# Shower Algorithms for Collider Physics

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Talk in http://moby.mib.infn.it/~nason/talks/

# Outline

- Shower Monte Carlo programs
- Large multiplicity events at LO: Matrix-Element programs
- Matching Matrix Elements and Showers
- Next-to-Leading (NLO) calculations
- Matching NLO and showers: `MC@NLO` and `POWHEG`
- `MC@NLO` and `POWHEG`: agreement and discrepancies
- Automation: the `POWHEG BOX`.
- Conclusions

# Shower Monte Carlo programs

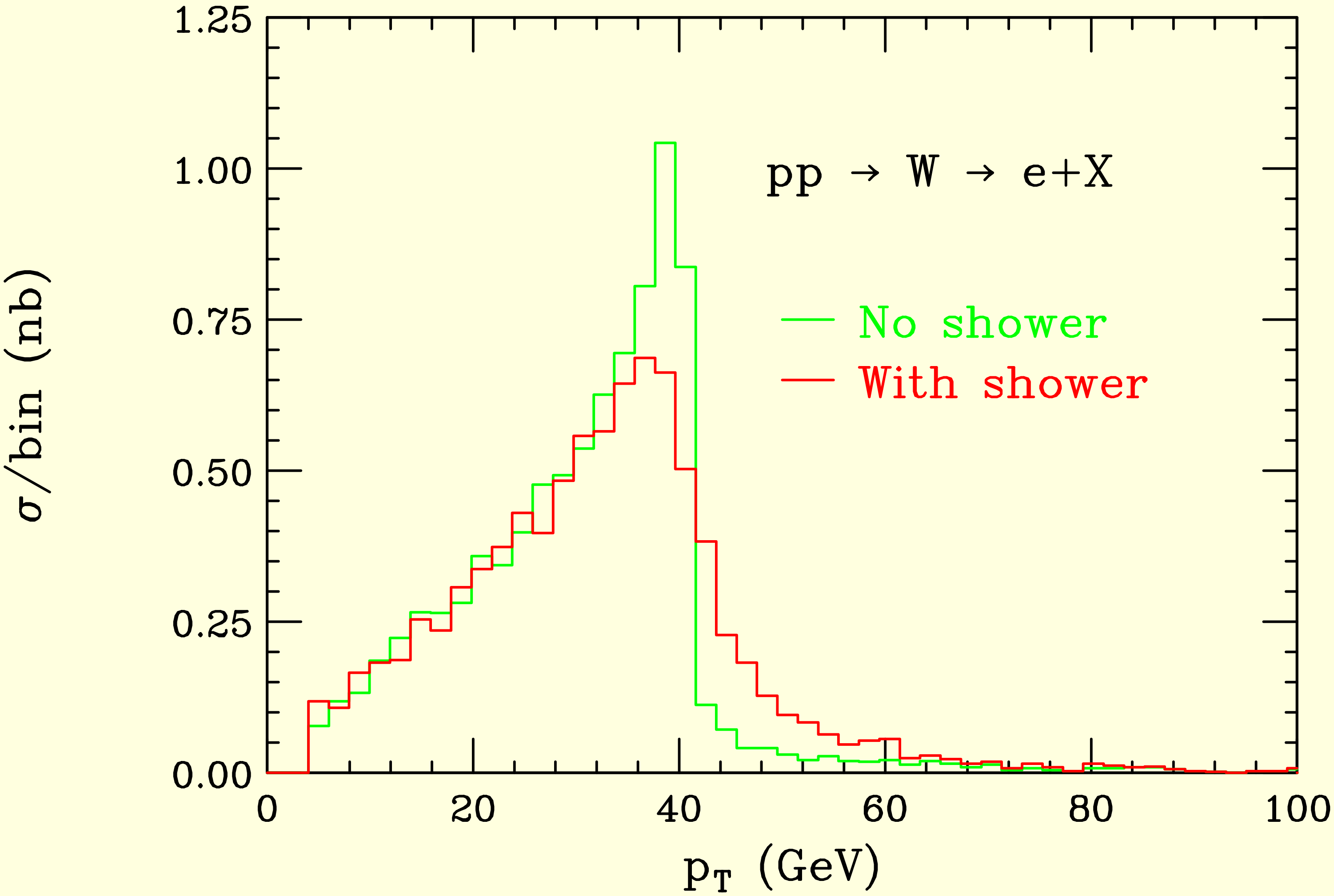
1. Large library of hard events cross sections (SM and BSM)
2. Dress hard events with QCD radiation: an algoritmic implementation of leading QCD corrections to all orders in perturbation theory.
3. Models for hadron formation
4. Models for underlying event, multi-parton collisions, minimum bias
5. Library for (spacetime) decays of unstable particles

The name SHOWER from item 2. The hope (and experience) is: the "Models" part is the same at all energies, and process independent. Once tuned at some energy, the SMC is predictive for all other energies.

HEP experiments feed this kind of output through their detector simulation software, and use it to determine efficiencies for signal detection and to perform background estimates. Analysis strategies are set up using these simulated data.

- In HEP (i.e. collider physics) not many questions can be answered without a Shower Monte Carlo (SMC). Heavily used since 1980's
- SMC's are forever (well, as long as HEP lives).
  Even if QCD was solved exactly, it is unlikely that complex
  high energy phenomena will be described better than in SMC models.
- SMC models have long been neglected in theoretical physics:
  Emphasis on QCD tests required more transparent theoretical methods.
  After LEP, QCD testing is less important.
  With LHC, QCD modeling is a primary issue: recent SMC revival.
- Thinking in terms of Shower algorithms gives us an easy to grasp,
  intuitive understanding of complex QCD phenomena
  (and a practical way to verify our ideas).

An example: (half an our of work)



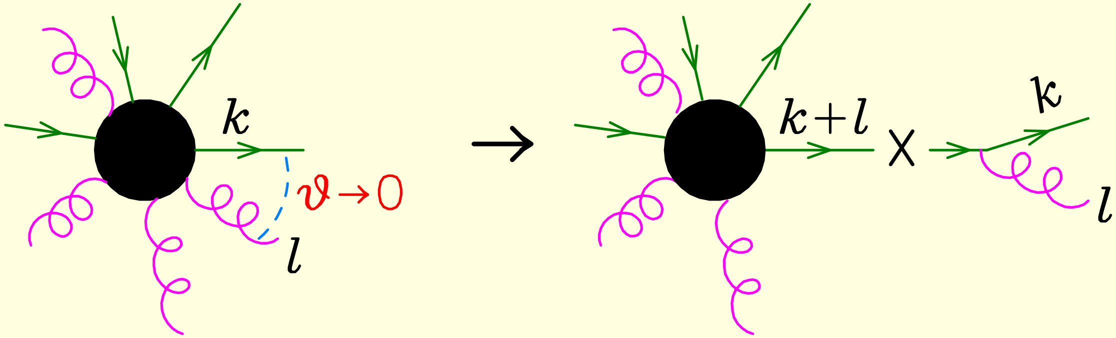
# Detailed description of the final state for each generated event:

| IHEP | ID | IDPDG | IST | MO1 | MO2 | DA1 | DA2 | P-X | P-Y | P-Z | ENERGY | MASS | V-X | V-Y | V-Z | V-C*T |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 30 | NU_E | 12 | 1 | 28 | 23 | 0 | 0 | 64.30 | 25.12 | -1194.4 | 1196.4 | 0.00 | 0.000E+00 | 0.000E+00 | 0.000E+00 | 0.000E+00 |
| 31 | E+ | -11 | 1 | 29 | 23 | 0 | 0 | -22.36 | 6.19 | -234.2 | 235.4 | 0.00 | 0.000E+00 | 0.000E+00 | 0.000E+00 | 0.000E+00 |
| 230 | PI0 | 111 | 1 | 155 | 24 | 0 | 0 | 0.31 | 0.38 | 0.9 | 1.0 | 0.13 | 4.209E-11 | 6.148E-11 | -3.341E-11 | 5.192E-10 |
| 231 | RHO+ | 213 | 197 | 155 | 24 | 317 | 318 | -0.06 | 0.07 | 0.1 | 0.8 | 0.77 | 4.183E-11 | 6.130E-11 | -3.365E-11 | 5.189E-10 |
| 232 | P | 2212 | 1 | 156 | 24 | 0 | 0 | 0.40 | 0.78 | 1.0 | 1.6 | 0.94 | 4.156E-11 | 6.029E-11 | -4.205E-11 | 5.250E-10 |
| 233 | NBAR | -2112 | 1 | 156 | 24 | 0 | 0 | -0.13 | -0.35 | -0.9 | 1.3 | 0.94 | 4.168E-11 | 6.021E-11 | -4.217E-11 | 5.249E-10 |
| 234 | PI- | -211 | 1 | 157 | 9 | 0 | 0 | 0.14 | 0.34 | 286.9 | 286.9 | 0.14 | 4.660E-13 | 8.237E-12 | 1.748E-09 | 1.749E-09 |
| 235 | PI+ | 211 | 1 | 157 | 9 | 0 | 0 | -0.14 | -0.34 | 624.5 | 624.5 | 0.14 | 4.056E-13 | 8.532E-12 | 2.462E-09 | 2.462E-09 |
| 236 | P | 2212 | 1 | 158 | 9 | 0 | 0 | -1.23 | -0.26 | 0.9 | 1.8 | 0.94 | -4.815E-11 | 1.893E-11 | 7.520E-12 | 3.252E-10 |
| 237 | DLTABR-- | -2224 | 197 | 158 | 9 | 319 | 320 | 0.94 | 0.35 | 1.6 | 2.2 | 1.23 | -4.817E-11 | 1.900E-11 | 7.482E-12 | 3.252E-10 |
| 238 | PI0 | 111 | 1 | 159 | 9 | 0 | 0 | 0.74 | -0.31 | -27.9 | 27.9 | 0.13 | -1.889E-10 | 9.893E-11 | -2.123E-09 | 2.157E-09 |
| 239 | RHO0 | 113 | 197 | 159 | 9 | 321 | 322 | 0.73 | -0.88 | -19.5 | 19.5 | 0.77 | -1.888E-10 | 9.859E-11 | -2.129E-09 | 2.163E-09 |
| 240 | K+ | 321 | 1 | 160 | 9 | 0 | 0 | 0.58 | 0.02 | -11.0 | 11.0 | 0.49 | -1.890E-10 | 9.873E-11 | -2.135E-09 | 2.169E-09 |
| 241 | KL_1- | -10323 | 197 | 160 | 9 | 323 | 324 | 1.23 | -1.50 | -50.2 | 50.2 | 1.57 | -1.890E-10 | 9.879E-11 | -2.132E-09 | 2.166E-09 |
| 242 | K- | -321 | 1 | 161 | 24 | 0 | 0 | 0.01 | 0.22 | 1.3 | 1.4 | 0.49 | 4.250E-11 | 6.333E-11 | -2.746E-11 | 5.211E-10 |
| 243 | PI0 | 111 | 1 | 161 | 24 | 0 | 0 | 0.31 | 0.38 | 0.2 | 0.6 | 0.13 | 4.301E-11 | 6.282E-11 | -2.751E-11 | 5.210E-10 |

# Shower basics: Collinear factorization

QCD emissions are enhanced near the collinear limit

Cross sections factorize near collinear limit




$$|M_{n+1}|^2 d\Phi_{n+1} \Longrightarrow |M_n|^2 d\Phi_n \, \frac{\alpha_s}{2\pi} \frac{dt}{t} P_{q,qg}(z) \, dz \, \frac{d\phi}{2\pi}$$

$t$ : hardness (either virtuality or $p_T^2$ or $E^2\theta^2$ etc.)

$z = k^0/(k^0 + l^0)$ : energy (or $p_\parallel$, or $p^+$) fraction of quark

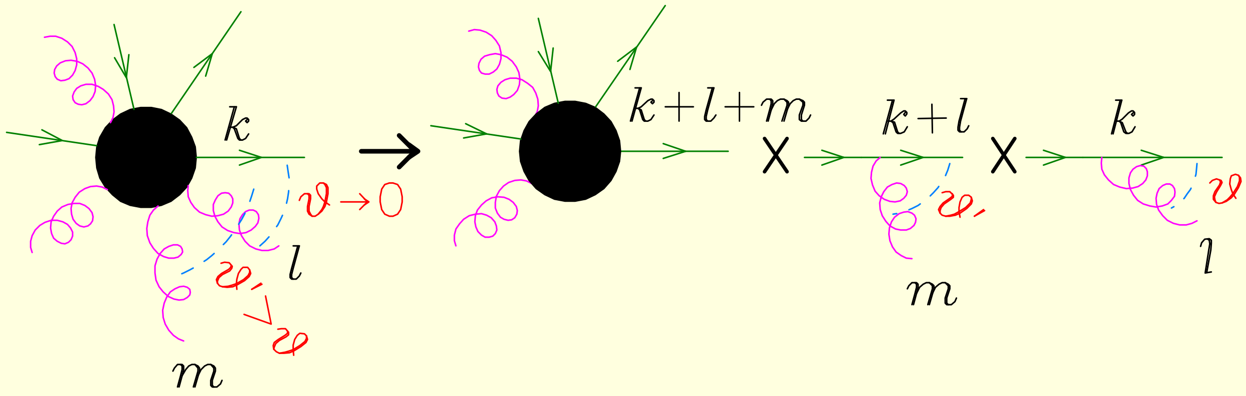
$P_{q,qg}(z) = C_F \frac{1+z^2}{1-z}$ : Altarelli – Parisi splitting function

(ignore $z \to 1$ IR divergence)

(the really difficult part is to deal with IR divergencies ... not discussed here)

If another gluon becomes collinear, iterate the previous formula:

$\theta', \theta \to 0$
with $\theta' > \theta$




$$|M_{n+1}|^2 d\Phi_{n+1} \Longrightarrow |M_{n-1}|^2 d\Phi_{n-1} \times \frac{\alpha_s}{2\pi}\frac{dt'}{t'} P_{q,qg}(z')\, dz' \frac{d\phi'}{2\pi} \times \frac{\alpha_s}{2\pi}\frac{dt}{t} P_{q,qg}(z)\, dz \frac{d\phi}{2\pi}\theta(t'-t)$$

Collinear partons can be described by a factorized formula ordered in $t = \theta E$.
For $m$ collinear emissions:

$$\left(\frac{\alpha_s}{2\pi}\right)^m \int_{\theta_{\rm min}} \frac{d\theta_1}{\theta_1} \int_{\theta_1} \frac{d\theta_2}{\theta_2} \dots \int_{\theta_{m-1}} \frac{d\theta_m}{\theta_m} \propto \frac{\log^m \frac{1}{\theta_{\rm min}^2}}{m!} \approx \left(\frac{\alpha_s}{2\pi}\right)^m \frac{\log^m \frac{Q^2}{\Lambda^2}}{m!}$$

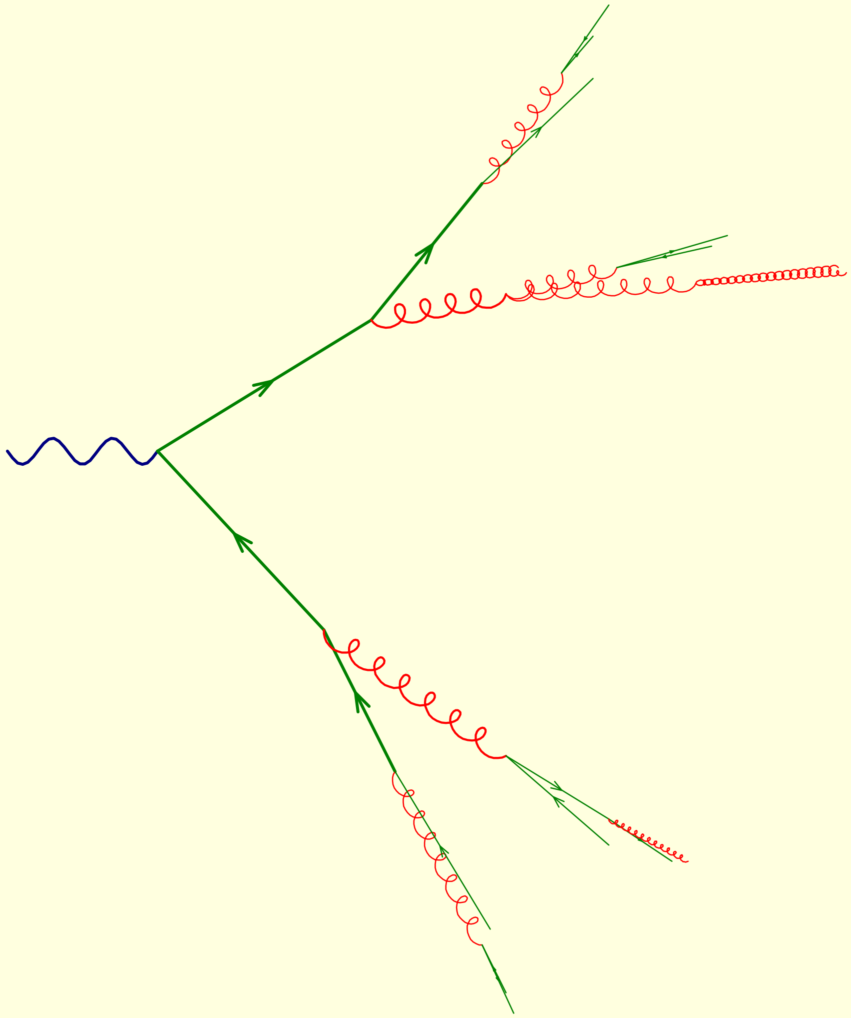
where we have taken $\theta_{\rm min} \approx \Lambda/Q$; (Leading Logs) This is of order 1!
Typical dominant configuration at very high $Q^2$

Besides $q \to qg$, also $g \to gg$, $g \to q\bar{q}$ come into play.

Typical configurations: intermediate angles of order of geometric average of upstream and downstream angles.

Each angle is $\mathcal{O}(\alpha_s)$ smaller than its upstream angle, and $\mathcal{O}(\alpha_s)$ bigger than its downstream angle.

As relative momenta become smaller $\alpha_s$ becomes bigger, and this picture breaks down.



## For a consistent description:
## include virtual corrections at same LL approximation

One can show that the effect of virtual corrections is given by

- Let $\alpha(\mu) \Longrightarrow \alpha(t)$ in each vertex, where $t$ is the hardness of the vertex (i.e. hardness of the incoming line)
- For each intermediate line include the factor

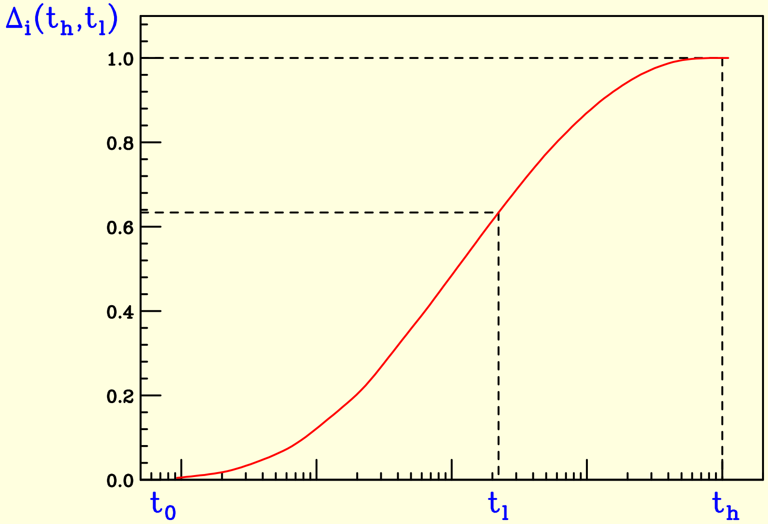
$$\Delta_i(t_h, t_l) = \exp\left[ -\sum_{(jk)} \int_{t_l}^{t_h} \frac{dt'}{t'} \int dz \, \frac{\alpha_s(t')}{2\pi} P_{i,jk}(z) \right]$$

where $t_h$ is the hardness of the vertex originating the line, and $t_l$ is the hardness of the vertex where the line ends.

# Sudakov form factor

$$\Delta_i(t_h, t_l) = \exp\left[ -\sum_{(jk)} \int_{t_l}^{t_h} \frac{dt'}{t'} \int dz \, \frac{\alpha_s(t')}{2\pi} P_{i,jk}(z) \right]$$

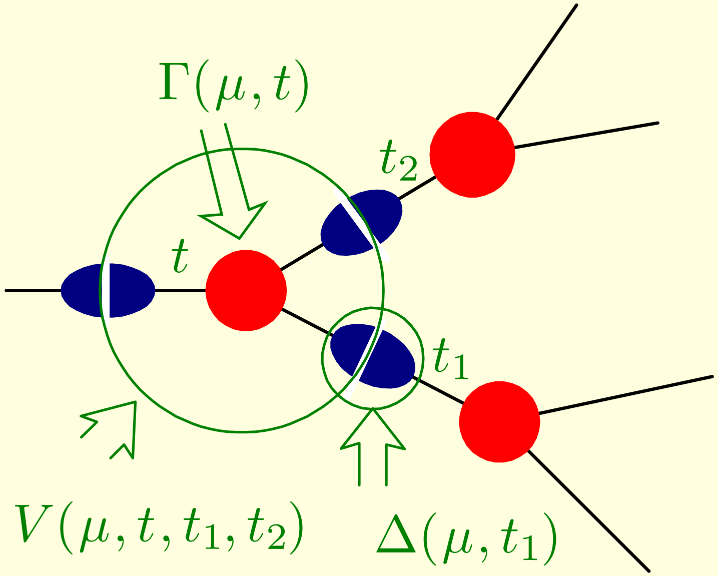
As $t_l$ becomes small the exponent tend to diverge, and $\Delta_i(t_h, t_l)$ approaches $0$.
In fact, because of $\alpha_s(t)$, we must stop at $t_0 \gtrsim \Lambda_{\rm QCD}$.




Physical interpretation:

Large radiation probability $\Rightarrow$ strong suppression of radiation gaps.

# Proof of effect of virtual corrections




Effective (RG invariant) splitting vertex:

$$V^2(\mu,t,t_1,t_2) = \underbrace{\Gamma^2(\mu,t)}_{\text{dominated by hardest scale!}}\Delta(\mu,t)\Delta(\mu,t_1)\Delta(\mu,t_2)$$

Choosing $\mu = t$ (using $\Delta(t,t)\approx 1$)

$$V^2(\mu,t,t_1,t_2) = V^2(t,t,t,t)\,\Delta(t,t_1)\Delta(t,t_2)$$

$V(t,t,t,t)$ is the three level vertex with $\alpha \to \alpha(t)$.
The form $\Delta(t,t_1)$ follows from RG arguments.

In fact: $\Delta_i(t,t_1) = \exp\left[ -\sum_{(jk)} \int_{t_1}^{t} \frac{dt'}{t'} \int dz \, \frac{\alpha_s(t')}{2\pi} P_{i,jk}(z) \right]$ Sudakov form factor

consistent with KLN cancellation of IR singularities, and with RG.

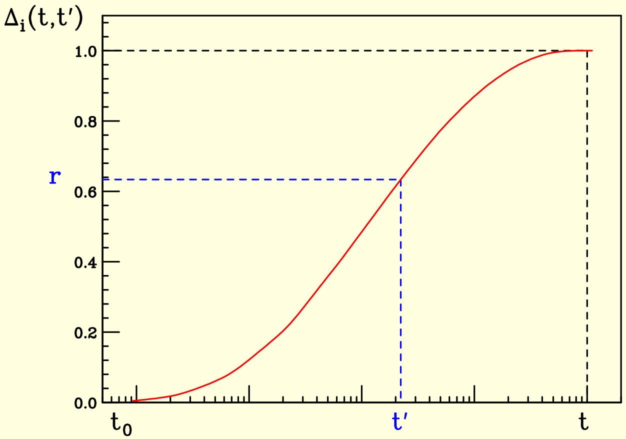
# Final Recipe

- Consider all tree graphs.
- Assign ordered hardness parameters $t$ to each vertex.
- Include a factor
$$\frac{\alpha_s(t)}{2\pi} P_{i,jk}(z) \frac{dt}{t} dz \frac{d\phi}{2\pi}$$
at each vertex $i \to jk$.
- Include a factor $\Delta_i(t_1, t_2)$ to each internal line with a parton $i$, from hardness $t_1$ to hardness $t_2$.
- Include a factor $\Delta_i(t, t_0)$ on final lines ($t_0$: IR cutoff)

## Most important: the shower recipe can be easily implemented as a computer code!

## Shower Algorithm:

- Generate the Born phase space with a probability $B(\Phi_B)\,d\Phi_B$.
- For each outgoing coloured leg:
  - → Determine its hardness $t$
  - → Generate a uniform random number $0 < r < 1$;
  - → Solve the equation $\Delta_i(t,t') = r$ for $t'$;
  - → If $t' < t_0$ stop here (final state line);
  - → generate $z, jk$ with probability $P_{i,jk}(z)$, and $0 < \phi < 2\pi$ uniformly;

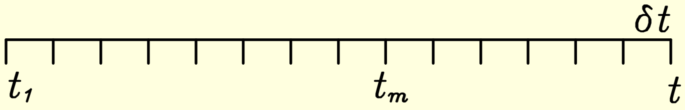



- restart from each branch, with hardness parameter $t'$

Probabilistic intepretation: branching probability of line of flavor $i$

$$dP(t_1,t) = \underbrace{\exp\left[-\sum_{(jk)}\int_t^{t_1}\frac{dt'}{t'}\int dz\,\frac{\alpha_s(t')}{2\pi}P_{i,jk}(z)\right]}_{\Delta(t_1,t)}\frac{\alpha_s(t)}{2\pi}P_{i,jk}(z)\,\frac{dt}{t}\,dz\,\frac{d\phi}{2\pi}$$

break up $t_1, t$ into small subintervals:




$$dP(t_1,t) = \left[\prod_m\left(\underbrace{1-\sum_{(jk)}\frac{\delta t}{t_m}\int dz\,\frac{\alpha_s(t_m)}{2\pi}P_{i,jk}(z)}_{\text{No emission prob. in } t_m, t_m+\delta t}\right)\right]\underbrace{\frac{\alpha_s(t)}{2\pi}P_{i,jk}(z)\,\frac{\delta t}{t}\,dz\,\frac{d\phi}{2\pi}}_{\text{emission prob. in } t, t+\delta t}$$

So: the probability for the first branching at hardness $t$ is the product of the non-emission probability $\Delta(t_1,\ t)$ in all hardness intervals between $t_1$ and $t$, times the emission probability at hardness $t$.

(more or less) obvious consequences:

- The total branching probability plus the no-branching probability is 1; mathematically

$$\int_{t_0}^{t_1} dP(t_1, t') = \int_{t_0}^{t_1} d\Delta_i(t_1, t') = 1 - \Delta_i(t_1, t_0)$$

- The Sudakov form factor $\Delta_i(t_1, t)$ is the no-branching probability from scale $t_1$ down to the scale $t$.

- The branching probability is independent of what happens next (because the total probability of what happens next is 1).

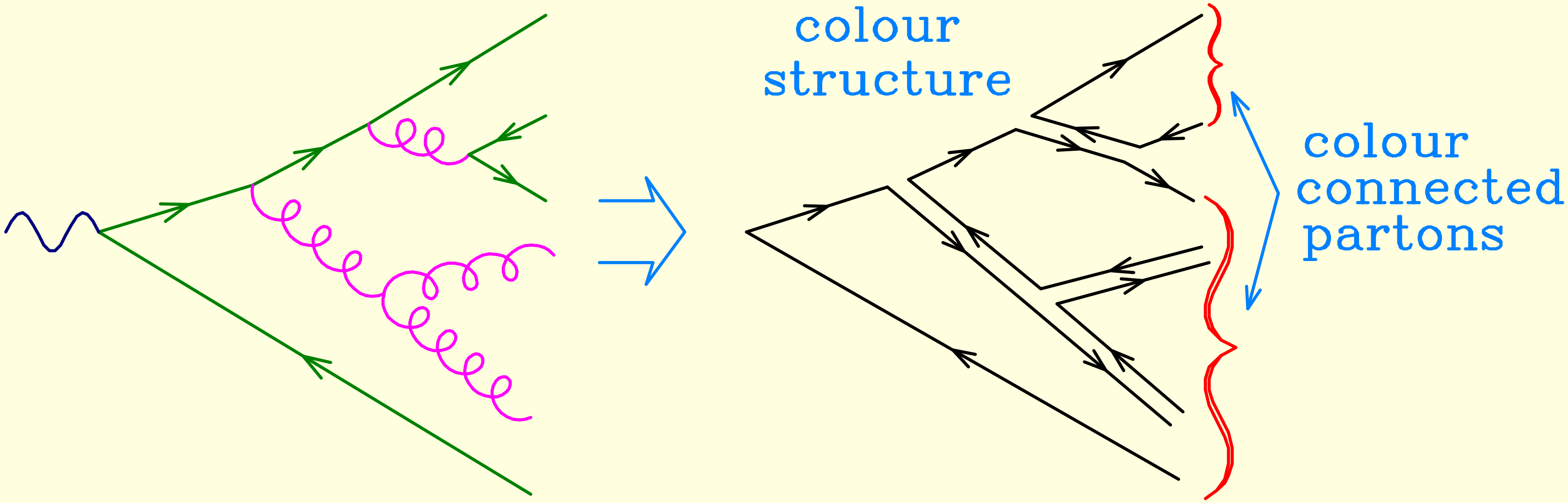
This property is often called unitarity of the shower. It is a consequence of the Kinoshita-Lee-Nauenberg theorem: collinear divergence must cancel in the inclusive cross section.

# COLOUR AND HADRONIZATION

SMC's assign colour labels to partons.

Only colour connections are recorded (as in large N limit).

Initial colour assigned according to hard cross section.




Colour assignements are used in the hadronization model.

Most popular models: Lund String Model, Cluster Model.

In all models, color singlect structures are formed out of colour connected partons, and are decayed into hadrons preserving energy and momentum.

# Implementation

- Origin: Fox+Wolfram (1980)
- `COJETS` Odorico (1984)
- `ISAJET` Page+Protopopescu (1986)
- `FIELDAJET` Field (1986)
- `JETSET` Sjöstrand (1986)
- `PYTHIA` Bengtsson+Sjöstrand (1987), Skands+Sjöstrand
  `PYTHIA 8` Mrenna+Skands+Sjöstrand (2007)
- `Ariadne` Lönnblad (1991)
- `HERWIG` Marchesini+Webber (1988)
  Marchesini+Webber+Abbiendi+Knowles+Seymour+Stanco (1992)
  `HERWIG++` Bahr+Gieseke+Gigg+Grellscheid+Hamilton+Platzer +Richardson+Seymour+Tully (2003)
- `SHERPA` Gleisberg+Hoche+Krauss+Schalicke+Schumann+Winter (2004)

# New developements

- Interfacing ME (Matrix-Elements) generators with Parton Showers (CKKW matching (Catani, Krauss, Küen, Webber), MLM matching)
- Interfacing NLO calculations to Parton Showers (NLO+S) (MC@NLO (Frixione, Webber), POWHEG (PN))

Several NLO+S approaches have appeared:

- Kramer, Mrenna, Soper ($e^+e^- \to 3\,\mathrm{partons}$)
- Shower by antenna factorization (Frederix,Giele,Kosower,Skands) (toy implementation for $H \to gg$)
- Shower by Catani-Seymour dipole factorization (Schumann)
- Shower with quantum interference (Nagy,Soper)
- Shower by Soft Collinear Effective Theory (Bauer,Schwartz)

Until now, complete results for hadron colliders only from MC@NLO and POWHEG

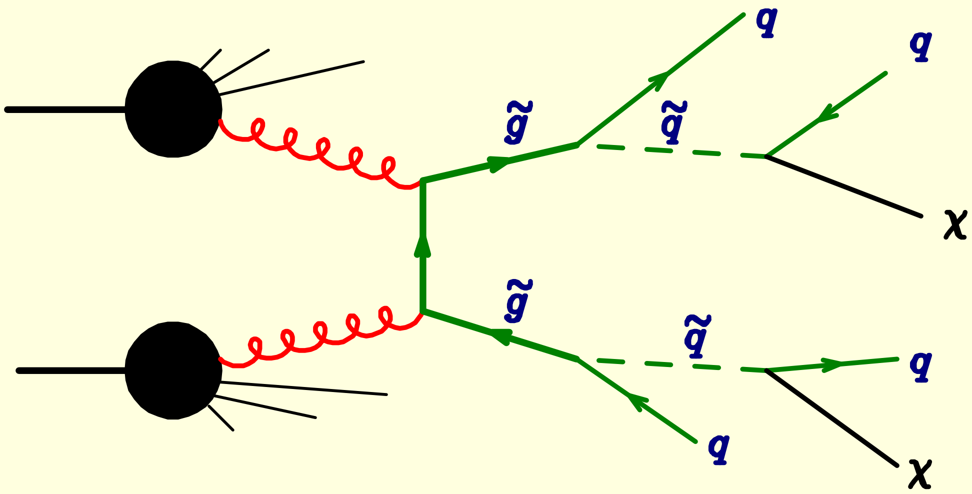
# Large angle emission

A disturbing feature of SMC's: the hardest jet generated in the shower is not really collinear in about 10% of the events (i.e. $\mathcal{O}(\alpha_s)$). Thus, the gross feature of the event is wrongly described by the SMC in 10% of the cases.
Most SMC deal with this problem, implementing a matrix element correction for the simplest processes.
Multi-parton processes: algorithms exist to compute them with high efficiency;

The 10% large angle events are most important for BSM backgrounds!

# Tree level processes for LHC

Discovery example: gluino production



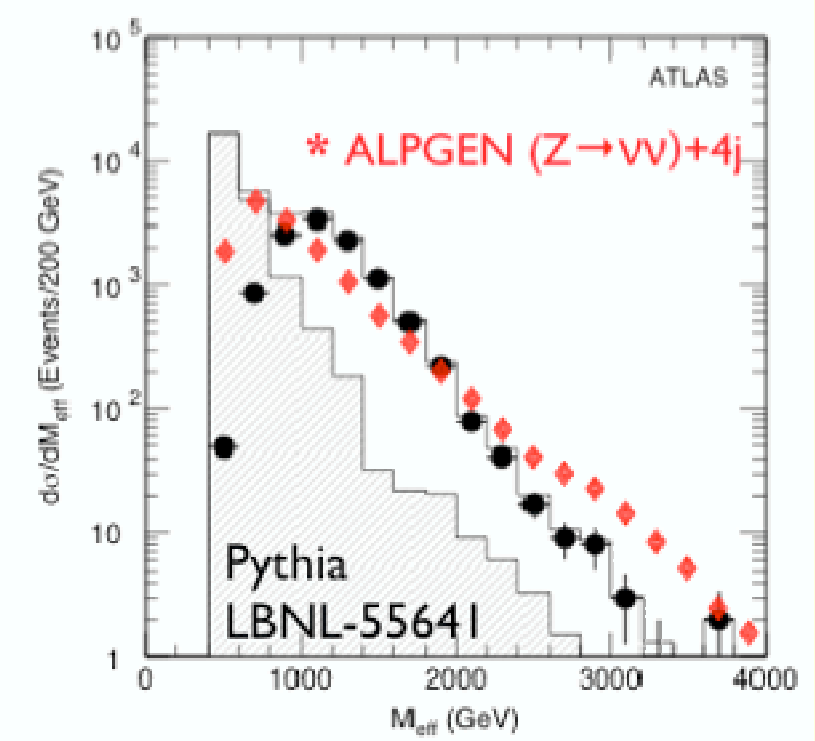

If $\chi = \chi^0$: MET+4 jets;
if $\chi = \chi^\pm \to W^\pm + \chi^0$:
MET+ up to 8 jets;

$W + \text{jets}$,
$Z + \text{jets}\ (Z \to \nu\bar{\nu})$,
$t\bar{t} + \text{jets}$
are all backgrounds;

But: signal jets are well separated (i.e. not collinear) because they come from the decay of heavy objects. Cannot really trust the SMC for the calculation of the jets accompanying the $W$, $Z$ or $t\bar{t}$ pair.

The use of exact ME is mandatory (Gianotti,Mangano, 05)

$M_{\text{eff}}$ distribution for a potential multijet+$E_T^{\text{miss}}$ SUSY signal
dark circles: signal
Shaded area: MC background



# Calculation of Complex Processes: LO (tree Level) Matrix Elements

Many available programs can do automatic evaluation of LO cross sections.

1. Helicity amplitudes (HELAS, Hagiwara, Kanzaki, Murayama, Watanabe; MadGraph, Maltoni, Stelzer)
2. Behrends-Giele recursion relations (VecBos)
3. other recursive methods, (ALPHA, Caravaglios, M.Moretti)
   - ALPGEN, Mangano, Moretti, Piccinini, Pittau, Polosa
   - HELAC, Kanaki, Papadopoulos
4. CSW recursion (from twistors), Cachazo, Svrček, Witten,2004, Dixon, Glover, Khoze, Badger, Bern, Forde, Kosower, Mastrolia
5. BCFW recursion, Britto, Cachazo, Feng, Witten,2004 +masses: Badger, Glover, Khoze, Svrček; Schwinn, Weinzierl

# Comparison of algorithms

CSW and BCF yield more compact expressions.
Comparison of automated algorithms by
Duhr, Hoche, F.Maltoni, Jun.06; also Dinsdale, Ternick, Weinzierl, Feb.06;

BG=Berends-Giele, CSW=Cachazo-Svrček-Witten, BCF=Britto-Cachazo-Feng
CO=Colour ordered, CD=Colour dressed (i.e. full amplitude)

| Final state | BG | | BCF | | CSW | |
|---|---|---|---|---|---|---|
| | CO | CD | CO | CD | CO | CD |
| $2g$ | 0.24 | 0.28 | 0.28 | 0.33 | 0.31 | 0.26 |
| $3g$ | 0.45 | 0.48 | 0.42 | 0.51 | 0.57 | 0.55 |
| $4g$ | 1.20 | 1.04 | 0.84 | 1.32 | 1.63 | 1.75 |
| $5g$ | 3.78 | 2.69 | 2.59 | 7.26 | 5.95 | 5.96 |
| $6g$ | 14.20 | 7.19 | 11.9 | 59.10 | 27.80 | 30.60 |
| $7g$ | 58.50 | 23.70 | 73.6 | 646.00 | 146.00 | 195.00 |
| $8g$ | 276.00 | 82.10 | 597 | 8690.00 | 919.00 | 1890.00 |
| $9g$ | 1450.00 | 270.00 | 5900 | 127000.00 | 6310.00 | 29700.00 |
| $10g$ | 7960.00 | 864.00 | 64000 | | 48900.00 | |

Berends-Giele (comparable to ALPGEN, HELAC) still faster ...

## summarizing (LO):

- General purpose ME generators for SM and MSSM tree level processes are available (example: Madgraph, any process, not very fast)
- Very fast generators, capable to add several gluons in the final state already available. Example: ALPGEN, processes added by authors

| | |
|---|---|
| $WQ\bar{Q}$ + up to 4 jets | $Q\bar{Q}\,H$ + up to 4 jets |
| $Z/\gamma + Q\bar{Q}$ + up to 4 jets | Inclusive $N$ jets, with $N$ up to 6 |
| $W$ + up to 6 jets | $N\gamma$ + $M$ jets |
| $W + c$ + up to 5 jets | Single top |
| $Z$ + up to 6 jets | $W$ + photons + jets |
| $nW + m\,Z + k\,H + l\gamma$ + up to 3 jets | $WQ\bar{Q}$ + photons + jet |
| $Q\bar{Q}$ + up to 6 jets | $Q\bar{Q}$ + $M$-photons + $N$-jets |
| $Q\bar{Q} + Q'\bar{Q}'$ + up to 4 jets | Higgs + up to 5 jets |

Total automation of fast techniques desirable (not far)

# Matrix elements and Shower algorithms

Exact matrix elements for high multiplicity emission are available.
Why use the collinear approximation in the shower algorithms?

Exact matrix elements can only be used for relatively wide angular separation of light partons. Shower algorithms remedy to this by including enhanced virtual corrections (i.e. Sudakov form factors), that suppress small angle emission.
Can we do this with matrix elements?

# Historical approach: CKKW

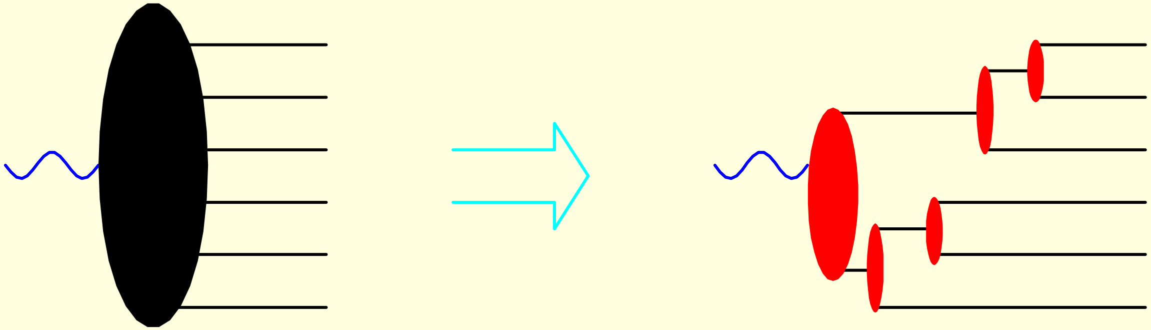
Catani, Krauss, Küen, Webber (2001), (in $e^+e^-$ annihilation).

## In a nut-shell:

- Correct ME calculations when they approach the collinear region, so that they reproduce Shower results
- Let the Shower take care of radiation with $M < M_{\rm cut}$, where $M$ is some cutoff on the jet separation

# In a better approximation

- Build a sample of ME events, generated with a probability proportional to the corresponding cross section. At this stage, use a fixed reference value of the strong coupling $\alpha_s(M)$. Events are generated with a cut $M$ on the $t$ of parton pairs, and on the $p_T$ of each parton.
- Clusterize ME partons to reconstruct a shower skeleton (by pairing up particles that yield smallest $t$ recursively)



Red blobs have decreasing $t$ values

You can think of $t$ as the virtuality of the pair, but other definitions are possible.

- Evaluate ME couplings $\alpha_s(t)$ at scales $t$ of vertices in shower skeleton
- Assign Sudakov form factors $\Delta(t,t')$ to the skeleton intermediate lines (as in Shower MC)
- Reject the event with a probability $\prod \frac{\alpha_s(t)}{\alpha_s(M)} \prod \Delta(t,t')$
- Pass the event to a shower Monte Carlo, with the instruction to shower each final state line, with shower initial condition equal to $M$.

Events generated in this way reduce to what a shower MC would do for small angles. Furthermore, the procedure should have only small $M$ dependence. By changing $M$, the amount of job performed by the ME and by the shower changes, but this should not make much difference if $M$ is small enough.

This is not yet the full content of the CKKW algorithm. The really difficult part has to do with the handling of soft-collinear radiation (not discussed in this talk ...)

# Variants

Several alternatives have been proposed:

- MLM matching (ALPGEN group)
- Pseudo showers (Mrenna and Richardson, 2003)
- CKKW-Lönnblad (Lönnblad, 2002)

mostly to avoid computing explicitly the Sudakov form factors;

It would be interesting to discuss in details the relation of these methods with CKKW.

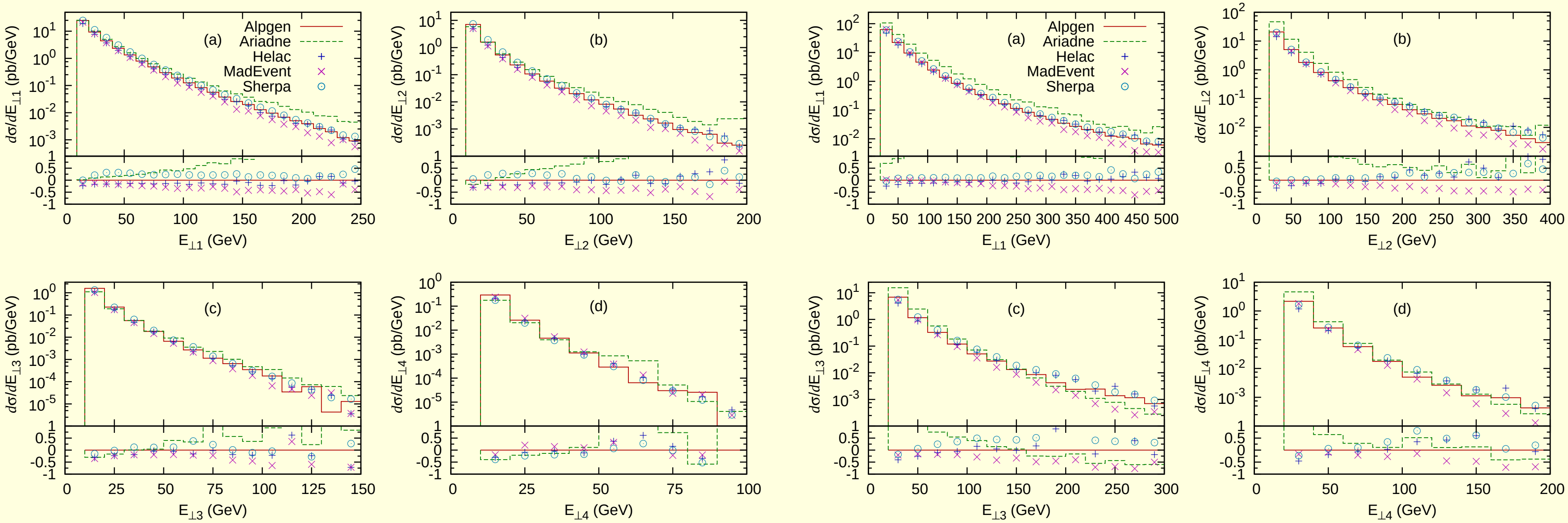
A critical comparison of the various methods is outside the scope of this talk ... However

# Comparison among different ME generators

(Alwall etal, Jul.07): compare Alpgen,Ariadne,Helac,MadEvent,Sherpa

## $W + n\,\mathrm{jets}$, jet $E_T$ spectra

TEVATRON LHC




THE MESSAGE:

good agreement among different ME implementation, in spite of different matching prescriptions (CKKW, MLM, and others)

# NLO Calculations

SMC with ME-corrections are only leading order accurate. Scale uncertainty

$$\alpha_s^n(2\mu) \approx \alpha_s^n(\mu)(1 - b_0\alpha_s(\mu)\log(4))^n \approx \alpha_s(\mu)(1 - n\alpha_s(\mu))$$

For $\mu = 100\,\mathrm{GeV}$, $\alpha_s = 0.12$;
uncertainty:

| $W+1J$ | $W+2J$ | $W+3J$ |
|---|---|---|
| $\pm 12\%$ | $\pm 24\%$ | $\pm 36\%$ |

This scale uncertainty can be considered as an estimate of the error due to missing higher order terms

To improve on this, need to go to NLO

Positive experience with NLO calculations at LEP, HERA, Tevatron
(we TRUST perturbative QCD after LEP!).

Huge NLO effort for the computation of signals and backgrounds for LHC.

## LHC priority wish list, Les Houches 2005 (hep-ph/0604120)

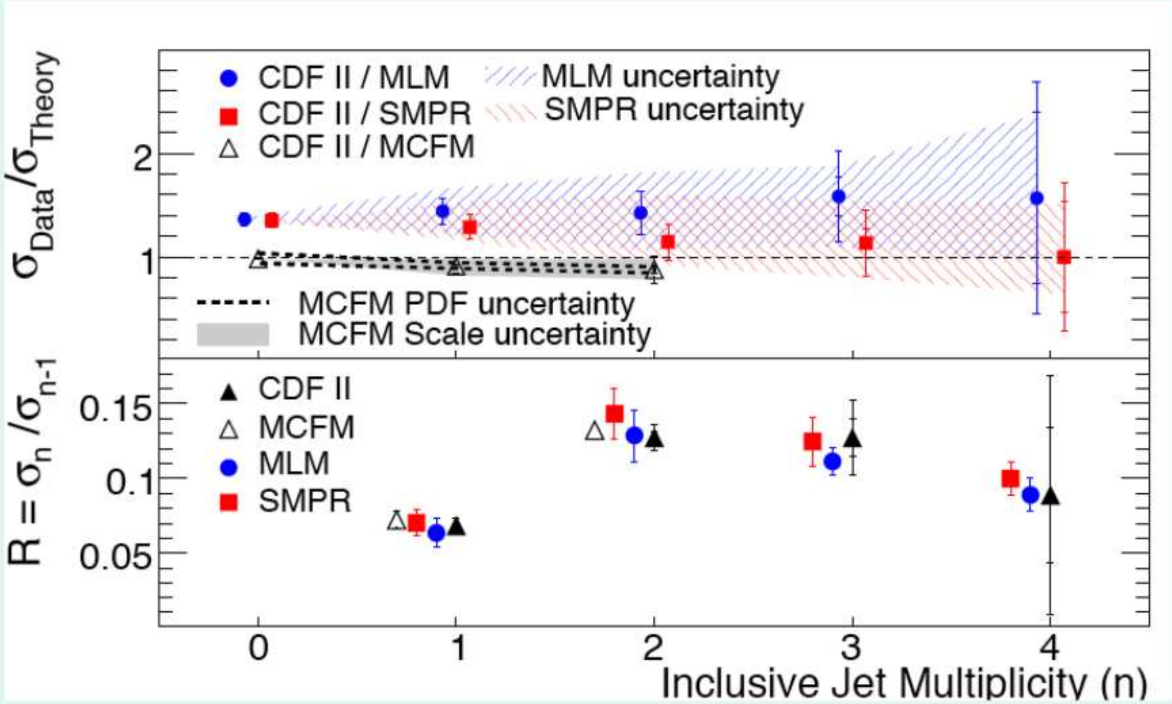
| process, $V \in \gamma, W^{\pm}, Z$ | background to | As of now |
|---|---|---|
| $pp \to VV + 1j$ | $t\bar{t}H$, BSM | $WW$ |
| $pp \to H + 2j$ | | * |
| $pp \to t\bar{t} + b\bar{b}$ | $t\bar{t}H$ | New! |
| $pp \to t\bar{t} + 2j$ | $t\bar{t}H$ | $t\bar{t} + 1j$ (2007) |
| $pp \to VV + b\bar{b}$ | $\mathrm{VBF} \to VV$, $t\bar{t}H$, BSM | * |
| $pp \to VV + 2j$ | $\mathrm{VBF} \to VV$ | * |
| $pp \to V + 3j$ | BSM signatures | New! |
| $pp \to VVV$ | SUSY trilepton | $ZZZ, WWZ$ |
| $pp \to b\bar{b}b\bar{b}$ | Higgs and BSM | |

Recent contributions:
$W + 3j$: Ellis, Melnikov, Zanderighi 2009; Berger etal, 2009;
$pp \to t\bar{t} + b\bar{b}$: Bredstein, Denner, Dittmaier, Pozzorini, 2009;

Unlike tree level processes, research groups still focus upon specific processes;
However, very complex calculations (like $W \to 3\,\mathrm{partons}$ at NLO) are possible.

Special techniques to compute loop graphs are needed;
In particular, a technique by Ossola, Papadopoulos and Pittau (2007) leads to hope that full automation of these calculations will become soon a reality,

# W+n jet rates from CDF

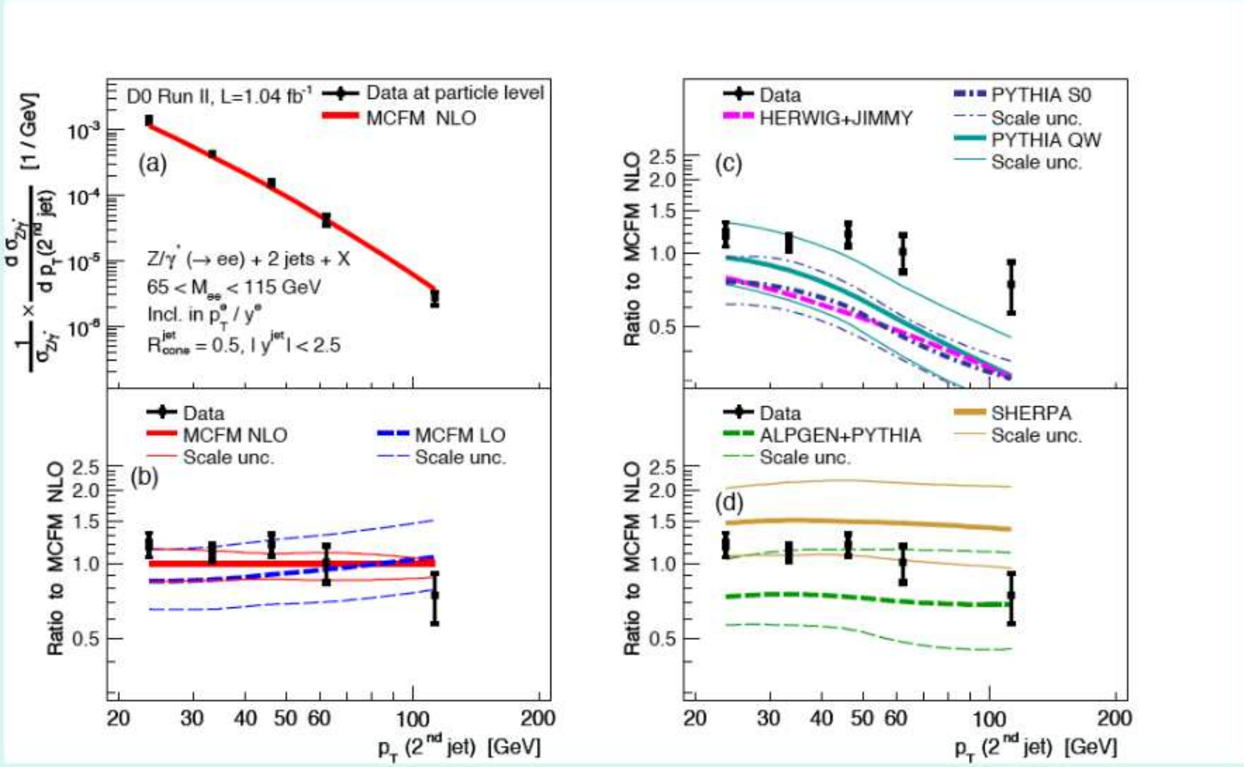



Both uncertainty on rates and deviation of Data/Theory from 1 are smaller than other calculations. The ratio R agrees well for all theory calculations, but only available with from MCFM with small error for n≤2.

Keith Ellis, Madison 2009

MCFM RKE, Campbell

# New Z + jets results from D0




- MCFM, LO and NLO agrees with data;
- shower-based generators show significant differences with data;
- matrix element + parton shower models agree in shape, but with larger normalization uncertainties.

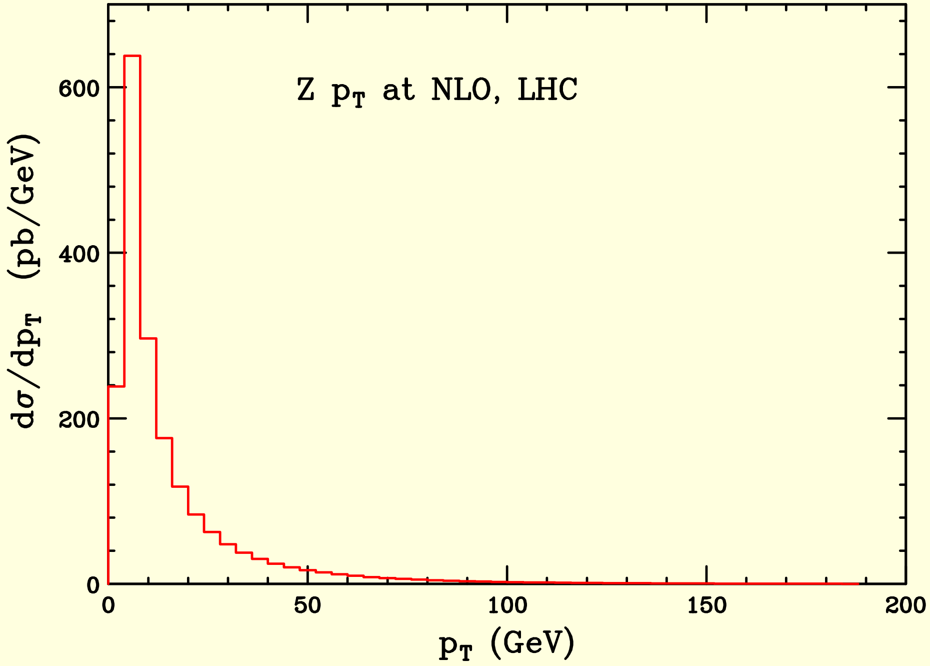
So: NLO calculations represent the data better;

But: NLO results are cumbersome and unfriendly: typically made up of an $n$-body (Born+Virtual+Soft and Collinear remnants) and $n+1$ body (real emission) terms, both divergent (finite only when summed up).

The same problems that we find with ME results are made worse when NLO corrections are included.

Divergent contributions to the cross section for $p_T^Z > 0$, compensated by negative divergences $p_T^Z = 0$, that arise from the virtual corrections.

$p_T^Z$ at NLO:




For small enough histogram bins the first bin will always turn negative!

A negative bin means: $\mathcal{O}(\alpha_s)$ corrections larger than Born term: cannot trust perturbation theory!

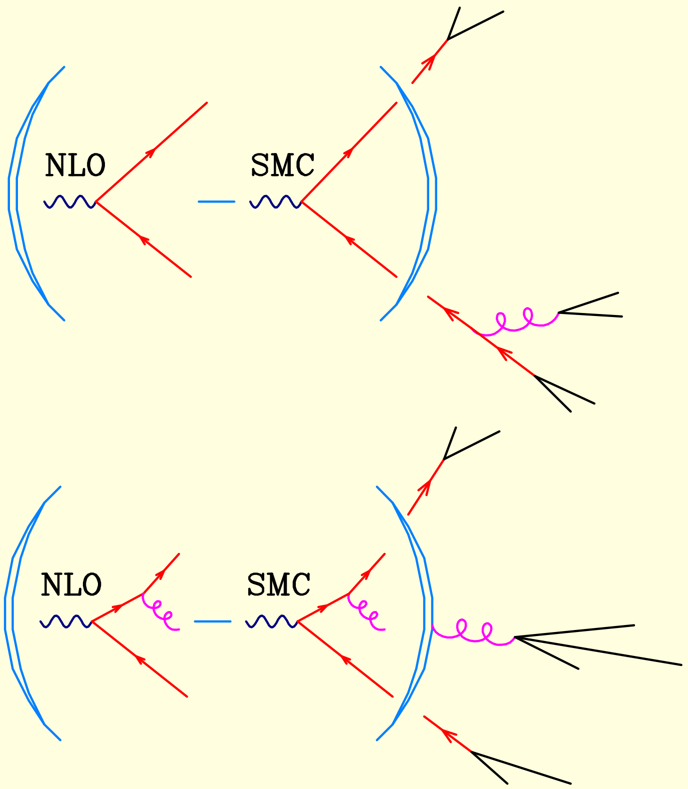
One should carefully decide the appropriate bin size around the origin.

For more complex processes this becomes a requirement on jet parameters.

Some sort of resummation of the diverging virtual corrections should be carried out, in order to get sensible results in the dangerous regions of collinear and soft emissions.

The problem of diverging negative virtual corrections is dealt with and solved in the Shower formalism. Can we apply the same solution in this context, by merging NLO calculations and Shower algorithms?

# MC@NLO (2002, Frixione+Webber)




Add difference between exact NLO and approximate (MC) NLO.

- Must use MC kinematics
- Difference should be regular (if the MC is OK)
- Difference may be negative

Several collider processes already there:
Vector Bosons, Vector Bosons pairs,
Higgs, Single Top.
Heavy Quarks

# How it works (roughly)

The cross section for the hardest event in `MC@NLO` is

$$d\sigma = \underbrace{\bar{B}^{\rm MC}(\Phi_B) d\Phi_B}_{S\ \rm event} \left[ \underbrace{\Delta^{\rm MC}_{t_0} + \Delta^{\rm MC}_{t} \frac{R^{\rm MC}(\Phi)}{B(\Phi_B)} d\Phi^{\rm MC}_r}_{\rm MC\ shower} \right] + \left[ \underbrace{R(\Phi) - R^{\rm MC}(\Phi)}_{H\ \rm event} \right] d\Phi$$

$$\bar{B}^{\rm MC}(\Phi_B) = B(\Phi_B) + \underbrace{\left[ \underbrace{V(\Phi_B)}_{\rm infinite} + \underbrace{\int R^{\rm MC}(\Phi)\, d\Phi^{\rm MC}_r}_{\rm infinite} \right]}_{\rm finite}$$

Imagine that soft and collinear singularities in $R^{\rm MC}$ are regulated as in $V$.

The full phase space $\Phi$ is parametrized in terms of the Born phase space $\Phi_B$ and the radiation variables of the MC $\Phi^{\rm MC}_r$ (typically $z$, $t$, $\phi$), according to the MC procedure (reshuffling) that yields $\Phi$ from $\Phi_B$ and $\Phi^{\rm MC}_r$.

$B$ : Born cross section; $V$ : exact virtual cross section.

$R^{\rm MC}$ : radiation cross section in the MC, typically: $R^{\rm MC} = B\,\frac{1}{t}\,\frac{\alpha}{2\pi} P(z)$

$R$ : exact radiation cross section;

# Recipe for MC@NLO

- Compute total cross section for $S$ and $H$ events:

$$\sigma_S = \int |\bar{B}^{\rm MC}(\Phi_B)| d\Phi_B, \;\; \sigma_H = \int |R - R^{\rm MC}| d\Phi$$

- Chose an $S$ or $H$ event with probability proportional to $\sigma_S$, $\sigma_H$
- For an $S$ event:
  - generate Born kinematics with probability

$$|\bar{B}^{\rm MC}(\Phi_B)| = \left| B(\Phi_B) + \left[ V(\Phi_B) + \int R^{\rm MC}(\Phi) \, d\Phi_r^{\rm MC} \right] \right|$$

  - Feed the Born kinematics to the MC for subsequent shower with weight $\pm 1$, same sign as $\bar{B}^{\rm MC}(\Phi_B)$ (mostly $+1$).
- For an H event:
  - generate Radiation kinematics with probability $|R - R^{\rm MC}|$.
  - Feed to the MC (with weight $\pm 1$, same sign as $R - R^{\rm MC}$)

Issues:

- Must use of the MC kinematic mapping $(\Phi_B, \Phi_r^{\rm MC}) \Rightarrow \Phi$.
- $R - R^{\rm MC}$ must be non singular: the MC must reproduce exactly the soft and collinear singularities of the radiation matrix element. (Many MC are not accurate in the soft limit)
- The cancellation of divergences in the expression of $\bar{B}^{\rm MC}$ is taken care of in the framework of the subtraction method (cancellation of divergences under the integral sign) so that the integral in $\bar{B}^{\rm MC}$ becomes in fact convergent.
- Negative weights in the output (not like standard MC's).

# POWHEG

Positive Weight Hardest Emission Generator

Method to generate the hardest emission first, with NLO accuracy, and independently of the SMC (P.N. 2004).

- SMC independent; no need of SMC expert; same calculation can be interfaced to several SMC programs with no extra effort
- SMC inaccuracies only affect next-to-hardest emissions; no matching problems
- As the name says, it generates events with positive weight

# How it works (roughly)

The cross section for the hardest event in MC@NLO is

$$d\sigma = \bar{B}^s(\Phi_B)d\Phi_B\left[\Delta^s_{t_0} + \Delta^s_t \frac{R_s(\Phi)}{B(\Phi_B)}d\Phi_r\right] + [R(\Phi) - R_s(\Phi)]d\Phi$$

$$\bar{B}^s(\Phi_B) = B(\Phi_B) + \underbrace{\left[\underbrace{V(\Phi_B)}_{\text{infinite}} + \underbrace{\int R^s(\Phi)\, d\Phi^s_r}_{\text{infinite}}\right]}_{\text{finite}}$$

Looks identical to MC@NLO with $R^{\mathrm{MC}} \to R^s$. However
$R_s = RF$, where $F \leqslant 1$, with $F = 1$ in the singular limit; also $F = 1$ possible.
$\Phi$ is arbitrarily parametrized in terms of a Born phase space $\Phi_B$ and a radiation phase space $\Phi_r$.
The hardest radiation is generated by POWHEG itself, and not by the SMC.
POWHEG is the collection of tricks used to implement this formula.

Advantages:

- Positive weighted events: $R - R_s = R(F - 1) \geqslant 0$!
- Independence on the Shower MC: The hardest emission is generated by POWHEG; less hard emissions are generated by the shower.

Accuracy: $d\sigma = d\Phi_{B}\, \bar{B}(\Phi_{B})\Big[\Delta_{t_0} + \Delta_t \frac{R_s(\Phi)}{B(\Phi_{B})} d\Phi_r\Big] + (R - R_s)\, d\Phi$

Small $k_T$: $\frac{R_s(\Phi)}{B(\Phi_{B})}\, d\Phi_{\rm rad} \approx \frac{\alpha_s(t)}{2\pi} P_{i,jk}(z)\, \frac{dt}{t}\, dz\, \frac{d\phi}{2\pi}$,

Also: $\bar{B} \approx B \times (1 + \mathcal{O}(\alpha_s))$

Thus: all features of SMC's are preserved at small $k_T$.

Large $k_T$: $\Delta \to 1, \quad d\sigma = \bar{B} \times \frac{R_s}{B}\, d\Phi + (R - R_s)\, d\Phi \approx R_s \times (1 + \mathcal{O}(\alpha_s)) d\Phi + (R - R_s)\, d\Phi$,
so: large $k_t$ accuracy is preserved.

NLO accuracy: since $\Delta_{t_0} + \int \; \Delta_t \frac{R_s(\Phi)}{B(\Phi_{B})} d\Phi_r = 1$, integrating in $d\Phi_r$ at fixed $\Phi_{B}$

$$\int \delta(\Phi_{B} - \bar{\Phi}_{B})\, d\sigma = \Big[\bar{B} + \int \; (R - R_s)\, d\Phi_r\Big]_{\Phi_{B} = \bar{\Phi}_{B}} = \Big[B + V + \int R\, d\Phi_r\Big]_{\Phi_{B} = \bar{\Phi}_{B}}$$

So: NLO accuracy is preserved for inclusive quantities.

Although MC@NLO and POWHEG yield the exact NLO cross section, differential distributions are affected by induced NNLO terms:

$$d\sigma = d\Phi_B \bar{B}\Big[\Delta_{t_0} + \Delta_t \tfrac{R_s}{B} d\Phi_r\Big] + (R - R_s)\, d\Phi, \qquad \bar{B} = B + \big[V + \textstyle\int\, R_s\, d\Phi_r\big]$$

The expression for $\Delta_{t_1,t} = \exp\Big[-\int\, \frac{R}{B}\, d\Phi_r\, \theta(k_T - t)\Big]$ generates terms of all orders, and suppresses the distributions at small $p_T$.

The square bracket term in $\bar{B}$, multiplied by $R_s/B$, generates NNLO terms, since $\bar{B}/B = 1 + \mathcal{O}(\alpha_s)$.
(in case of positive NLO corrections, it typically enhances the distributions)

# POWHEG: Interfacing to SMC's

POWHEG is completely detached from the SMC to which it is interfaced.
It uses the standard Les Houches Interface for User's Processes (LHI):

The LHI provides a facility to pass the $p_T$ of the event to the SMC, so that no radiation harder than $p_T$ will be generated by the MC.

Caveat to implement correctly soft radiation:
For angular ordered showers (i.e. HERWIG), to preserve double log accuracy one should provide truncated showers (P.N. 2004), now implemented in HERWIG++.

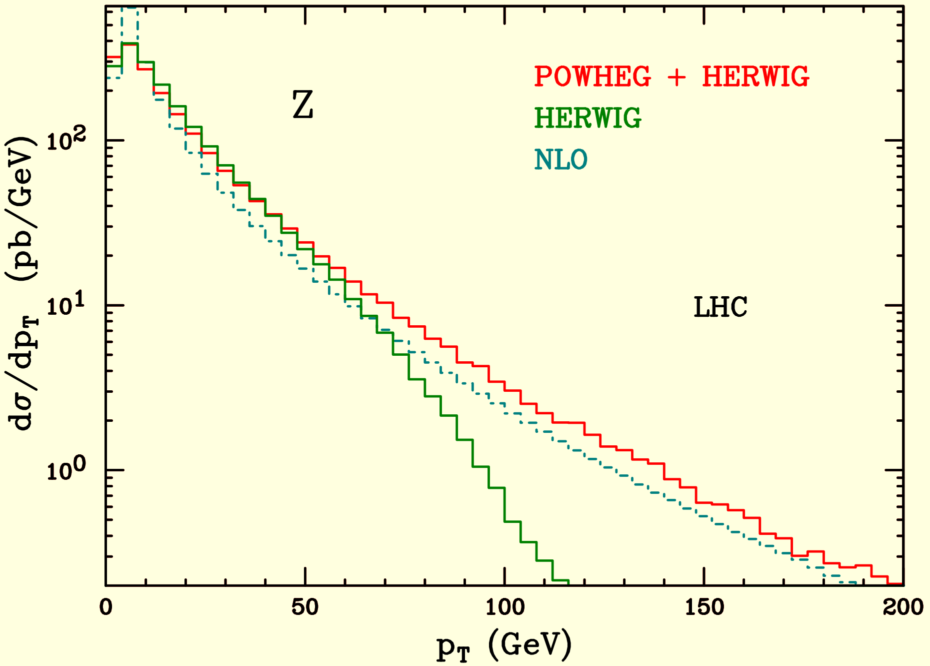
# Status of POWHEG

Up to now, the following processes have been implemented in POWHEG:

- $hh \to ZZ$ (Ridolfi, P.N., 2006)
- $e^+e^- \to \text{hadrons}$, (Latunde-Dada,Gieseke,Webber, 2006),
  $e^+e^- \to t\bar{t}$, including top decays at NLO (Latunde-Dada,2008),
- $hh \to Q\bar{Q}$ (Frixione, Ridolfi, P.N., 2007)
- $hh \to Z/W$ (Alioli, Oleari, Re, P.N., 2008; )
  (Hamilton,Richardson,Tully, 2008;)
- $hh \to H$ (gluon fusion) (Alioli, Oleari, Re, P.N., 2008; Herwig++)
- $hh \to H$, $hh \to HZ/W$ (Hamilton,Richardson,Tully, 2009;)
- $hh \to t + X$ (single top) NEW (Alioli, Oleari, Re, P.N., 2009)
- $hh \to Z + \text{jet}$, Very preliminary (Alioli, Oleari, Re, P.N., 2009)
- The POWHEG BOX, Very preliminary, (Alioli, Oleari, Re, P.N., 2009)
- VBF Higgs production (Oleari,P.N.) in preparation

# Examples: Z production

HERWIG alone fails at large $p_T$;
NLO alone fails at small $p_T$;
MC@NLO and POWHEG work
in both regions;



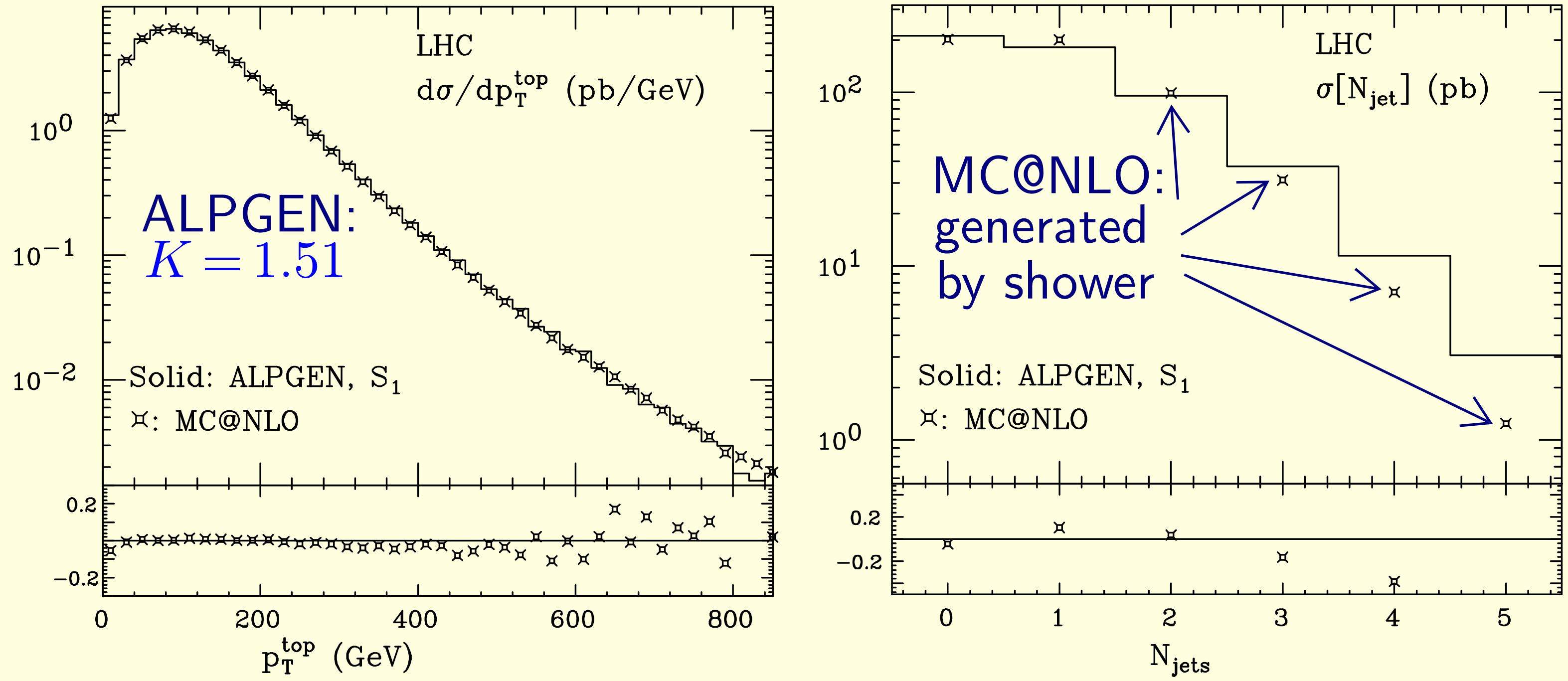

Notice:
HERWIG with ME corrections or any ME program, give the same NLO shape at large $p_T$
However: Normalization around small $p_T$ region is incorrect (i.e. only LO).

# NLO+PS compared with ME programs: ALPGEN and MC@NLO in $t\bar{t}$ production

expect:

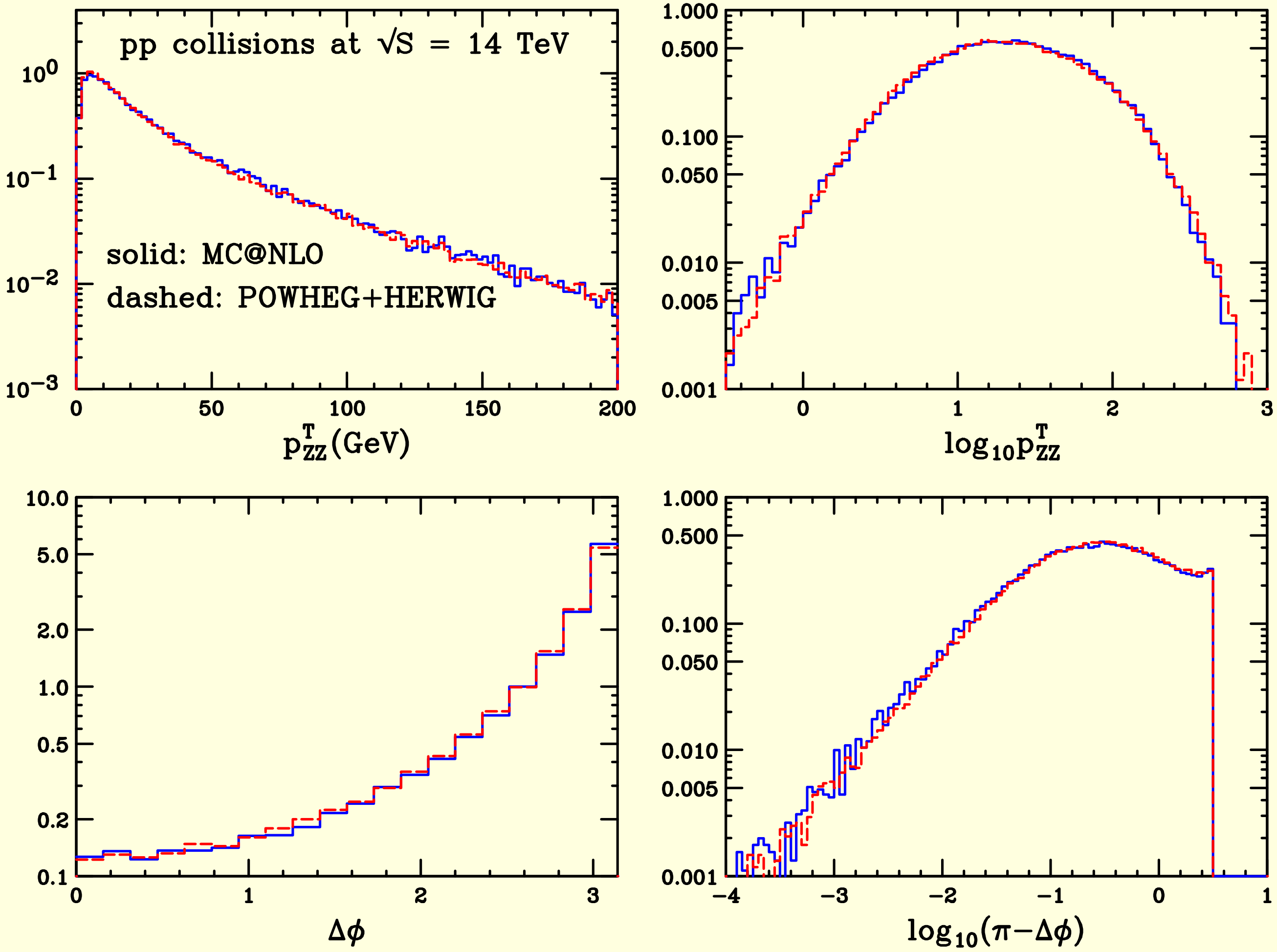
- Disadvantage: worse normalization (no NLO)
- Advantage: better high jet multiplicities (exact ME)

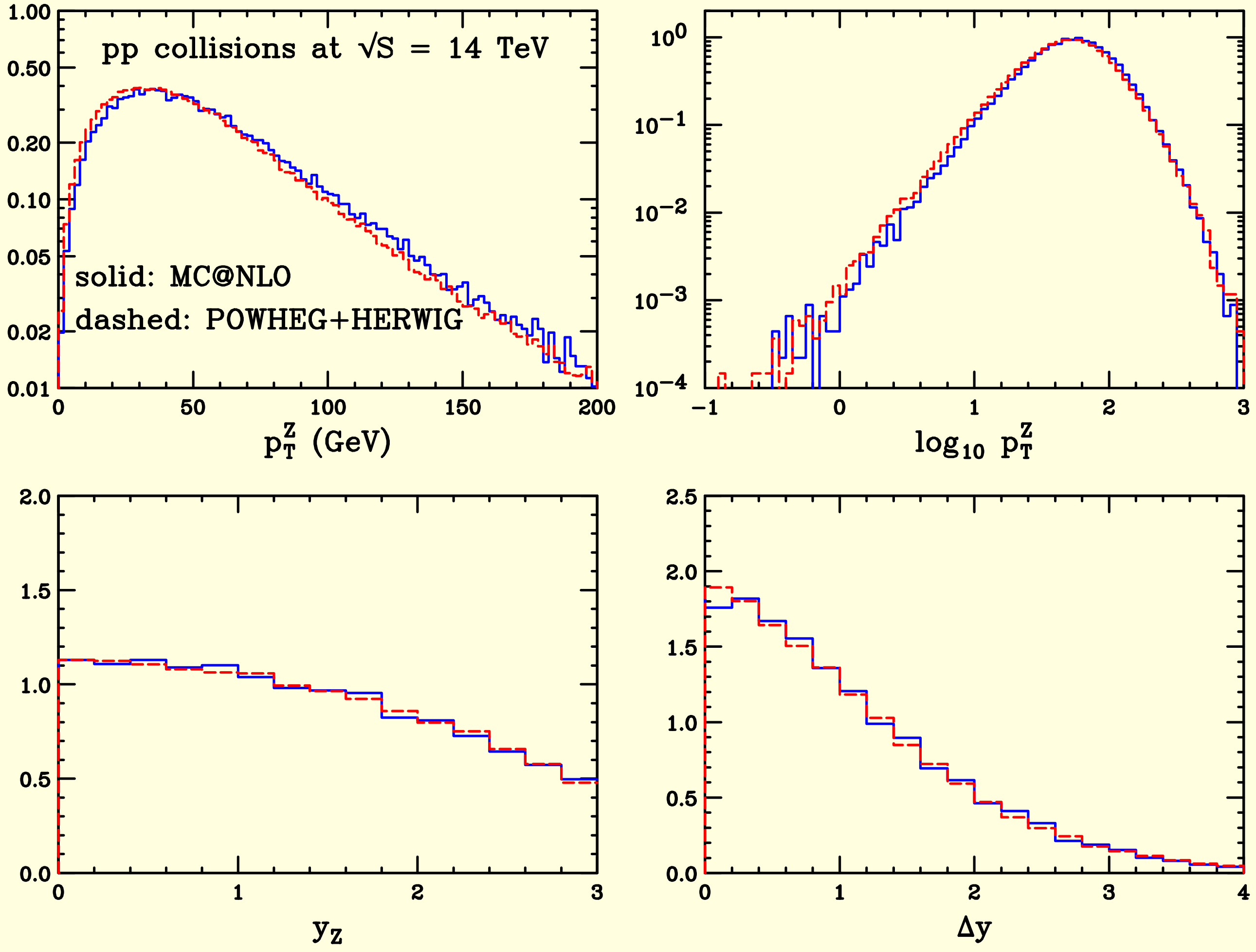
(Mangano, Moretti,Piccinini,Treccani, Nov.06)

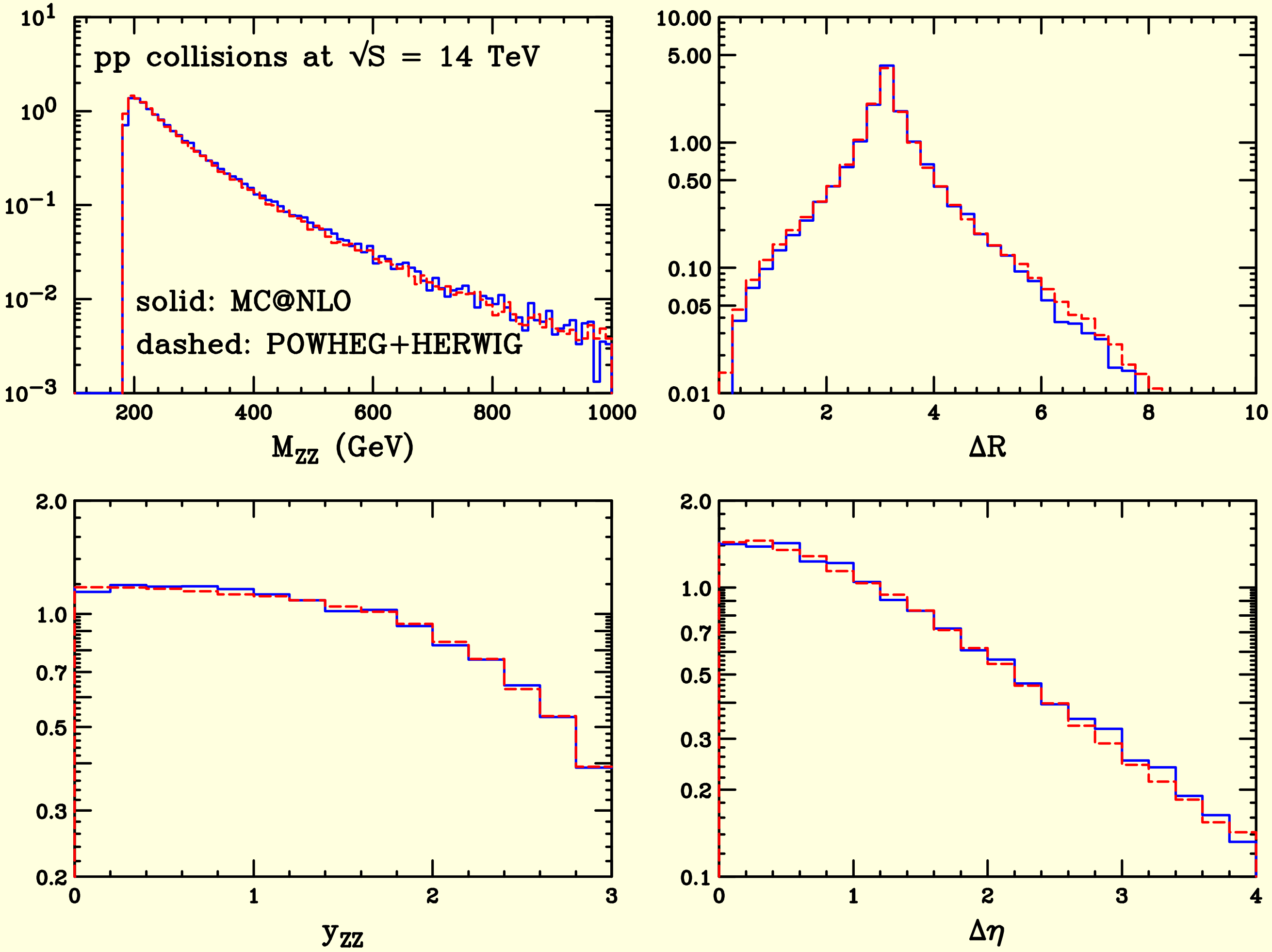


# Comparisons of POWHEG+HERWIG vs. MC@NLO

# Z pair production

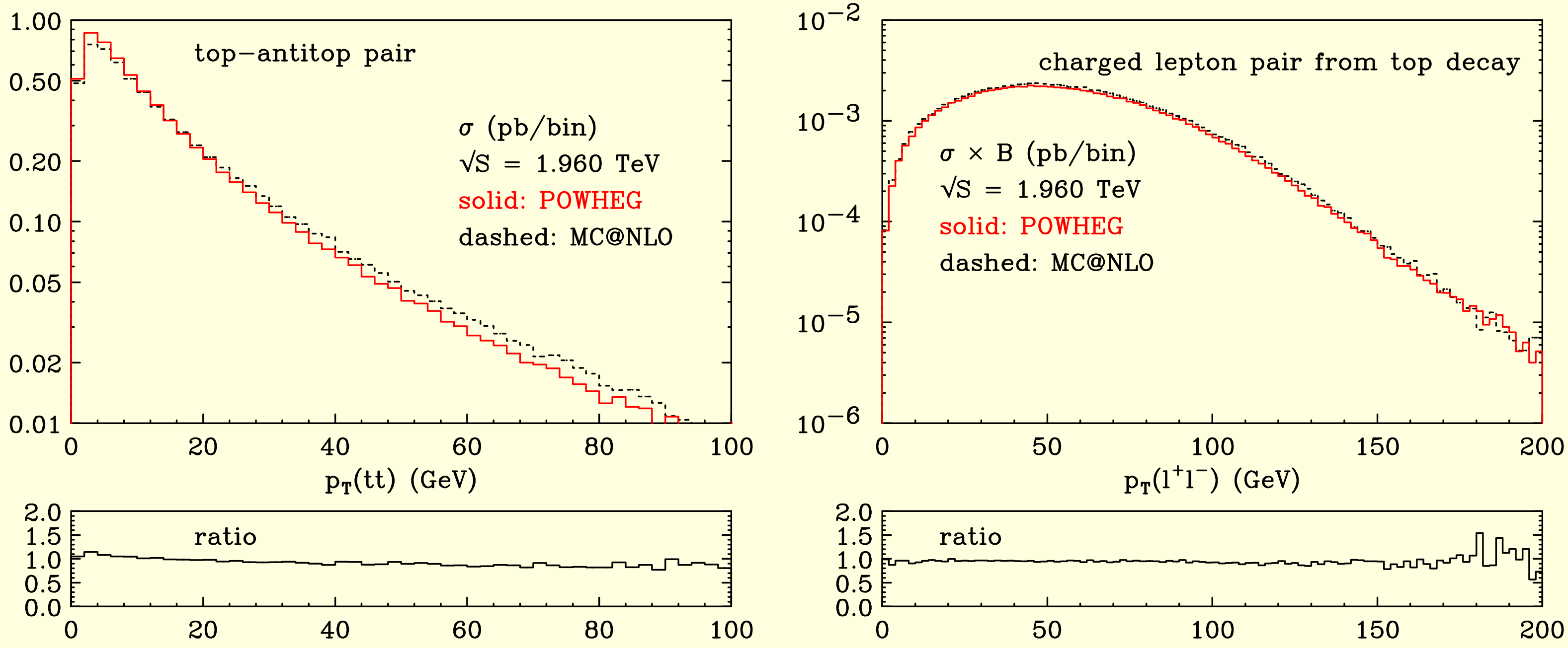



pp collisions at √S = 14 TeV
solid: MC@NLO
dashed: POWHEG+HERWIG
$p_T^Z$ (GeV)
$\log_{10} p_T^Z$
$y_Z$
$\Delta y$



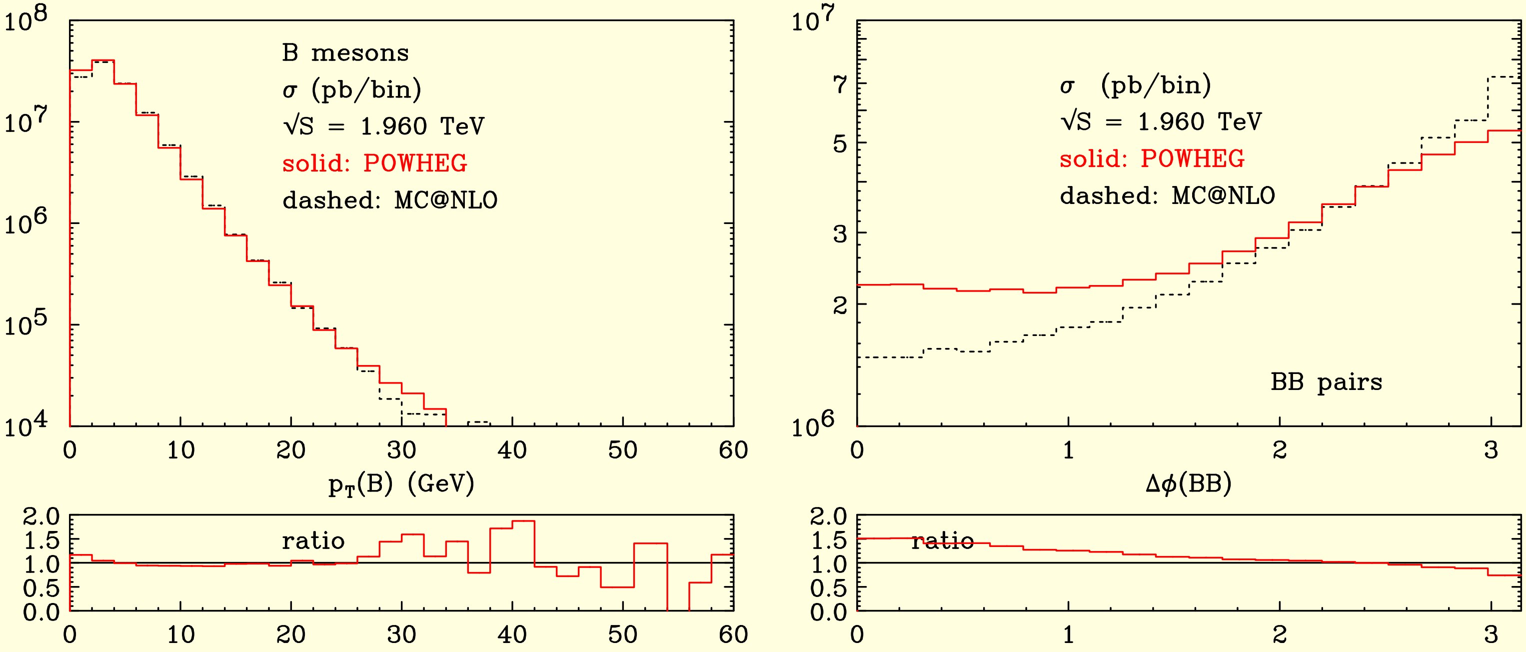

Remarkable agreement for most quantities;

# POWHEG and MC@NLO comparison: Top pair production



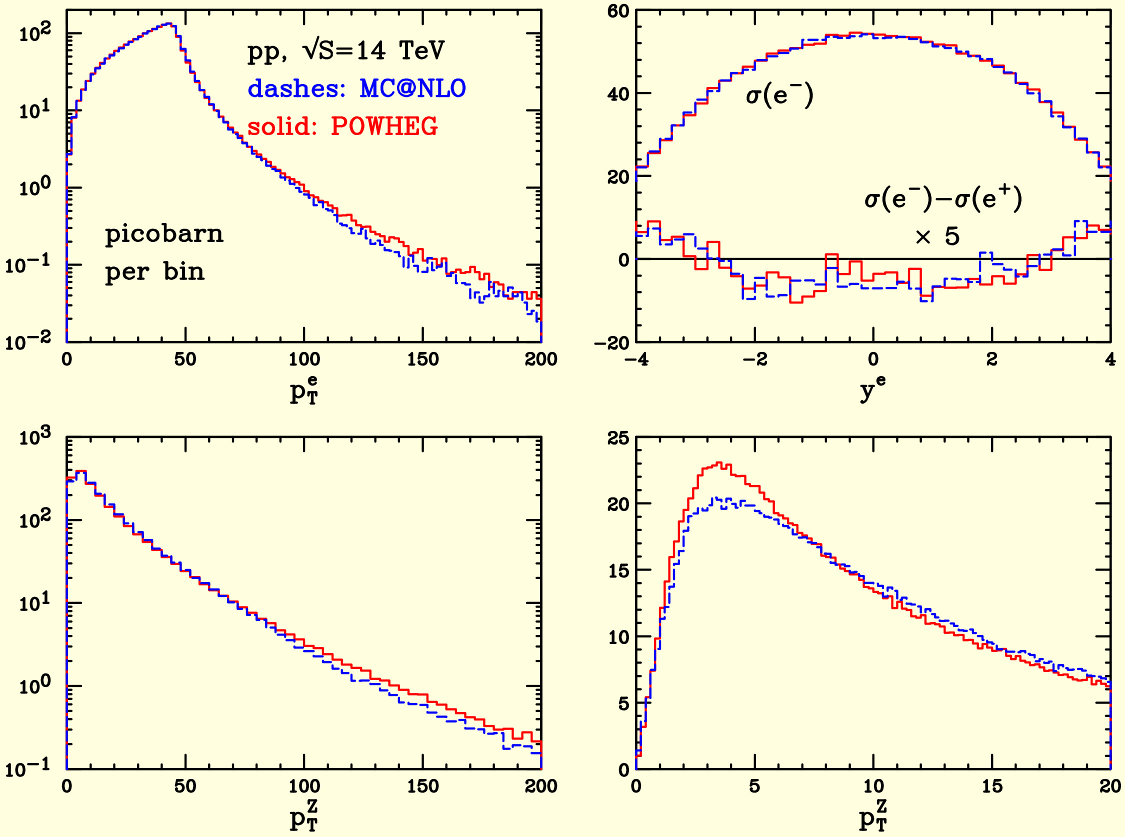

Good agreement for most observables considered
(differences can be ascribed to different treatment of higher order terms)

# Bottom pair production



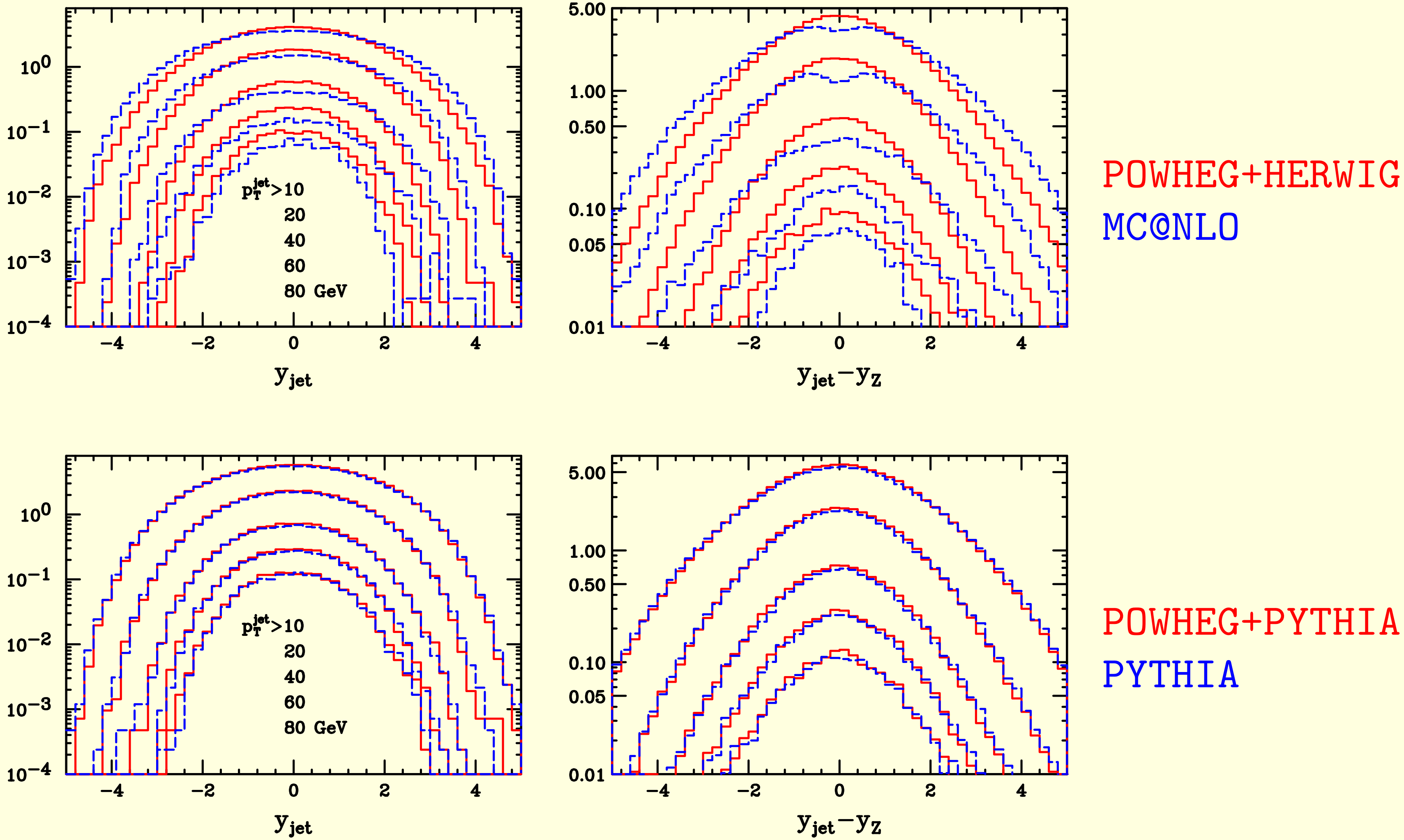

- Very good agreement For large scales ($ZZ$, $t\bar{t}$ production)
- Differences at small scales ($b\bar{b}$ at the Tevatron)
- POWHEG more reliable in extreme cases like $b\bar{b}, c\bar{c}$ at LHC
(yields positive results, MC@NLO has problems with negative weights)

# Z production: POWHEG+HERWIG vs. MC@NLO

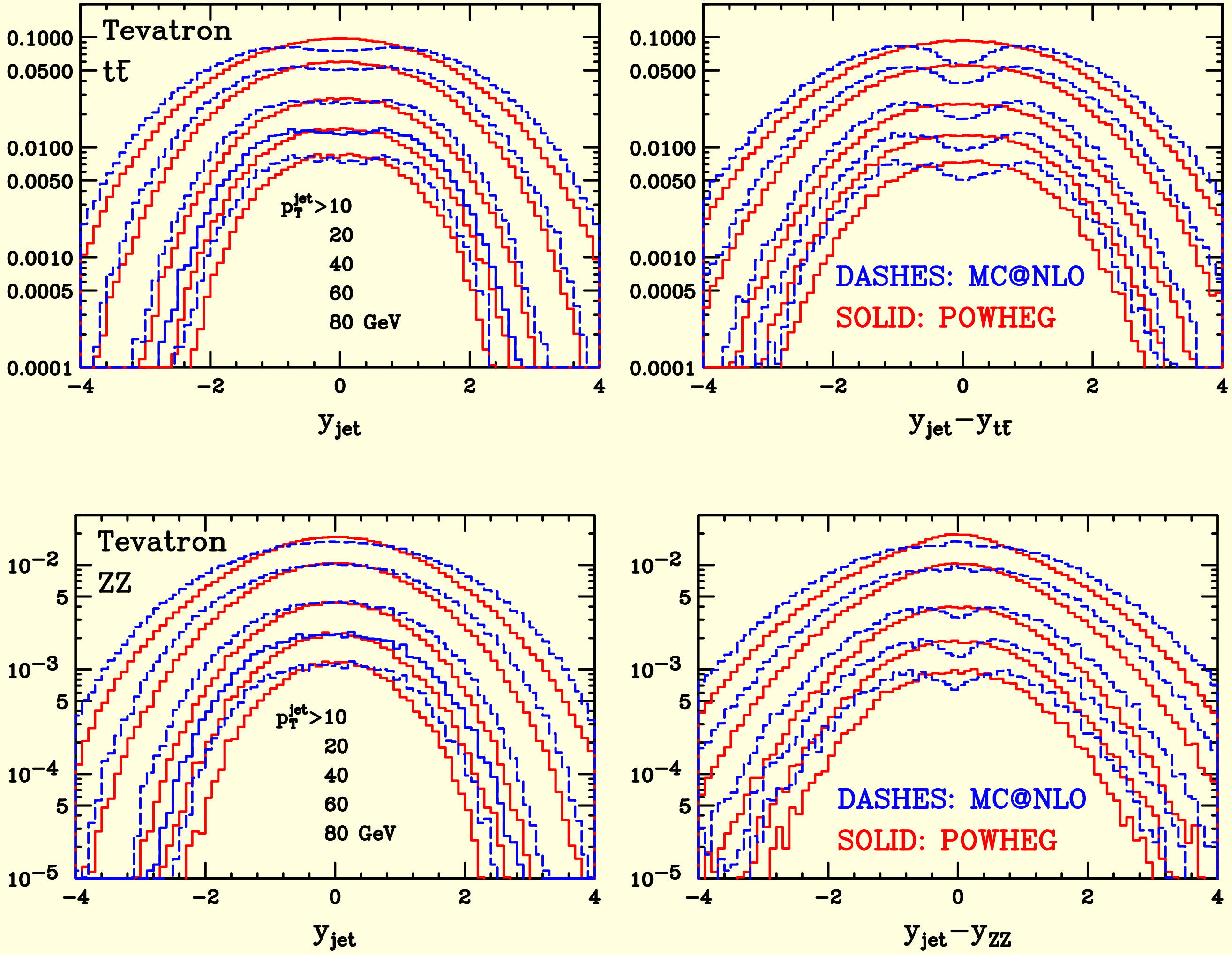



Small differences in high and low $p_T$ region

# Z production: rapidity of hardest jet (TEVATRON)



# Dip in central region in MC@NLO also in $t\bar{t}$ and $ZZ$



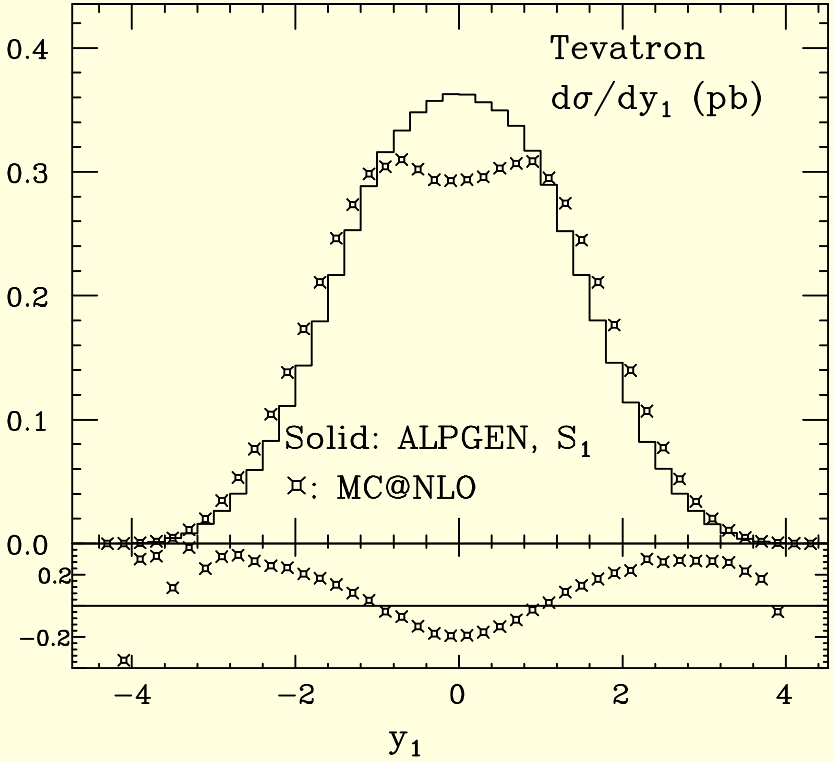

POWHEG+HERWIG

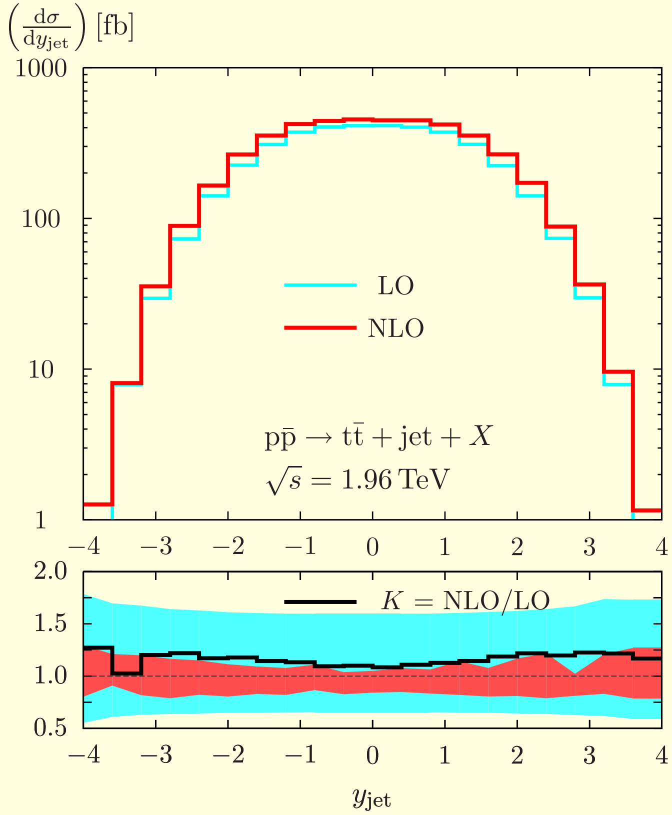
MC@NLO

POWHEG+HERWIG

MC@NLO

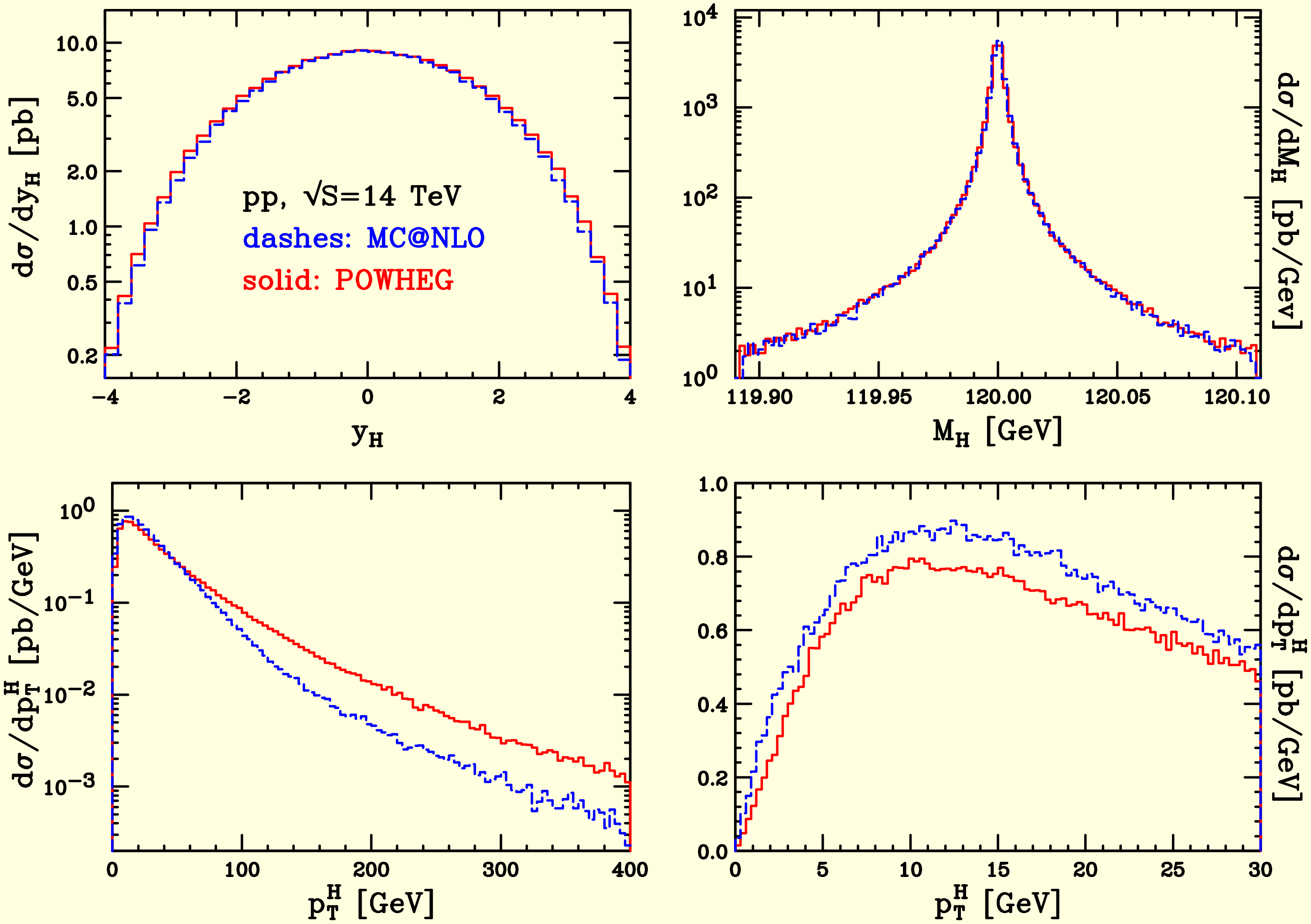
# ALPGEN and $t\bar{t}+\text{jet}$ at NLO vs. MC@NLO



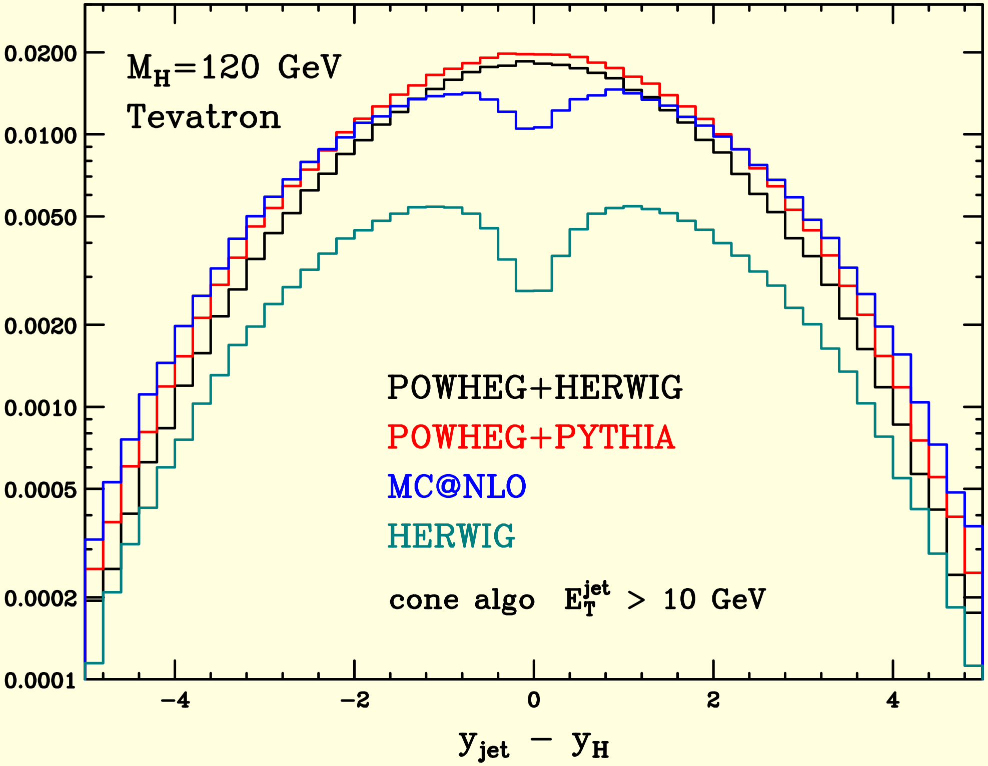




POWHEG distribution as in ALPGEN (Mangano,Moretti,Piccinini,Treccani,Nov.06) and in $t\bar{t}+\text{jet}$ at NLO (Dittmaier, Uwer, Weinzierl) : no dip present.

# Higgs boson via gluon fusion at LHC



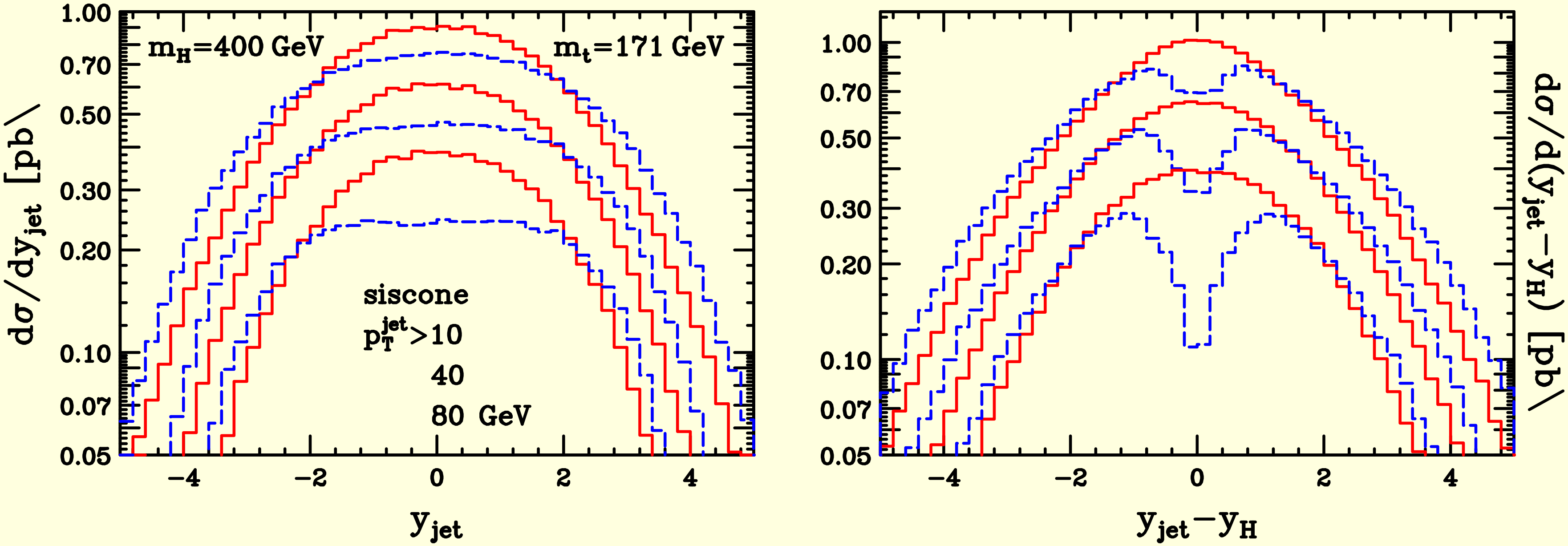
# Jet rapidity in $h$ production




Dip in MC@NLO inerithed from even deeper dip in HERWIG

(MC@NLO tries to fill dead regions in HERWIG, a mismatch remains).

Gets worse for larger $E_T$ cuts:



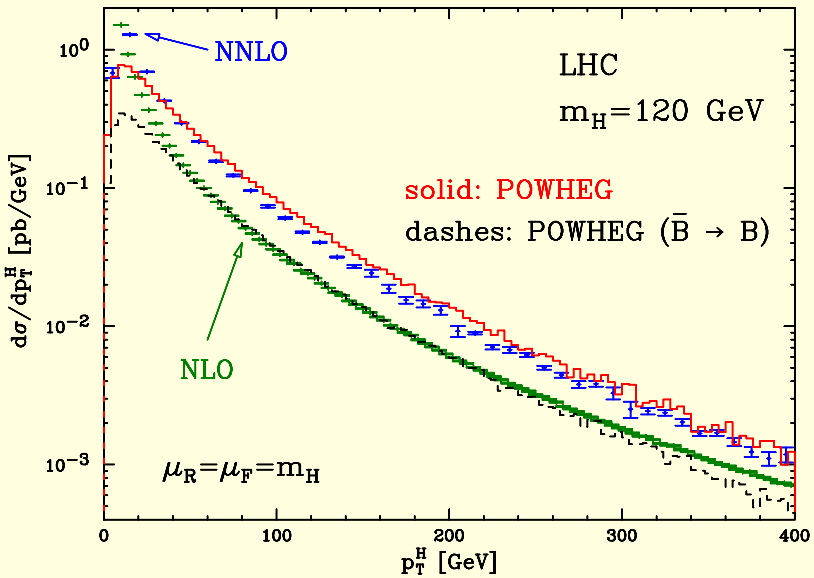

Questions:

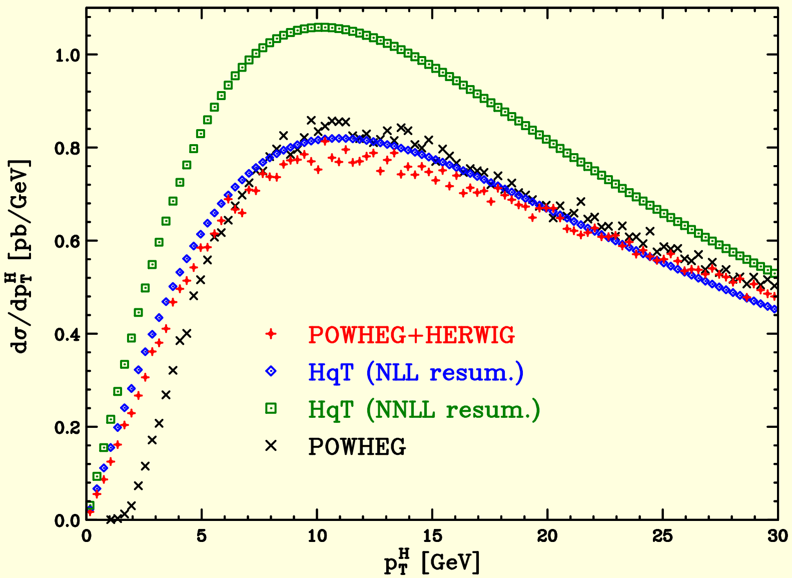
Why MC@NLO has a dip in the hardest jet rapidity?

Why POWHEG has no dip? Is that because of the hardest $p_T$ spectrum?

# Hard $p_T$ spectrum in POWHEG

## POWHEG vs. NNLO vs. NNLL




$$d\sigma = \bar{B}\, d\Phi_B \left\{ \Delta_{t^0} + \Delta_t \frac{R}{B} d\Phi_r \right\} \approx \frac{\bar{B}}{B} R\, d\Phi_B\, d\Phi_r = \underbrace{\{1 + \mathcal{O}(\alpha_s)\}}_{\approx 2 \text{ for here!}} R\, d\Phi$$

Large enhancement because of the large $K$ factor in Higgs production.
Better agreement with NNLO this way.

There is enough flexibility in POWHEG to get rid of it (if one wants)!!!

In the POWHEG cross section:

$$d\sigma' = d\Phi_B \, \bar{B}^s \left[ \Delta^s_{t_0} + \Delta^s_t \frac{R_s}{B} d\Phi_r \right] + R_f \, d\Phi$$

with:

$$\Delta^s_t = \exp\left[ - \int \theta(t_r - t) \frac{R_s}{B} d\Phi_r \right].$$

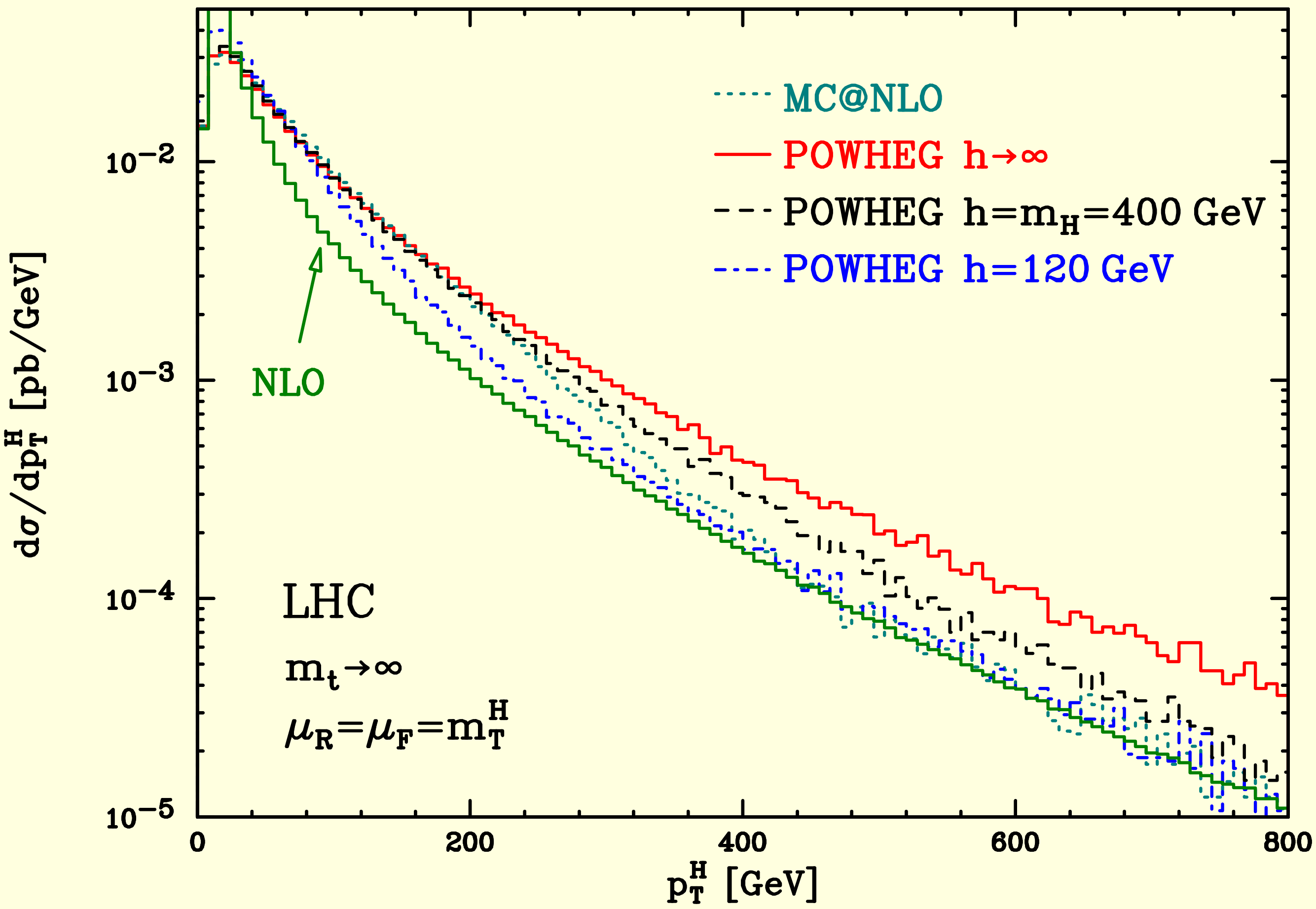
Rather than choosing $R_f = 0, \; R_s = R$, choose

$$R_f = R \frac{k_T^2}{k_T^2 + h^2}, \quad R_f = R \frac{h^2}{k_T^2 + h^2}\,;$$

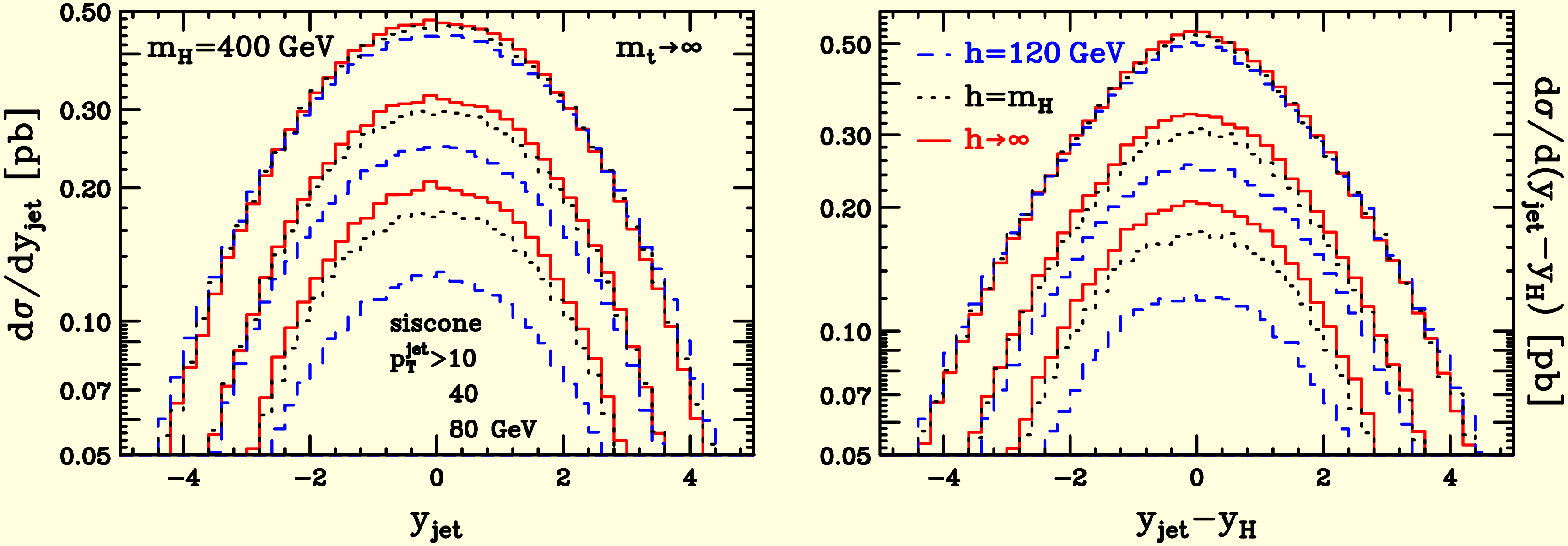
$$R_s = R\frac{h^2}{k_T^2 + h^2}$$

$$R_f = R\frac{k_T^2}{k_T^2 + h^2}$$

Agrees with NLO at high $p_T$.



No new features (dips and the like) arise in the other distributions:




So: high $k_T$ cross section and dips are unrelated issues.

# Why is there a dip in MC@NLO?

$$d\sigma = \underbrace{d\Phi_B \bar{B}^{\rm MC}}_{S\ \text{event}} \left[ \underbrace{\Delta^{\rm MC}_{t_0} + \Delta^{\rm MC}_{t} \frac{R^{\rm MC}}{B} d\Phi^{\rm MC}_{r}}_{\text{HERWIG shower}} \right] + \left[ \underbrace{R - R^{\rm MC}}_{H\ \text{event}} \right] d\Phi$$

$$\bar{B}^{\rm MC} = B + \left[ V + \int R^{\rm MC}(\Phi_B, \Phi_r)\, d\Phi_r \right]$$

For large $k_T$:

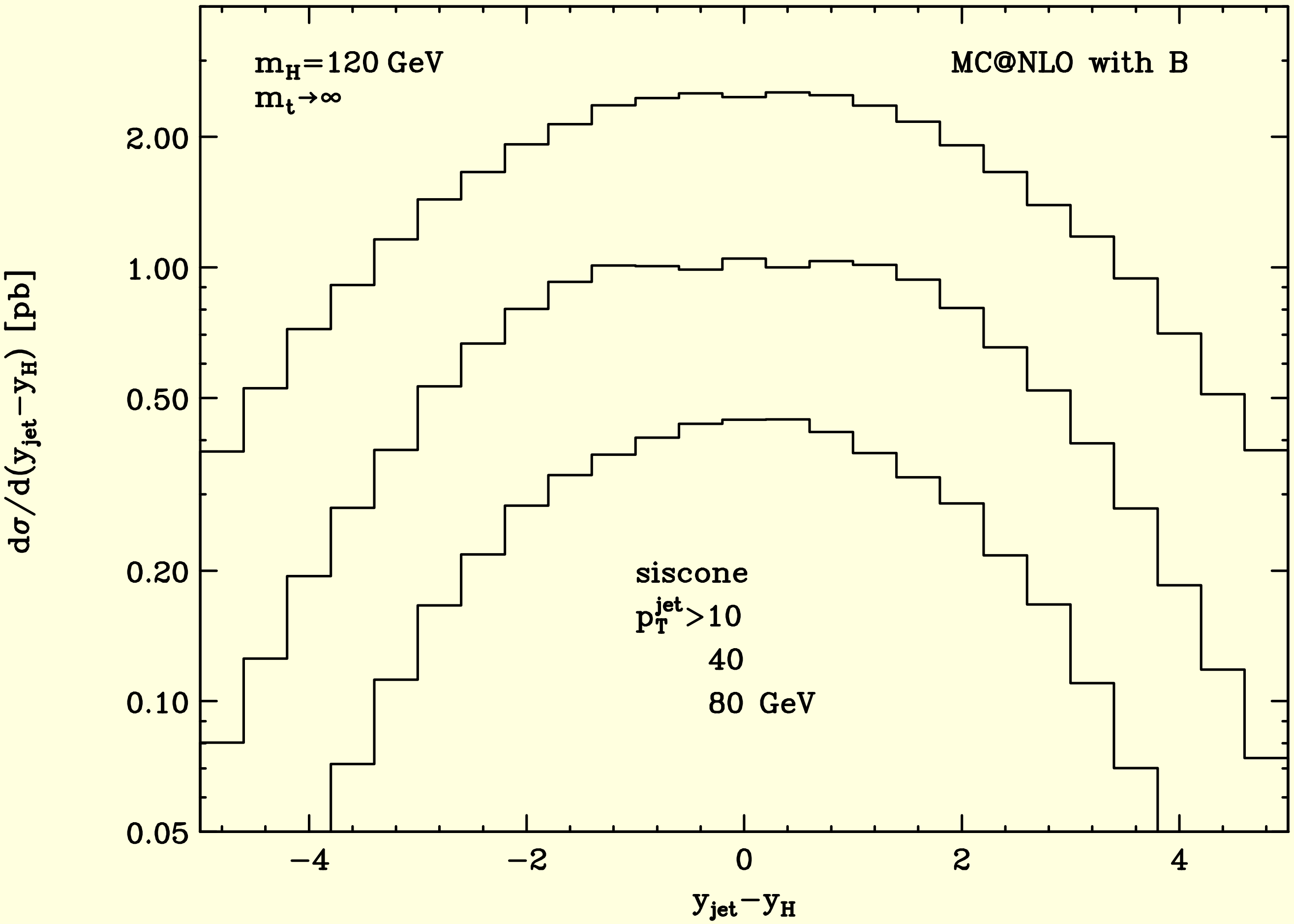
$$d\sigma = \frac{\bar{B}^{\rm MC}}{B} R^{\rm MC}\, d\Phi_B d\Phi^{\rm MC}_{r} + [R - R^{\rm MC}] d\Phi$$

$$= \underbrace{R d\Phi}_{\text{no dip}} + \underbrace{\left( \frac{\bar{B}^{\rm HW}}{B} - 1 \right)}_{\mathcal{O}(\alpha_s),\ \text{but large for Higgs}} \times \underbrace{R^{\rm HW}}_{\text{Pure Herwig dip}} d\Phi$$

So: a contribution with a dip is added to the exact NLO result;
The contribution is $\mathcal{O}(\alpha_s R)$, i.e. NNLO!

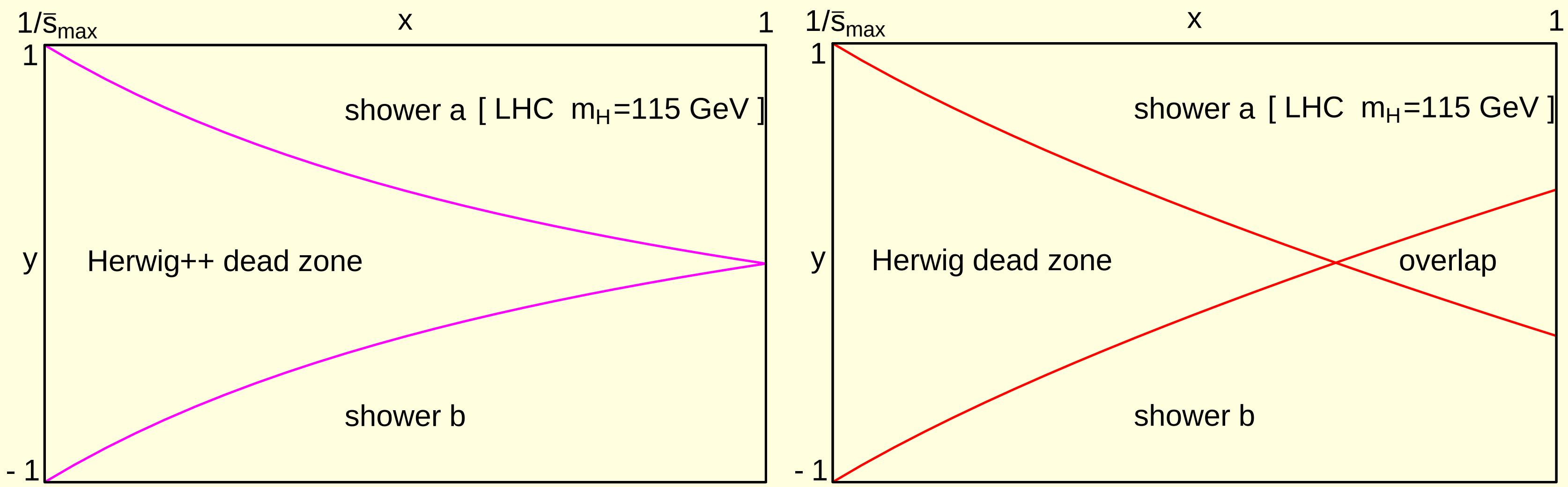
Can we test this hypothesis? Replace $\bar{B}^{\rm HW}(\Phi_n) \Rightarrow B(\Phi_n)$ in MC@NLO!
the dip should disappear ...

# MC@NLO with $B^{\mathrm{HW}}$ replaced by $B$

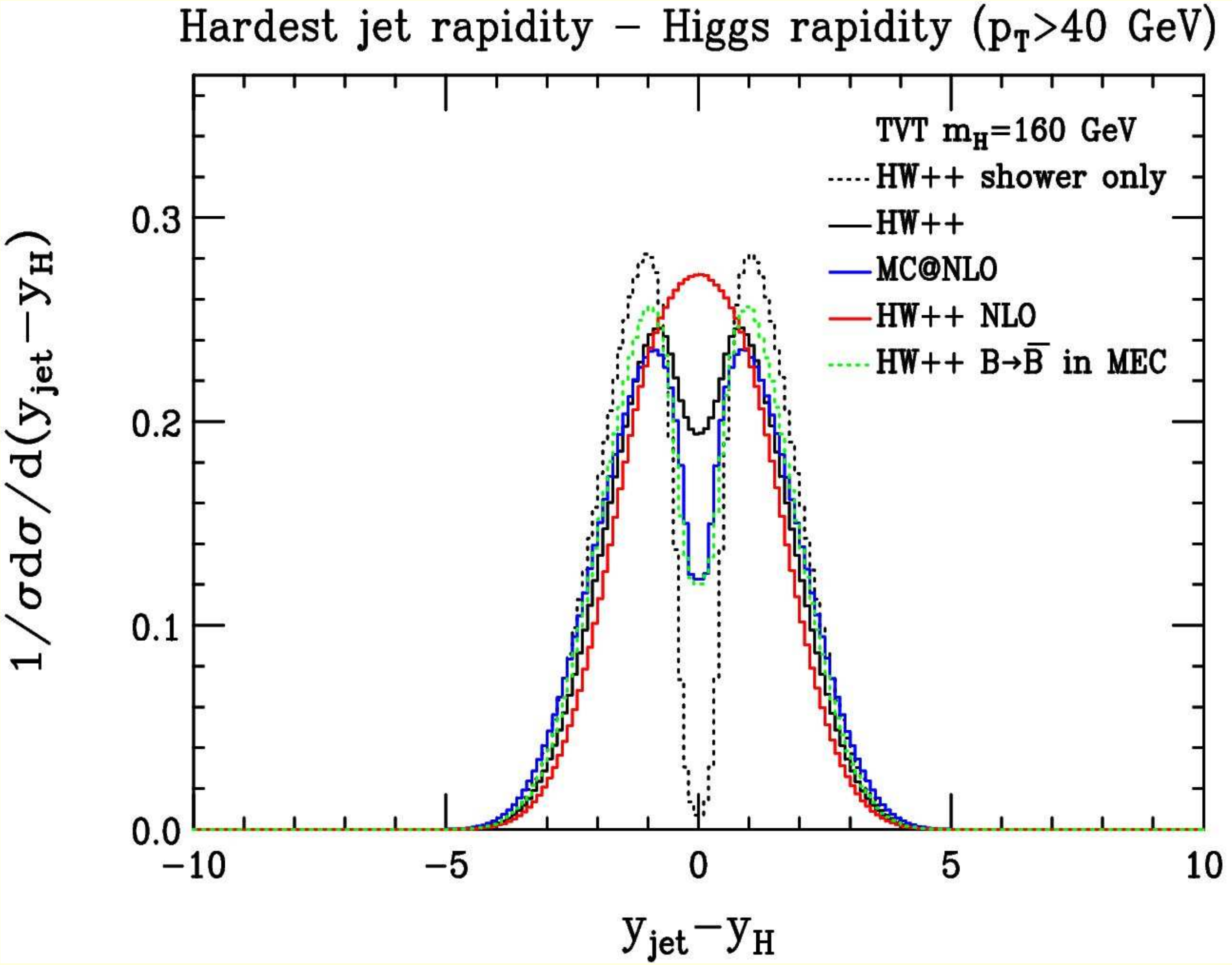



No visible dip is present! (on the right track ...)

Detailed study of the problem also by Hamilton,Richardson,Tully, 2009




Both HERWIG and HERWIG++ have a dead radiation region corresponding to central rapidity and high energy. Dip in central region in HERWIG can be attributed to the dead zone.



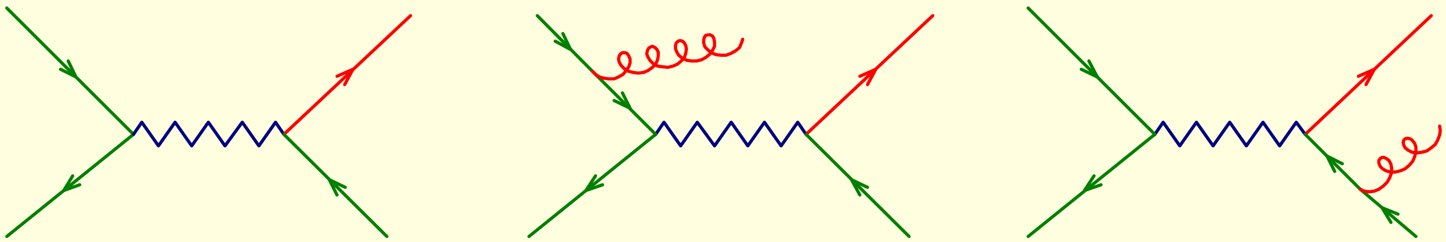
Hardest jet rapidity – Higgs rapidity ($p_T$>40 GeV)
TVT $m_H$=160 GeV
HW++ shower only
HW++
MC@NLO
HW++ NLO
HW++ B→$\overline{B}$ in MEC
$1/\sigma d\sigma/d(y_{jet}-y_H)$
0.0
0.1
0.2
0.3
−10
−5
0
5
10
$y_{jet}-y_H$

# Summary of MC@NLO and POWHEG comparisons

- Fairly good agreement on most distributions
- Areas of disagreement can be tracked back to NNLO terms, arising mostly because of the use of an NLO inclusive cross section (the $\bar{B}$ function) to shower out the hardest radiation.
- In POWEG, since the hardest radiation is generated by POWHEG itself, one has high flexibility in tuning the magnitude of these NNLO terms.
- For MC@NLO, these NNLO terms can generate unphysical behaviour in physical distributions, reflecting the dead zones structure of the underlying shower Monte Carlo. Since MC@NLO uses the underlying MC to generate the hardest emission, it is difficult to remedy to these problems without intervening on the MC itself

# Single Top

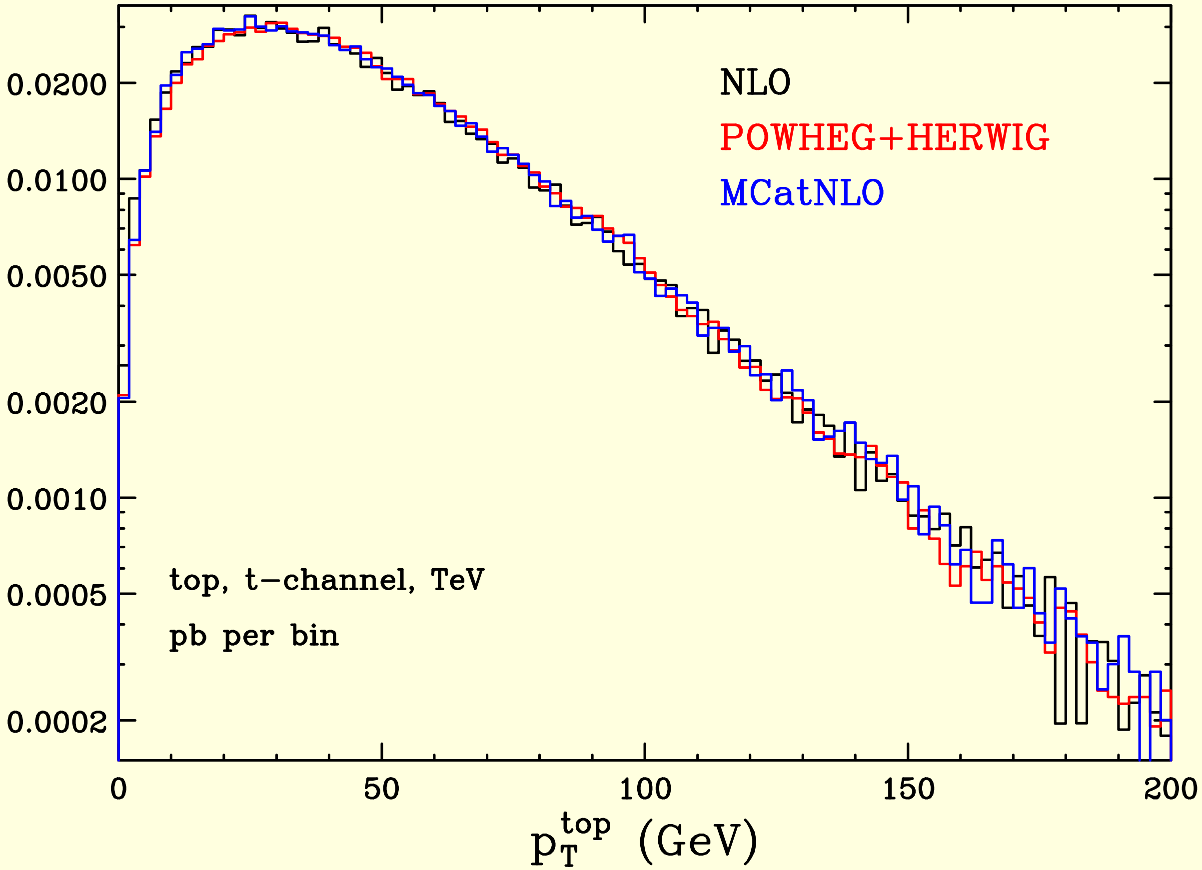
Both initial state and final state radiation is present;

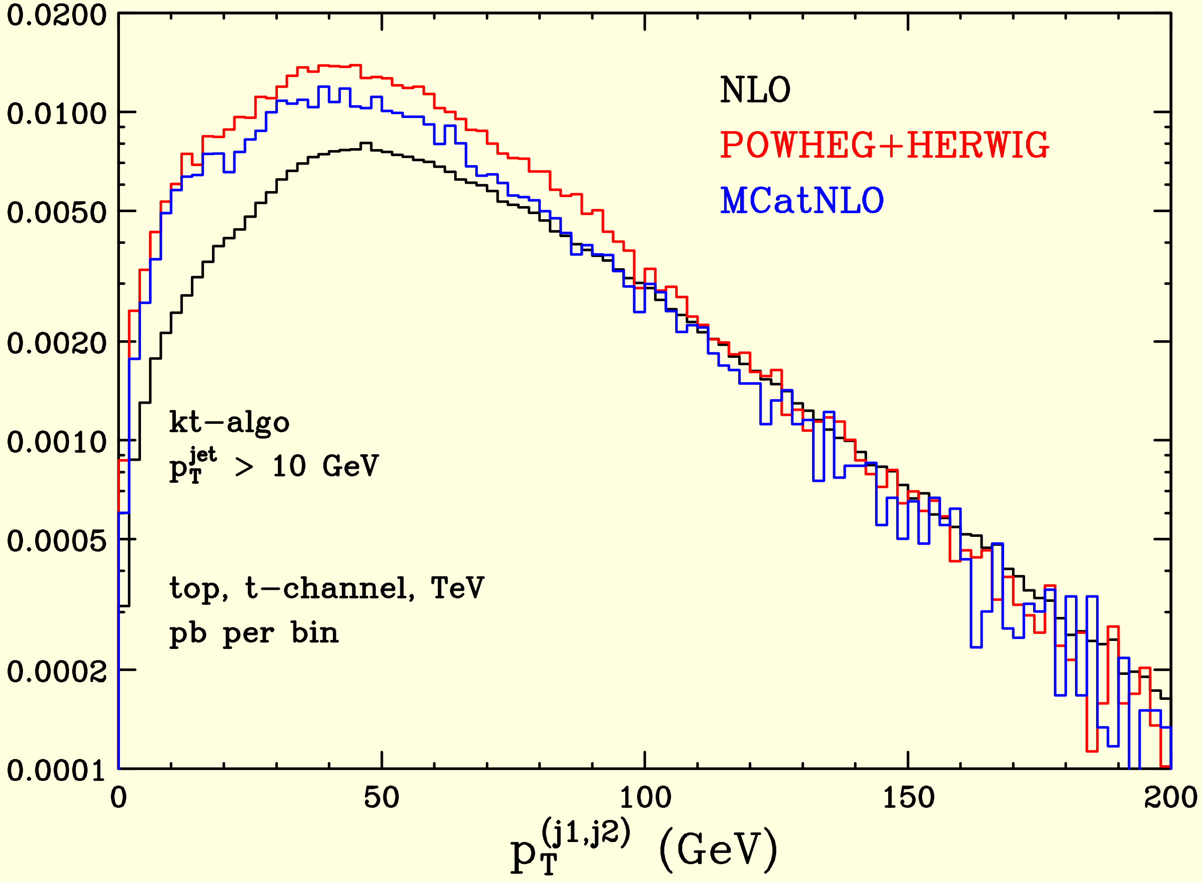


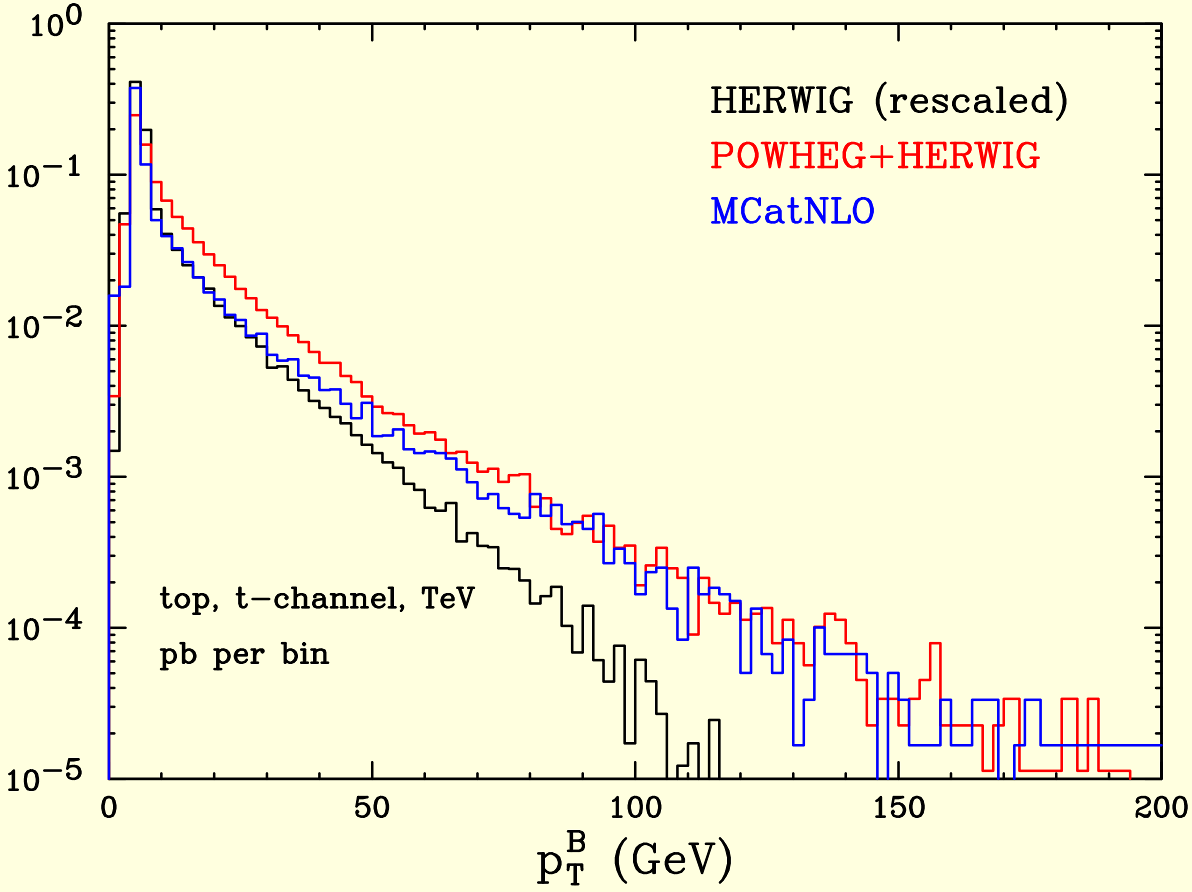
Born Initial state radiation final state radiation

Simplest process with ISR and FSR (simplest because finite without cuts)

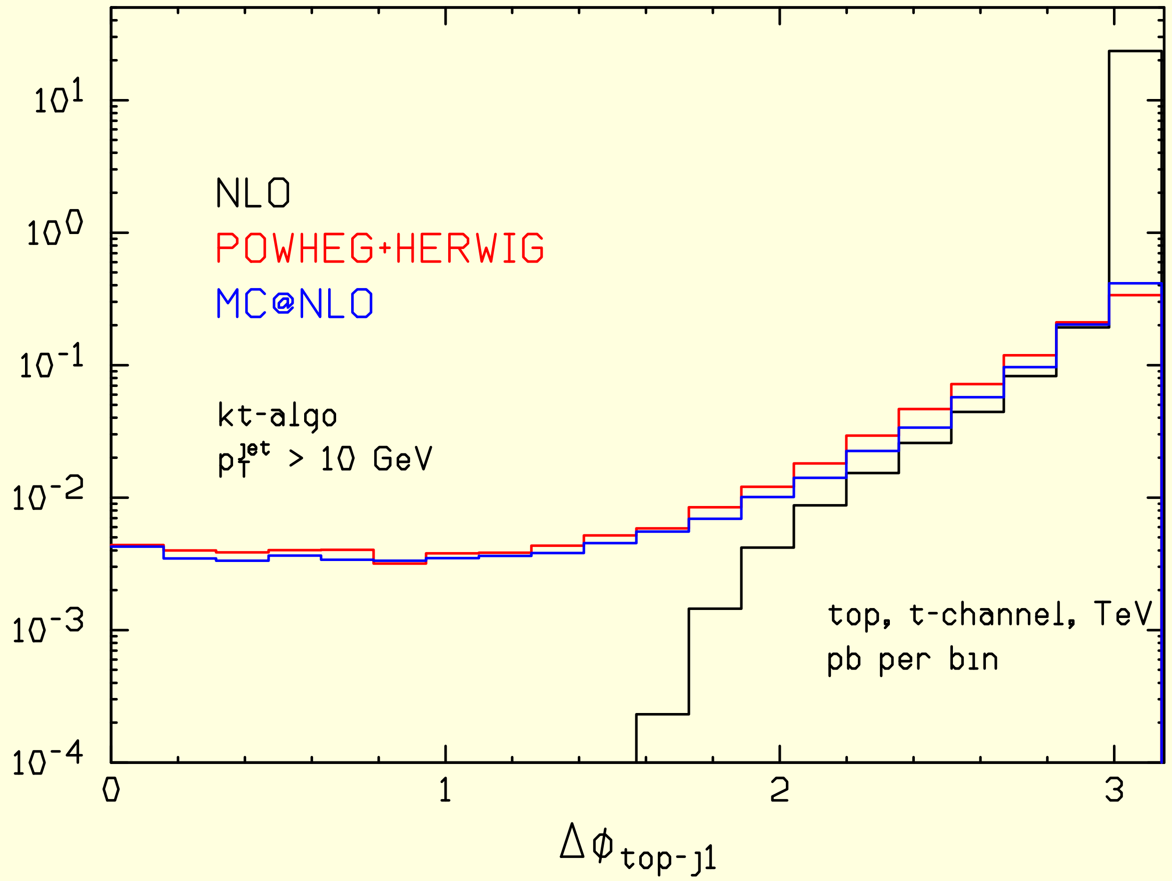
We have applied the general formalism given in Frixione, Oleari, P.N. 2007 to single top production (Alioli, Oleari, Re, P.N. 2009).

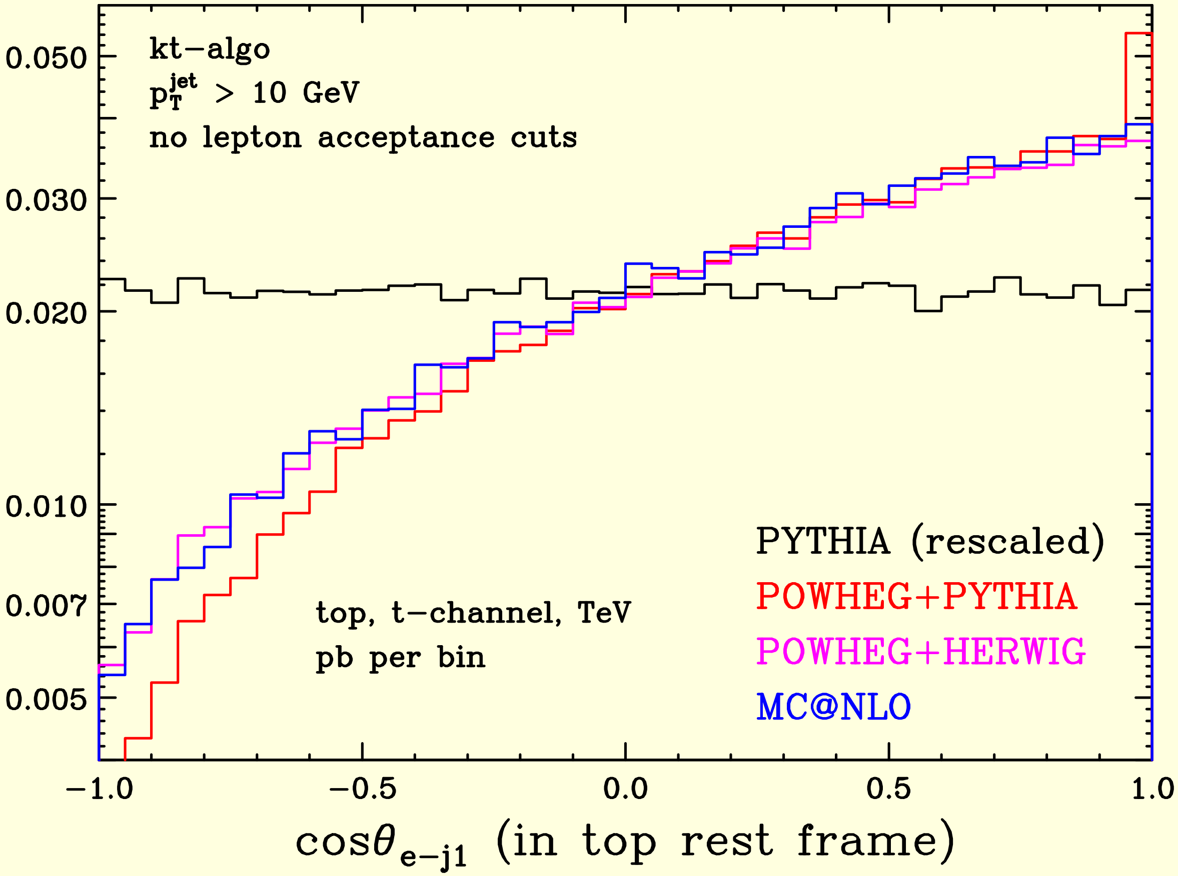

NLO
POWHEG+HERWIG
MCatNLO
top, t−channel, TeV
pb per bin
0.0200
0.0100
0.0050
0.0020
0.0010
0.0005
0.0002
0
50
100
150
200
$p_T^{top}$ (GeV)

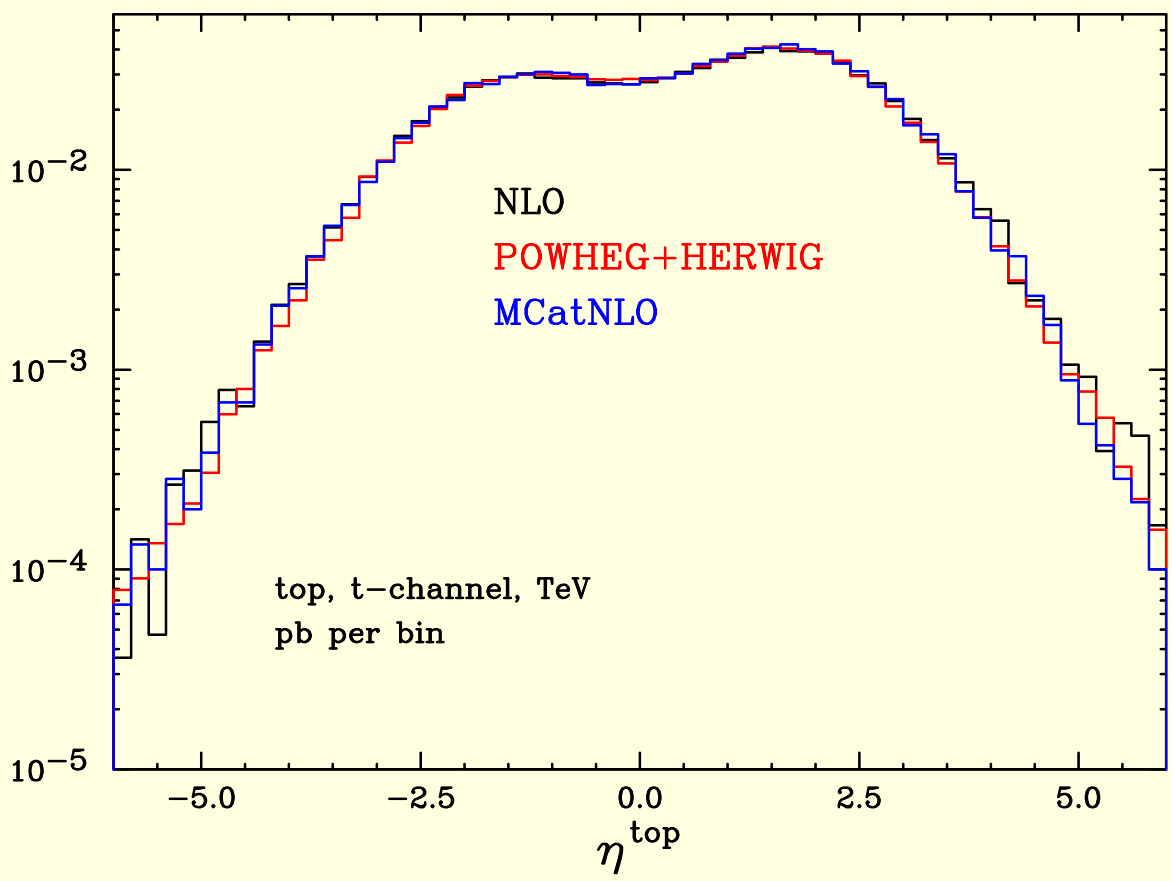

NLO
POWHEG+HERWIG
MCatNLO
kt−algo
$p_T^{jet} > 10$ GeV
top, t−channel, TeV
pb per bin
0.0200
0.0100
0.0050
0.0020
0.0010
0.0005
0.0002
0.0001
0
50
100
150
200
$p_T^{(j1,j2)}$ (GeV)



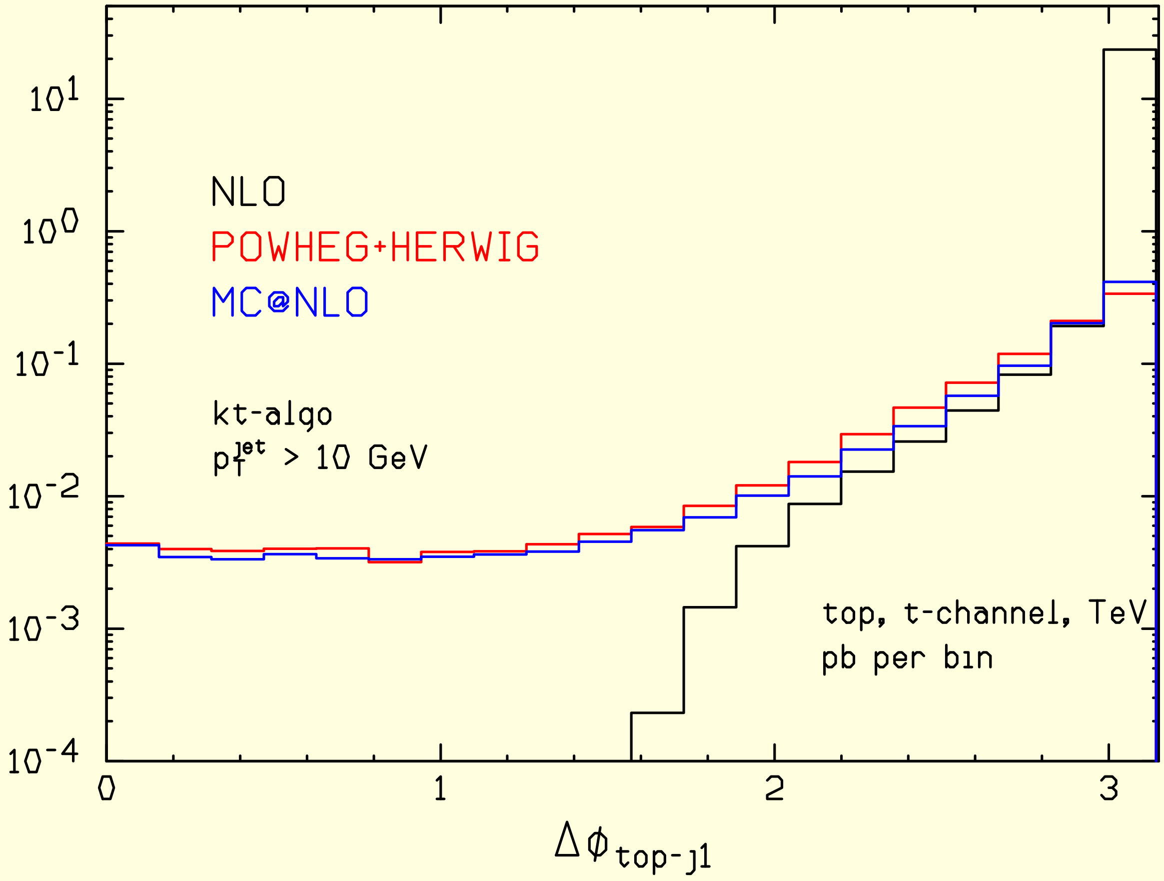
HERWIG (rescaled)
POWHEG+HERWIG
MCatNLO
top, t−channel, TeV
pb per bin
$10^0$
$10^{-1}$
$10^{-2}$
$10^{-3}$
$10^{-4}$
$10^{-5}$
0
50
100
150
200
$p_T^B$ (GeV)


NLO
POWHEG+HERWIG
MC@NLO
kt-algo
$p_T^{jet}$ > 10 GeV
top, t-channel, TeV
pb per bin
$10^1$
$10^0$
$10^{-1}$
$10^{-2}$
$10^{-3}$
$10^{-4}$
0
1
2
3
$\Delta\phi_{top-j1}$


kt−algo
$p_T^{jet}$ > 10 GeV
no lepton acceptance cuts
top, t−channel, TeV
pb per bin
PYTHIA (rescaled)
POWHEG+PYTHIA
POWHEG+HERWIG
MC@NLO
0.050
0.030
0.020
0.010
0.007
0.005
−1.0
−0.5
0.0
0.5
1.0
$\cos\theta_{e-j1}$ (in top rest frame)



NLO
POWHEG+HERWIG
MCatNLO
top, t−channel, TeV
pb per bin
$10^{-2}$
$10^{-3}$
$10^{-4}$
$10^{-5}$
−5.0
−2.5
0.0
2.5
5.0
$\eta^{top}$



NLO
POWHEG+HERWIG
MC@NLO
kt-algo
$p_T^{jet}$ > 10 GeV
top, t-channel, TeV
pb per bin
$\Delta\phi_{top-j1}$

# Towards automation: the POWHEG BOX

The MIB (Milano-Bicocca) group (Alioli, Oleari, Re, P.N.) is working on an automatic implementation of POWHEG for generic NLO processes.

This framework is being tested in the process $h\,h \to Z + 1\mathrm{jet}$.

# The POWHEG BOX

Build a computer code framework, such that, given the Born cross section, the finite part of the virtual corrections, and the real graph cross section, one builds immediately a POWHEG generator. More precisely, the user must supply:

- The Born phase space
- The lists of Born and Real processes (i.e. $u\,\bar{s} \to W^+ c\,\bar{c}$, etc.)
- The Born squared amplitudes $\mathcal{B} = |\mathcal{M}|^2$, $\mathcal{B}_{ij}$, $\mathcal{B}_{j,\mu_j,\mu'_j}$, for all relevant partonic processes; $\mathcal{B}_{ij}$ is the colour ordered Born amplitude squared, $\mathcal{B}_{j,\mu\nu}$ is the spin correlated amplitude, where $j$ runs over all external gluons in the amplitude. All these amplitudes are common ingredient of an NLO calculation.
- The Real squared amplitude, for all relevant partonic processes.
- The finite part of the virtual amplitude contribution, for all relevant partonic processes.

# Strategy

Use the FKS framework according to the general formulation of POWHEG given in (Frixione, Oleari, P.N. 2007), hiding all FKS implementation details.
In other words, we use FKS, but the user needs not to understand it.
(Attempts to use the Catani-Seymour method did not work ...)
It includes:

- The phase space for ISR and FSR, according to FNO2006.
- The combinatorics, the calculation of all $R_\alpha$, the soft and coll. limits
- The calculation of $\tilde{B}$
  (spinoff: NLO implementation using the FKS method)
- The calculation of the upper bounds for the generation of radiation
- The generation of radiation
- Writing the event to the Les Houches interface

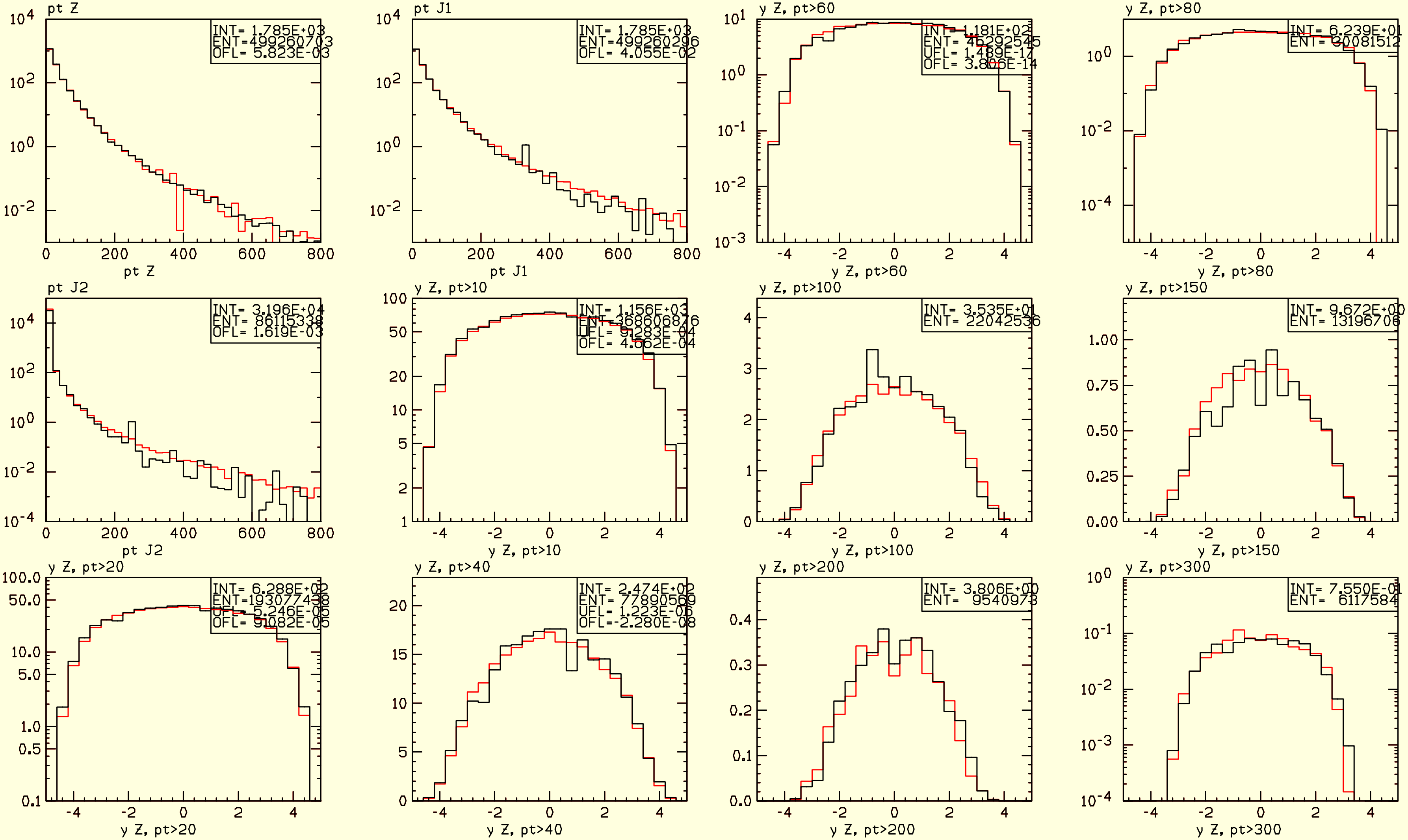
It works! Lots of more testing needed now ...

## Case study: $Z + \mathrm{jet}$ production

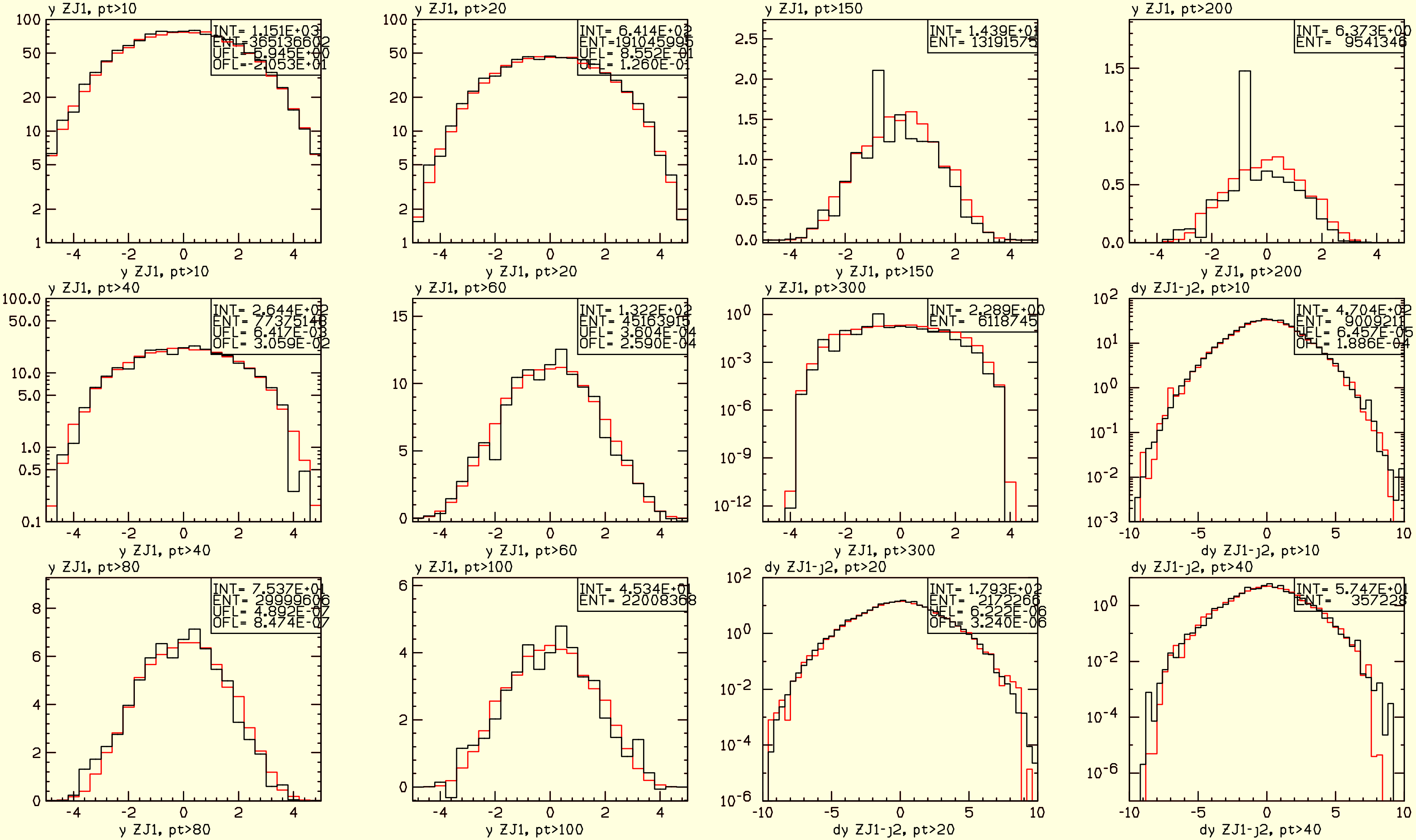
Got virtual matrix elements from `MCFM`;

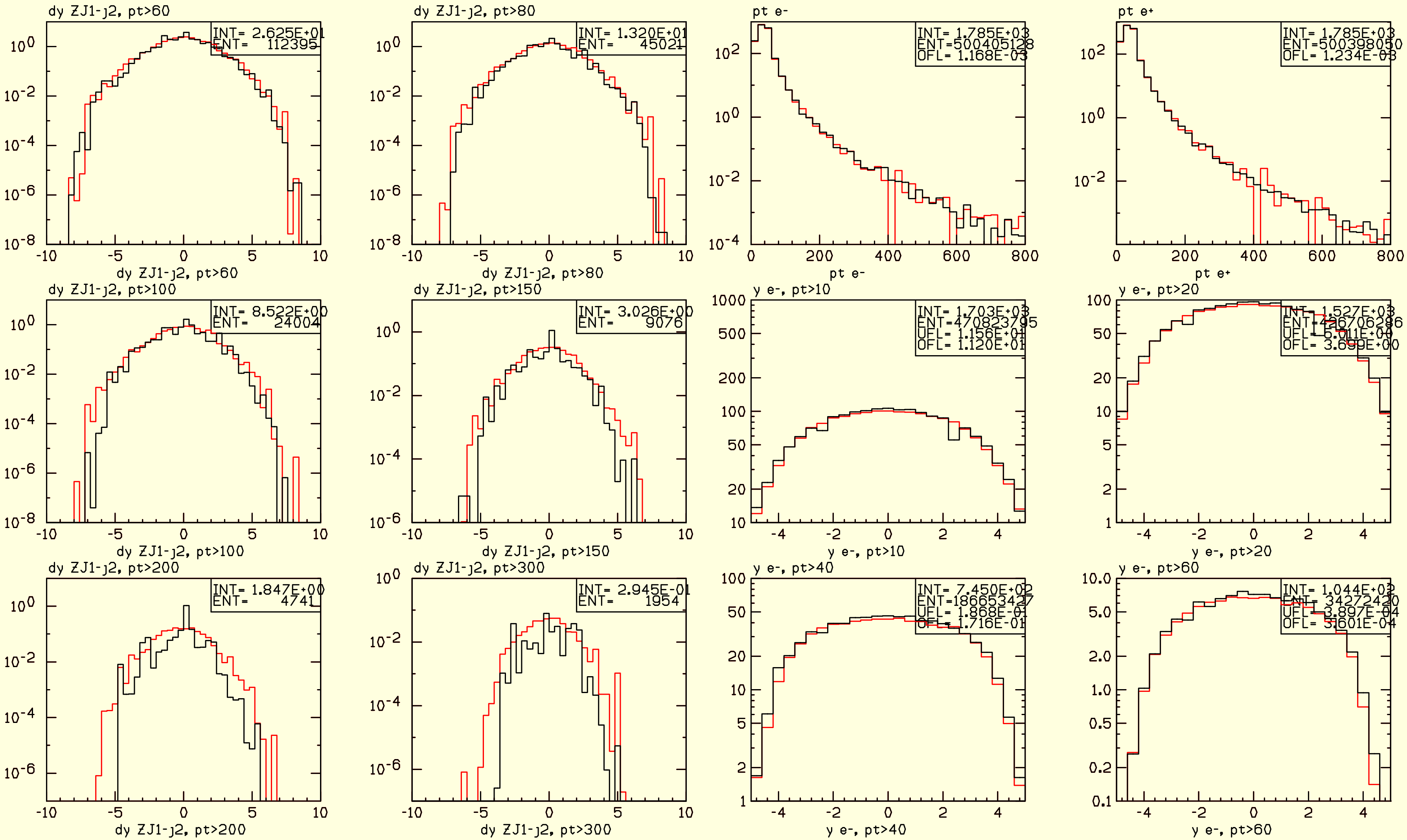
Compare NLO predictions obtained with `MCFM` and the `POWHEG BOX`

Virtual corrections are the same, but subtraction terms, soft and collinear remnants are all different; non trivial test;

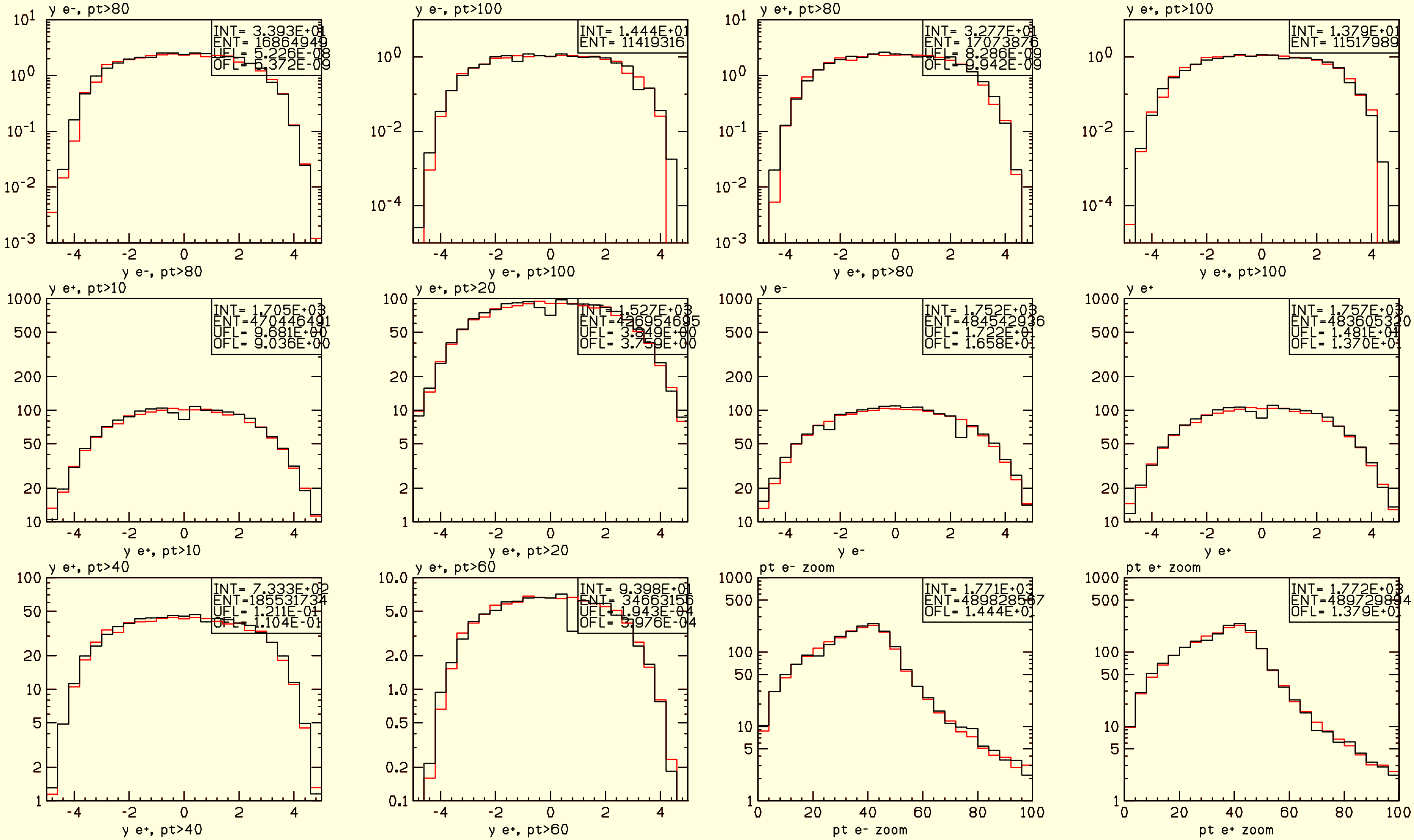


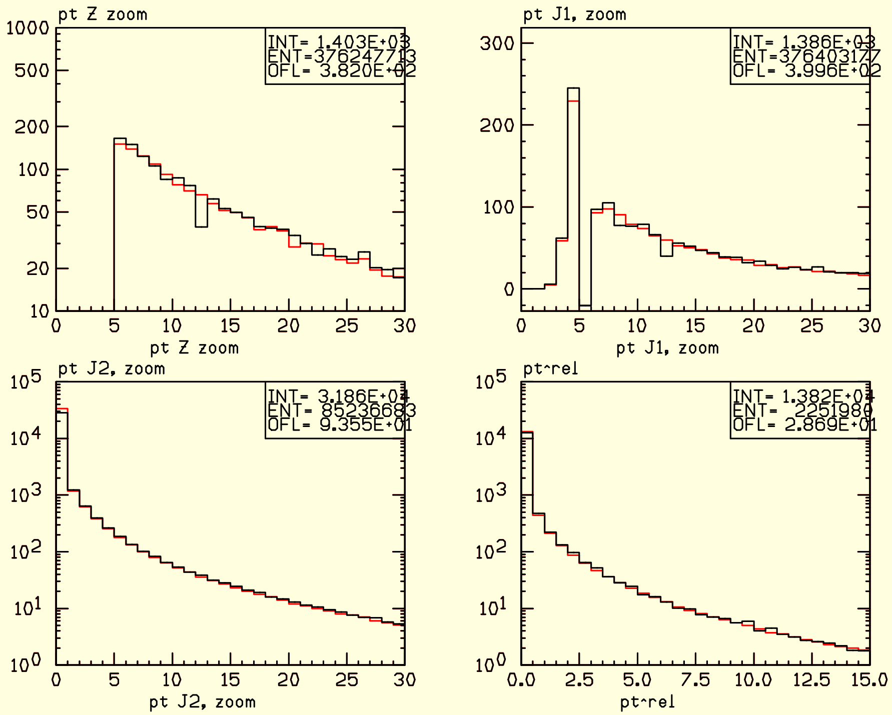
pt Z
INT= 1.785E+03
ENT=499260703
OFL= 5.823E-03
pt J1
INT= 1.785E+03
ENT=499260296
OFL= 4.055E-02
y Z, pt>60
INT= 1.181E+02
ENT= 46292545
UFL= 1.489E-17
OFL= 3.806E-14
y Z, pt>80
INT= 6.239E+01
ENT= 30081512
pt J2
INT= 3.196E+04
ENT= 86115338
OFL= 1.619E-03
y Z, pt>10
INT= 1.156E+03
ENT= 368606876
UFL= 9.283E-04
OFL= 4.662E-04
y Z, pt>100
INT= 3.535E+01
ENT= 22042536
y Z, pt>150
INT= 9.672E+00
ENT= 13196706
y Z, pt>20
INT= 6.288E+02
ENT=193077438
UFL= 5.246E-05
OFL= 9.1082E-05
y Z, pt>40
INT= 2.474E+02
ENT= 77890569
UFL= 1.223E-06
OFL=-2.280E-08
y Z, pt>200
INT= 3.806E+00
ENT= 9540973
y Z, pt>300
INT= 7.550E-01
ENT= 6117584


y ZJ1, pt>10
INT= 1.151E+03
ENT=365136602
UFL= 6.945E+00
OFL= 2.053E+01
y ZJ1, pt>20
INT= 6.414E+02
ENT=191045995
UFL= 8.552E-01
OFL= 1.260E-01
y ZJ1, pt>150
INT= 1.439E+01
ENT= 13191575
y ZJ1, pt>200
INT= 6.373E+00
ENT= 9541346
y ZJ1, pt>40
INT= 2.644E+02
ENT= 77375148
UFL= 6.417E-03
OFL= 3.059E-02
y ZJ1, pt>60
INT= 1.322E+02
ENT= 45163915
UFL= 3.604E-04
OFL= 2.590E-04
y ZJ1, pt>300
INT= 2.289E+00
ENT= 6118745
dy ZJ1-j2, pt>10
INT= 4.704E+02
ENT= 9009211
UFL= 6.457E-05
OFL= 1.886E-04
y ZJ1, pt>80
INT= 7.537E+01
ENT= 29999606
UFL= 4.892E-07
OFL= 8.474E-07
y ZJ1, pt>100
INT= 4.534E+01
ENT= 22008368
dy ZJ1-j2, pt>20
INT= 1.793E+02
ENT= 2172266
UFL= 6.222E-06
OFL= 3.240E-06
dy ZJ1-j2, pt>40
INT= 5.747E+01
ENT= 357228



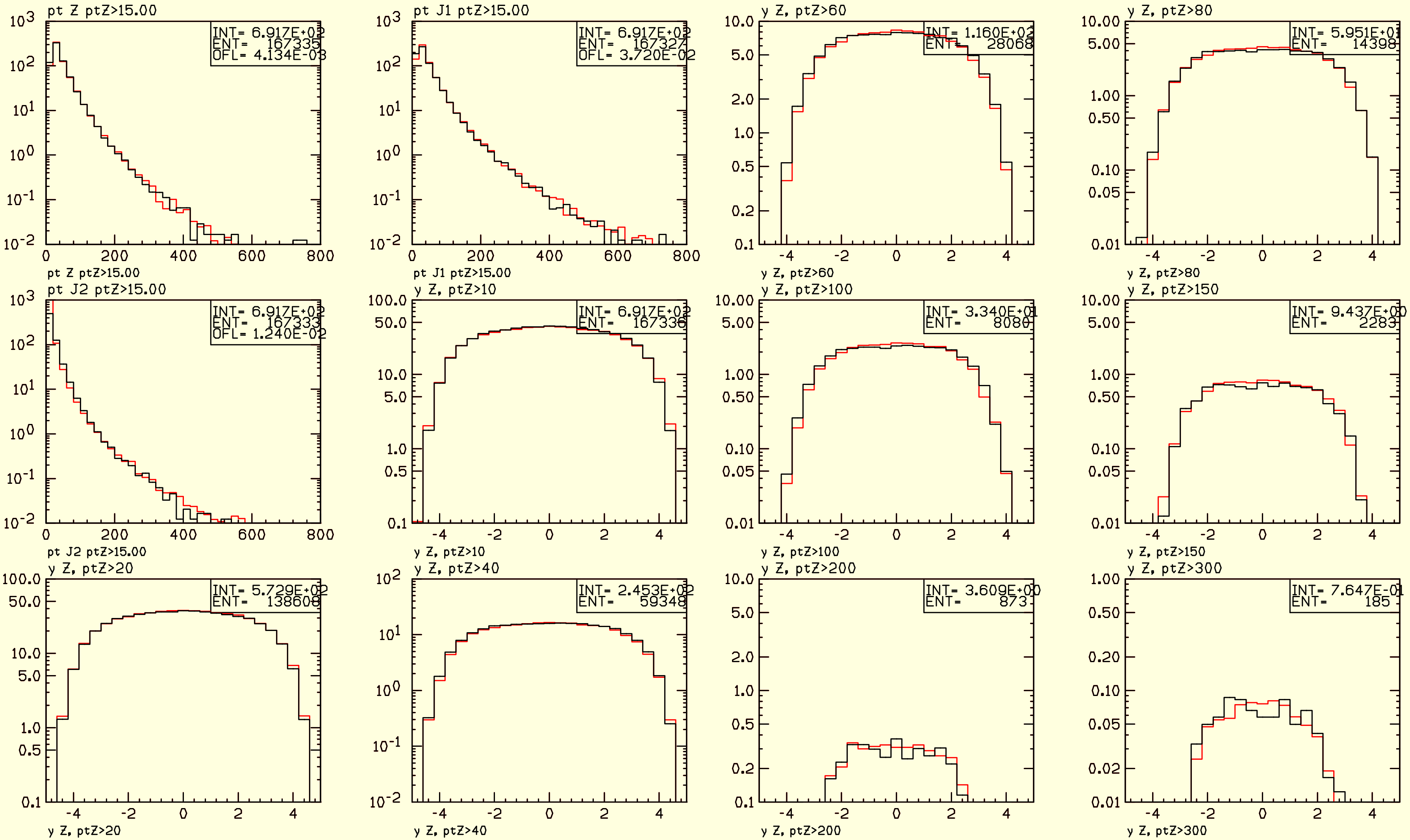
dy ZJ1-j2, pt>60
INT= 2.625E+01
ENT= 112395
dy ZJ1-j2, pt>80
INT= 1.320E+01
ENT= 45021
pt e-
INT= 1.785E+03
ENT=500405128
OFL= 1.168E-03
pt e+
INT= 1.785E+03
ENT=500398050
OFL= 1.234E-03
dy ZJ1-j2, pt>100
INT= 8.522E+00
ENT= 24004
dy ZJ1-j2, pt>150
INT= 3.026E+00
ENT= 9076
y e-, pt>10
INT= 1.703E+03
ENT=470823795
UFL= 1.156E+01
OFL= 1.120E+01
y e-, pt>20
INT= 1.527E+03
dy ZJ1-j2, pt>200
INT= 1.847E+00
ENT= 4741
dy ZJ1-j2, pt>300
INT= 2.945E-01
ENT= 1954
y e-, pt>40
INT= 7.450E+02
ENT=186653427
UFL= 1.868E-01
y e-, pt>60
INT= 1.044E+02

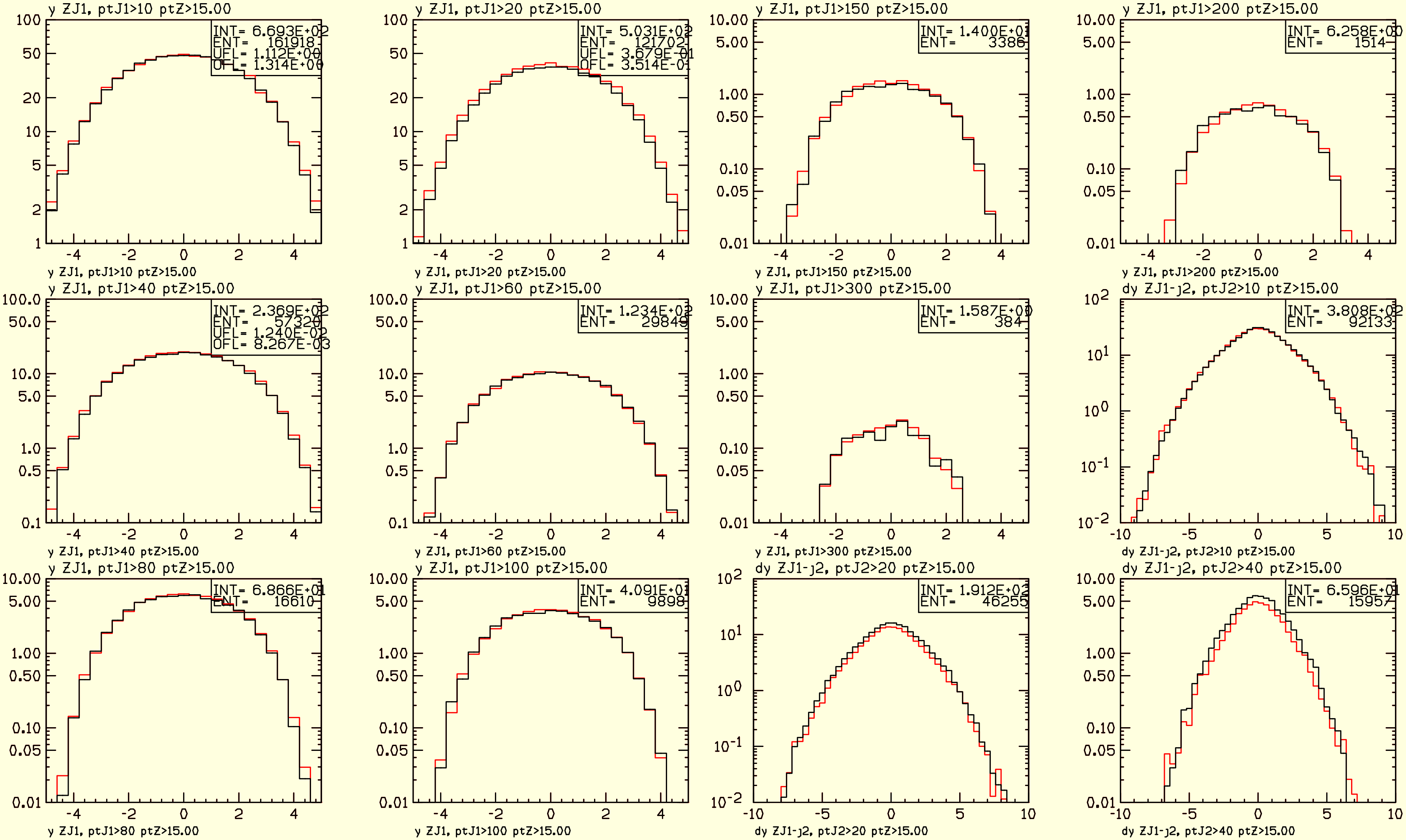

y e-, pt>80
INT= 3.393E+01
ENT= 16864949
UFL= 5.226E-08
OFL= 6.372E-09
y e-, pt>100
INT= 1.444E+01
ENT= 11419316
y e+, pt>80
INT= 3.277E+01
ENT= 17073876
UFL= 8.286E-09
OFL= 9.942E-09
y e+, pt>100
INT= 1.379E+01
ENT= 11517989
y e+, pt>10
INT= 1.705E+03
ENT=470446491
UFL= 9.681E+00
OFL= 9.036E+00
y e+, pt>20
INT= 1.527E+03
ENT=426954695
y e-
INT= 1.752E+03
ENT=484542936
UFL= 1.722E+01
OFL= 1.658E+01
y e+
INT= 1.757E+03
ENT=483605320
UFL= 1.481E+01
OFL= 1.370E+01
y e+, pt>40
INT= 7.333E+02
ENT=185531734
UFL= 1.211E-01
OFL= 1.104E-01
y e+, pt>60
INT= 9.398E+01
ENT= 34663156
UFL= 1.943E-04
OFL= 5.976E-04
pt e- zoom
INT= 1.771E+03
ENT=489828567
OFL= 1.444E+01
pt e+ zoom
INT= 1.772E+03
ENT=489729894
OFL= 1.379E+01

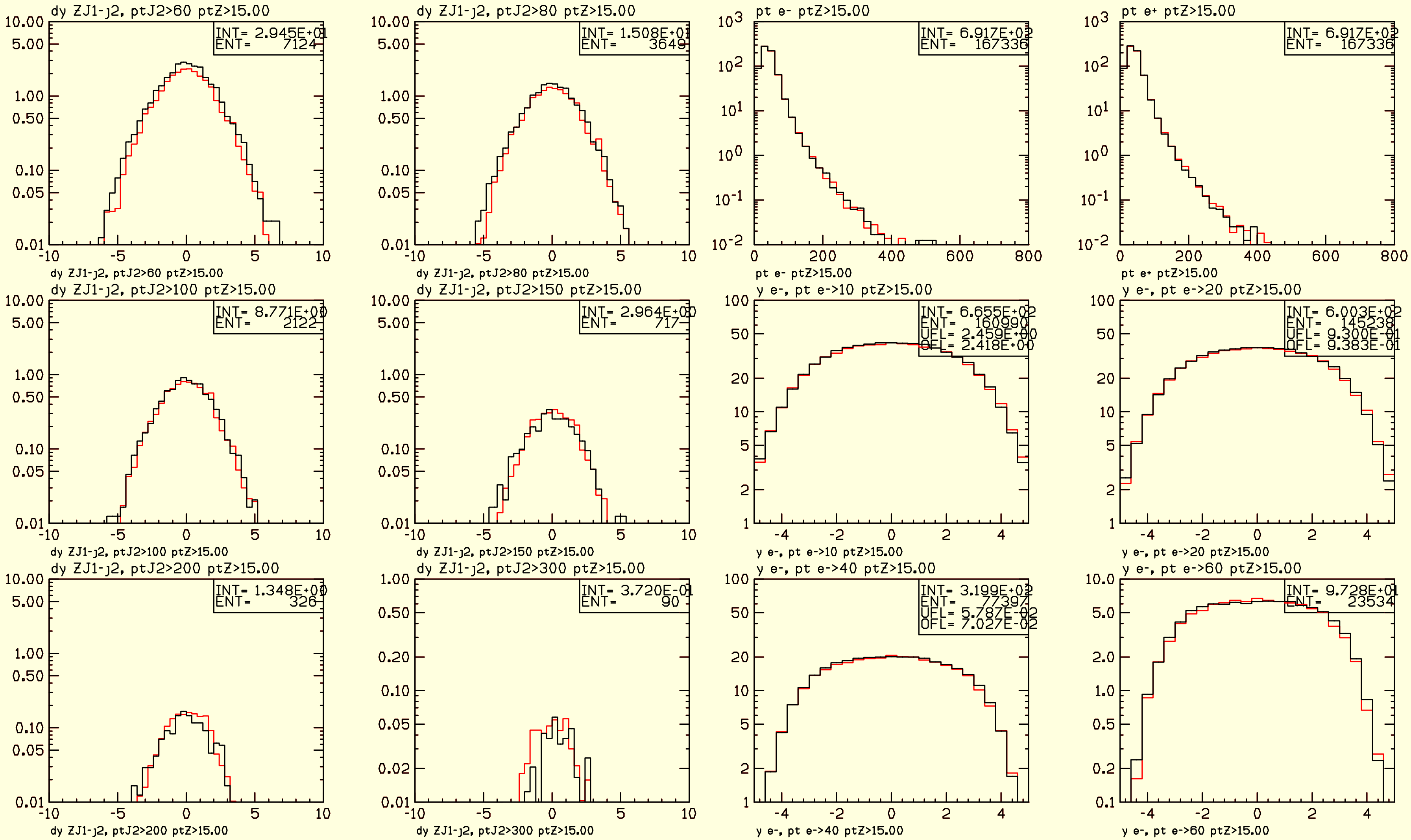


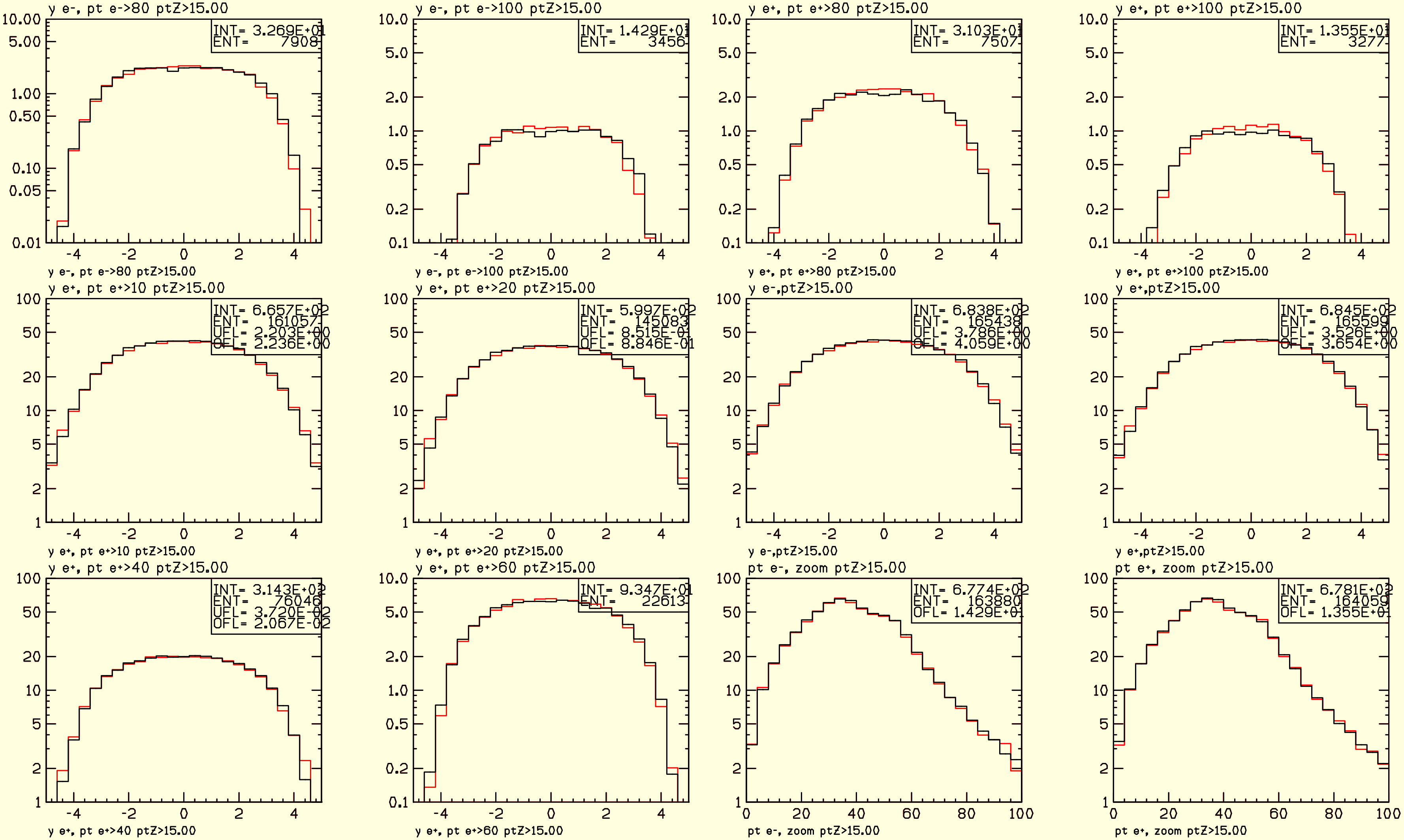

Everything seems to work ...

# Now compare POWHEG+HERWIG with NLO (red)

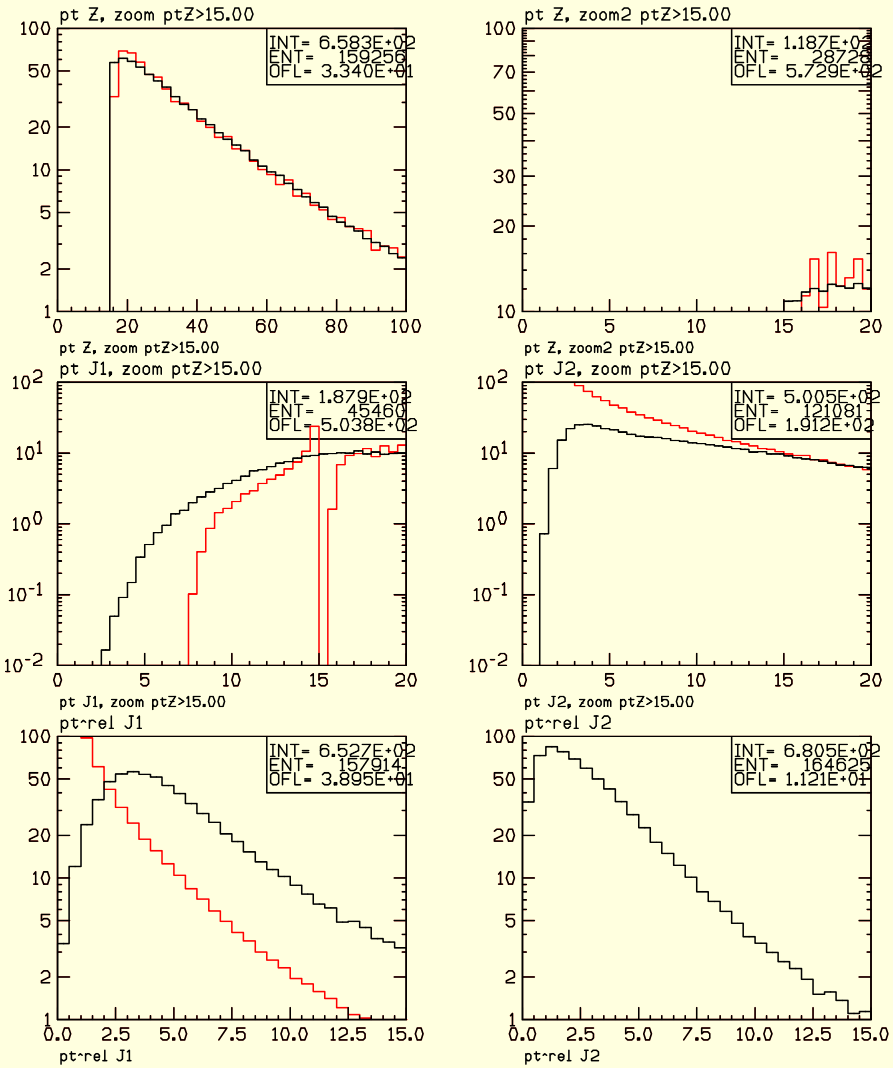



y ZJ1, ptJ1>10 ptZ>15.00
INT= 6.693E+02
ENT= 161918
UFL= 1.112E+00
OFL= 1.314E+00
y ZJ1, ptJ1>20 ptZ>15.00
INT= 5.031E+02
ENT= 121702
UFL= 3.679E-01
OFL= 3.514E-01
y ZJ1, ptJ1>150 ptZ>15.00
INT= 1.400E+01
ENT= 3386
y ZJ1, ptJ1>200 ptZ>15.00
INT= 6.258E+00
ENT= 1514
y ZJ1, ptJ1>40 ptZ>15.00
INT= 2.369E+02
ENT= 57320
UFL= 1.240E-02
OFL= 8.267E-03
y ZJ1, ptJ1>60 ptZ>15.00
INT= 1.234E+02
ENT= 29849
y ZJ1, ptJ1>300 ptZ>15.00
INT= 1.587E+00
ENT= 384
dy ZJ1-j2, ptJ2>10 ptZ>15.00
INT= 3.808E+02
ENT= 92133
y ZJ1, ptJ1>80 ptZ>15.00
INT= 6.866E+01
ENT= 16610
y ZJ1, ptJ1>100 ptZ>15.00
INT= 4.091E+01
ENT= 9898
dy ZJ1-j2, ptJ2>20 ptZ>15.00
INT= 1.912E+02
ENT= 46255
dy ZJ1-j2, ptJ2>40 ptZ>15.00
INT= 6.596E+01
ENT= 15957


dy ZJ1-j2, ptJ2>60 ptZ>15.00
INT= 2.945E+01
ENT= 7124
dy ZJ1-j2, ptJ2>80 ptZ>15.00
INT= 1.508E+01
ENT= 3649
pt e- ptZ>15.00
INT= 6.917E+02
ENT= 167336
pt e+ ptZ>15.00
INT= 6.917E+02
ENT= 167336
dy ZJ1-j2, ptJ2>100 ptZ>15.00
INT= 8.771E+00
ENT= 2122
dy ZJ1-j2, ptJ2>150 ptZ>15.00
INT= 2.964E+00
ENT= 717
y e-, pt e->10 ptZ>15.00
INT= 6.655E+02
ENT= 160990
UFL= 2.459E+00
OFL= 2.418E+00
y e-, pt e->20 ptZ>15.00
INT= 6.003E+02
ENT= 145238
UFL= 9.300E-01
OFL= 9.383E-01
dy ZJ1-j2, ptJ2>200 ptZ>15.00
INT= 1.348E+00
ENT= 326
dy ZJ1-j2, ptJ2>300 ptZ>15.00
INT= 3.720E-01
ENT= 90
y e-, pt e->40 ptZ>15.00
INT= 3.199E+02
ENT= 77397
UFL= 5.787E-02
OFL= 7.027E-02
y e-, pt e->60 ptZ>15.00
INT= 9.728E+01
ENT= 23534


y e-, pt e->80 ptZ>15.00
INT= 3.269E+01
ENT= 7908
y e-, pt e->100 ptZ>15.00
INT= 1.429E+01
ENT= 3456
y e+, pt e+>80 ptZ>15.00
INT= 3.103E+01
ENT= 7507
y e+, pt e+>100 ptZ>15.00
INT= 1.355E+01
ENT= 3277
y e+, pt e+>10 ptZ>15.00
INT= 6.657E+02
ENT= 161057
UFL= 2.203E+00
OFL= 2.236E+00
y e+, pt e+>20 ptZ>15.00
INT= 5.997E+02
ENT= 145083
UFL= 8.515E-01
OFL= 8.846E-01
y e-,ptZ>15.00
INT= 6.838E+02
ENT= 165438
UFL= 3.786E+00
OFL= 4.059E+00
y e+,ptZ>15.00
INT= 6.845E+02
ENT= 165599
UFL= 3.526E+00
OFL= 3.654E+00
y e+, pt e+>40 ptZ>15.00
INT= 3.143E+02
ENT= 76046
UFL= 3.720E-02
OFL= 2.067E-02
y e+, pt e+>60 ptZ>15.00
INT= 9.347E+01
ENT= 22613
pt e-, zoom ptZ>15.00
INT= 6.774E+02
ENT= 163880
OFL= 1.429E+01
pt e+, zoom ptZ>15.00
INT= 6.781E+02
ENT= 164059
OFL= 1.355E+01

Distributions sensitive to more than two jet show noticeably different. All others in agreement with NLO



# Conclusions

- NLO accuracy with Shower MC has become a reality in recent years.
- The POWHEG method is progressing, with new processes being included
- Progress in understanding agreement and differences between `MC@NLO` and `POWHEG`
- A path to full automation of POWHEG implementations of arbitrary NLO calculation is open
- Many interesting problems remain to be addressed, and the NLO+Shower community is steadily growing.