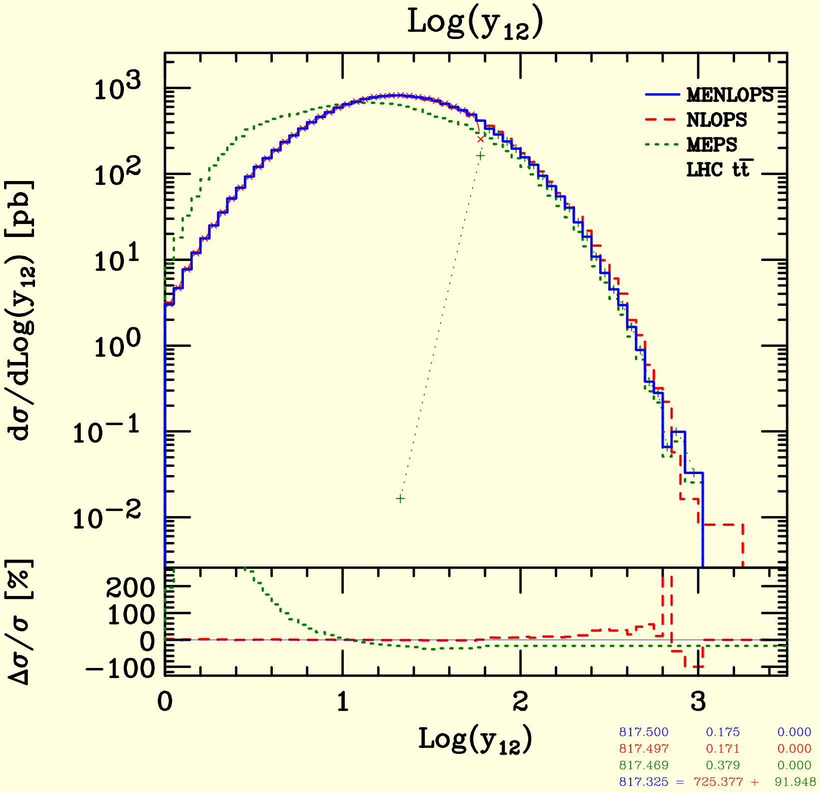


Rapidity of second jet corrected according to the MEPS result.

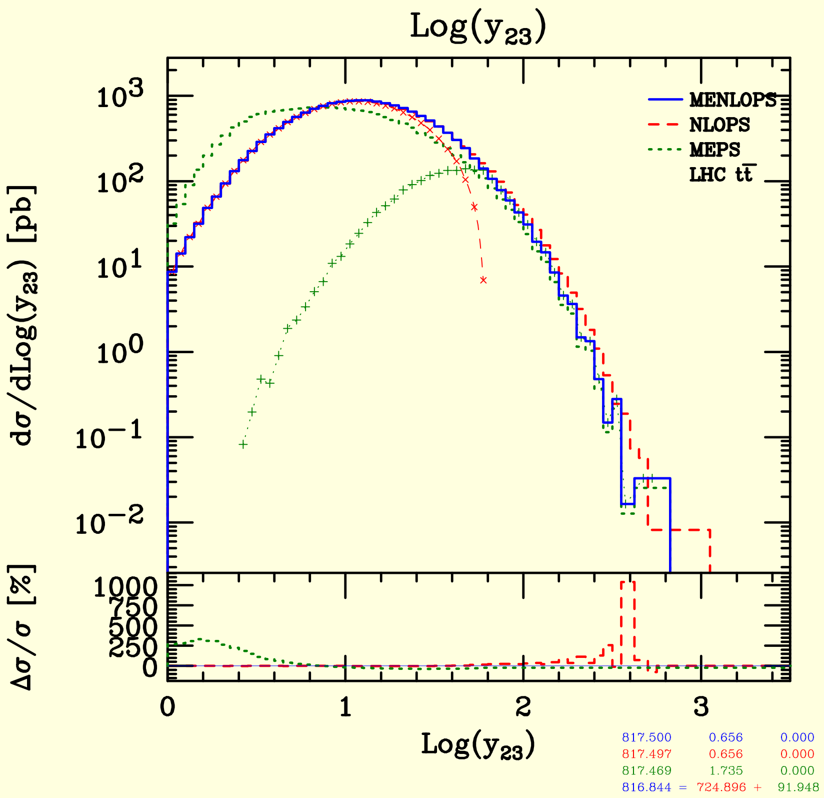
Azimuthal distance between $t\bar{t}$ system and hardest jet controlled by NLOPS in the back-to-back region, MEPS in the multijet region.

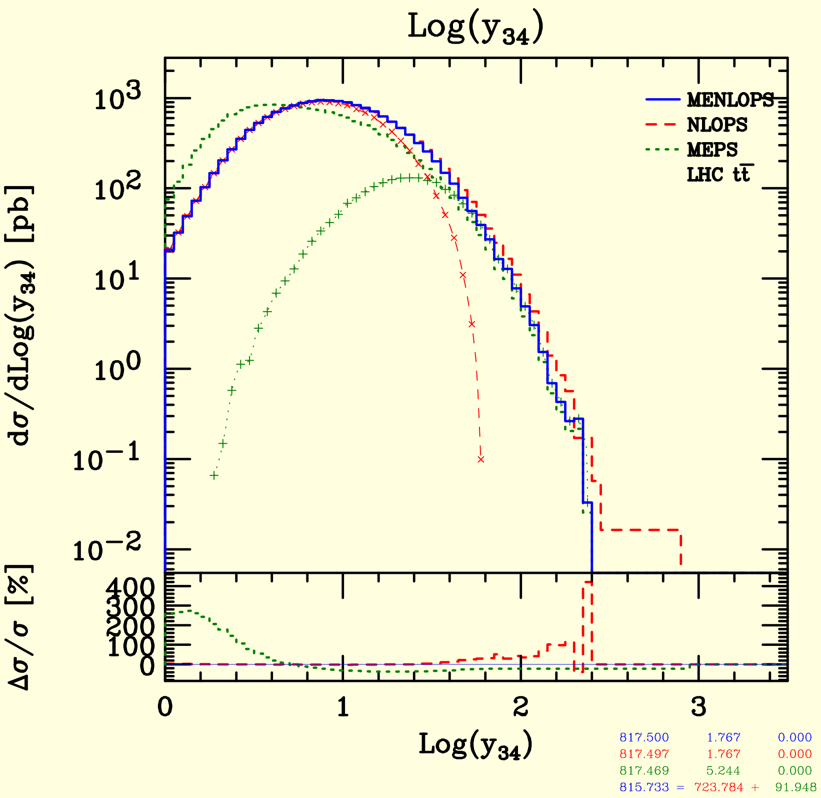


Log(y_{01})
MENLOPS
NLOPS
MEPS
LHC $t\bar{t}$
$d\sigma/d\text{Log}(y_{01})$ [pb]
$\Delta\sigma/\sigma$ [%]
Log(y_{01})
817.500 0.017 0.000
817.497 0.017 0.000
817.469 0.040 0.000
817.483 = 725.534 + 91.948



Log(y_{12})
MENLOPS
NLOPS
MEPS
LHC $t\bar{t}$
$d\sigma/d\text{Log}(y_{12})$ [pb]
$\Delta\sigma/\sigma$ [%]
Log(y_{12})
817.500 0.175 0.000
817.497 0.171 0.000
817.469 0.379 0.000
817.325 = 725.377 + 91.948





No kinks observed at the boundary of the merging parameters.

So:

- Merging NLO+PS and ME+PS does not look easy
- However: a simple practical approach leads to sensible results

Conclusions

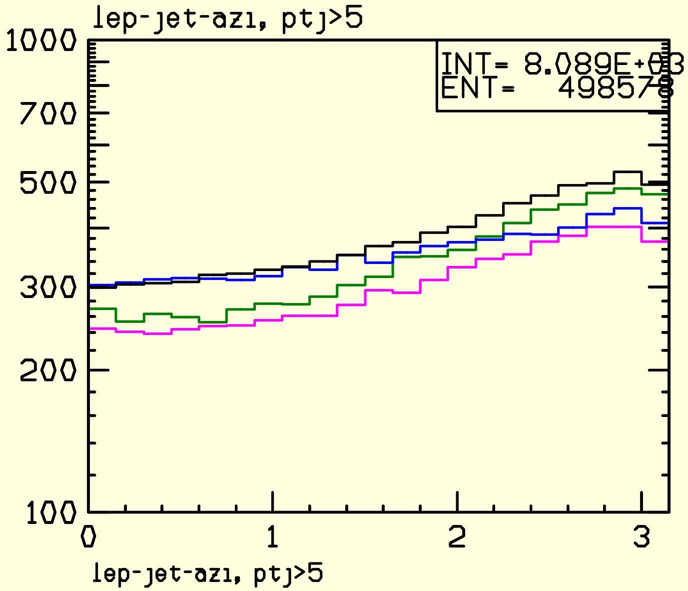
- Basic Higgs production processes available in NLO+PS implementations
- Several important backgrounds already there
- Automation: new processes should come in faster
- Merging NLO+PS and ME+PS under study: a promising practical approach has been demonstrated.

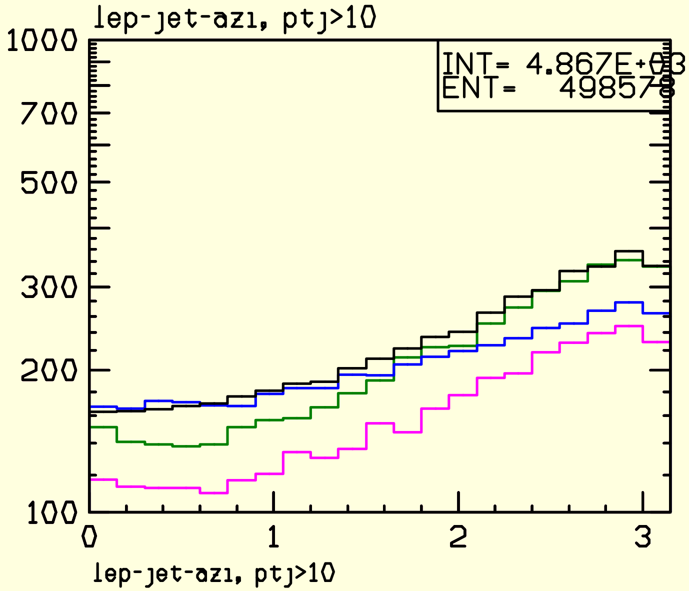
Do we expect differences between MC@NLO and POWHEG at NLO?

In MC@NLO: $R - R^{\rm MC}$ difference in $\mathcal{H}$ events is explicitly suppressed in the collinear and soft region. This may cause inaccuracies of NLO order when describing relatively soft jets.

Preliminary investigation in W production: look at the relative azimuth of the hardest jet and lepton. Expect flatter distributions in MC@NLO for small jet k_T.

Observed, but not (yet) fully understood:





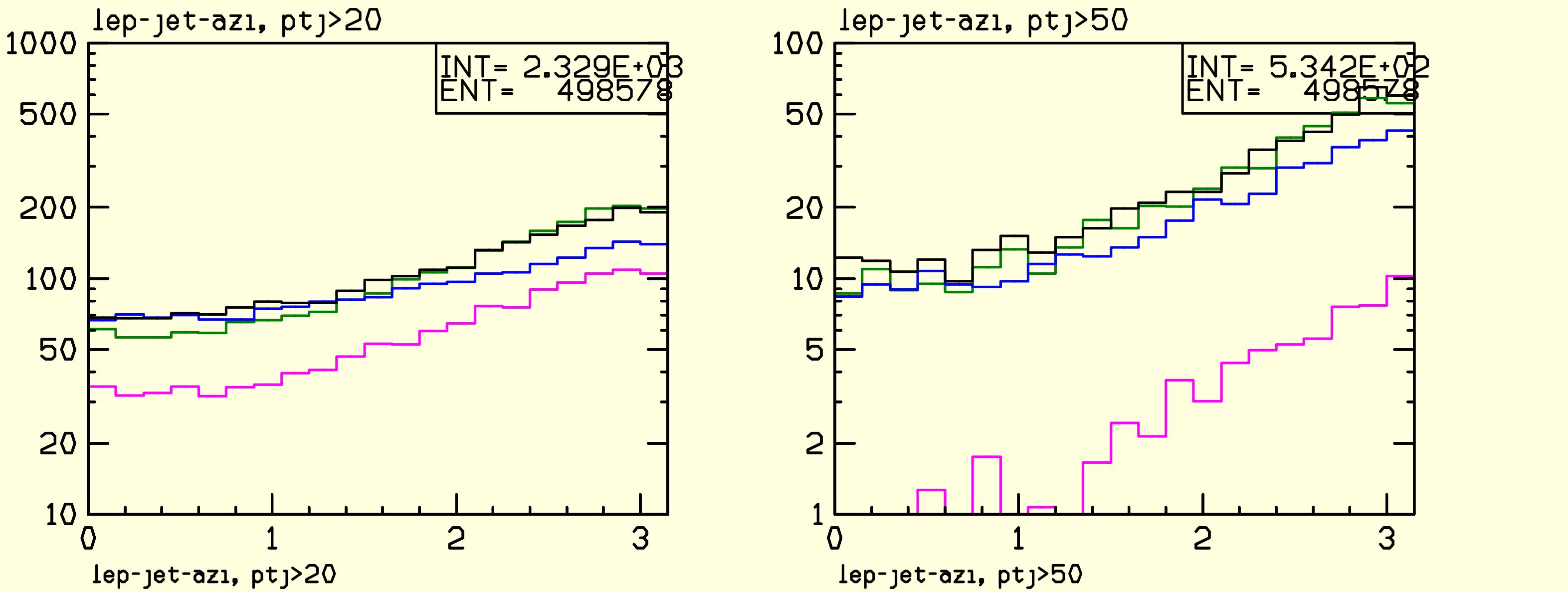
W production, LHC, 7 TeV; $\Delta\phi$ between the lepton and the hardest jet.

Black: POWHEG

Blue: MC@NLO

Green, HERWIG with soft and hard ME corrections

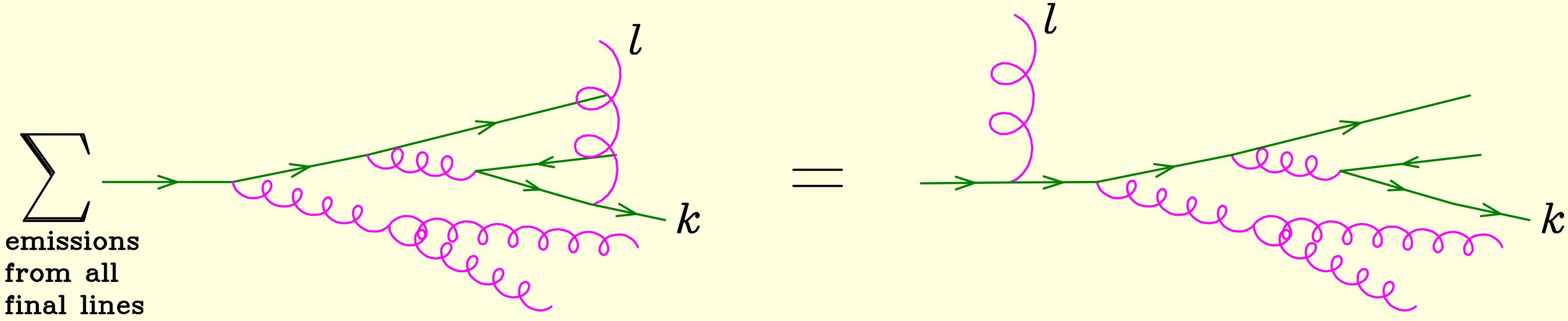
Magenta: HERWIG without soft and hard ME



MC@NLO is flatter (but HERWIG without ME is not flat). Needs more study.

Thuncated Showers

In angular ordered PS (HERWIG, HERWIG++) the hardest radiation may not be the first. Earlier radiations account for coherent emission of final state partons.



In P.N. 2004 (1st POWHEG paper), it was shown that, in order to recover coherence in cases where the hardest radiation is generated first (POWHEG, but also all ME+PS generators), one should add truncated vetoed showers to the event.

Truncated showers have been implemented in HERWIG++ POWHEG for Drell-Yan processes (Hamilton, Richardson and Tully, 2008), where only minor effects were found.

Truncated showers are also needed in relatively simple processes in the basic LO shower (all processes involving more than two coloured partons).

Summarizing:

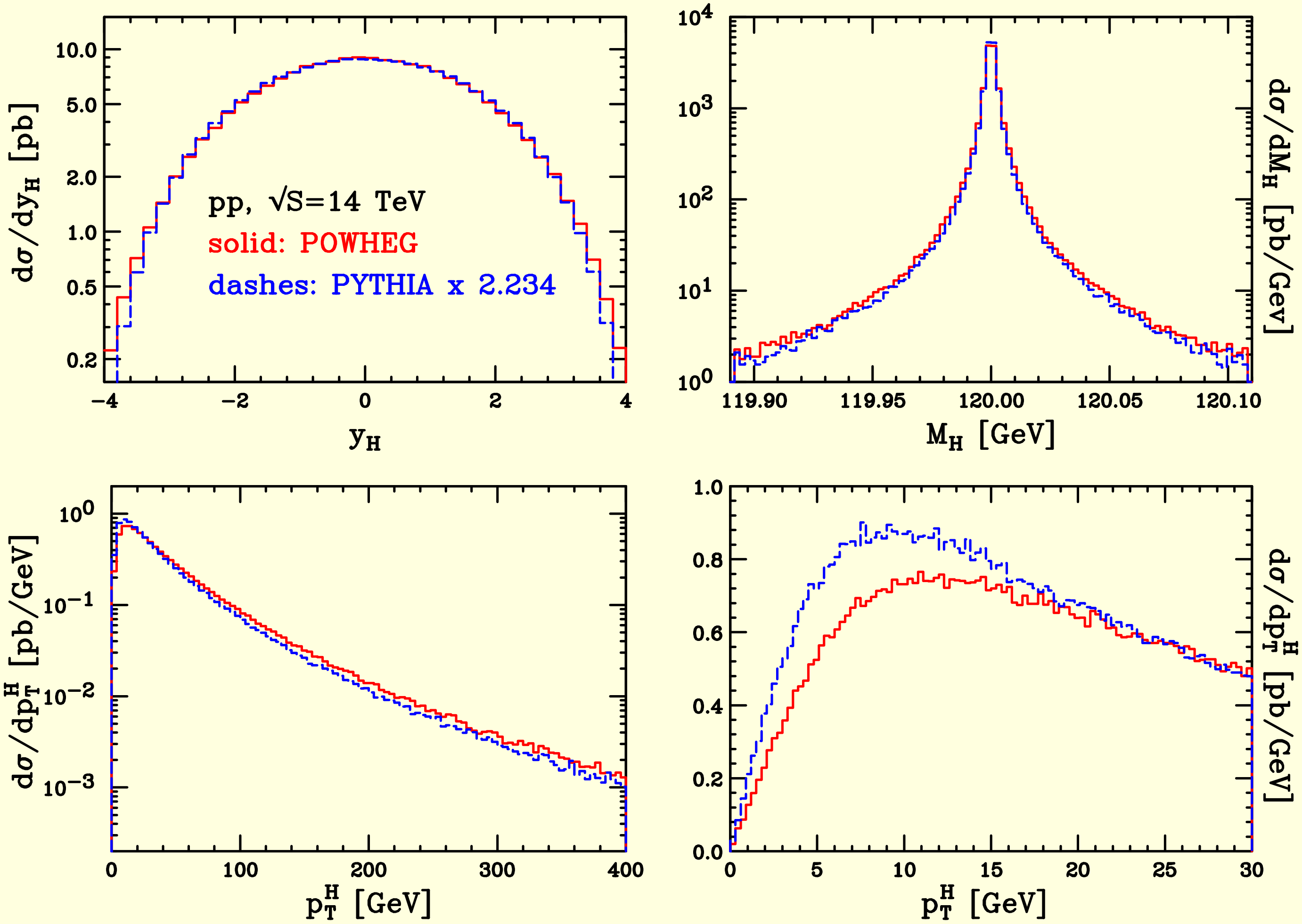
- Truncated showers should be implemented in conjunction with angular ordered shower Monte Carlo, if they are to be used interfaced to ME ot POWHEG generators, in order to preserve soft radiation coherence

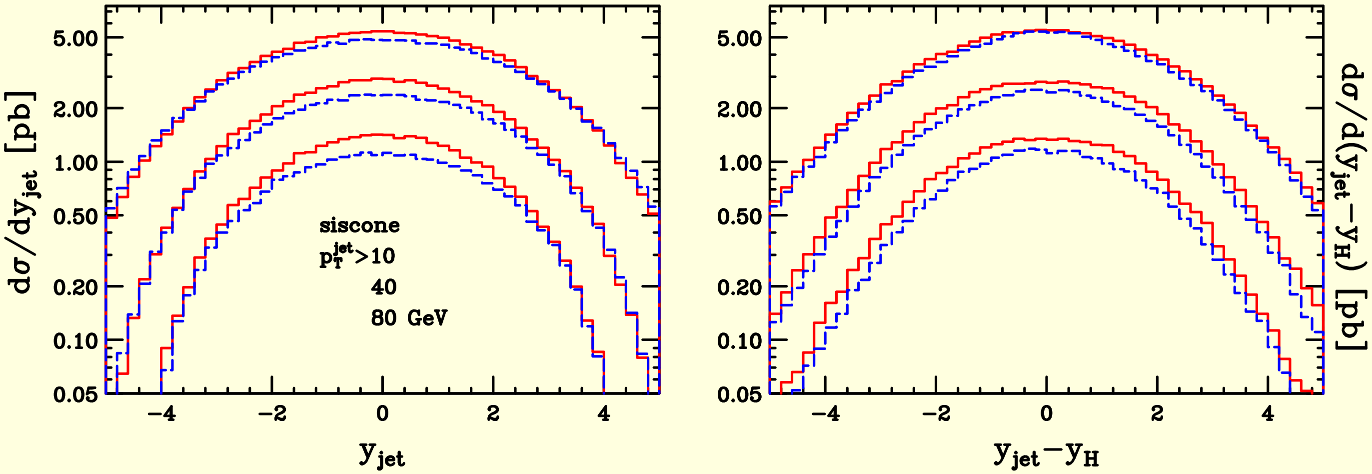
- Truncated shower are also needed in HERWIG or MC@NLO for elementary processes, like parton-parton scattering or heavy flavour production, that involve more than 2 coloured partons.

- Truncated showers are irrelevant for Monte Carlo that do not implement coherence correctly (virtuality ordered showers), or that implement coherence via p_T ordered dipole showers (new PYTHIA, SHERPA)

- Implementation of truncated showers for some processes have been studied by the HERWIG++ team. Up to now, no visible effects have been found

POWHEG and PYTHIA (with ME corrections)





Good agreement very well understood. PYTHIA Matrix-Element correction formalism very similar to POWHEG. Only misses y_H dependemt K factor to achieve full NLO accuracy.