Shower Monte Carlo at Next-to-Leading Order

P. Nason
INFN, Sez. of Milano Bicocca

Talk in `http://moby.mib.infn.it/~nason/talks/`

Outline

Introduction:

- Shower Monte Carlo programs

Shower improvements

- NLO and showers:
  - MC@NLO
  - POWHEG
- Results:
  - POWHEG vs. LO SMC's
  - POWHEG vs. MC@NLO: Higgs production as a case study
- Perspective
- Conclusions
- Further issues and subtleties

Shower Monte Carlo programs

1. Large library of hard events cross sections (SM and BSM)
2. Dress hard events with QCD radiation
3. Models for hadron formation
4. Models for underlying event, multi-parton collisions, minimum bias
5. Library for (spacetime) decays of unstable particles

The name SHOWER from item 2.

The hope (and the experience) is:
the "Models" part is the same at all energies, and process independent
Once tuned at some energy, the SMC is predictive for all other energies.

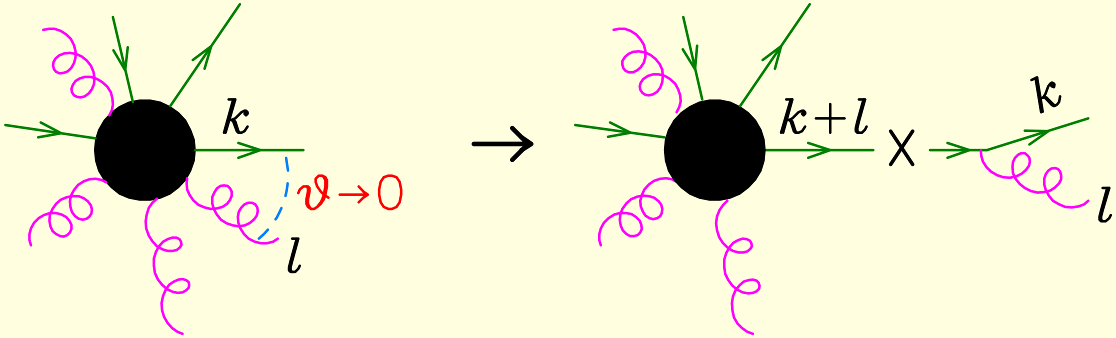
HEP experiments feed this kind of output through their detector simulation software, and use it to determine efficiencies for signal detection and to perform background estimates. Analysis strategies are set up using these simulated data.

- In HEP (i.e. collider physics) not many questions can be answered without a Shower Monte Carlo (SMC). Heavily used since 1980's
- SMC's are forever (well, as long as HEP lives).
  Even if QCD was solved exactly, it is unlikely that complex
  high energy phenomena will be described better than in SMC models.
- SMC models have long been neglected in theoretical physics:
  Emphasis on QCD tests required more transparent theoretical methods.
  After LEP, QCD testing is less important.
  With LHC, QCD modeling is a primary issue: recent SMC revival.
- Thinking in terms of Shower algorithms gives us an easy to grasp,
  intuitive understanding of complex QCD phenomena
  (and a practical way to verify our ideas).

Shower basics: Collinear factorization

QCD emissions are enhanced near the collinear limit

Cross sections factorize near collinear limit



$$|M_{n+1}|^2 d\Phi_{n+1} \Longrightarrow |M_n|^2 d\Phi_n \, \frac{\alpha_s}{2\pi} \, \frac{dt}{t} \, P_{q,qg}(z) \, dz \, \frac{d\phi}{2\pi}$$

t : hardness (either virtuality or p_T^2 or $E^2\theta^2$ etc.)

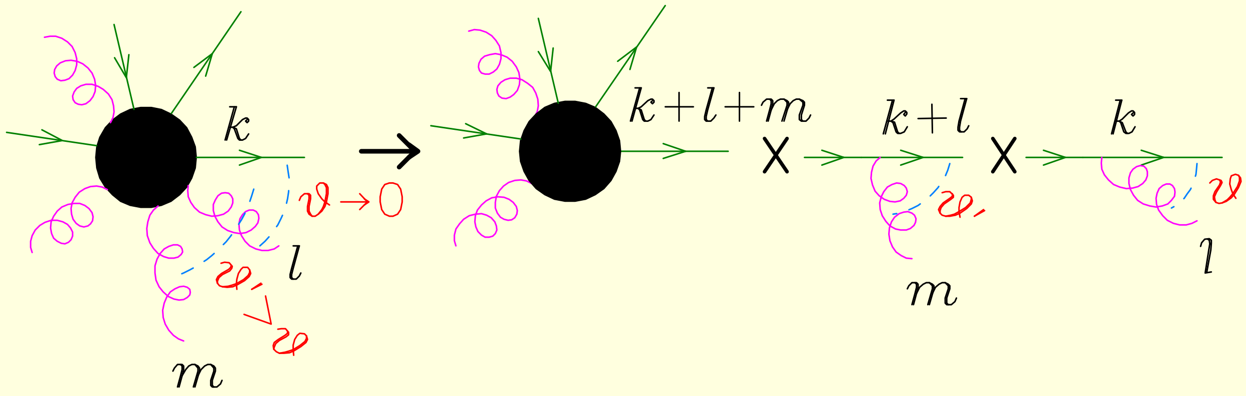
$z = k^0/(k^0 + l^0)$: energy (or $p_{\parallel}$, or p^+) fraction of quark

$P_{q,qg}(z) = C_F \frac{1+z^2}{1-z}$: Altarelli – Parisi splitting function

(ignore $z \to 1$ IR divergence for now)

If another gluon becomes collinear, iterate the previous formula:

$\theta', \theta \to 0$
with $\theta' > \theta$



$$|M_{n+1}|^2 d\Phi_{n+1} \Longrightarrow |M_{n-1}|^2 d\Phi_{n-1} \times \frac{\alpha_s}{2\pi} \frac{dt'}{t'} P_{q,qg}(z')\, dz' \frac{d\phi'}{2\pi} \times \frac{\alpha_s}{2\pi} \frac{dt}{t} P_{q,qg}(z)\, dz \frac{d\phi}{2\pi} \theta(t'-t)$$

Collinear partons can be described by a factorized integral ordered in t.
For m collinear emissions:

$$\left(\frac{\alpha_s}{2\pi}\right)^m \int_{\theta_{\rm min}} \frac{d\theta_1}{\theta_1} \int_{\theta_1} \frac{d\theta_2}{\theta_2} \cdots \int_{\theta_{m-1}} \frac{d\theta_m}{\theta_m} \quad \propto \frac{\log^m \frac{1}{\theta^2_{\rm min}}}{m!} \approx \left(\frac{\alpha_s}{2\pi}\right)^m \frac{\log^m \frac{Q^2}{\Lambda^2}}{m!}$$

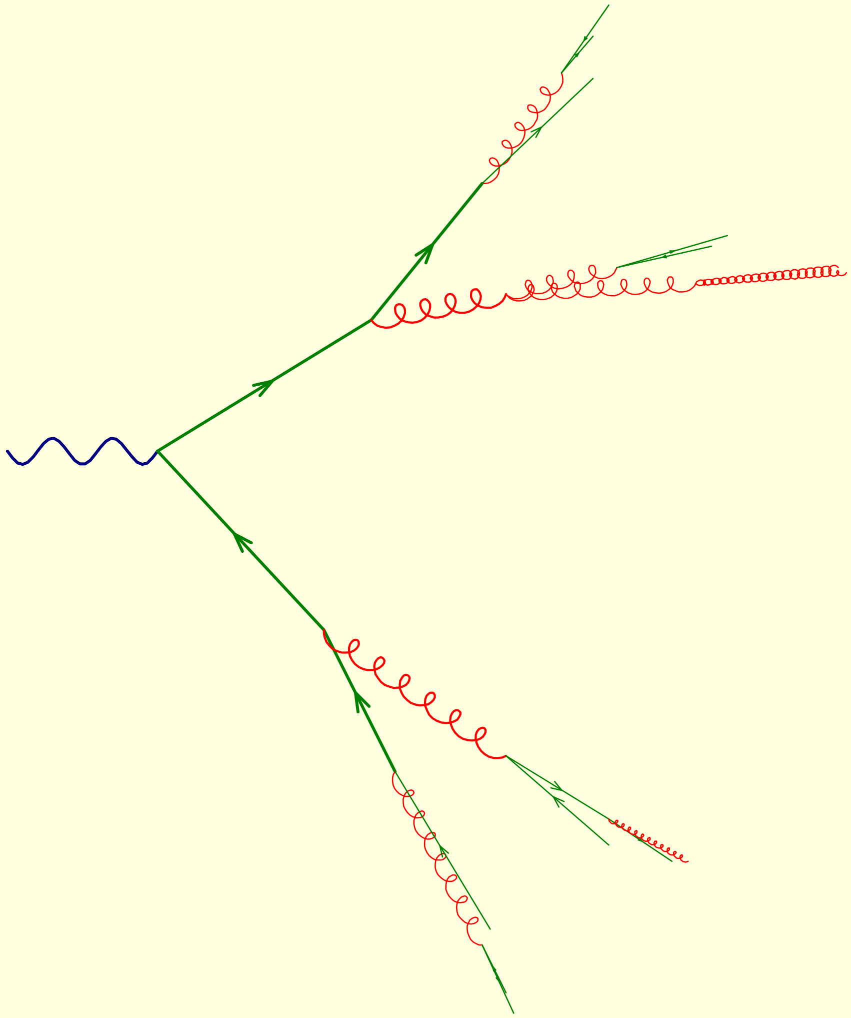
where we have taken $\theta_{\rm min} \approx \Lambda/Q$; (Leading Logs) This is of order 1!
Typical dominant configuration at very high Q^2

Besides $q \to qg$, also $g \to gg$, $g \to q\bar{q}$ come into play.

Typical configurations: intermediate angles of order of geometric average of upstream and downstream angles.

Each angle is $\mathcal{O}(\alpha_s)$ smaller than its upstream angle, and $\mathcal{O}(\alpha_s)$ bigger than its downstream angle.

As relative momenta become smaller α_s becomes bigger, and this picture breaks down.



For a consistent description:

include virtual corrections to same LL approximation

One can show that the effect of virtual corrections is given by

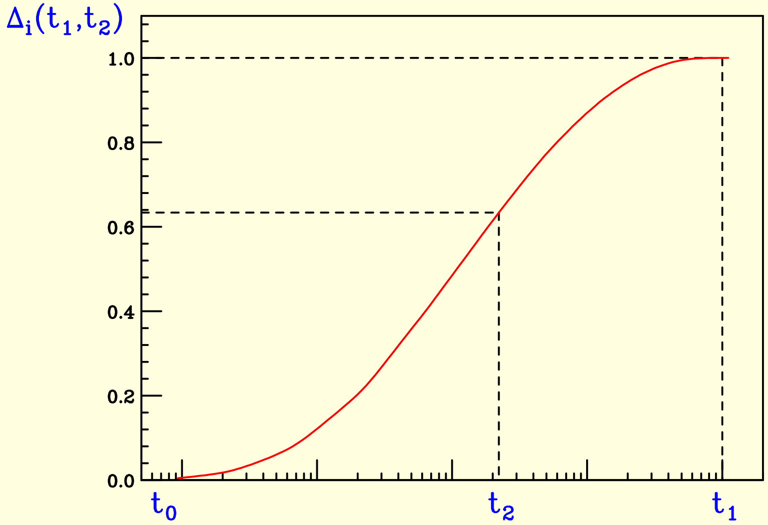
- Let $\alpha(\mu) \Longrightarrow \alpha(t)$ in each vertex, where t is the hardness of the vertex (i.e. hardness of the incoming line)
- For each intermediate line include the factor

$$\Delta_i(t_h, t_l) = \exp\left[-\sum_{(jk)} \int_{t_l}^{t_h} \frac{dt'}{t'} \int dz \, \frac{\alpha_s(t')}{2\pi} P_{i,jk}(z) \right]$$

where t_h is the hardness of the vertex originating the line, and t_l is the hardness of the vertex where the line ends.

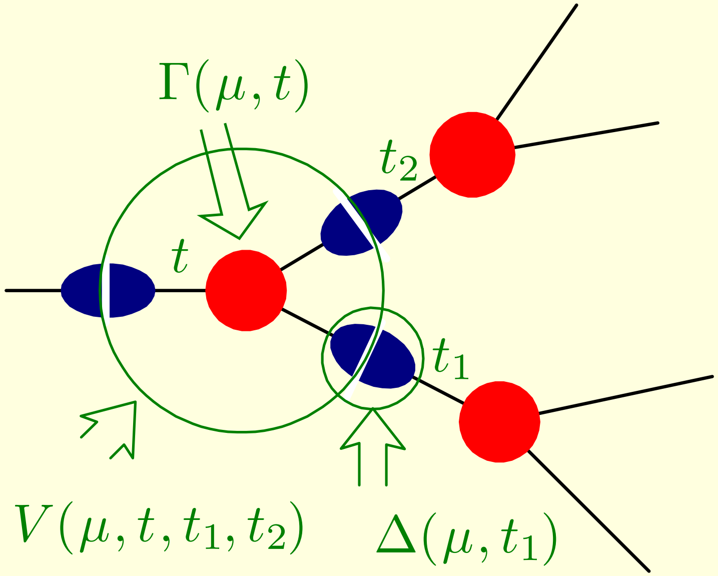
Sudakov form factor

$$\Delta_i(t_h, t_l) = \exp\left[-\sum_{(jk)} \int_{t_l}^{t_h} \frac{dt'}{t'} \int dz \, \frac{\alpha_s(t')}{2\pi} P_{i,jk}(z) \right]$$



As t_l becomes small the exponent tend to diverge, and $\Delta_i(t_h, t_l)$ approaches 0. In fact, because of $\alpha_s(t)$, we must stop at $t_0 \gtrsim \Lambda_{\rm QCD}$.

Proof of effect of virtual corrections



Effective (RG invariant) splitting vertex:

$$V^2(\mu,t,t_1,t_2) = \underbrace{\Gamma^2(\mu,t)}_{\text{dominated by hardest scale!}}\Delta(\mu,t)\Delta(\mu,t_1)\Delta(\mu,t_2)$$

Choosing $\mu = t$ (using $\Delta(t,t) \approx 1$)

$$V^2(\mu,t,t_1,t_2) = V^2(t,t,t,t)\,\Delta(t,t_1)\Delta(t,t_2)$$

$V(t,t,t,t)$ is the three level vertex with $\alpha \to \alpha(t)$.
The form $\Delta(t,t_1)$ follows from RG arguments.

In fact: $\Delta_i(t,t_1) = \exp\left[-\sum_{(jk)} \int_{t_1}^{t} \frac{dt'}{t'} \int dz\, \frac{\alpha_s(t')}{2\pi} P_{i,jk}(z)\right]$ Sudakov form factor

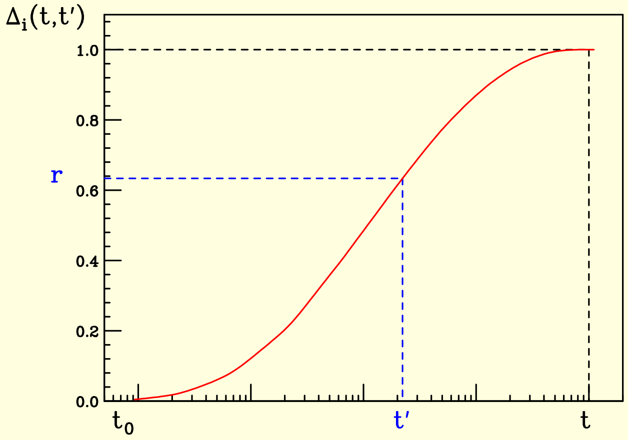
consistent with KLN cancellation of IR singularities, and with RG.

Final Recipe

- Consider all tree graphs.
- Assign ordered hardness parameters t to each vertex.
- Include a factor
$$\frac{\alpha_s(t)}{2\pi} P_{i,jk}(z) \frac{dt}{t} dz \frac{d\phi}{2\pi}$$
at each vertex $i \to jk$.
- Include a factor $\Delta_i(t_1, t_2)$ to each internal line with a parton i, from hardness t_1 to hardness t_2.
- Include a factor $\Delta_i(t, t_0)$ on final lines (t_0: IR cutoff)

Most important: the shower recipe can be easily implemented as a computer code!
Shower Algorithm:

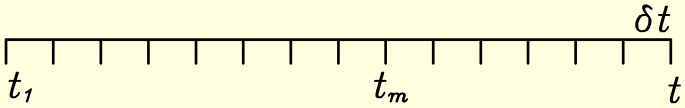
- Generate a uniform random number $0 < r < 1$;
- Solve the equation $\Delta_i(t,t') = r$ for t';
- If $t' < t_0$ stop here (final state line);
- generate z, jk with probability $P_{i,jk}(z)$, and $0 < \phi < 2\pi$ uniformly;
- restart from each branch, with hardness parameter t'.



Probabilistic intepretation: branching probability of line of flavor i

$$dP(t_1,t) = \underbrace{\exp\left[-\sum_{(jk)} \int_t^{t_1} \frac{dt'}{t'} \int dz \, \frac{\alpha_s(t')}{2\pi} P_{i,jk}(z)\right]}_{\Delta(t_1,t)} \frac{\alpha_s(t)}{2\pi} P_{i,jk}(z) \, \frac{dt}{t} \, dz \, \frac{d\phi}{2\pi}$$

break up t_1, t into small subintervals:



$$dP(t_1,t) = \left[\prod_m \left(\underbrace{1 - \sum_{(jk)} \frac{\delta t}{t_m} \int dz \, \frac{\alpha_s(t_m)}{2\pi} P_{i,jk}(z)}_{\text{No emission prob. in } t_m, t_m+\delta t}\right)\right] \underbrace{\frac{\alpha_s(t)}{2\pi} P_{i,jk}(z) \, \frac{\delta t}{t} \, dz \, \frac{d\phi}{2\pi}}_{\text{emission prob. in } t, t+\delta t}$$

So: the probability for the first branching at hardness t is the product of the non-emission probability $\Delta(t_1, t)$ in all hardness intervals between t_1 and t, times the emission probability at hardness t.

(more or less) obvious consequences:

- The total branching probability plus the no-branching probability is 1; mathematically

$$\int_{t_0}^{t_1} dP(t_1, t') = \int_{t_0}^{t_1} d\Delta_i(t_1, t') = 1 - \Delta_i(t_1, t_0)$$

- The Sudakov form factor $\Delta_i(t_1, t)$ is the no-branching probability from scale t_1 down to the scale t.

- The branching probability is independent of what happens next (because the total probability of what happens next is 1).

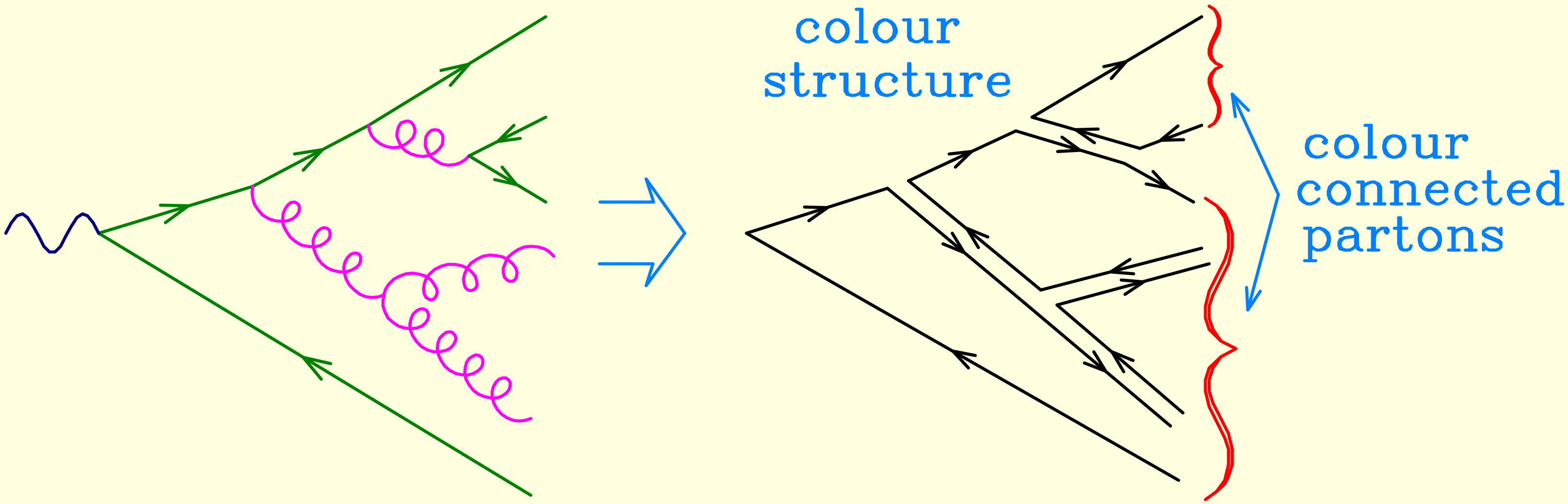
This property is often called unitarity of the shower. It is a consequence of the Kinoshita-Lee-Nauenberg theorem: collinear divergence must cancel in the inclusive cross section.

COLOUR AND HADRONIZATION

SMC's assign colour labels to partons.

Only colour connections are recorded (as in large N limit).

Initial colour assigned according to hard cross section.



Colour assignements are used in the hadronization model.

Most popular models: Lund String Model, Cluster Model.

In all models, color singlect structures are formed out of colour connected partons, and are decayed into hadrons preserving energy and momentum.

Implementation

- Origin: Fox+Wolfram (1980)
- COJETS Odorico (1984)
- ISAJET Page+Protopopescu (1986)
- FIELDAJET Field (1986)
- JETSET Sjöstrand (1986)
- PYTHIA Bengtsson+Sjöstrand (1987), Skands+Sjöstrand
  PYTHIA 8 Mrenna+Skands+Sjöstrand (2007)
- Ariadne Lönnblad (1991)
- HERWIG Marchesini+Webber (1988)
  Marchesini+Webber+Abbiendi+Knowles+Seymour+Stanco (1992)
  HERWIG++ Bahr+Gieseke+Gigg+Grellscheid+Hamilton+Platzer +Richardson+Seymour+Tully (2003)
- SHERPA Gleisberg+Hoche+Krauss+Schalicke+Schumann+Winter (2004)

Accuracy

	Collinear	Soft-Collinear	Soft-large N_c	Soft
PYTHIA	Leading	Partial	No	No
HERWIG	Leading	Leading	No	No
ARIADNE	Partial	Partial	Leading	No
PYTHIA 6.4	Partial	Partial	Leading	No
SHERPA	Leading	Partial	No	No

One can realistically aim at:

Leading Collinear, Leading double log, Leading soft in large N_c limit

(Soft effects for finite N_c require matrix exponentiation in the Sudakov FF)

Not much progress in shower accuracy since the 80's.

New developements

- Interfacing ME (Matrix-Elements) generators with Parton Showers (CKKW matching (Catani, Krauss, Küen, Webber), MLM matching)
- Interfacing NLO calculations to Parton Showers (MC@NLO (Frixione, Webber), POWHEG (PN))

Several other approaches have appeared:

- Kramer, Mrenna, Soper ($e^+e^- \to 3\,\mathrm{partons}$)
- Shower by antenna factorization (Frederix,Giele,Kosower,Skands) (toy implementation for $H \to gg$)
- Shower by Catani-Seymour dipole factorization (Schumann)
- Shower with quantum interference (Nagy,Soper)
- Shower by Soft Collinear Effective Theory (Bauer,Schwartz)

Until now, complete results for hadron colliders only from MC@NLO and POWHEG

NLO+Shower

LO-ME good for shapes; uncertain absolute normalizations.

$$\alpha_s^n(2\mu) \approx \alpha_s^n(\mu)(1 - b_0\alpha_s(\mu)\log(4))^n \approx \alpha_s(\mu)(1 - n\alpha_s(\mu))$$

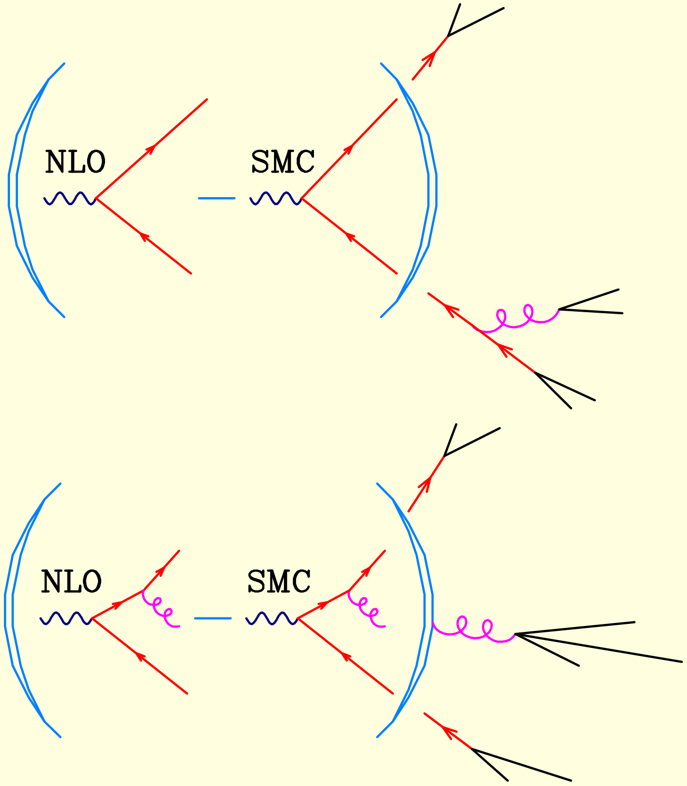
For $\mu = 100\,\text{GeV}$, $\alpha_s = 0.12$;
Normalization uncertainty:

$W+1J$	$W+2J$	$W+3J$
$\pm 12\%$	$\pm 24\%$	$\pm 36\%$

To improve on this, need to go to NLO

- Positive experience with NLO calculations at LEP, HERA, Tevatron (we TRUST perturbative QCD after LEP!)
- NLO results are cumbersome to use: typically made up of an n body (Born+Virtual+Soft and Collinear remnants) and $n + 1$ body (real emission) terms, both divergent (finite only when summed up).
- Merging NLO with shower: a natural extension of present approaches

MC@NLO (2002, Frixione+Webber)



Add difference between exact NLO and approximate (MC) NLO.

- Must use MC kinematics
- Difference should be regular (if the MC is OK)
- Difference may be negative

Several collider processes already there:
Vector Bosons, Vector Bosons pairs,
Higgs, Single Top.
Heavy Quarks

POWHEG

Positive Weight Hardest Emission Generator

Method to generate the hardest emission first, with NLO accuracy, and independently of the SMC (P.N. 2004).

- SMC independent; no need of SMC expert; same calculation can be interfaced to several SMC programs with no extra effort
- SMC inaccuracies only affect next-to-hardest emissions; no matching problems
- As the name says, it generates events with positive weight

How it works (roughly)

In words: works like a standard Shower MC for the hardest radiation, with care to maintain higher accuracy.

In a standard MC, the hardest radiation cross section is

$$d\sigma = d\Phi_n\, B(\Phi_n) \left(\underbrace{\Delta_{t_I,t_0}}_{\text{No radiation}} + \underbrace{\Delta_{t_I,t}\, \frac{\alpha_s(t)}{2\pi} P_{i,jk}(z)\, \frac{dt}{t}\, dz\, \frac{d\phi}{2\pi}}_{\text{radiation}} \right)$$

- t_I is the maximum hardness allowed initially
- $\Delta_{t_I,t}$ in the no-radiation probability with hardness $> t$

SMC algorithm reconstructs from Born kinematics Φ_n and radiation variables t, z, ϕ, the full $n+1$ body phase space Φ_{n+1} (momentum reshuffling)

We say that Φ_n is the underlying Born configuration of Φ_{n+1} according to the mapping defined by the MC algorithm

Steps to go NLO:

$$(\Phi_n, t, z, \phi) \Leftrightarrow \Phi_{n+1} \quad \Longrightarrow \quad (\Phi_n, \Phi_r) \Leftrightarrow \Phi_{n+1},\ d\Phi_{n+1} = d\Phi_n \, d\Phi_r$$

$$B(\Phi_n) \quad \Longrightarrow \quad \bar{B}(\Phi_n) = B(\Phi_n) + \underbrace{\left[\overbrace{V(\Phi_n)}^{\text{INFINITE}} + \overbrace{\int R(\Phi_n, \Phi_r)\, d\Phi_r}^{\text{INFINITE}} \right]}_{\text{FINITE!}}$$

$$\frac{\alpha_s(t)}{2\pi} P_{i,jk}(z) \frac{dt}{t} dz \frac{d\phi}{2\pi} \quad \Longrightarrow \quad \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} d\Phi_{\text{rad}}$$

POWHEG cross section:

$$d\sigma = d\Phi_n \bar{B}(\Phi_n) \left[\Delta_{t_0} + \Delta_t \frac{R(\Phi_{n+1})}{B(\Phi_n)} d\Phi_r \right], \qquad \Delta_t = \exp\left[- \underbrace{\int \theta(t_r - t) \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} d\Phi_r}_{\text{FINITE because of } \theta \text{ function}} \right]$$

with $t_r = k_T(\Phi_n, \Phi_r)$, the transverse momentum for the radiation.
In the collinear limit, k_t^2 must be of the order of t.

How does it work: $d\sigma = d\Phi_n \, \bar{B}(\Phi_n)\left[\Delta_{t_0} + \Delta_t \frac{R(\Phi_{n+1})}{B(\Phi_n)} d\Phi_r\right],$

For small k_T, the factorization theorem yields

$$\frac{R(\Phi_{n+1})}{B(\Phi_n)} d\Phi_{\rm rad} \approx \frac{\alpha_s(t)}{2\pi} P_{i,jk}(z) \frac{dt}{t} dz \frac{d\phi}{2\pi}$$

and

$$\bar{B} \approx B \times (1 + \mathcal{O}(\alpha_s))$$

Thus: all features of SMC's are preserved at small k_T.
For large k_T, $\Delta \to 1$,

$$d\sigma = \bar{B} \times \frac{R}{B} \approx R \times (1 + \mathcal{O}(\alpha_s)),$$

so large k_t accuracy is preserved. Integrating in $d\Phi_r$ at fixed Φ_n

$$\int \delta(\Phi_n - \bar{\Phi}_n) \, d\sigma = \bar{B}(\bar{\Phi}_n)$$

So NLO accuracy is preserved for inclusive quantities.

Example of mapping $\Phi_{n+1} \Leftrightarrow (\Phi_n, \Phi_r)$: Z pair production

Φ_2 variables: choose $M_{\rm zz}$, $Y_{\rm zz}$ and θ, where

- $M_{\rm zz}$: invariant mass of the ZZ pair
- $Y_{\rm zz}$: rapidity of ZZ pair
- θ: go in the (longitudinally) boosted frame where $Y_{\rm zz} = 0$.
  go to the ZZ rest frame with a transverse boost
  In this frame θ is the angle of a Z to the longitudinal direction.

Φ_r variables:

- $x = M_{\rm zz}/s$, (s is the invariant mass of the incoming parton system)
  $x \to 1$ is the soft limit
- y: cosine of the angle of the radiated parton to the beam direction
  in the partonic CM frame.
- ϕ: radiation azimuth.

Few tricks to do it

$$\bar{B}(\Phi) = B(\Phi) + V(\Phi) + \int d\Phi_r \left[R(\Phi,\Phi_r) - C(\Phi,\Phi_r)\right]$$

Seems to need one Φ_r integrations to get weight of each Φ point.

In fact, write

$$\tilde{B}(\Phi,\Phi_r) = N[B(\Phi) + V(\Phi)] + R(\Phi,\Phi_r) - C(\Phi,\Phi_r), \qquad N = \frac{1}{\int d\Phi_r}.$$

so that

$$\bar{B}(\Phi) = \int \tilde{B}(\Phi,\Phi_r) d\Phi_r .$$

Use standard procedures (SPRING-BASES, Kawabata; MINT, P.N.)
to generate unweighted events for $\tilde{B}(\bar{\Phi},\Phi_r) d\Phi_r d\bar{\Phi}$.
discard Φ_r (same as integrating over it!).

Radiation: $$\Delta(\Phi, p_T) = \exp\left[-\int \frac{R(\Phi,\Phi_r)}{B(\Phi)}\theta(k_T(\Phi,\Phi_r) - p_T)d\Phi_r\right],$$

Look for an upper bounding function;

$$\frac{R(\Phi,\Phi_r)}{B(\Phi)} \leq U(\Phi) = N\frac{\alpha_S(k_T)}{(1-x)(1-y^2)}$$

Generate x, y according to

$$\exp\left[-\int U(\Phi)\theta(k_T(\Phi,\Phi_r) - p_T)d\Phi_r\right]$$

accept the event with a probability

$$\frac{R(\Phi,\Phi_r)}{B(\Phi)U(\Phi)}\,.$$

If the event is rejected generate a new one for smaller p_T, and so on
(This procedure reconstructs the exact emission probability).
In the ZZ case, an event is generated with about six calls ro $R(\Phi,\Phi_r)$.

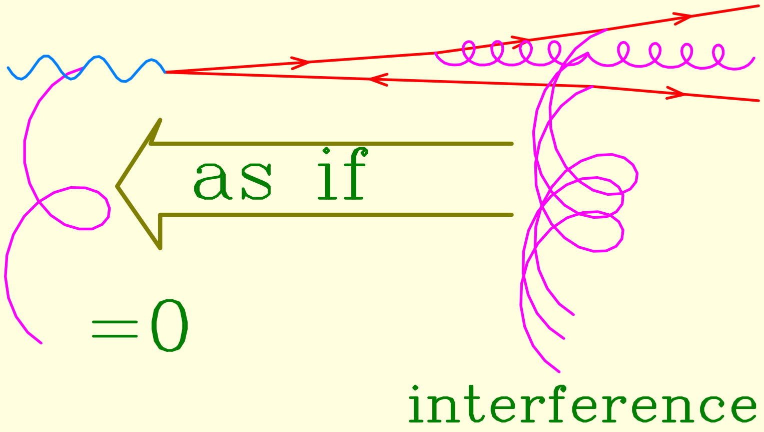
Interfacing to SMC's

For a p_T ordered SMC, nothing else needs to be done.
Use the standard Les Houches Interface for User's Processes (LHI):
put partonic event generated by POWHEG on the LHI;
Run the SMC in the LHI mode.
The LHI provides a facility to pass the p_T of the event to the SMC (SCALUP).
As far as the hardest emission is concerned, POWHEG can reach:

- NLO accuracy of (integrated) shape variables
- Collinear, double-log, soft (large N_c) accuracy of the Sudakov FF.
  (In fact, corrections that exponentiates are obviously OK)

As far as subsequent (less hard) emissions, the output has the accuracy of the SMC one is using.

For angular ordered SMC's (i.e. HERWIG):



Angular ordering accounts for soft gluon interference.

Intensity for photon jets $= 0$

Intensity for gluon jets $= C_A$

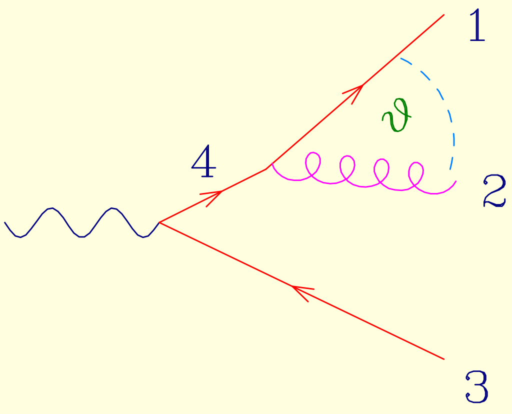
instead of $2C_F + C_A$

Consistent with a boosted jet pair, in the case of a photon jet.
In angular ordered SMC large angle soft emission is generated first.
Hardest emission (i.e. highest p_T) happens later.
Difficult to correct it explicitly.

Recipe for angular ordered showers

- Generate event with harderst emission
- Generate all subsequent emissions with a p_T veto equal to the hardest emission p_T
- Pair up the partons that are nearest in p_T
- Generate an angular ordered shower associated with the paired parton, stopping at the angle of the paired partons:
  Truncated shower, (P.N., 2004)
- Generate all subsequent (vetoed) showers

Example of truncated shower: e^+e^-



Nearby partons: 1,2

Truncated shower: 1,2 pair,

from maximum angle to θ

1 and 2 shower from θ to cutoff

3 showers from maximum to cutoff

The truncated shower reintroduces coherent soft radiation from 1,2 at angles larger than θ (Angular ordered SMC's generate those earlier).

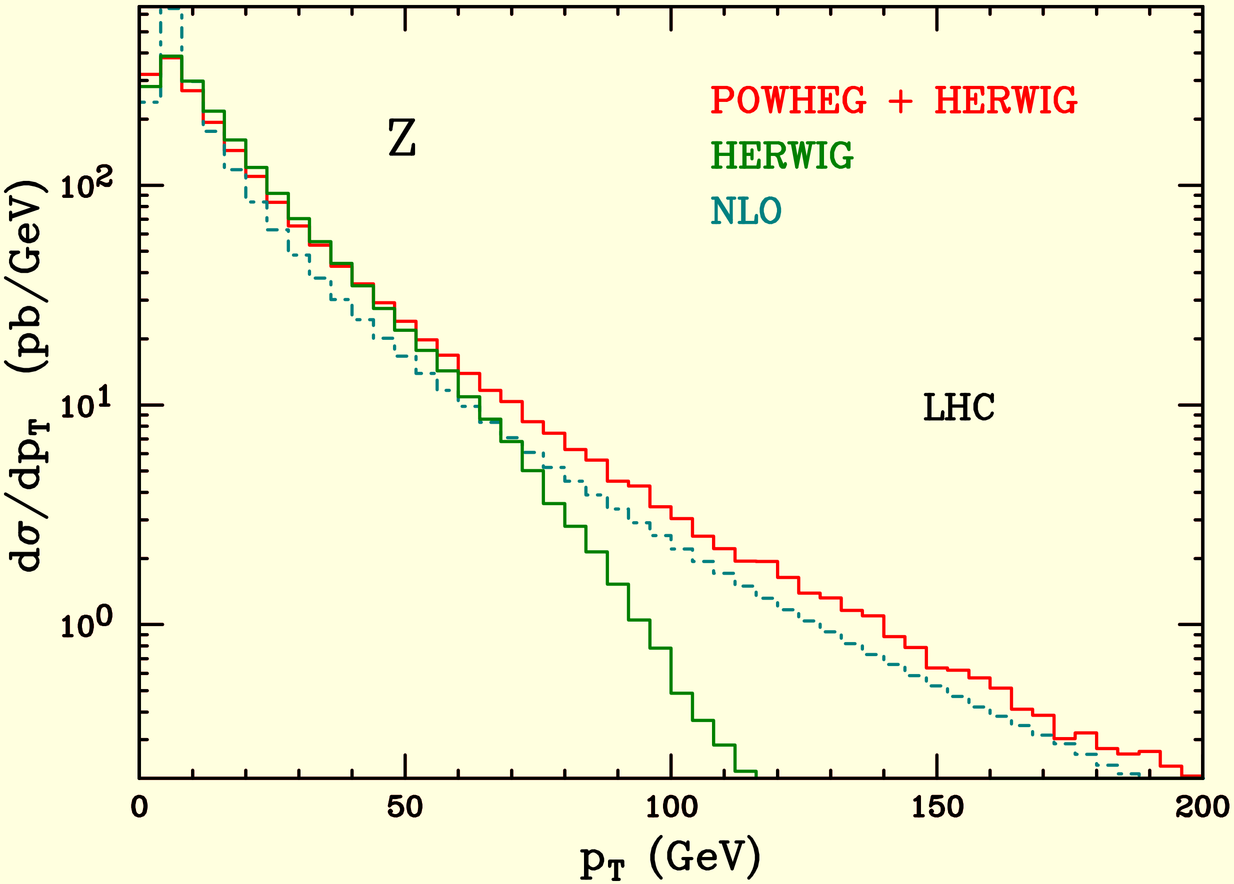
Truncated shower are generally needed in angular ordered MC;
They are not a specific problem of `POWHEG`.
They are now being implemented in `HERWIG++` (as of next release)

Status

Up to now, the following processes have been implemented:

- $hh \to ZZ$ (Ridolfi, P.N., 2006)
- $e^+e^- \to \mathrm{hadrons}$, (Latunde-Dada,Gieseke,Webber, 2006),
  $e^+e^- \to t\bar{t}$, including top decays at NLO (Latunde-Dada,2008),
- $hh \to Q\bar{Q}$ (Frixione, Ridolfi, P.N., 2007)
- $hh \to Z/W$ (Alioli, Oleari, Re, P.N., 2008;)
  (Hamilton,Richardson,Tully, 2008;)
- $hh \to H$ (gluon fusion) (Alioli, Oleari, Re, P.N., 2008; Herwig++)
- $hh \to t + X$ (single top) NEW! (Alioli, Oleari, Re, P.N., 2009)
- Truncated showers have being studied in the $e^+e^- \to \mathrm{hadrons}$ work (Latunde-Dada,Gieseke,Webber, 2006), and are being included in the `HERWIG++` framework (Bahr,Gieseke,Gigg,Grellscheid,Hamilton, Latunde-Dada,Platzer,Richardson,Seymour,Sherstnev,Webber)

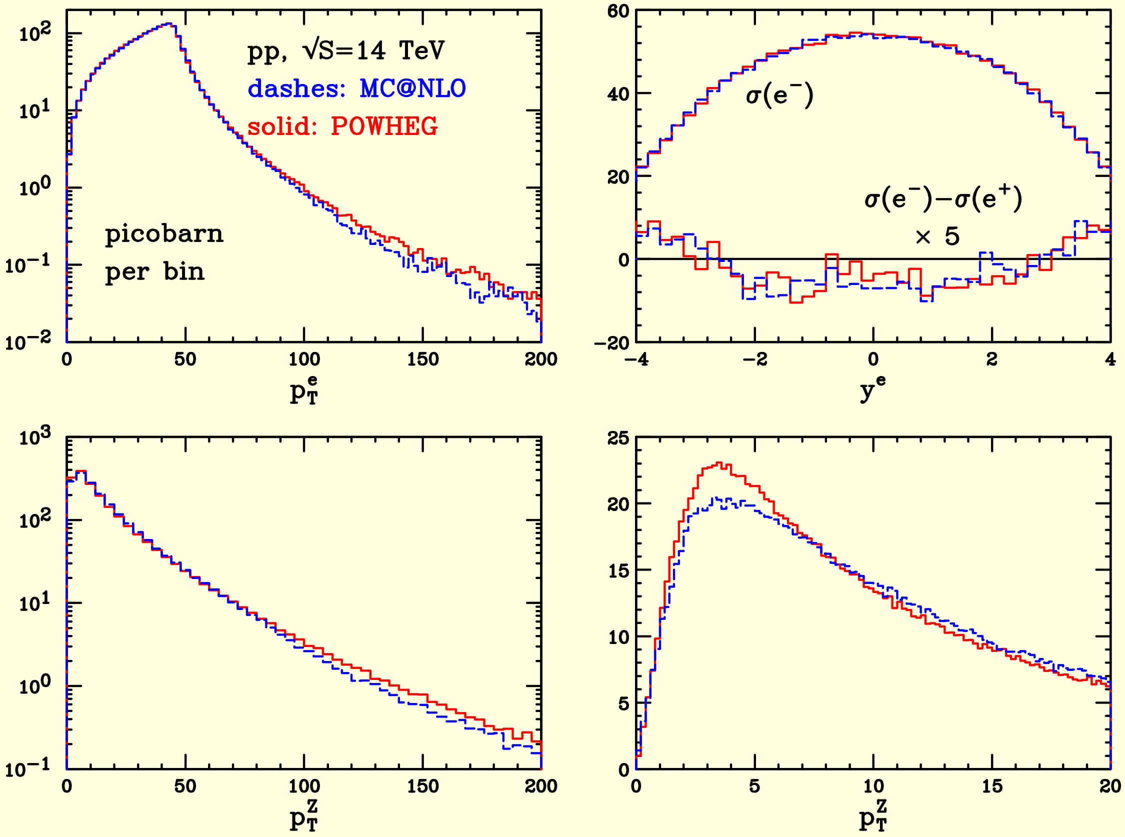
Examples: Z production: POWHEG vs. HERWIG vs. NLO



HERWIG alone fails ar large p_T; NLO alone fails at small p_T;

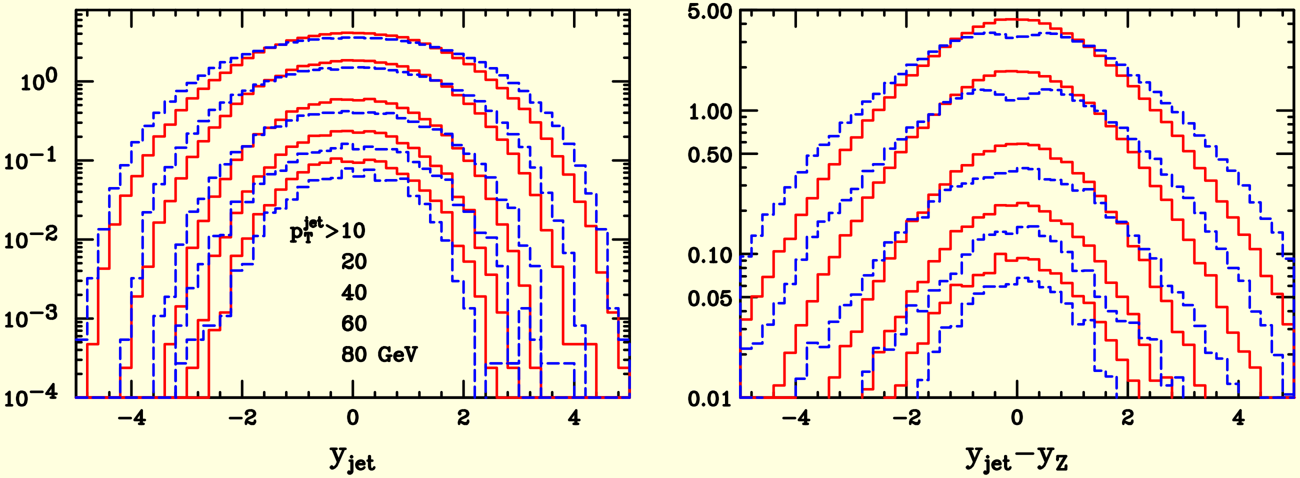
POWHEG works in both regions;

Z production: POWHEG+HERWIG vs. MC@NLO



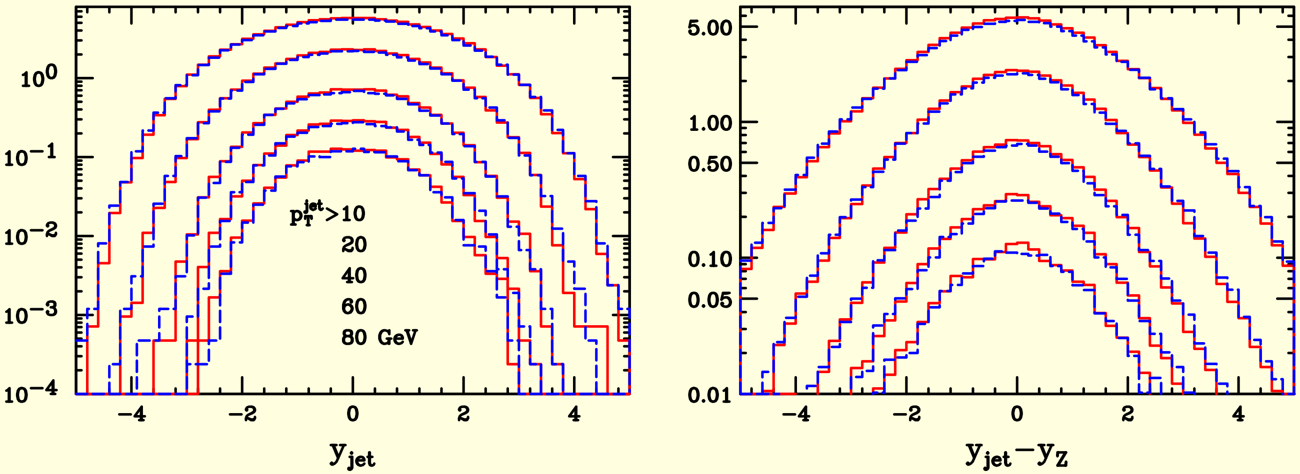
Small differences in high and low p_T region

Z production: rapidity of hardest jet (TEVATRON)



POWHEG+HERWIG

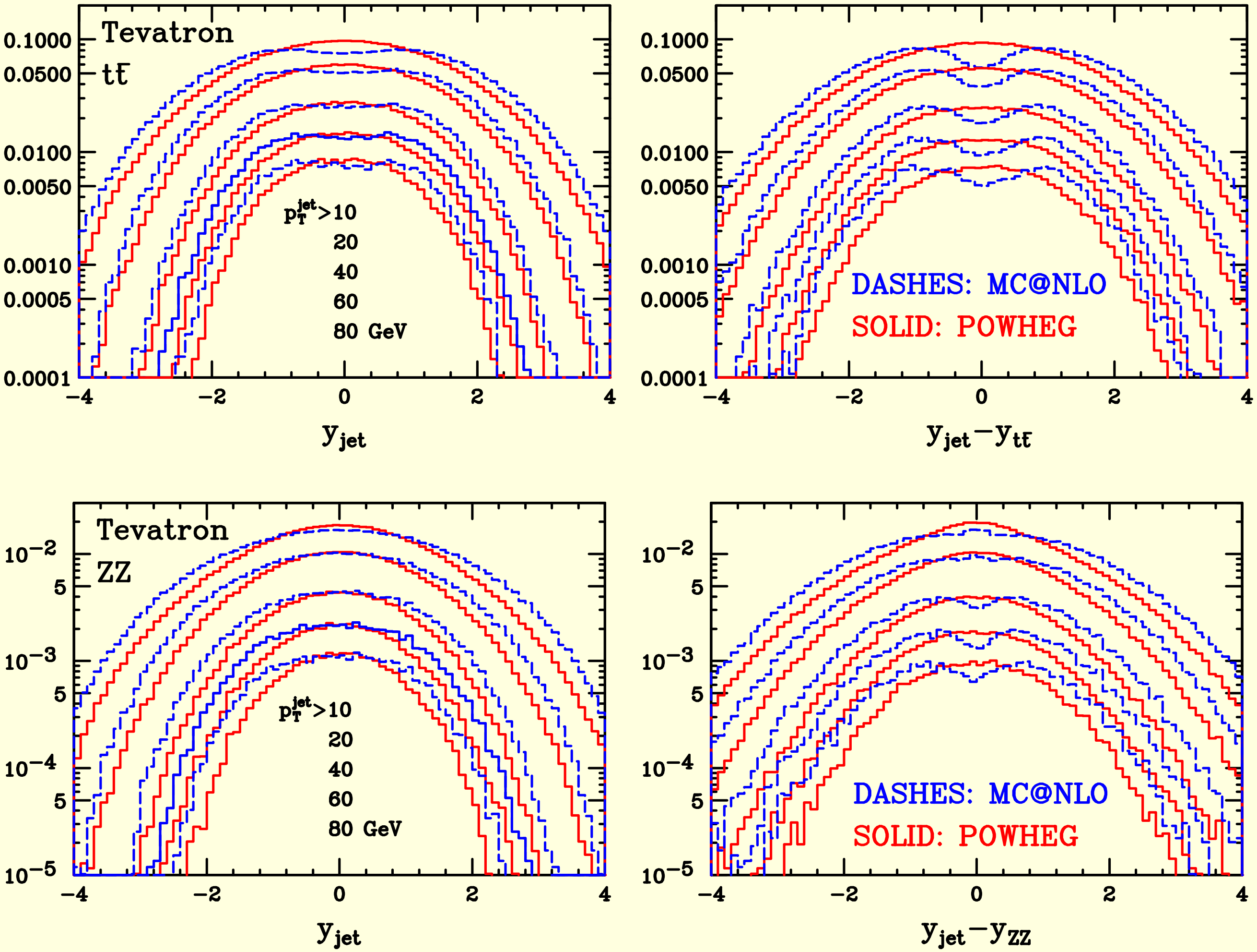
MC@NLO



POWHEG+PYTHIA

PYTHIA

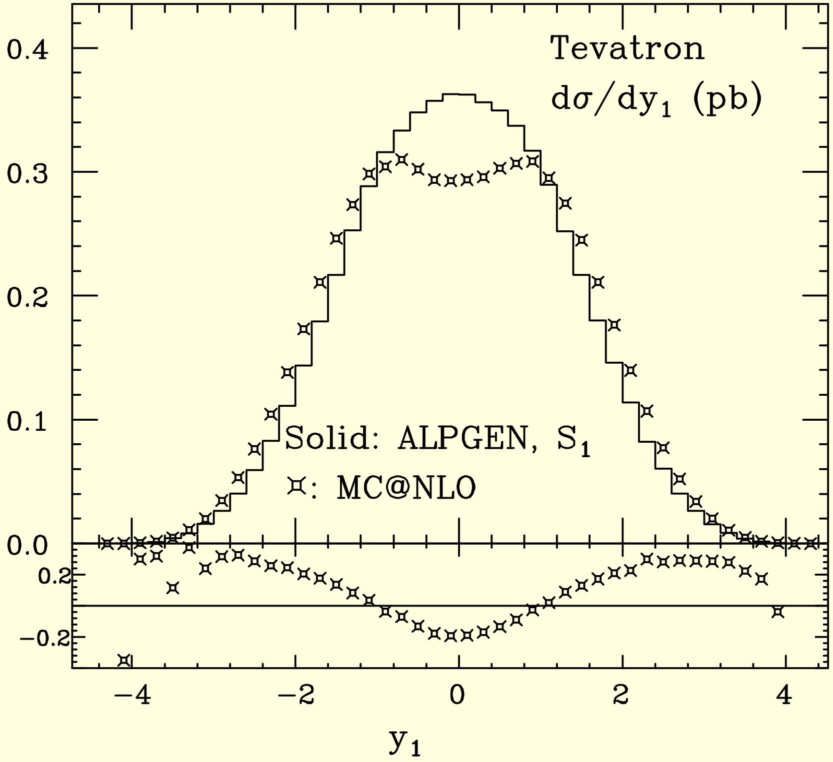
Dip in central region in MC@NLO also in $t\bar{t}$ and ZZ

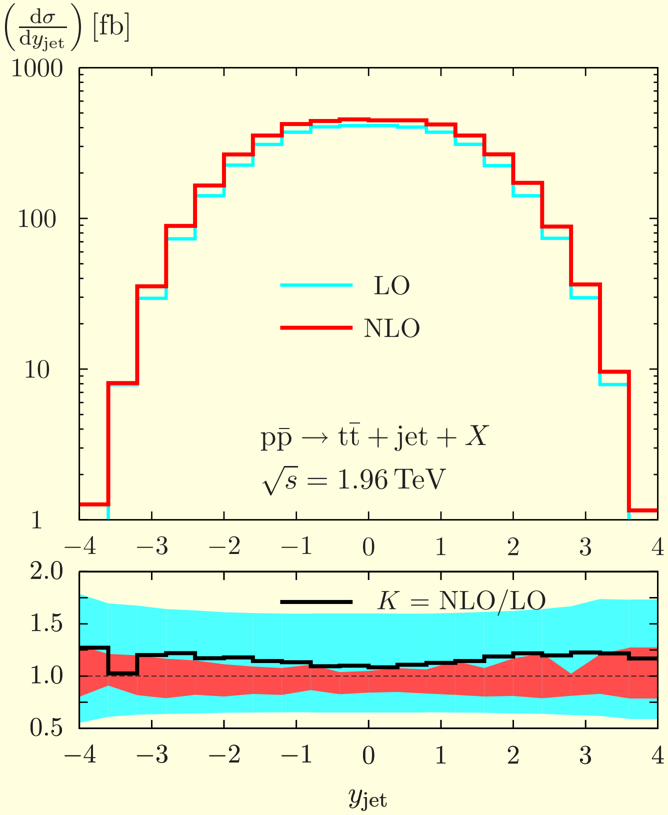


POWHEG+HERWIG
MC@NLO

POWHEG+HERWIG
MC@NLO

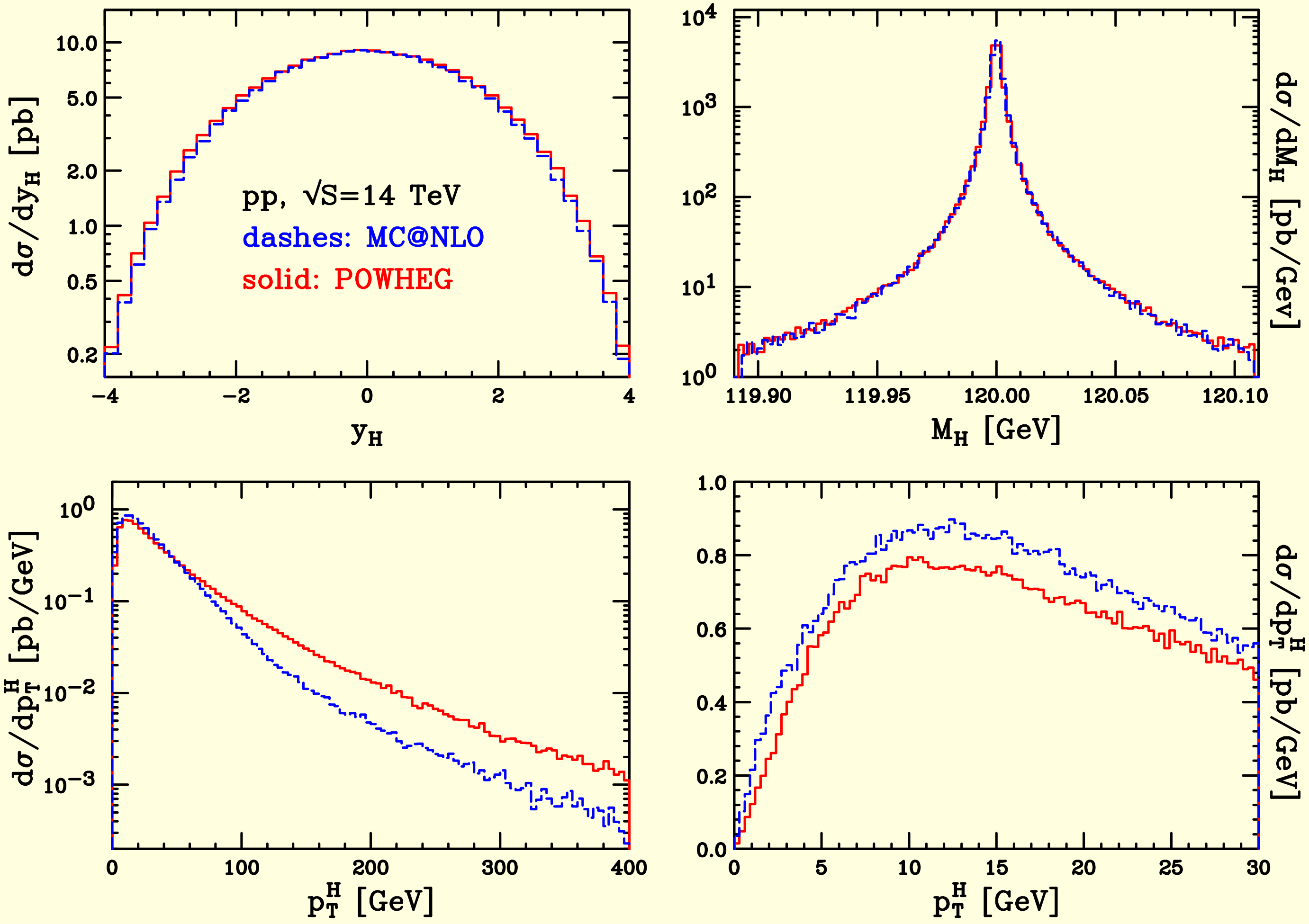
ALPGEN and $t\bar{t}$ + jet at NLO vs. MC@NLO



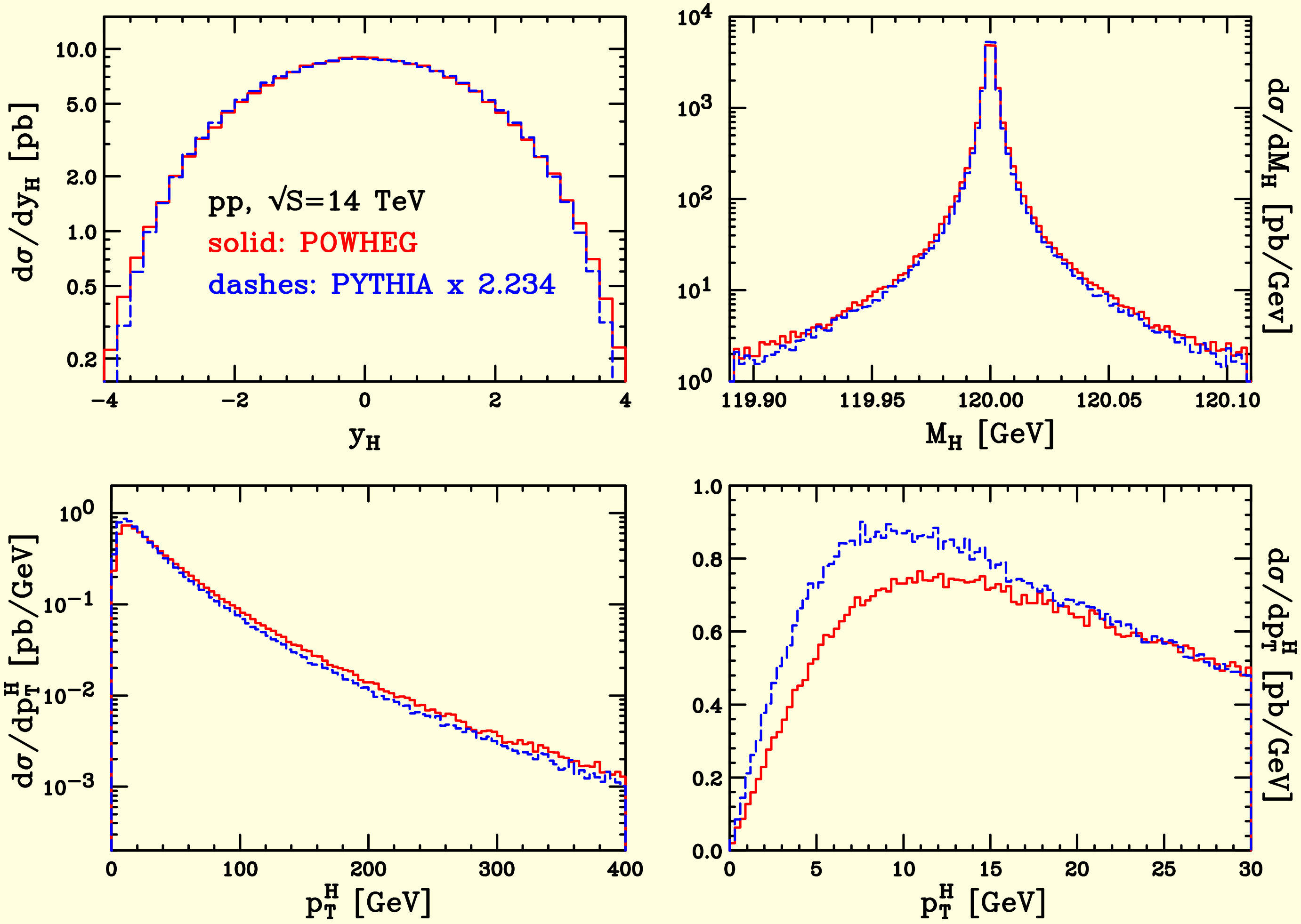


POWHEG distribution as in ALPGEN (Mangano,Moretti,Piccinini,Treccani,Nov.06) and in $t\bar{t}$ + jet at NLO (Dittmaier, Uwer, Weinzierl) : no dip present.

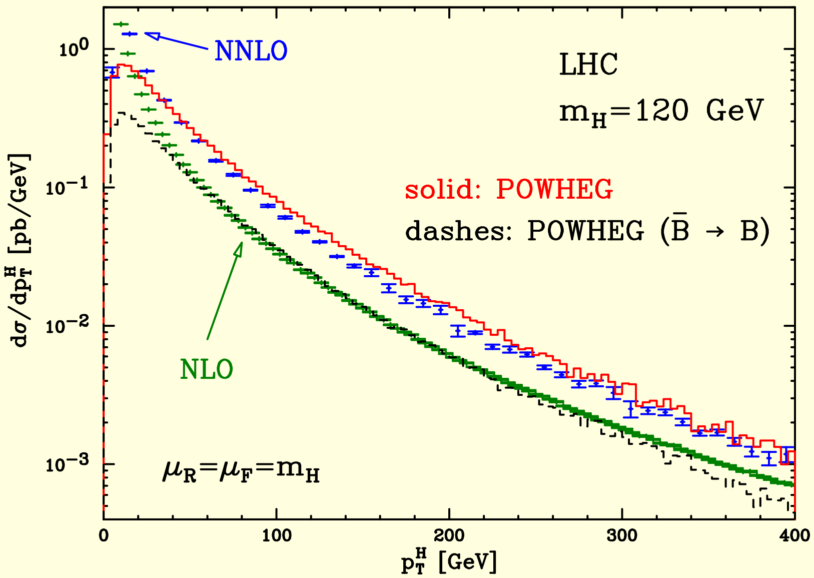
Higgs boson via gluon fusion at LHC

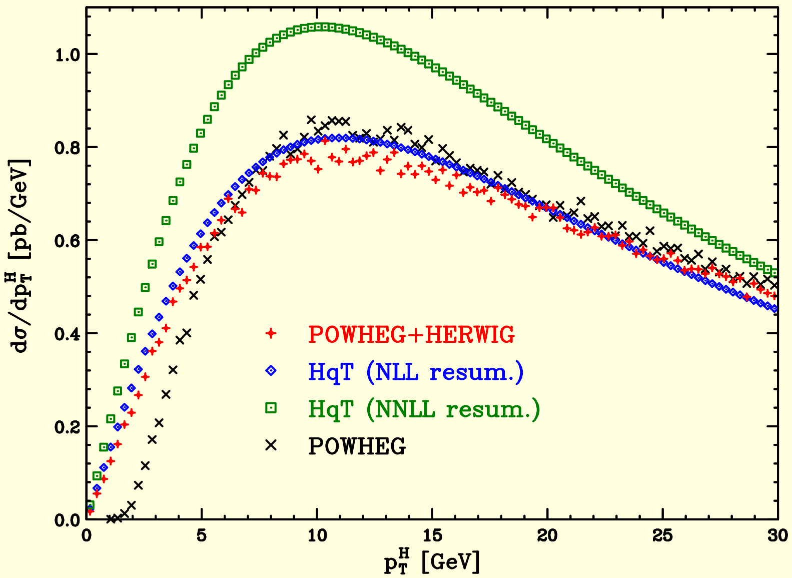


Higgs boson via gluon fusion at LHC



POWHEG vs. NNLO vs. NNLL



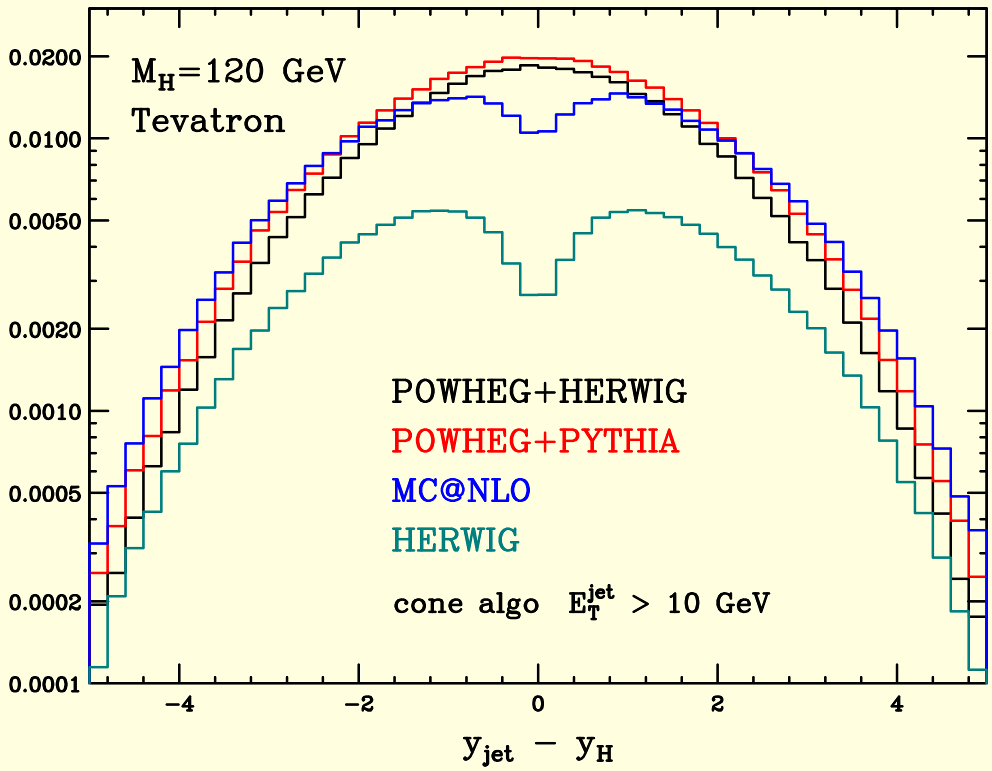


$$d\sigma = \bar{B}(\Phi_n)\, d\Phi_n \left\{ \Delta(\Phi_n, p_T^{\min}) + \Delta(\Phi_n, p_T) \frac{R(\Phi_n, \Phi_{\rm rad})}{B(\Phi_n)} d\Phi_r \right\}$$

$$\approx \frac{\bar{B}(\Phi_n)}{B(\Phi_n)} R(\Phi_n, \Phi_r)\, d\Phi_r = \{1 + \mathcal{O}(\alpha_s)\} R(\Phi_{n+1})\, d\Phi_{n+1}$$

Better agreement with NNLO this way.

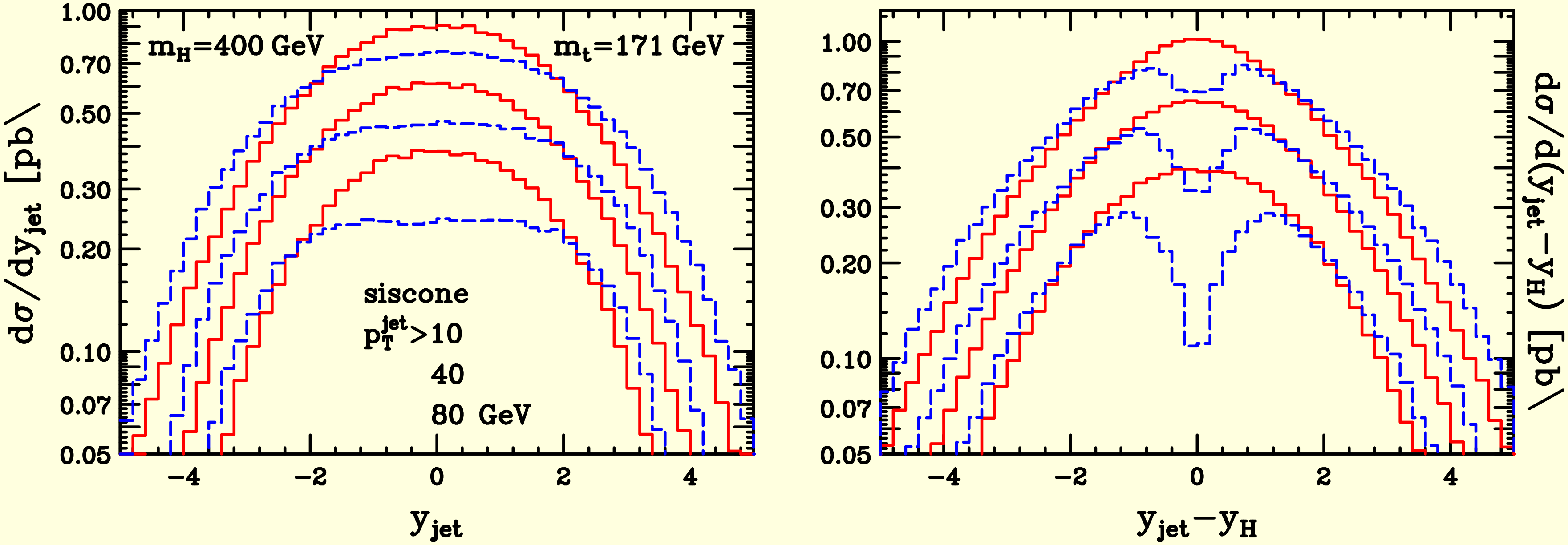
Jet rapidity in h production



Dip in MC@NLO inerithed from even deeper dip in HERWIG

(MC@NLO tries to fill dead regions in HERWIG, a mismatch remains).

Gets worse for larger E_T cuts:



Questions:

Why MC@NLO has a dip in the hardest jet rapidity?

Why POWHEG has no dip? Is that because of the hardest p_T spectrum?

Hard p_T spectrum in POWHEG

We understand the cause; we keep it because yields results closer to NNLO;
we have enough flexibility to get rid of it, if we want!
Go back to the POWHEG cross section:

$$d\sigma = \bar{B}(\Phi_n)\left[\Delta_{t_0} + \Delta_t \frac{R(\Phi_{n+1})}{B(\Phi_n)} d\Phi_r\right], \qquad \Delta_t = \exp\left[-\int \theta(t_r - t)\frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} d\Phi_r\right]$$

Break $R = R_s + R_f$, with R_f finite in collinear and soft limit, define

$$d\sigma' = \bar{B}^s(\Phi_n)\left[\Delta^s_{t_0} + \Delta^s_t \frac{R_s(\Phi_{n+1})}{B(\Phi_n)} d\Phi_r\right] + R_f(\Phi_{n+1}) d\Phi_{n+1}$$

with:

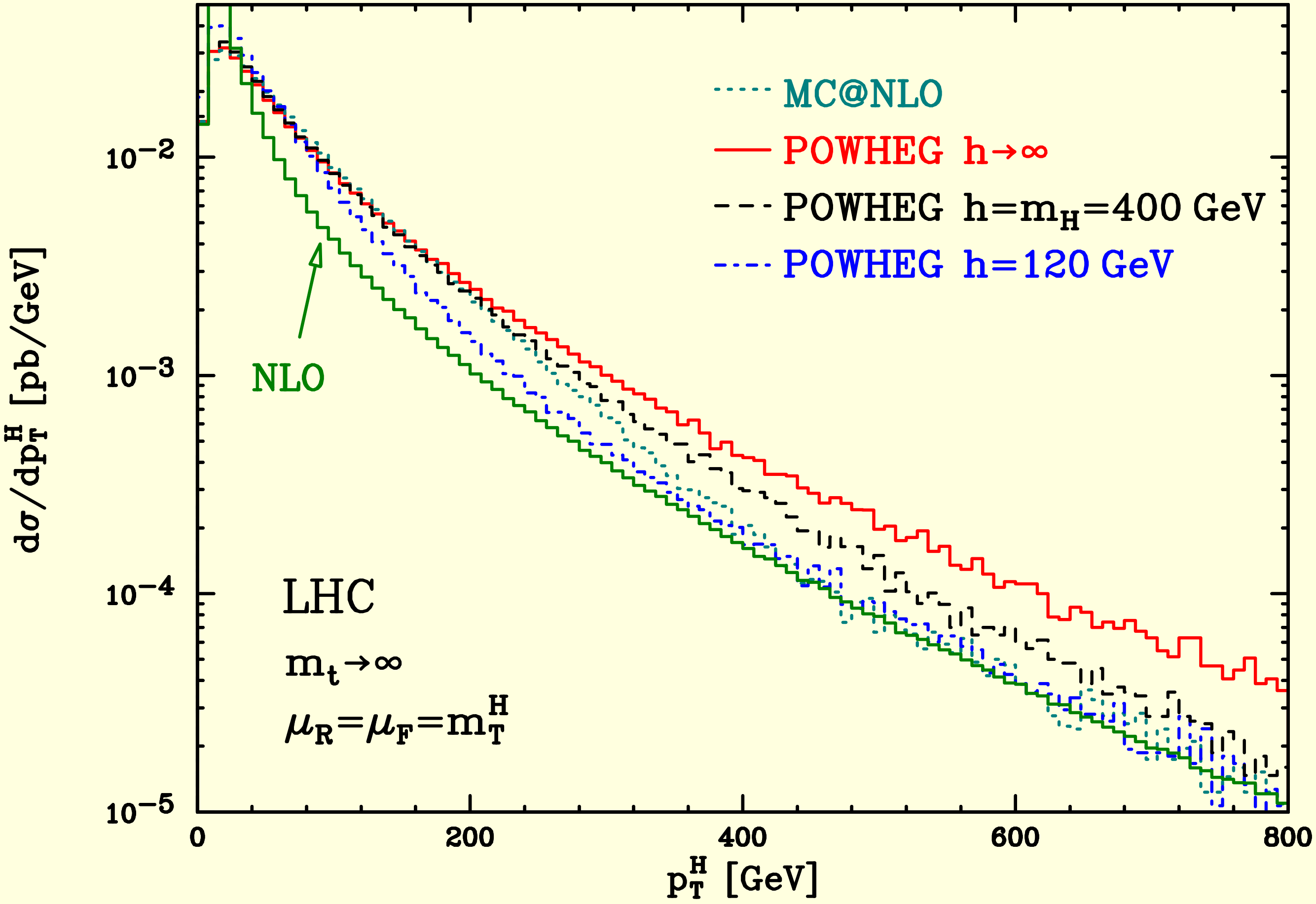
$$\Delta^s_t = \exp\left[-\int \theta(t_r - t)\frac{R^s(\Phi_n, \Phi_r)}{B(\Phi_n)} d\Phi_r\right].$$

Easy to prove that: $d\sigma'$ is equivalent to $d\sigma$.
In other words, the part of the real cross section that is treated with the Shower technique can be varied.

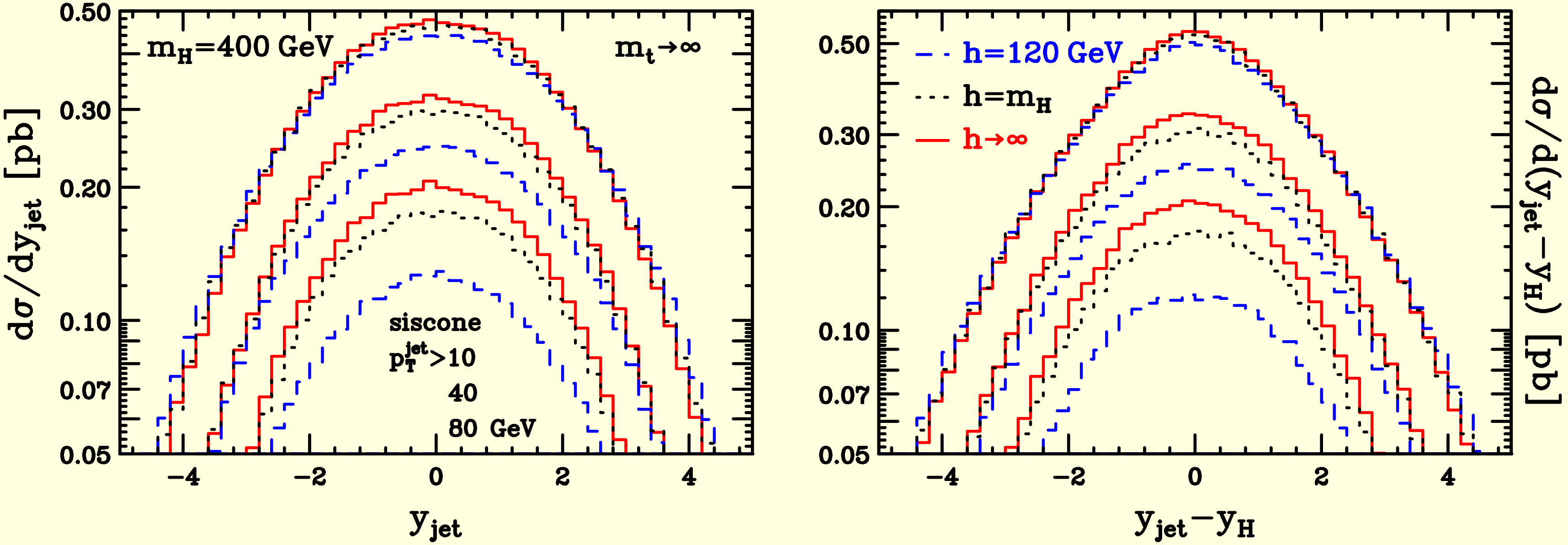
$$R_s = R\frac{h^2}{k_T^2 + h^2}$$

$$R_f = R\frac{k_T^2}{k_T^2 + h^2}$$

Agrees with NLO at high p_T.



No new features (dips and the like) arise in the other distributions:



So: high k_T cross section and dips are unrelated issues.

Why is there a dip in MC@NLO?

Write the MC@NLO hardest jet cross section in the POWHEG language;
Hardest emission (P.N., 2004) can be written as

$$d\sigma = \underbrace{\bar{B}^{\rm HW}(\Phi_n) d\Phi_n}_{S\ \text{event}} \left[\underbrace{\Delta^{\rm HW}_{t_0} + \Delta^{\rm HW}_{t} \frac{R^{\rm HW}(\Phi_{n+1})}{B(\Phi_n)} d\Phi^{\rm HW}_r}_{\text{HERWIG shower}} \right] + \left[\underbrace{R(\Phi_{n+1}) - R^{\rm HW}(\Phi_{n+1})}_{H\ \text{event}} \right] d\Phi_{n+1}$$

$$\bar{B}^{\rm HW}(\Phi_n) = B(\Phi_n) + \underbrace{\left[\underbrace{V(\Phi_n)}_{\text{infinite}} + \underbrace{\int R^{\rm HW}(\Phi_n, \Phi_r)\, d\Phi_r}_{\text{infinite}} \right]}_{\text{finite}}$$

(Imagine that soft and collinear singularities in $R^{\rm HW}$ are regulated as in V!).
Like POWHEG with $R_s = R^{\rm HW}$! But now $R_f = R - R^{\rm HW}$ can be negative!
This formula illustrates why MC@NLO and POWHEG are equivalent at NLO!
But differences can arise at NNLO ...

For large k_T:

$$d\sigma = \left[\frac{\bar{B}^{\rm HW}(\Phi_n)}{B(\Phi_n)} R^{\rm HW}(\Phi_{n+1}) + R(\Phi_{n+1}) - R^{\rm HW}(\Phi_{n+1}) \right] d\Phi_n \, d\Phi_r^{\rm HW}$$

$$= \underbrace{R(\Phi_{n+1}) d\Phi_{n+1}}_{\rm no\ dip} + \underbrace{\left(\frac{\bar{B}^{\rm HW}(\Phi_n)}{B(\Phi_n)} - 1 \right)}_{\mathcal{O}(\alpha_s),\ {\rm but\ large\ for\ Higgs}} \underbrace{R^{\rm HW}(\Phi_{n+1})}_{\rm Pure\ Herwig\ dip} d\Phi_{n+1}$$

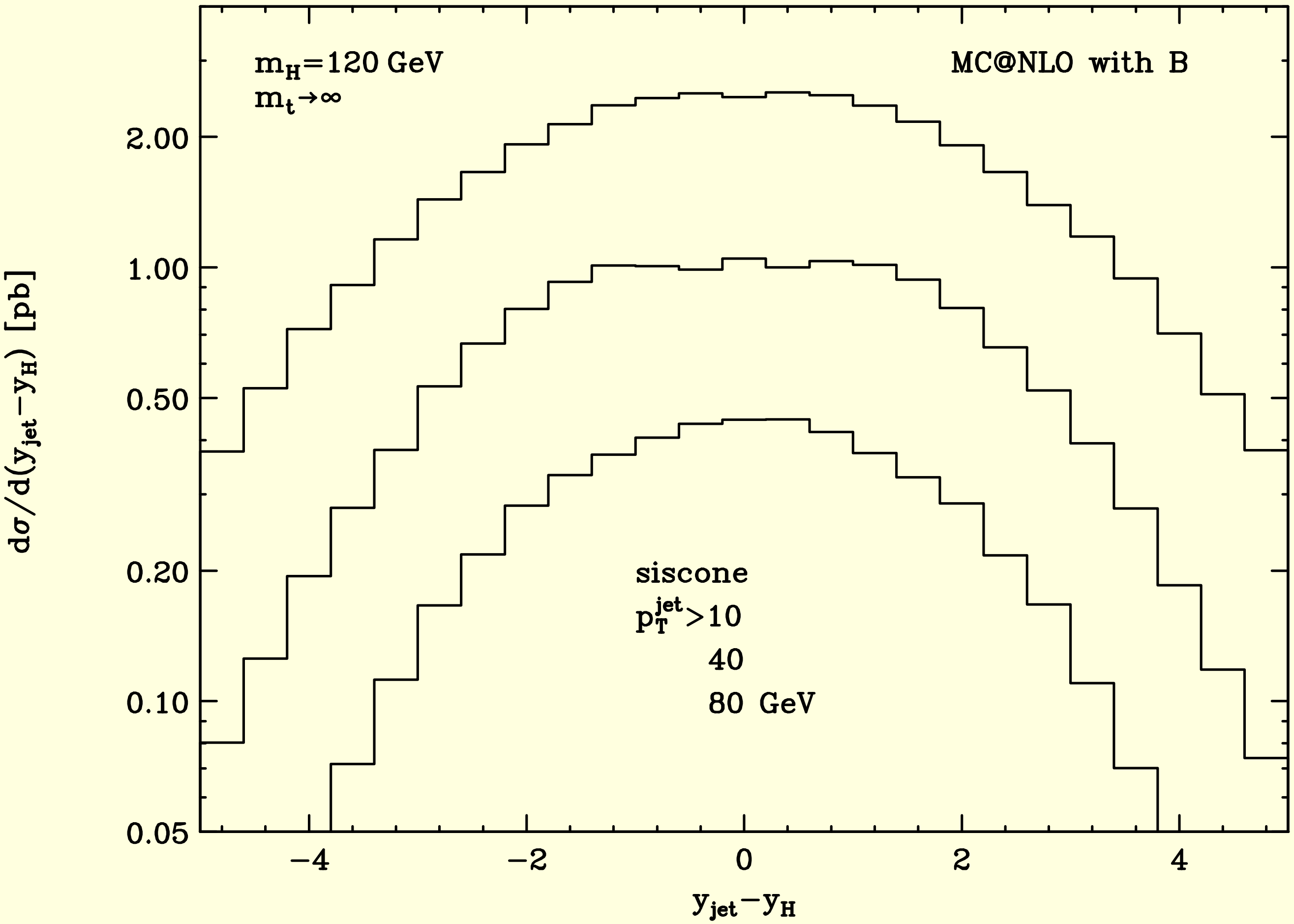
So: a contribution with a dip is added to the exact NLO result;

The contribution is $\mathcal{O}(\alpha_s R)$, i.e. NNLO!

Can we test this hypothesis? Replace $\bar{B}^{\rm HW}(\Phi_n) \Rightarrow B(\Phi_n)$ in MC@NLO!

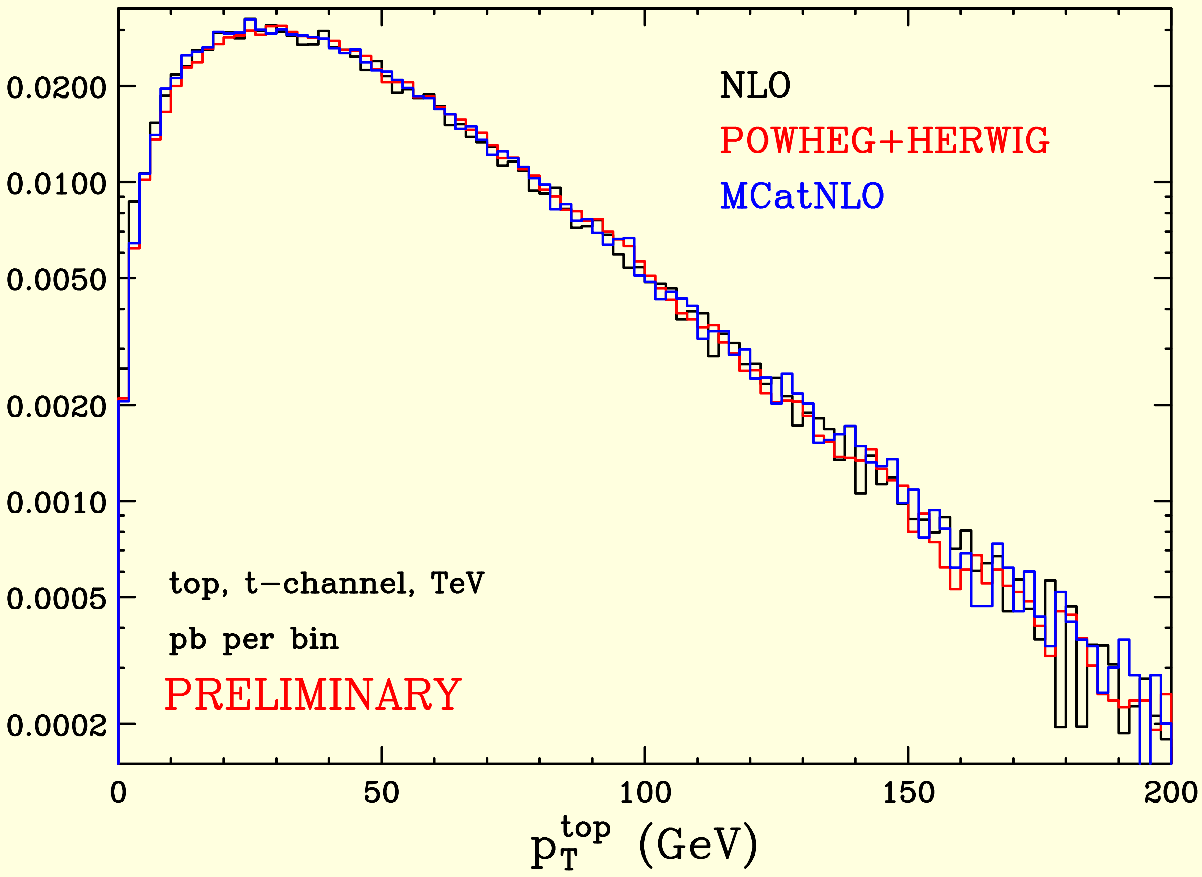
the dip should disappear ...

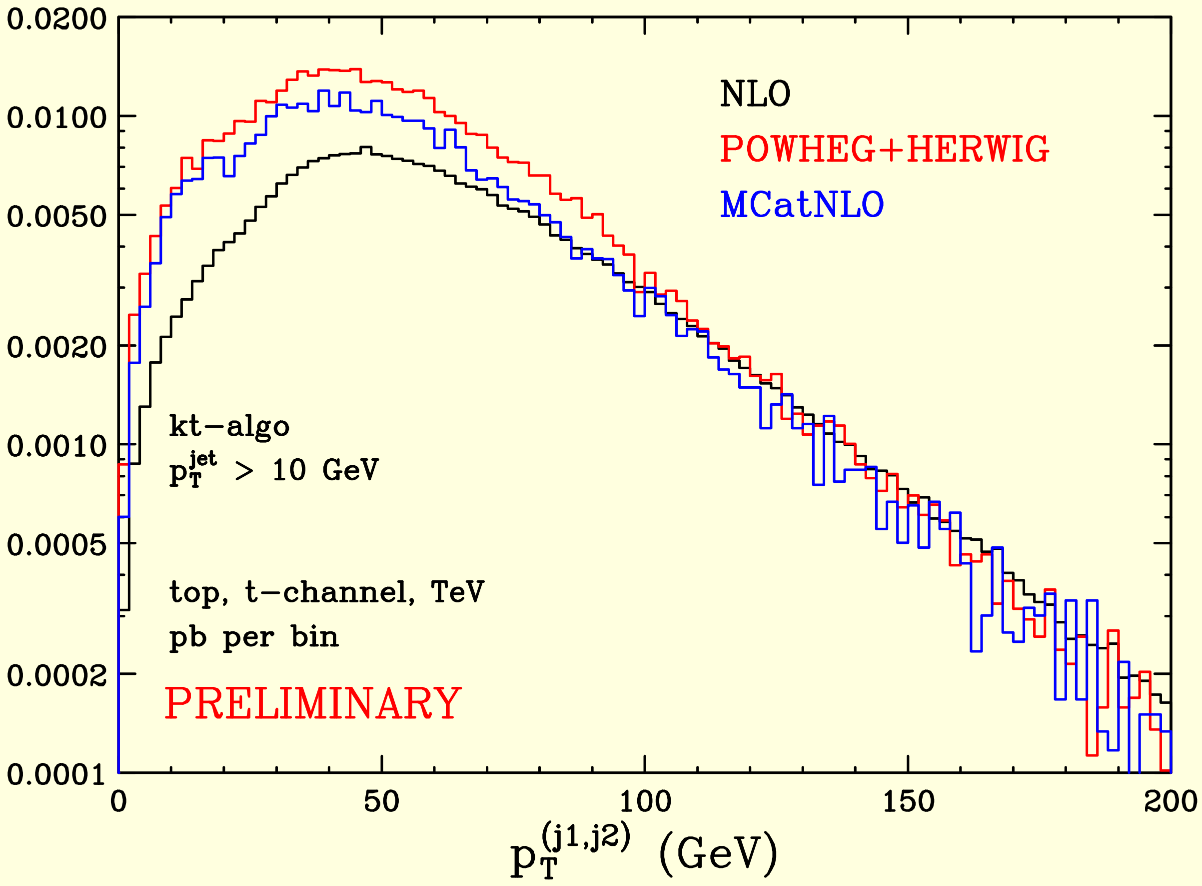
MC@NLO with B^{HW} replaced by B



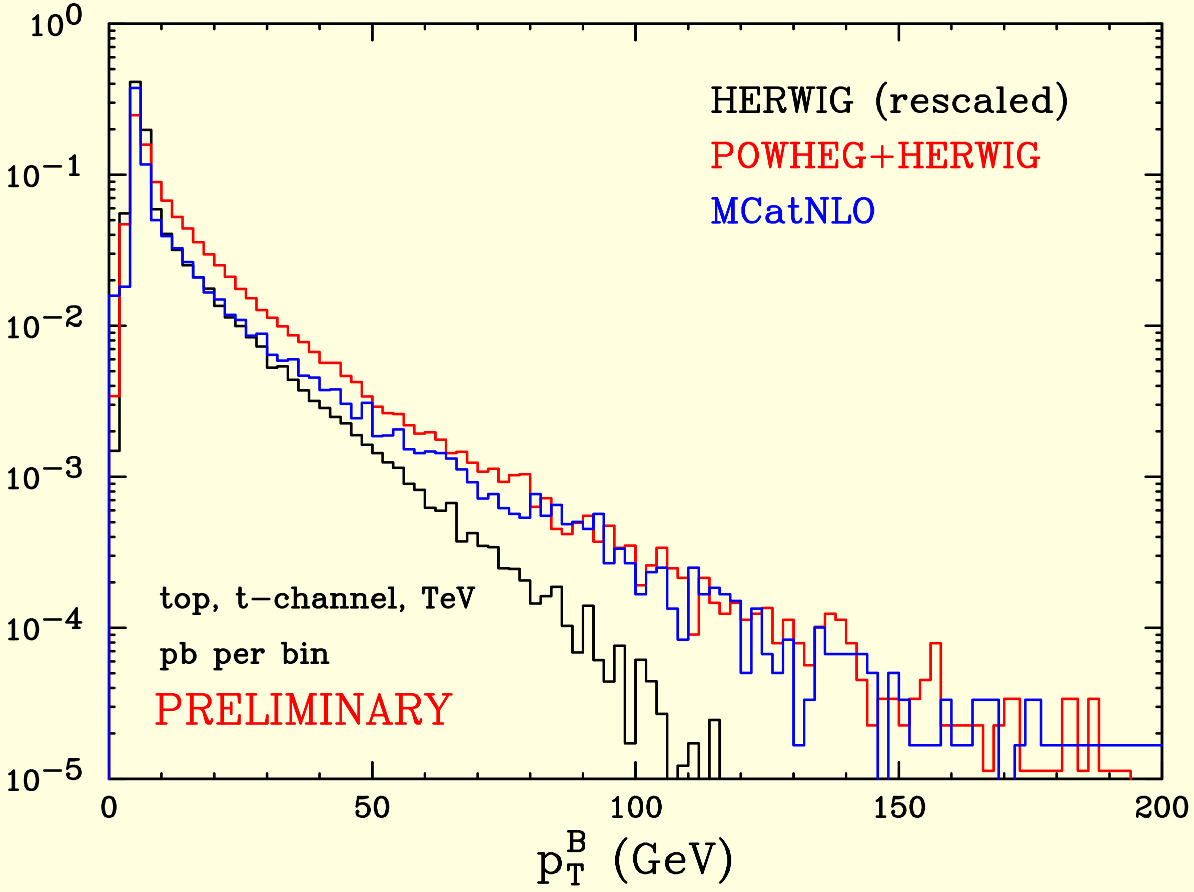
No visible dip is present! (more studies needed to clear the issue ...)

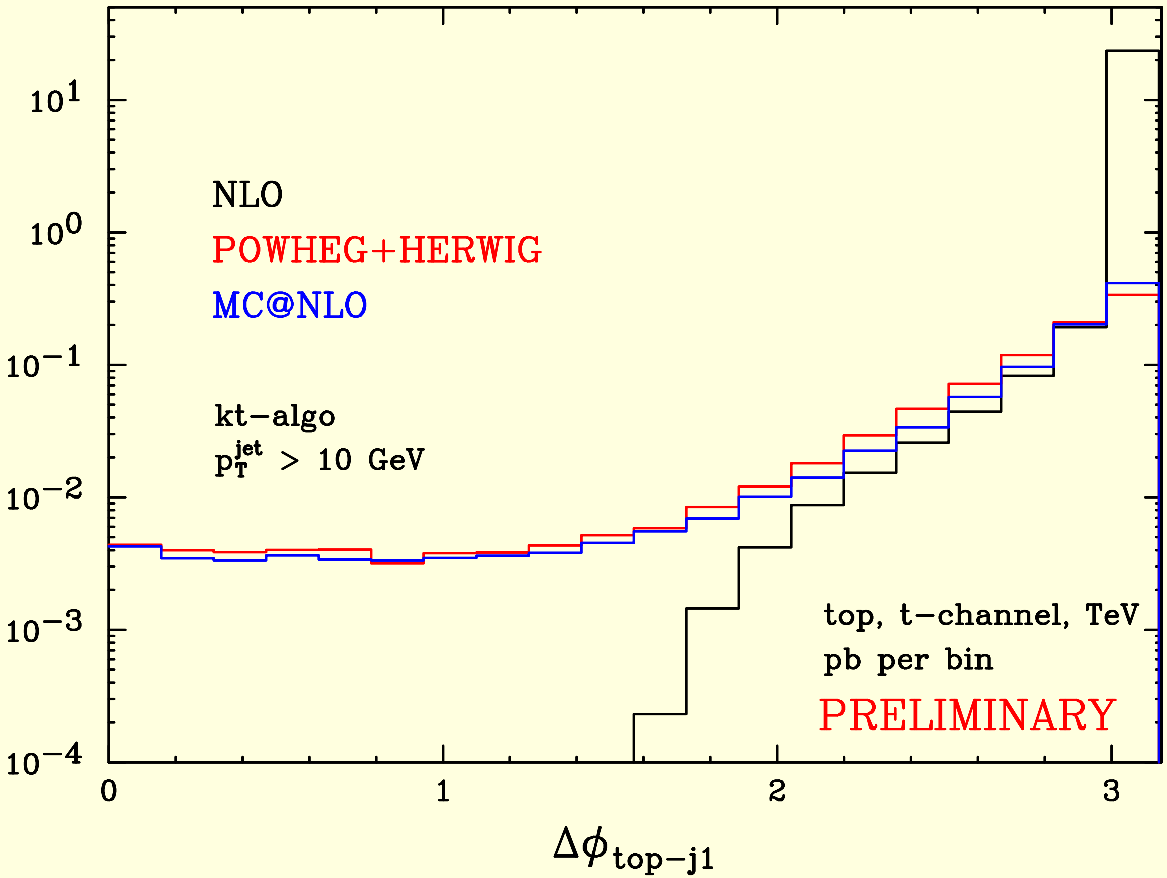
Single Top (PRELIMINARY!)

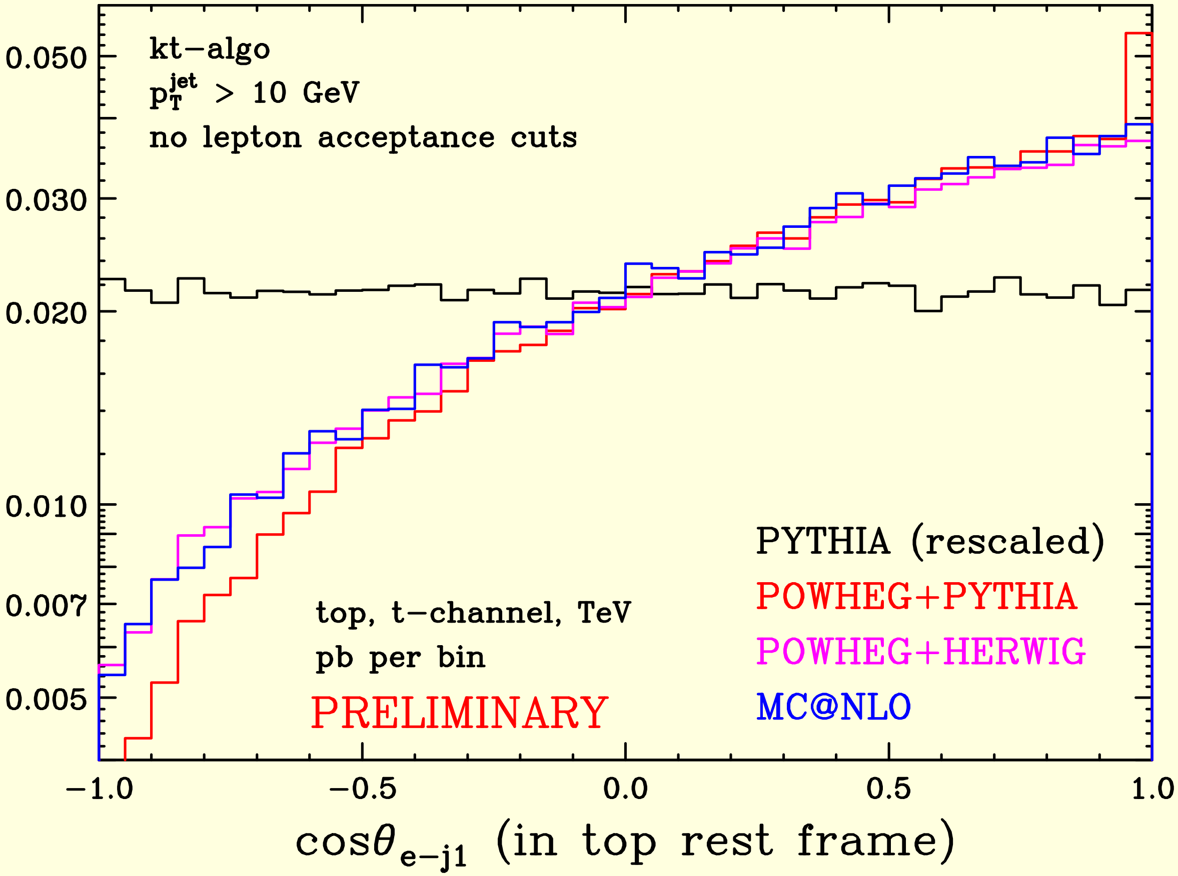


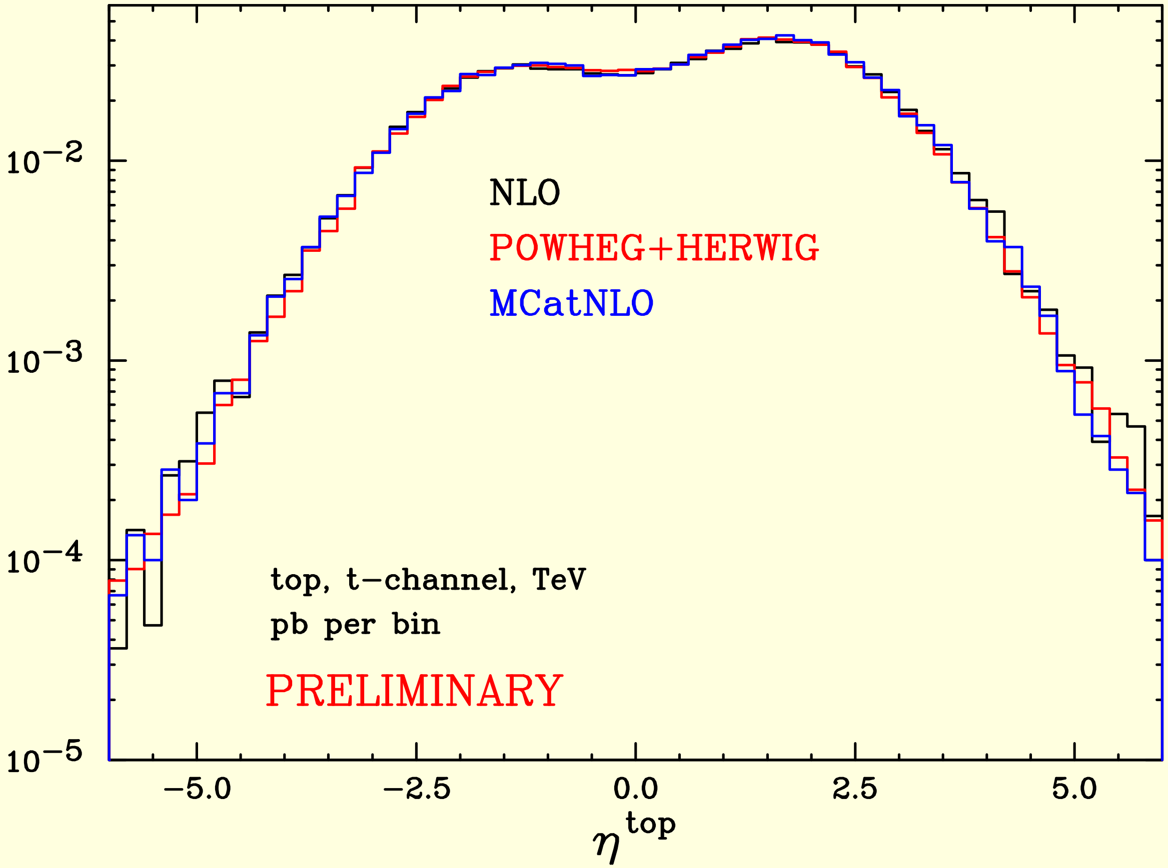


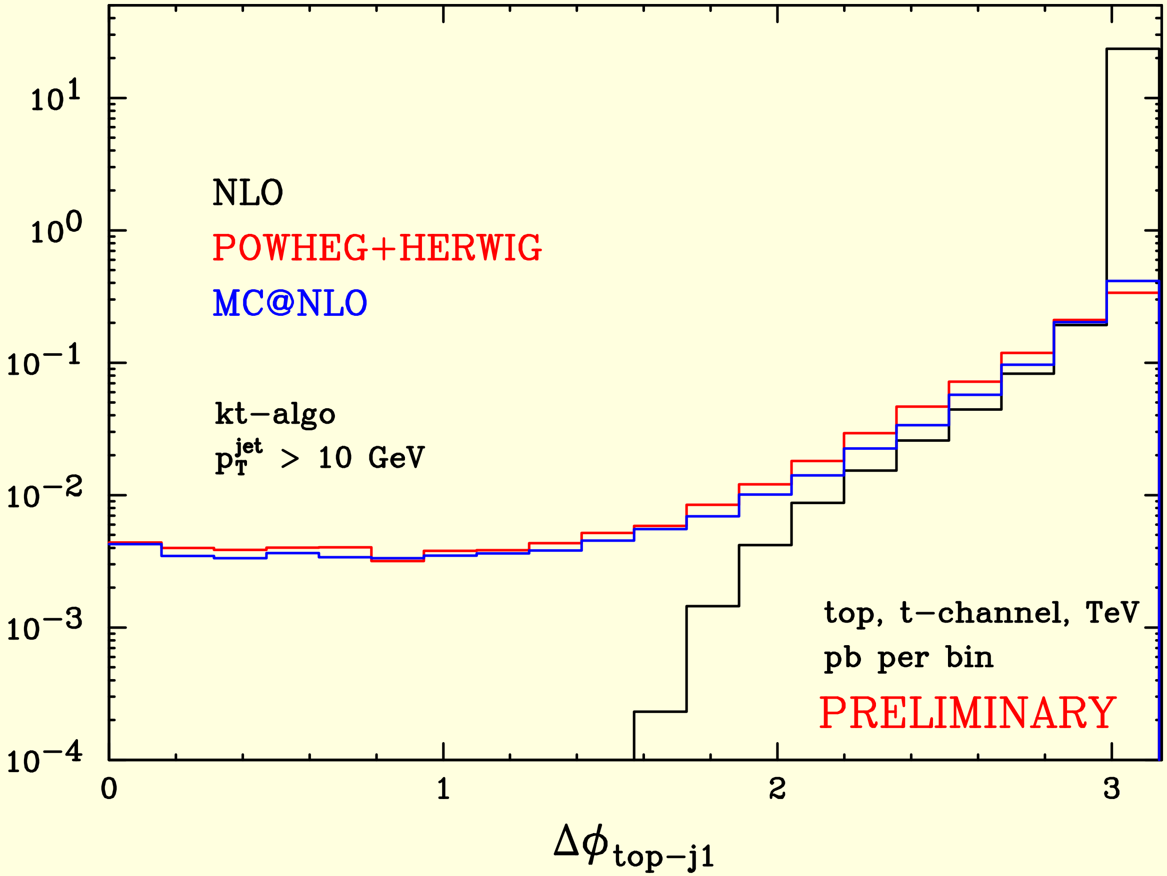
NLO
POWHEG+HERWIG
MCatNLO
kt−algo
p_T^{jet} > 10 GeV
top, t−channel, TeV
pb per bin
PRELIMINARY
0.0200
0.0100
0.0050
0.0020
0.0010
0.0005
0.0002
0.0001
0
50
100
150
200
$p_T^{(j1,j2)}$ (GeV)


HERWIG (rescaled)
POWHEG+HERWIG
MCatNLO
top, t-channel, TeV
pb per bin
PRELIMINARY
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
50
100
150
200
p_T^B (GeV)


NLO
POWHEG+HERWIG
MC@NLO
kt−algo
p_T^{jet} > 10 GeV
top, t−channel, TeV
pb per bin
PRELIMINARY
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
1
2
3
$\Delta\phi_{top-j1}$


kt−algo
p_T^{jet} > 10 GeV
no lepton acceptance cuts
0.050
0.030
0.020
0.010
0.007
0.005
top, t−channel, TeV
pb per bin
PRELIMINARY
PYTHIA (rescaled)
POWHEG+PYTHIA
POWHEG+HERWIG
MC@NLO
−1.0
−0.5
0.0
0.5
1.0
$\cos\theta_{e-j1}$ (in top rest frame)


NLO
POWHEG+HERWIG
MCatNLO
top, t−channel, TeV
pb per bin
PRELIMINARY
10^{-2}
10^{-3}
10^{-4}
10^{-5}
−5.0
−2.5
0.0
2.5
5.0
η^{top}


NLO
POWHEG+HERWIG
MC@NLO
kt−algo
p_T^{jet} > 10 GeV
top, t−channel, TeV
pb per bin
PRELIMINARY
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
1
2
3
$\Delta\phi_{top-j1}$

Prospects

The following processes are being worked on

- Single top production (Alioli Oleari, Re, P.N.): the code is there, studies and comparisons in progress
- $hh \to Z/W + 1\mathrm{jet}$ (Alioli, Oleari, Re, P.N.)

While working on $hh \to Z/W + 1\mathrm{jet}$, we realized that this process is already complex enough, so that a general framework for the implementation of a POWHEG generator for any NLO process can be setup;

Goal

Build a computer code framework, such that, given the Born cross section, the finite part of the virtual corrections, and the real graph cross section, one builds immediately a POWHEG generator.
More precisely, the user must supply:

- The Born phase space
- The lists of Born and Real processes (i.e. $u\,\bar{s} \to W^+ c\,\bar{c}$, etc.)
- The Born squared amplitudes $\mathcal{B} = |\mathcal{M}|^2$, $\mathcal{B}_{ij}$, $\mathcal{B}_{j,\mu_j,\mu_j'}$, for all relevant partonic processes; $\mathcal{B}_{ij}$ is the colour ordered Born amplitude squared, $\mathcal{B}_{j,\mu\nu}$ is the spin correlated amplitude, where j runs over all external gluons in the amplitude. All these amplitudes are common ingredient of an NLO calculation.
- The Real squared amplitude, for all relevant partonic processes.
- The finite part of the virtual amplitude contribution, for all relevant partonic processes.

Strategy

Initially, we tried to implement our calculation using the Catani-Seymour subtraction approach, because of its wide popularity. This turned out to be too cumbersome. We realized that we could use the FKS framework, hiding all FKS implementation details. In other words, we use FKS, but the user needs not to understand it.

All the needed code is there:

- The phase space for ISR and FSR, according to FNO2006.
- The combinatorics, the calculation of all R_α, the soft and coll. limits
- The calculation of $\tilde{B}$ is completely implemented (coll. and soft remnants included). This is the hardest part of the implementation.
- The calculation of the upper bounds for the generation of radiation
- The generation of radiation
- Writing the event to the Les Houches interface

Lots of testing needed now ...

This work should be fairly close to a full automation of a POWHEG implementation for arbitrary processes.

It cannot yet be claimed to be a fully automated procedure: problems may arise, and so they will (thinking about the Born zeros problem, for example).

It is likely, however, that after dealing with a few complex problems, full automation will be reached.

Conclusions

- POWHEG is a viable method for interfacing NLO and SMC
- It is easy to implement, does not require new NLO computations
- Does not require committment to specific SMC implementations
- Its output is as in traditional SMC's: positive, constant weight events
- Several processes already available, more to come
- We have competitors (the Cambridge group!). Anybody can work on it!

  POWHEG is not a code: it is a method.

- We collect and publish material to make it easy for others to implement POWHEG with their NLO calculation.
- A general framework for implementing arbitrary processes is being worked on.

ISSUES

Some topics on general formulation of POWHEG

FNO2007: Frixione, Oleari, P.N. 2007

Extension to the general case only a matter of bookkeeping;
POWHEG is fully general, can be applied in any subtraction framework.

We look in details at POWHEG in

- the FKS (Frixione, Kunszt, Signer)
- the CS (Catani, Seymour) subtraction frameworks.

Flavour separation

There are several allowed flavour structures in the n body process. A flavour structure is a flavour assignment to the incoming and outgoing partons. The B and V contributions are labelled by the flavour structure index f_b.

There are several allowed flavour structures in the $n+1$ body process. Thus R is labelled by a flavour structure index f_r. Each component R_{f_r} has several singularity regions. We thus write

$$R = \sum_{\alpha_r} R^{\alpha_r}$$

where each R^{α_r} has a specific flavour structure, and is singular in only one singular region. This partition of R is trivial to perform:

- FKS provides specific kinematic functions S_{α_r}, with $\sum_{\alpha_r} S_{\alpha_r} = 1$ that suppress all but one singular regions.
- in CS one can use instead $S_{\alpha_r} = C_{\alpha_r} / (\sum_{\alpha_r} C_{\alpha_r})$ where C_{α_r} are the dipole subtraction terms.

$\bar{B}$ carries an f_b index;
Sudakov FF also carries an f_b index:

$$\Delta^{f_b}(\Phi_n, p_T) = \exp\left\{ -\sum_{\alpha_r \in \{\alpha_r | f_b\}} \int \frac{[d\Phi_r \, R(\Phi_n, \Phi_r)\theta(k_T - p_T)]_{\alpha_r}}{B^{f_b}(\Phi_n)} \right\}$$

or

$$\Delta^{f_b}(\Phi_n, p_T) = \prod_{\alpha_r \in \{\alpha_r | f_b\}} \exp\left\{ -\sum \int \frac{[d\Phi_r \, R(\Phi_n, \Phi_r)\theta(k_T - p_T)]_{\alpha_r}}{B^{f_b}(\Phi_n)} \right\}$$

where

- $\{\alpha_r | f_b\}$ is the set of all singular regions having the underlying Born configuration with flavour structure f_b.
- $[\ldots]_{\alpha_r}$ means that everything inside is relative to the α_r singular term: thus R is R_{α_r}, the parametrization (Φ_n, Φ_r) is the one appropriate to the α_r singular region

The last expression is closer to typical SMC's, with each emission considered independently.

Accuracy of SMC's

Soft divergences and double log region

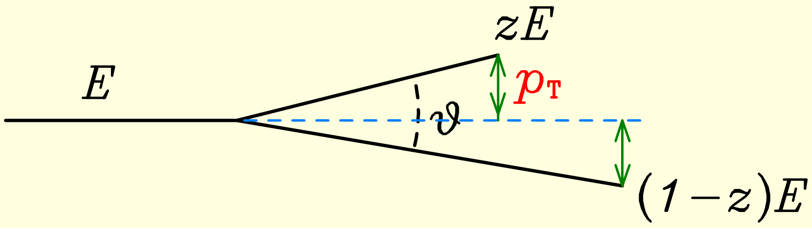
$z \to 1$ ($z \to 0$) region problematic: for $z \to 1$: $P_{qq}, P_{gg} \propto \frac{1}{1-z}$

Choice of hardness variable makes a difference

virtuality: $t \equiv E^2 z(1-z) \overbrace{\theta^2}^{1-\cos\theta}$

p_T^2: $t \equiv E^2 z^2 (1-z)^2 \, \theta^2$

angle: $t \equiv E^2 \, \theta^2$

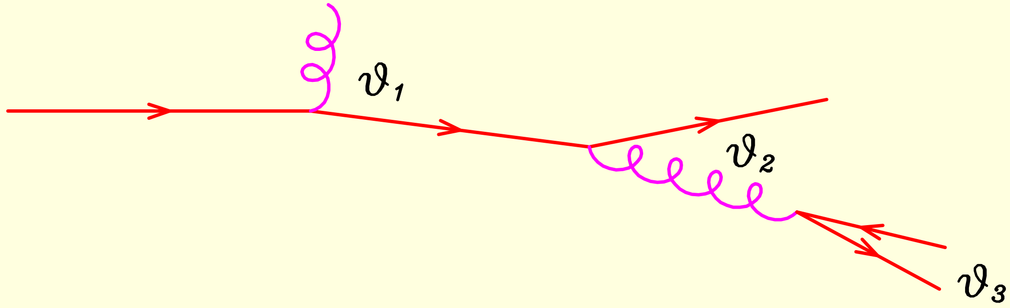


$$\underbrace{\int \frac{dt}{t} \int_{\sqrt{t}/E}^{1-\sqrt{t}/E} \frac{dz}{1-z}}_{\text{virtuality}: z(1-z) > t/E^2} \approx \frac{\log^2 \frac{t}{E^2}}{4}; \underbrace{\int \frac{dt}{t} \int_{t/E^2}^{1-t/E^2} \frac{dz}{1-z}}_{p_T^2: z^2(1-z)^2 > t/E} \approx \frac{\log^2 \frac{t}{E^2}}{2}; \underbrace{\int \frac{dt}{t} \int_0^1 \frac{dz}{1-z}}_{\text{angle}} \approx \log t \log \Lambda$$

Sizeable difference in double log structure!

Angular ordering is the correct choice (Mueller 1981)

$$\frac{d\theta}{\theta}\frac{\alpha_s(p_T^2)}{2\pi}P(z)\mathrm{dz}$$



$$\theta_1 > \theta_2 > \theta_3 \dots$$

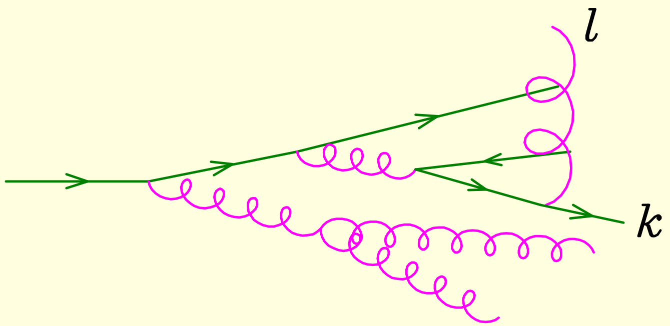
$$p_T^2 = E^2 z^2 (1-z)^2\,\theta^2$$

$\alpha_s(p_T^2)$ for a correct treatment of charge renormalization in soft region.

$$\Delta_i(t,t') = \exp\left[-\int_{t'}^{t}\frac{dt}{t}\int_{\sqrt{\frac{t_0}{t}}}^{1-\sqrt{\frac{t_0}{t}}} dz\,\frac{\alpha_s(p_T^2)}{2\pi}\sum_{(jk)} P_{i,jk}(z)\right]$$

$$\approx \exp\left[-\frac{c_i}{4\pi b_0}\left\{\log\frac{t}{\Lambda^2}\log\frac{\log\frac{t}{\Lambda^2}}{\log\frac{t_0}{\Lambda^2}} - \log\frac{t}{t_0}\right\}_{t'}^{t}\right] \qquad (c_q = C_F,\, c_g = 2C_A)$$

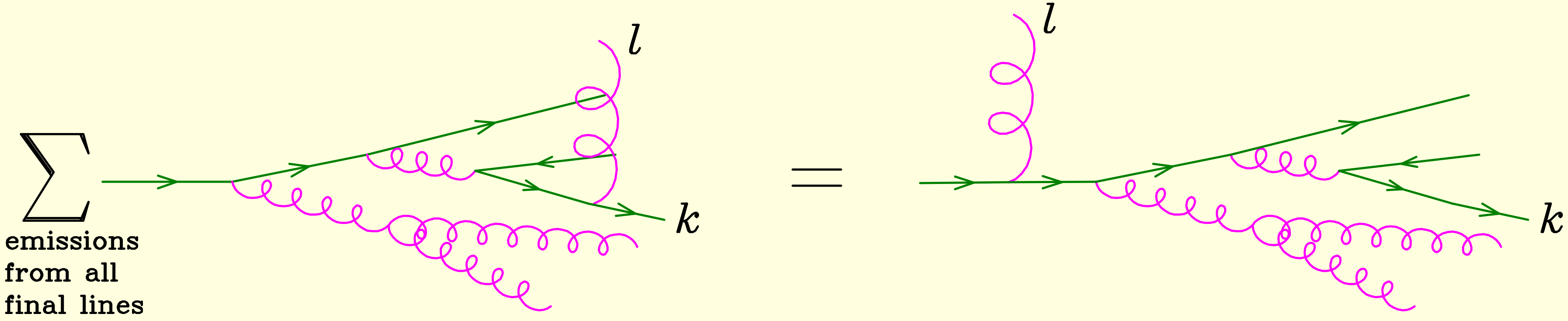
Sudakov damping stronger than any power of t.



With virtuality ordering:

Soft emissions give small virtuality.
At end of shower, large amount of
unrestricted (all angles) soft radiation

But soft gluons emitted at large angles from final state partons add coherently!



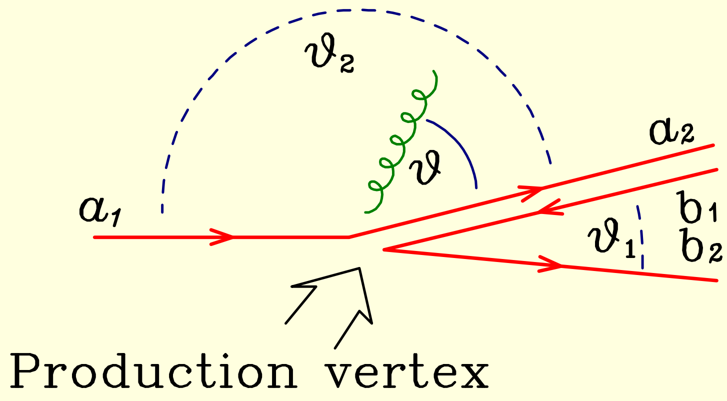
large angle, high energy: already ordered in angle
large angle, small energy: should be reordered by angle;

Thus: order in angle

Issue of truncated showers

Truncated shower are generally needed in angular ordered SMC's

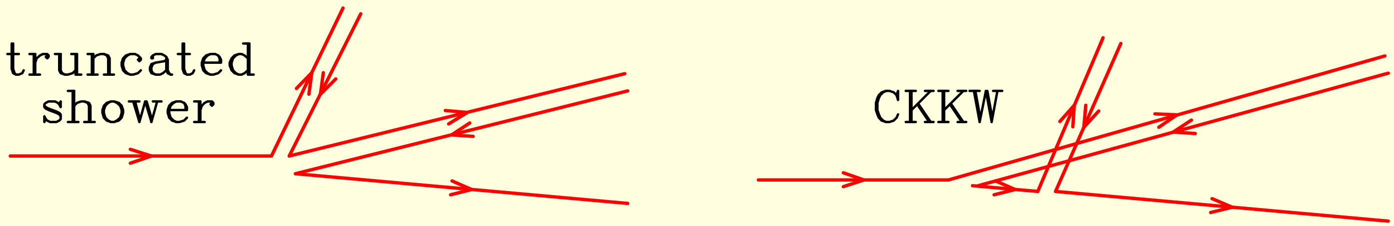
- Every time the shower is initiated by a relatively complex matrix element a truncated shower is needed
- CKKW mocks the effect of truncated shower with a trick (but it misses the correct colour flow)



Consider $e^+e^- \rightarrow q\,\bar{q}\,g$.
Assume θ_1 small. Consider gluon emission with angle $\theta \gg \theta_1$, $\theta \ll \theta_2$.
Coherence requires that the emission strength is C_F (gluon and quark coherently)

In HERWIG: initial angle for gluon radiation is θ_1 or θ_2 with a 50% probability. Thus (in the above region) strength is $C_A/2 \approx C_F$ (but only in the average!!)

In CKKW: radiation from gluon restricted to $\theta < \theta_1$, quark radiates with angle up to θ_2. Thus only the quark radiates in the above region, with strength C_F. However, the colour connection is incorrect! Large colour gap in CKKW!



So: coherent showers are always needed when doing ME-Shower matching with angular ordered showers.

Caveats in POWHEG

Born zeros

- Singularities in B
- Zeros in B

Both cause problems, but they are easily fixed.
For example, zeros in B: further separate

$$R_{\alpha_r} = \frac{k_T^2}{k_T^2 + B} R_{\alpha_r} + \frac{B}{k_T^2 + B} R_{\alpha_r}$$

The first term in non-singular (can be generated directly without Sudakov), while in the second term the zero in B cancels in the Sudakov exponent.

Accuracy of the Sudakov Form Factor

POWHEG's Sudakov FF has the form (with $c \approx 1$)

$$\Delta_t = \exp\left[-\int_t^{Q^2} \frac{d k_T^2}{k_T^2} \frac{\alpha_S(c\, k_T^2)}{\pi} \left\{ A \log\frac{M^2}{k_T^2} + B \right\} \right]$$

We know that the NLL Sudakov form factor has the form

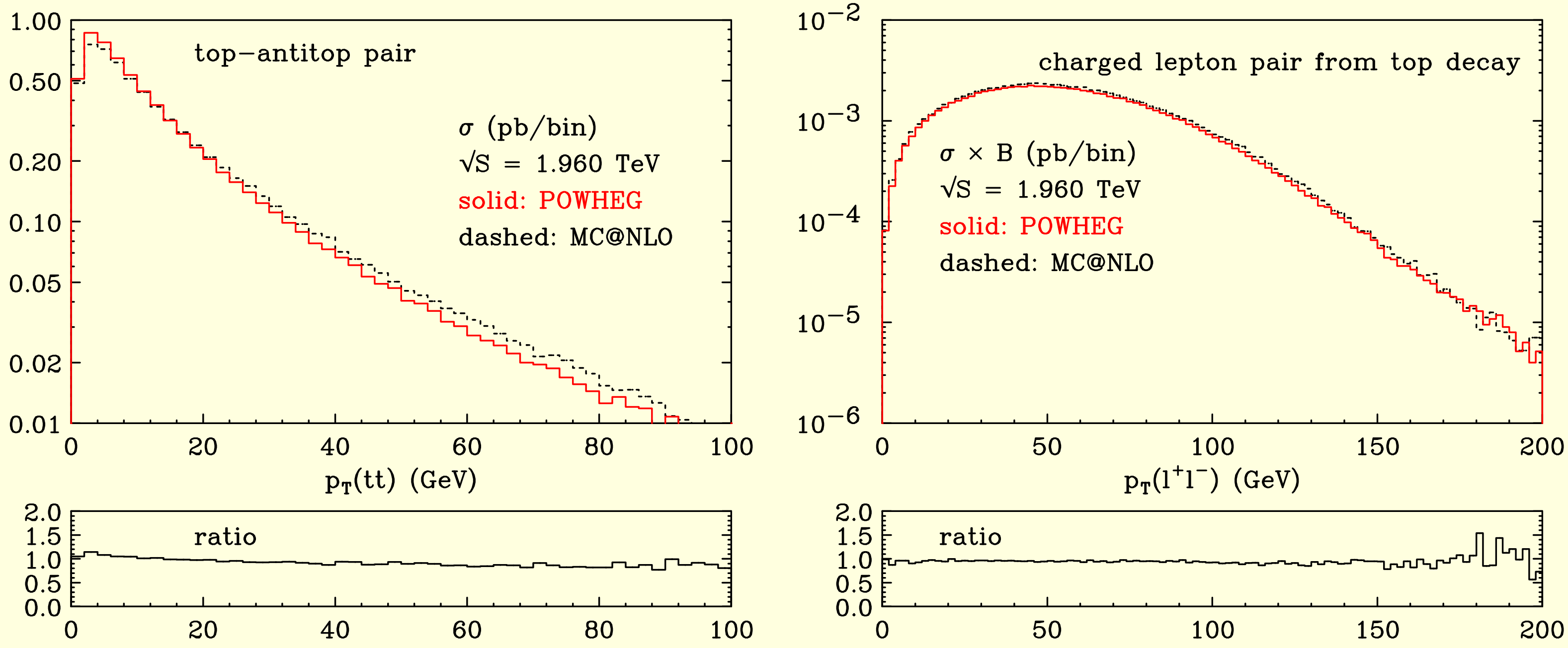
$$\Delta_t^{\mathrm{NLL}} = \exp\left[-\int_t^{Q^2} \frac{d k_T^2}{k_T^2} \frac{\alpha_S(k_T^2)}{\pi} \left\{ \left(A_1 + A_2 \frac{\alpha_s(k_T^2)}{\pi} \right) \log\frac{M^2}{k_T^2} + B \right\} \right]$$

provided the colour structure of the process is sufficiently simple ($\leqslant 3$ coloured legs). Can use this to fix c in POWHEG's Sudakov FF. (Suggested in (Catani, Webber, Marchesini, 1991) for HERWIG)

$\geqslant 4$ coloured legs: exponentiation only holds in LL,
or LL + (NLL large N_c) if planar colour structures are suitably separated

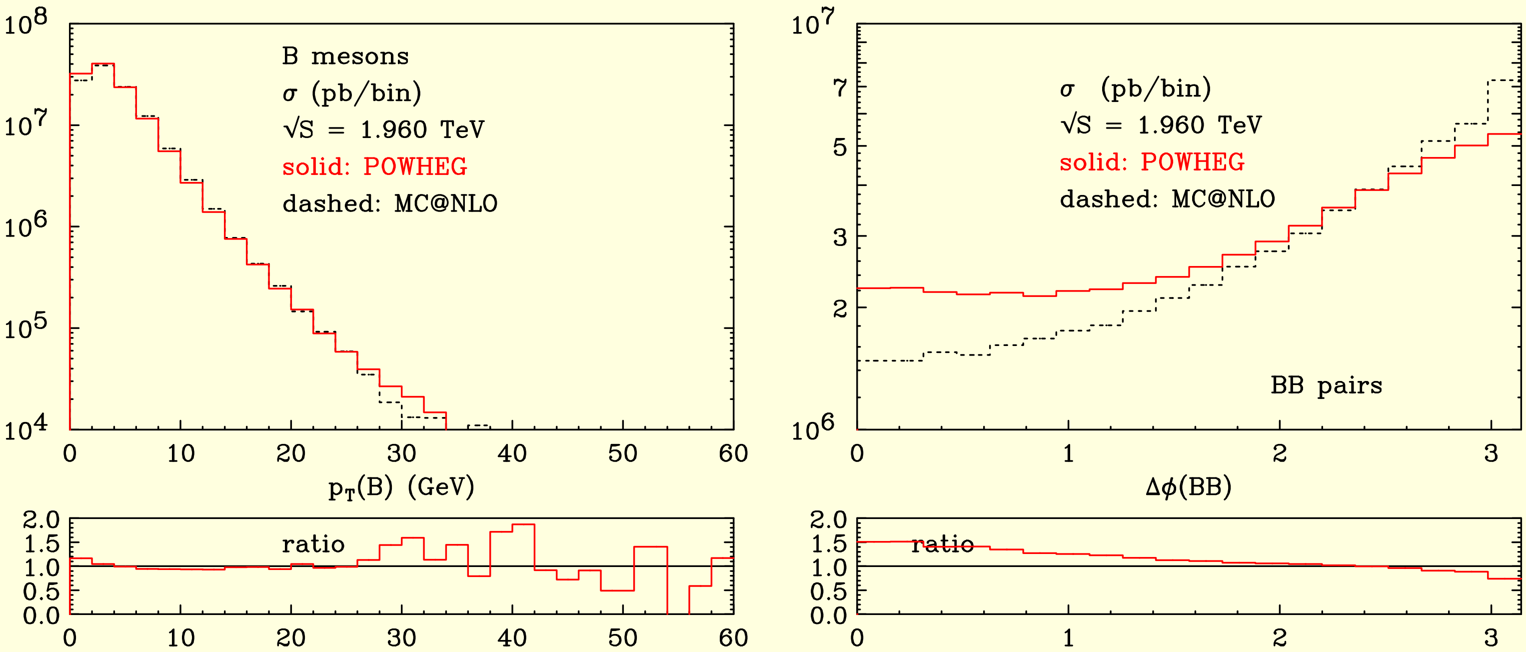
Summarizing:
POWHEG Sudakov is: always LL accurate,
NLL accurate for $\leqslant 3$ coloured legs, NLL accurate in leading N_c in all cases.

POWHEG and MC@NLO comparison: Top pair production



Good agreement for all observable considered
(differences can be ascribed to different treatment of higher order terms)

Bottom pair production



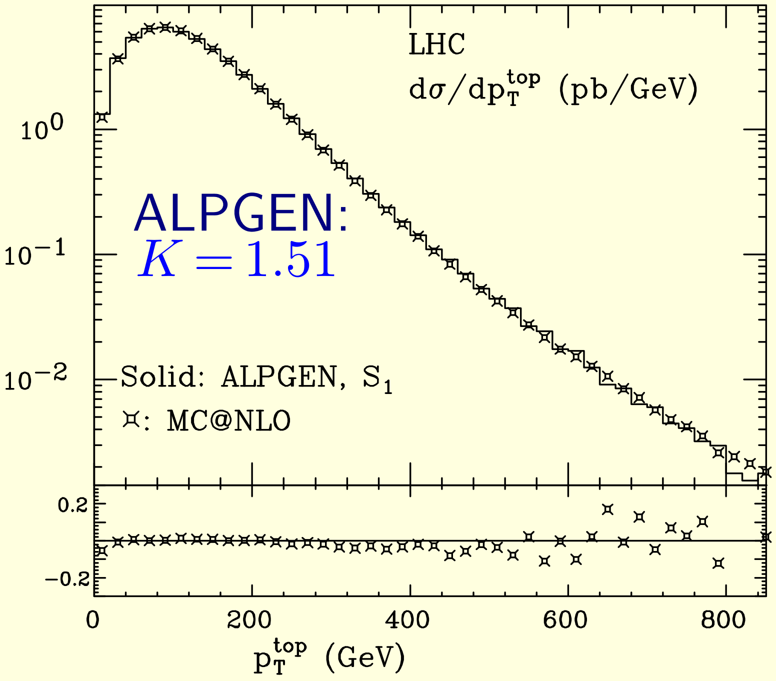
- Very good agreement For large scales (ZZ, $t\bar{t}$ production)
- Differences at small scales ($b\bar{b}$ at the Tevatron)
- POWHEG more reliable in extreme cases like $b\bar{b}, c\bar{c}$ at LHC
  (yields positive results, MC@NLO has problems with negative weights)

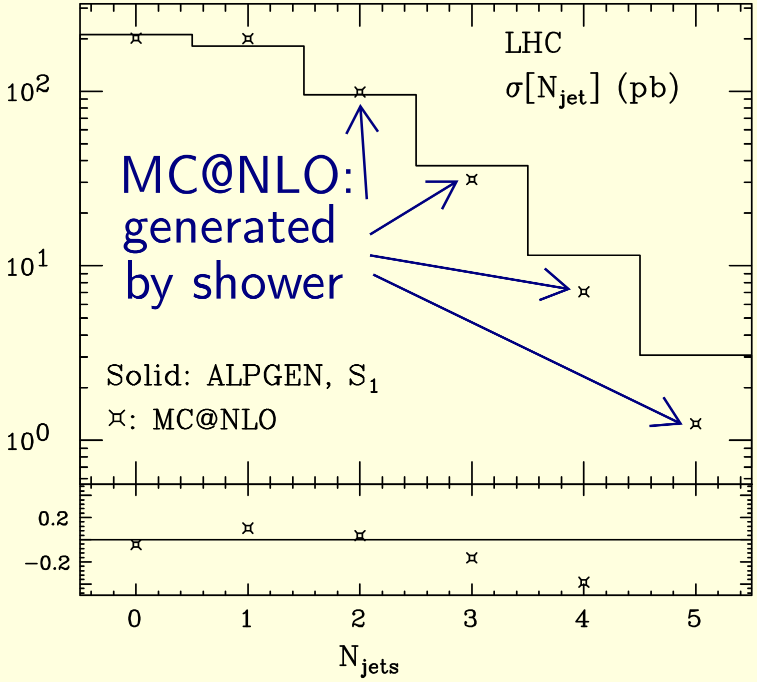
ALPGEN can generate samples of $t\bar{t} + n\,\mathrm{jets}$; can be compared to NLO+PS;

expect:

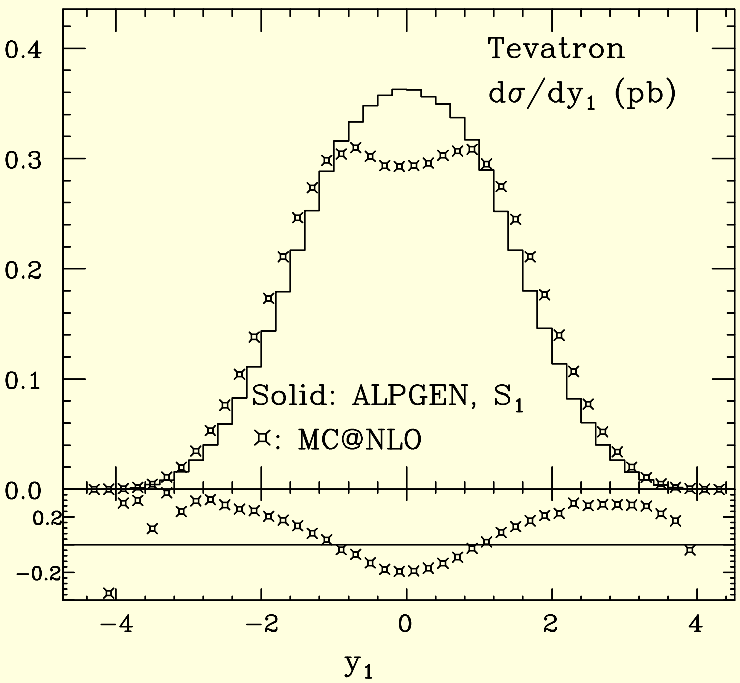
- Disadvantage: worse normalization (no NLO)
- Advantage: better high jet multiplicities (exact ME)

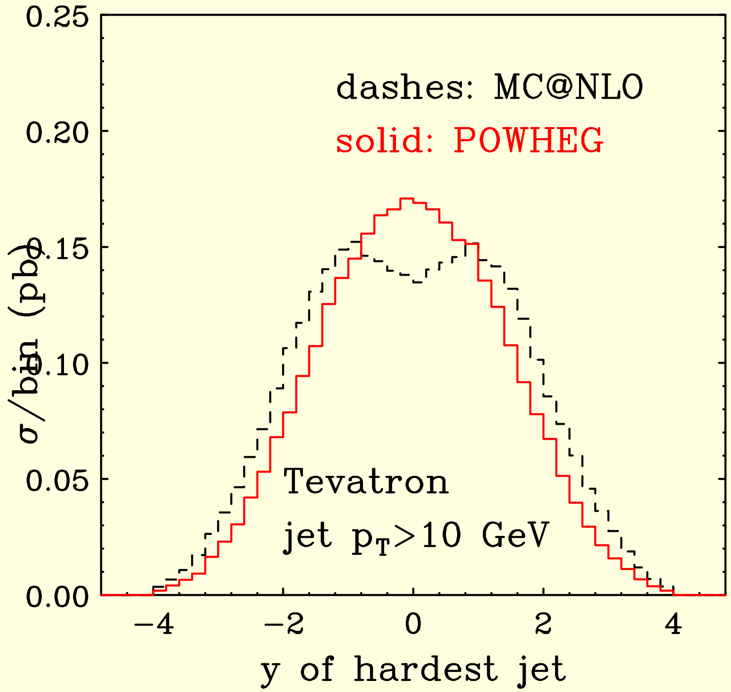
Comparison ALPGEN-MC@NLO carried out in detail
(Mangano, Moretti,Piccinini,Treccani, Nov.06)





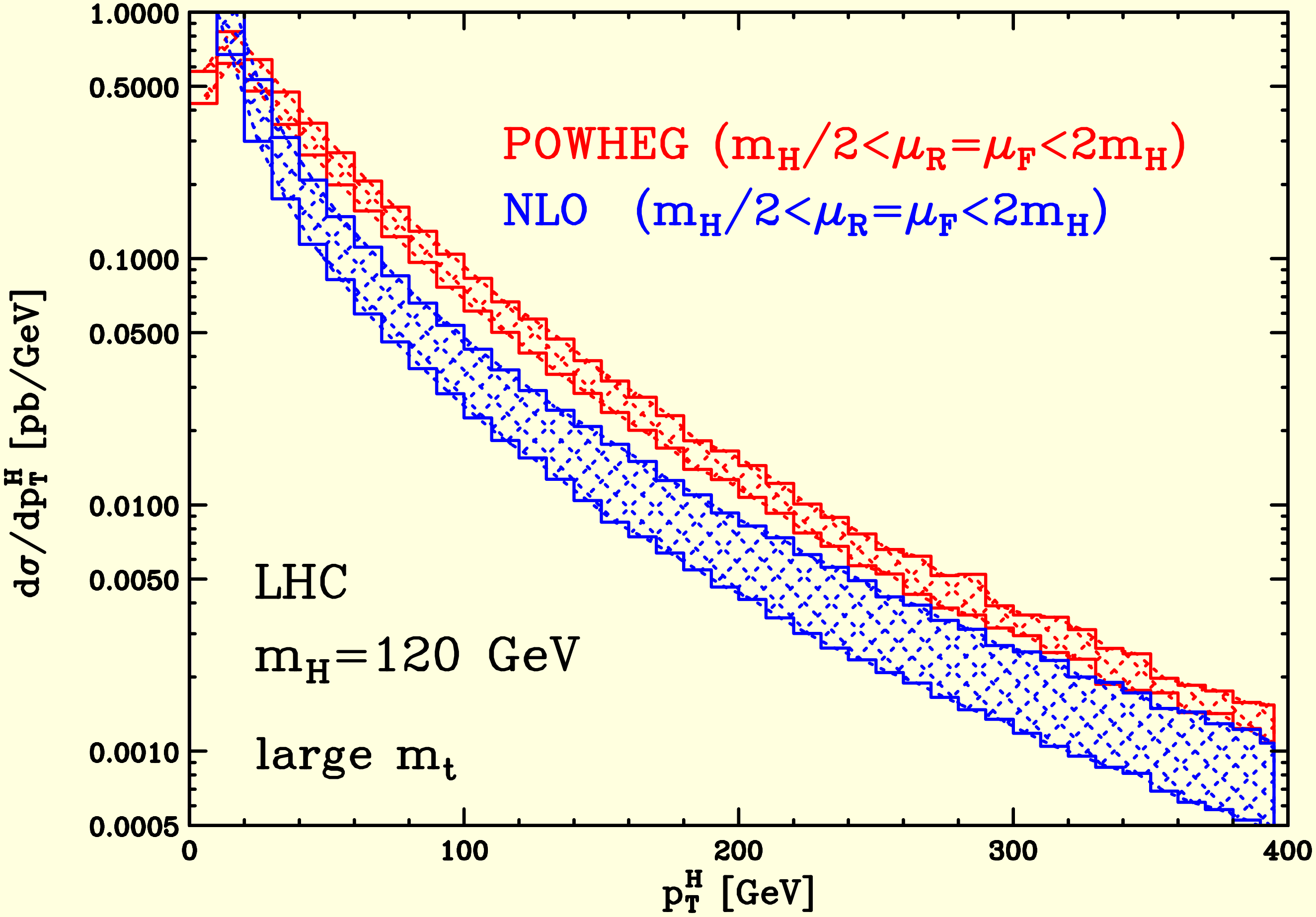
Results as expected but for 1 observable

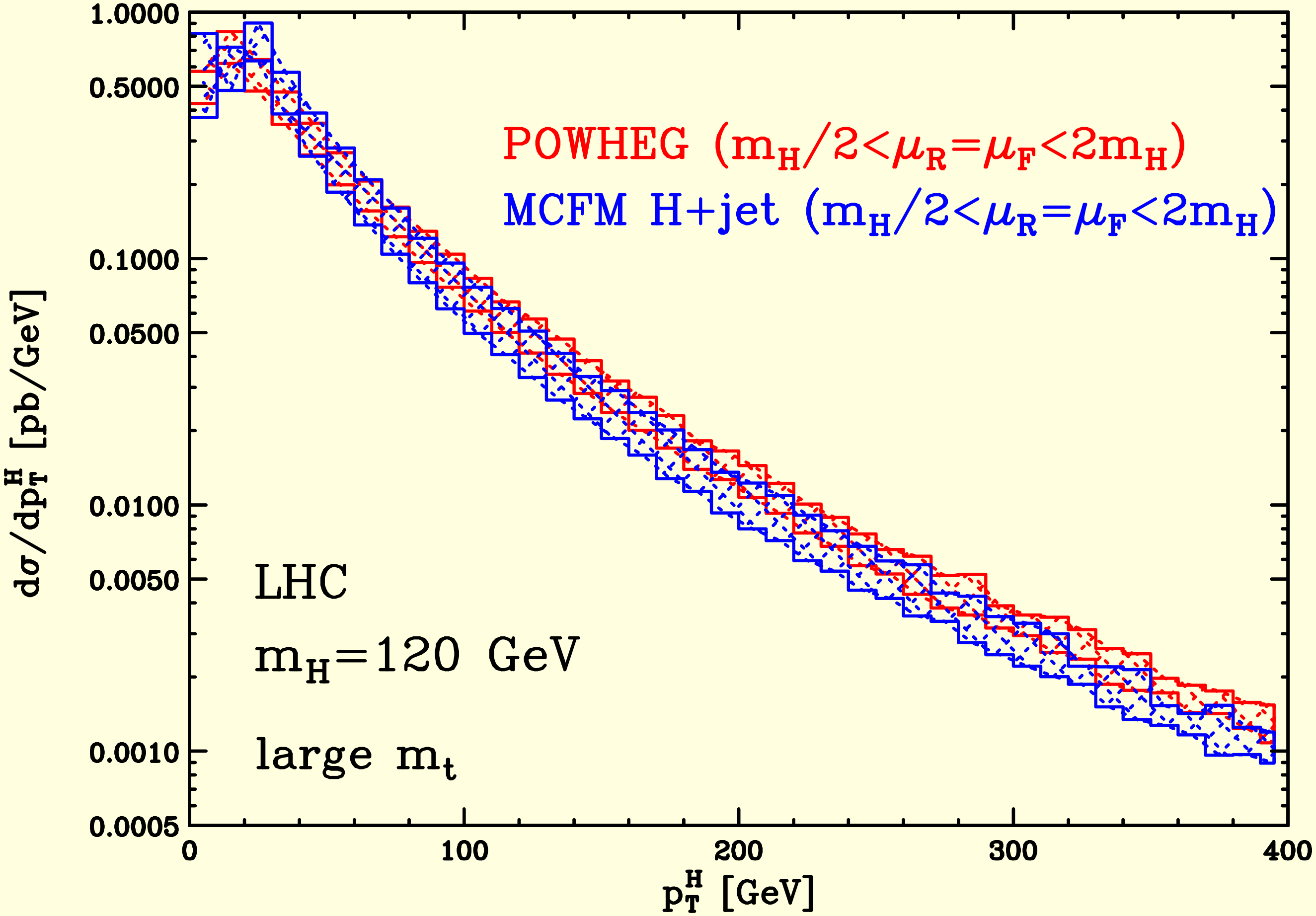


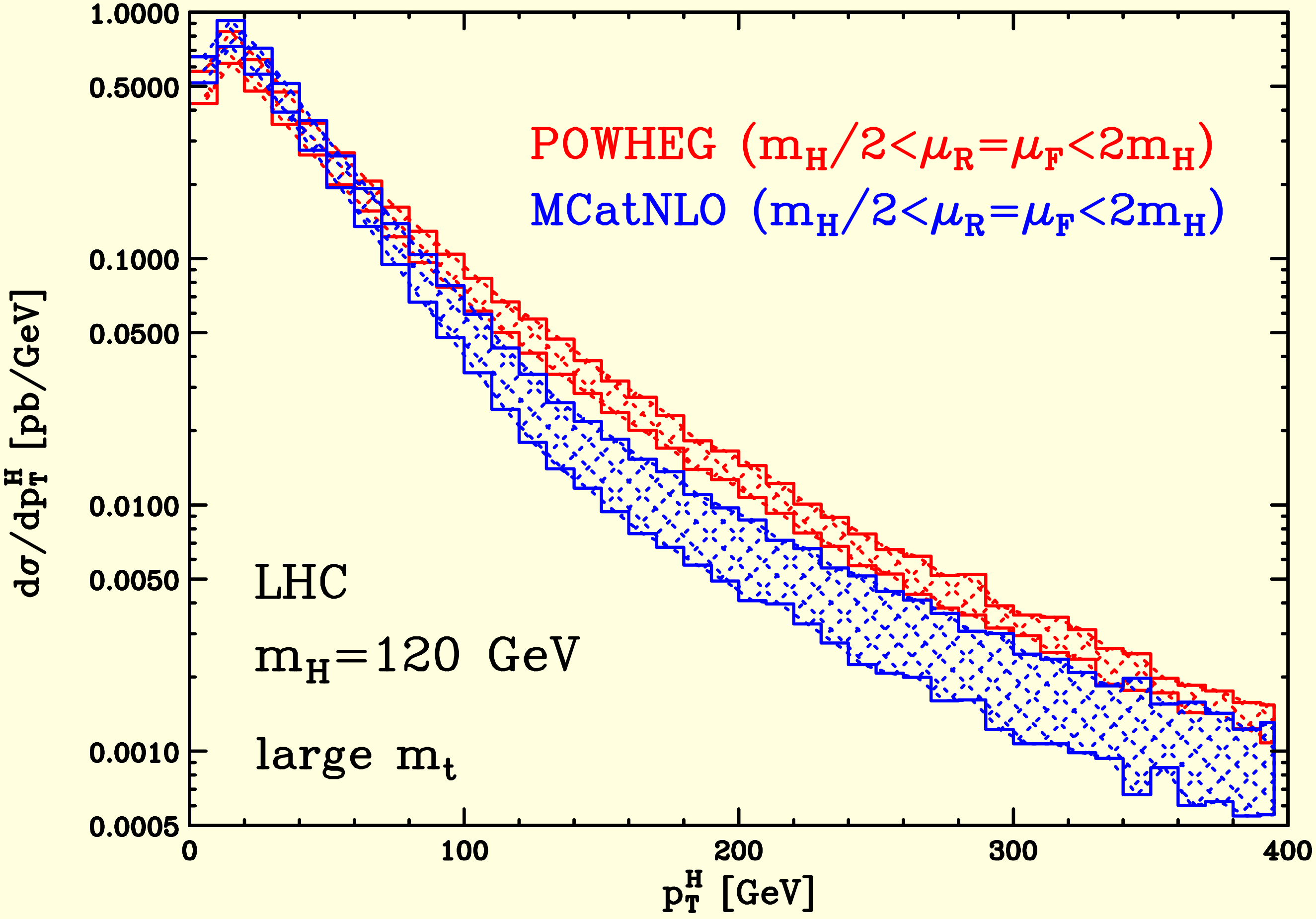


POWHEG's distribution as in ALPGEN (i.e., no dip);
Notice: size of discrepancy can be attributed to different treatment of higher order terms. Is this "feature" really there?
$pp \to t\bar{t} + \mathrm{Jet}$ at NLO (Dittmaier, Uwer, Weinzierl) agrees with ALPGEN and POWHEG

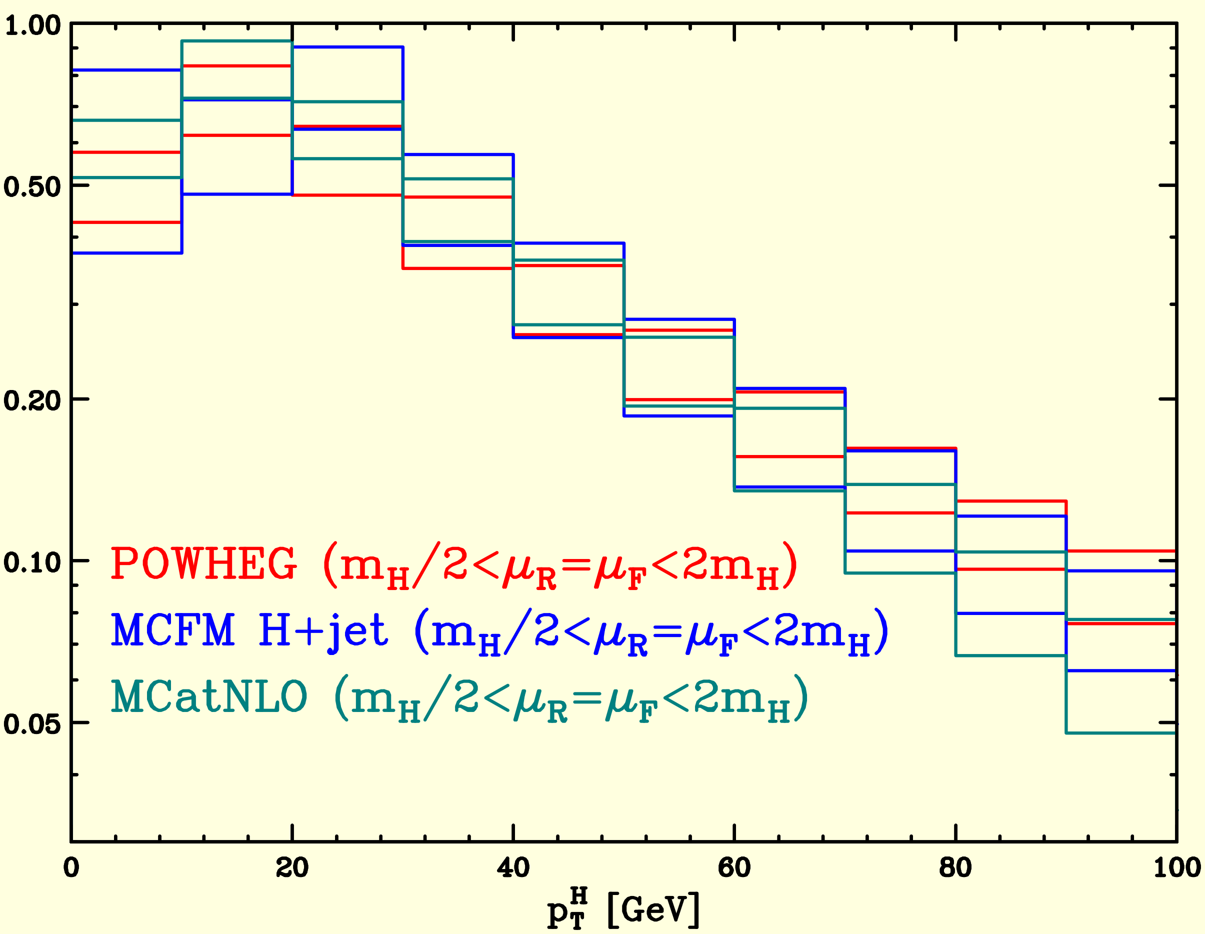
Scale dependence

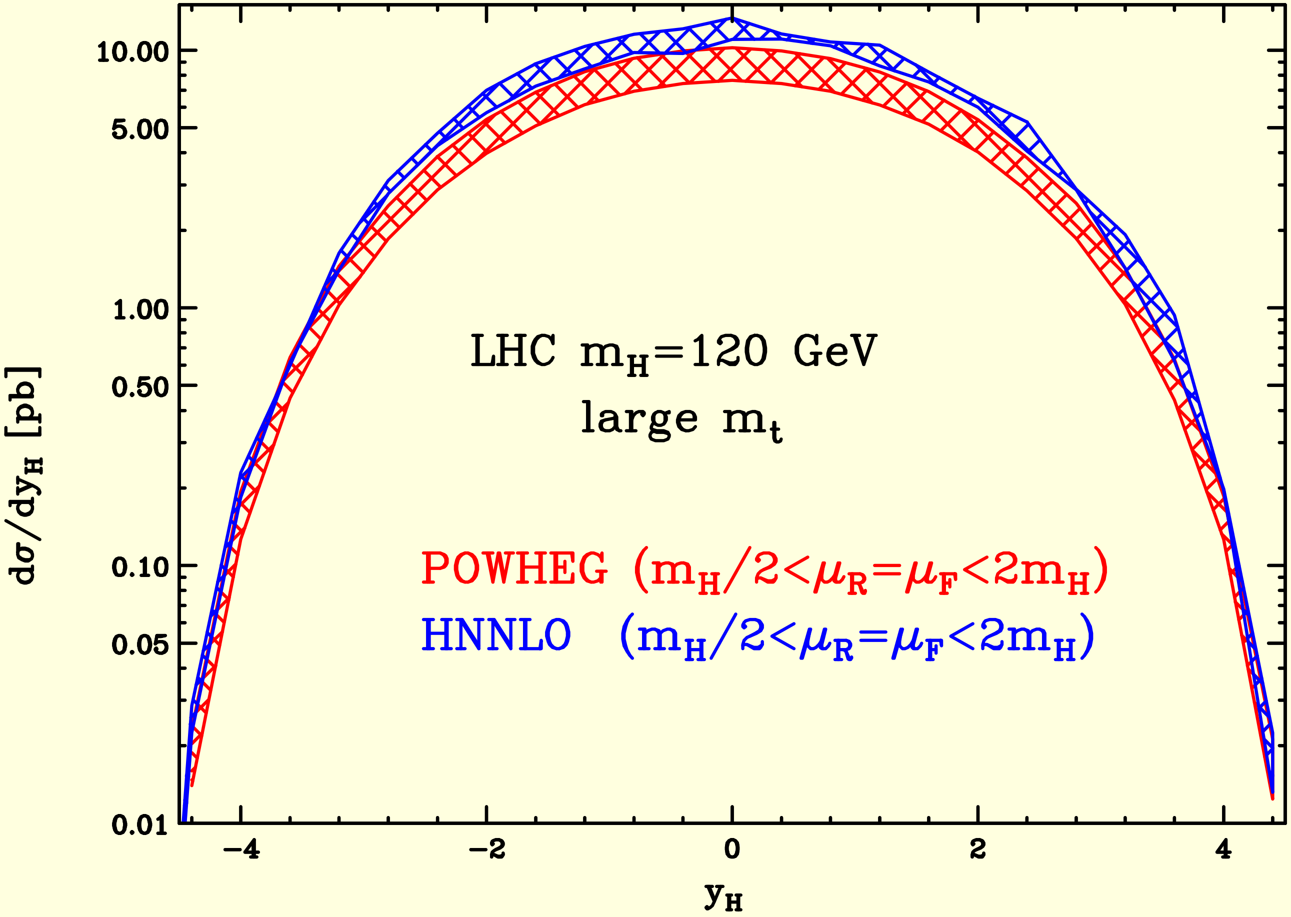



POWHEG ($m_H/2<\mu_R=\mu_F<2m_H$)
MCFM H+jet ($m_H/2<\mu_R=\mu_F<2m_H$)
LHC
m_H=120 GeV
large m_t
$d\sigma/dp_T^H$ [pb/GeV]
p_T^H [GeV]
1.0000
0.5000
0.1000
0.0500
0.0100
0.0050
0.0010
0.0005
0
100
200
300
400

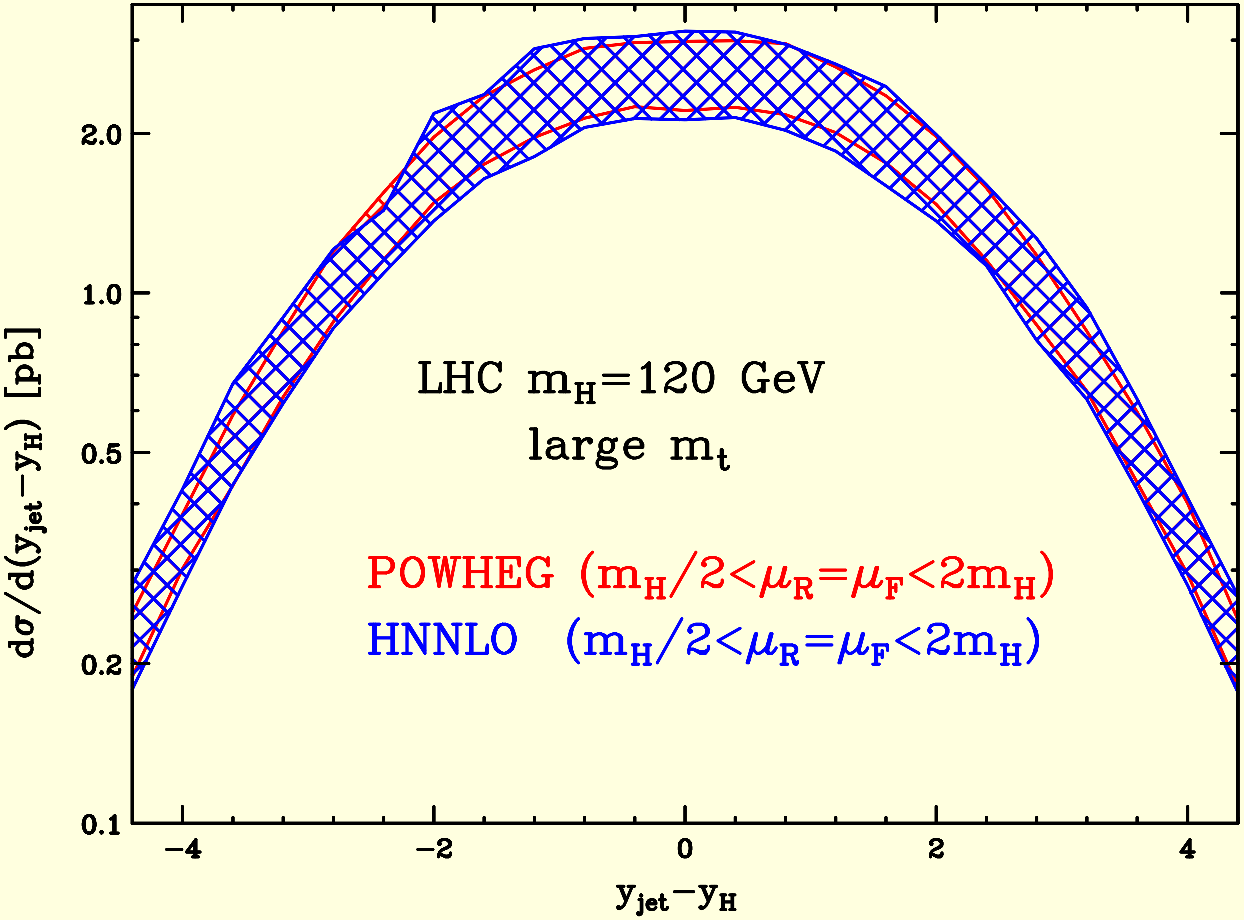


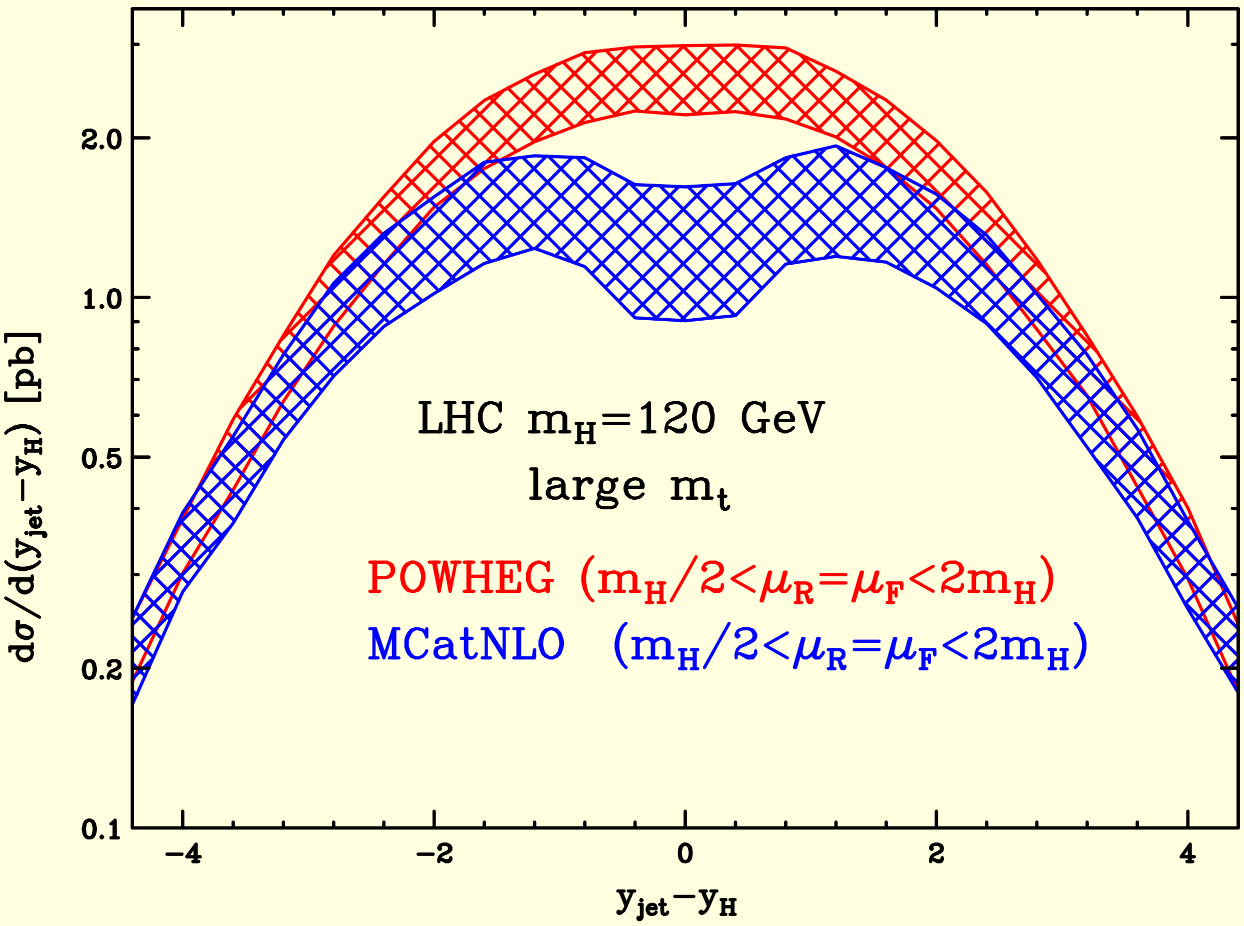
POWHEG ($m_H/2<\mu_R=\mu_F<2m_H$)
MCatNLO ($m_H/2<\mu_R=\mu_F<2m_H$)
LHC
m_H=120 GeV
large m_t
$d\sigma/dp_T^H$ [pb/GeV]
p_T^H [GeV]
1.0000
0.5000
0.1000
0.0500
0.0100
0.0050
0.0010
0.0005
0
100
200
300
400


1.00
0.50
0.20
0.10
0.05
0
20
40
60
80
100
POWHEG ($m_H/2<\mu_R=\mu_F<2m_H$)
MCFM H+jet ($m_H/2<\mu_R=\mu_F<2m_H$)
MCatNLO ($m_H/2<\mu_R=\mu_F<2m_H$)
p_T^H [GeV]



LHC m_H=120 GeV
large m_t
POWHEG ($m_H/2<\mu_R=\mu_F<2m_H$)
HNNLO ($m_H/2<\mu_R=\mu_F<2m_H$)
$d\sigma/dy_H$ [pb]
y_H
10.00
5.00
1.00
0.50
0.10
0.05
0.01
−4
−2
0
2
4


LHC m_H=120 GeV
large m_t
POWHEG ($m_H/2<\mu_R=\mu_F<2m_H$)
HNNLO ($m_H/2<\mu_R=\mu_F<2m_H$)
$d\sigma/d(y_{jet}-y_H)$ [pb]
$y_{jet}-y_H$
2.0
1.0
0.5
0.2
0.1
−4
−2
0
2
4


LHC m_H=120 GeV
large m_t
POWHEG ($m_H/2<\mu_R=\mu_F<2m_H$)
MCatNLO ($m_H/2<\mu_R=\mu_F<2m_H$)
$d\sigma/d(y_{jet}-y_H)$ [pb]
$y_{jet}-y_H$
0.1
0.2
0.5
1.0
2.0
−4
−2
0
2
4