

# NLO + PARTON SHOWER: A NEW APPROACH

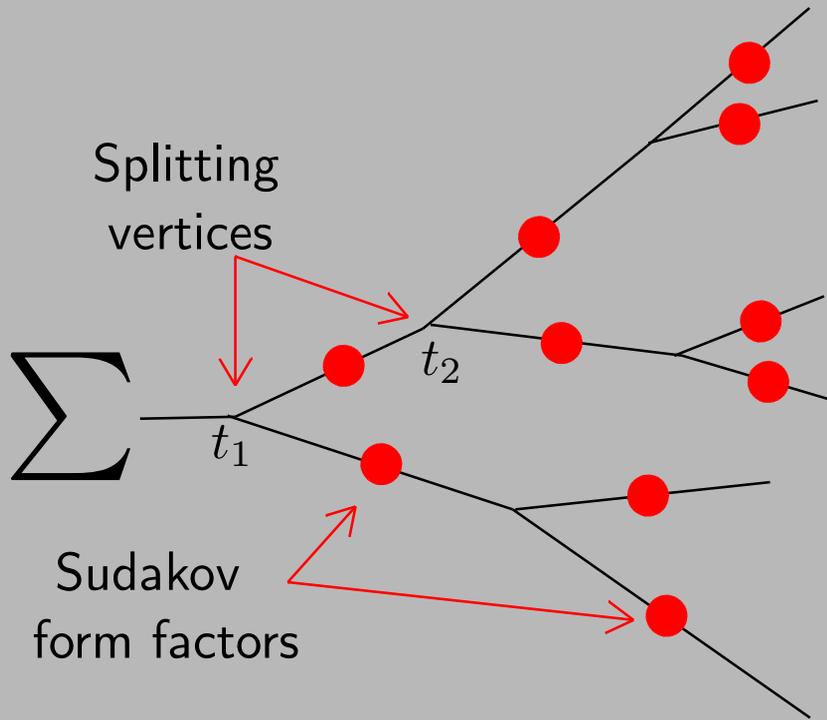
P. Nason

# Motivations

SMC (**Shower Monte Carlo**) programs: normally accurate at LL level  
Naturally interfaced to Born processes (i.e. **Matrix Elements** generators)  
NLO calculations: many available results for collider processes

Positive NLO experience with QCD tests at colliders;  
PS predictive for multiplicities and multiplicity distributions;  
Natural extension: NLO+PS

# What is a SMC



Splitting vertices are given, in the collinear approximation, by

$$d p = \frac{\alpha}{2\pi} P(z) \frac{d t}{t} d z d \varphi$$

The Sudakov form factors are

$$\Delta = \exp \left[ - \int_{t_2}^{t_1} \frac{\alpha}{2\pi} P(z) \frac{d t}{t} d z d \varphi \right]$$

Notice:  $\Delta d p = - d \Delta$

Sudakov form factors are **uniformly distributed**.

Uniformity of Sudakov form factors used for numerical implementations.

# What is an NLO calculation

Typically (subtraction method) for an observable  $O$

$$\begin{aligned}\langle O \rangle &= \int O d\sigma = \int O(\Phi_n) [B(\Phi_n) + V(\Phi_n)] d\Phi_n \\ &+ \int \left[ O(\Phi_{n+1})R(\Phi_{n+1}) - O(\tilde{\Phi}_{n+1})C(\Phi_{n+1}) \right] d\Phi_{n+1}\end{aligned}$$

$\tilde{\Phi}_{n+1}$ : singular  $n+1$  body phase space point, function of  $\Phi_{n+1}$

- Soft case:  $\tilde{\Phi}_{n+1}$  has a **zero momentum parton**
- Collinear case:  $\tilde{\Phi}_{n+1}$  has **two massless partons with parallel momenta**

Near the singular limit  $\tilde{\Phi}_{n+1} \approx \Phi_{n+1}$ .

(In general, several singular regions, counterterms,  $\tilde{\Phi}_{n+1}$  functions)

The singular configurations  $\tilde{\Phi}_{n+1}$  have an underlying Born kinematics  $\bar{\Phi}_n$ , obtained by:

- Soft case: removing the zero momentum parton
- Final state collinear: merging the collinear partons
- Initial state collinear: removing the radiated collinear parton, and subtracting its momentum from initial state parton

Assume that we can parametrize the  $\Phi_{n+1}$  phase space in terms of  $(\bar{\Phi}_n, \Phi_r)$  where  $\Phi_r$  are three more radiation variables. We have

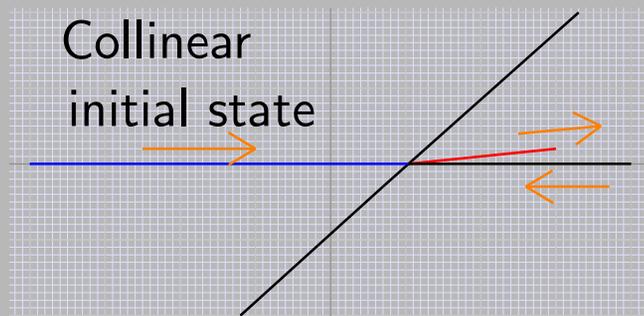
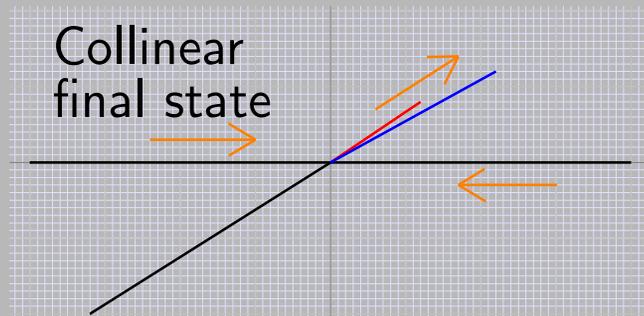
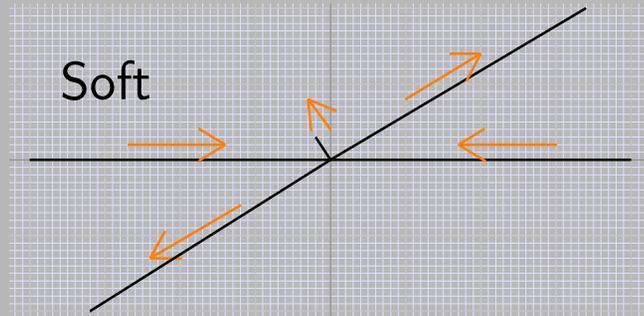
$$\begin{aligned} \langle O \rangle &= \int O(\Phi_n) [B(\Phi_n) + V(\Phi_n)] d\Phi_n \\ &+ \int [O(\bar{\Phi}_n, \Phi_r)R(\bar{\Phi}_n, \Phi_r) - O(\bar{\Phi}_n)C(\bar{\Phi}_n, \Phi_r)] d\bar{\Phi}_n d\Phi_r \end{aligned}$$

Defining the unregularized virtual  $V_b(\Phi_n) = V(\Phi_n) - \int C(\Phi_n, \Phi_r) d\Phi_r$ :

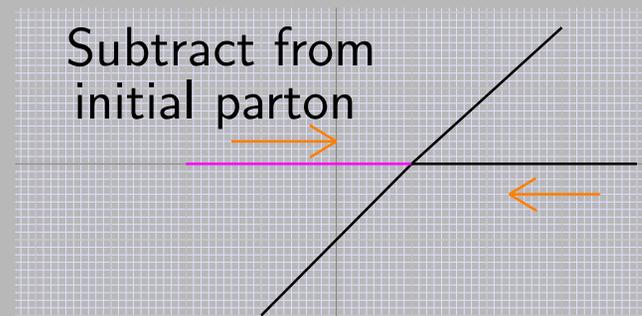
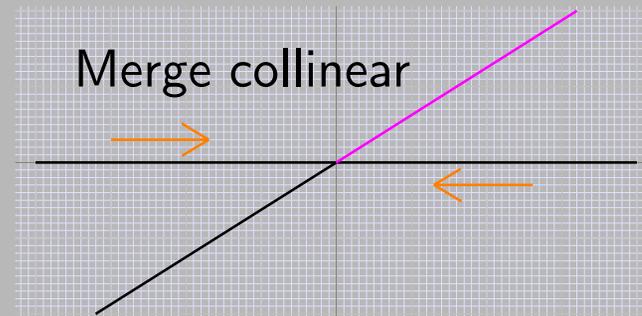
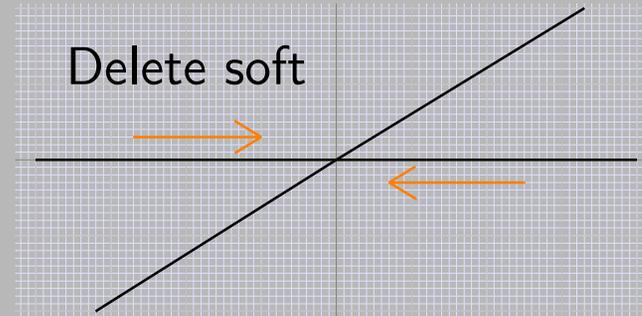
$$\langle O \rangle = \int O(\Phi_n) [B(\Phi_n) + V_b(\Phi_n)] d\Phi_n + \int O(\bar{\Phi}_n, \Phi_r)R(\bar{\Phi}_n, \Phi_r) d\bar{\Phi}_n d\Phi_r$$

which is the starting formula with divergent real and virtual corrections.

## Singular configuration



## Underlying Born



## NLO in SMC

In SMC's:  $\Phi_{n+1} = \Phi_{n+1}(\Phi_n, \Phi_r)$ , with  $\Phi_r = (t, z, \varphi)$  (**momentum reshuffling**).  
The form of the mapping depends upon the implementation.  
SMC cross section for first emission:

$$\langle O \rangle = \int B(\Phi_n) d\Phi_n \left\{ O(\Phi_n) \Delta_{t_{\text{min}}} + \int_{t_{\text{min}}} O(\Phi_n, \Phi_r) \Delta_t \frac{\alpha}{2\pi} P(z) \frac{dt}{t} dz d\varphi \right\}$$

with

$$\Delta_t = \exp \left[ - \int_t \frac{\alpha}{2\pi} P(z) \frac{dt'}{t'} dz d\varphi \right]$$

at NLO<sub>SMC</sub> order (**expand the above formula to  $O(\alpha_S)$** )

$$\langle O \rangle = \int B(\Phi_n) d\Phi_n \left\{ O(\Phi_n) + \int_{t_{\text{min}}} [O(\Phi_n, \Phi_r) - O(\Phi_n)] \frac{\alpha}{2\pi} P(z) \frac{dt}{t} dz d\varphi \right\}$$

(of course, this is the inexact NLO correction implemented by the SMC)

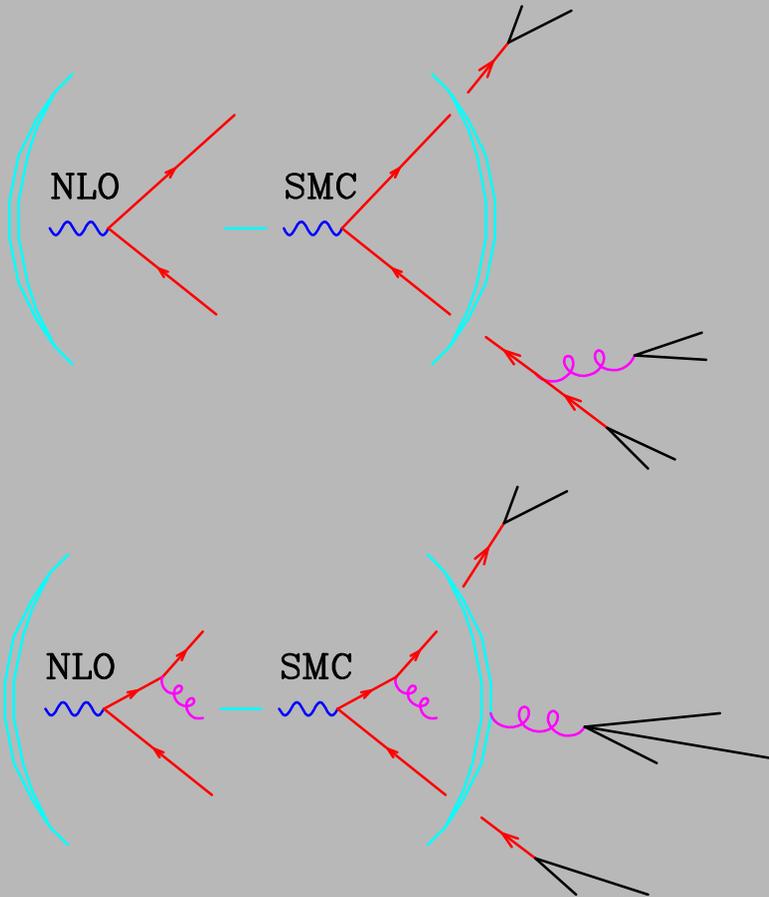
**How do we reach exact NLO accuracy?**

# MC@NLO (2002, Frixione+Webber)

Add difference between **exact** NLO and **approximate (MC) NLO**.

- Must use  $(\bar{\Phi}, \Phi_r)$  of the MC
- Difference should be regular (if the MC is OK)
- Difference may be **negative**

Several collider processes already implemented: **Vector Bosons, Vector Bosons pairs, Higgs, Heavy Quarks, Single Top.**



# MC@NLO

First **viable** (and useful) implementation of **SMC+NLO**.

No need to modify the shower program: **straightforward to implement**.

Drawbacks:

- Must use  $(\bar{\Phi}, \Phi_r)$  of the SMC: **SMC specific**, needs the authors...
- $\text{NLO} - \text{NLO}_{\text{SMC}}$  must be regular in the singular limit. Therefore:
  - SMC must implement correctly collinear singularities, also in the azimuthal dependence
  - SMC must implement correctly soft emission.  
**If not, one must worry about the importance of the left over.**
  - The matching in the subtraction is crucial and delicate
- Events with negative weights may be generated (undesirable feature). Physical distributions should turn out positive. However this may be difficult to prove for all cases.

# POWHEG

Positive Weight Hardest Emission Generator

Method to generate the hardest emission first, with NLO accuracy, and independently of the SMC (P.N. 2004).

- SMC independent; no need of SMC expert; same calculation can be interfaced to several SMC programs with no extra effort
- SMC inaccuracies only affect next-to-hardest emissions; no matching problems
- As the name says, it generates events with positive weight

## How it works

$$\begin{aligned}
 \langle O \rangle &= \int O(\Phi_n) [B(\Phi_n) + V(\Phi_n)] d\Phi_n \\
 &+ \int [O(\bar{\Phi}_n, \Phi_r) R(\bar{\Phi}_n, \Phi_r) - O(\bar{\Phi}_n) C(\bar{\Phi}_n, \Phi_r)] d\bar{\Phi}_n d\Phi_r \\
 &= \int O(\Phi_n) \left[ B(\Phi_n) + V(\Phi_n) + \int \{R(\bar{\Phi}_n, \Phi_r) - C(\bar{\Phi}_n, \Phi_r)\} d\Phi_r \right] d\Phi_n \\
 &+ \int R(\Phi_n, \Phi_r) [O(\Phi_n, \Phi_r) - O(\Phi_n)] d\Phi_n d\Phi_r
 \end{aligned}$$

Define:  $\bar{B}(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \int \{R(\bar{\Phi}_n, \Phi_r) - C(\bar{\Phi}_n, \Phi_r)\} d\Phi_r$ , get:

$$\langle O \rangle = \int O(\Phi_n) \bar{B}(\Phi_n) d\Phi_n + \int R(\Phi_n, \Phi_r) [O(\Phi_n, \Phi_r) - O(\Phi_n)] d\Phi_n d\Phi_r$$

In NLO<sub>SMC</sub> was

$$\langle O \rangle = \int O(\Phi_n) B(\Phi_n) d\Phi_n + \int B(\Phi_n) \frac{\alpha}{2\pi} P(z) \frac{1}{t} [O(\Phi_n, \Phi_r) - O(\Phi_n)] d\Phi_n d\Phi_r$$

Thus:  $\text{NLO}_{\text{SMC}} \leftrightarrow \text{NLO}$ :  $B(\Phi_n) \leftrightarrow \bar{B}(\Phi_n)$ ,  $B(\Phi_n) \frac{\alpha}{2\pi} P(z) \frac{1}{t} \leftrightarrow R(\Phi_n, \Phi_r)$

All order emission probability in SMC:

$$\langle O \rangle = \int B(\Phi_n) d\Phi_n \left\{ O(\Phi_n) \Delta_{t_{\text{min}}} + \int_{t_{\text{min}}} O(\Phi_n, \Phi_r) \Delta_t \frac{\alpha}{2\pi} P(z) \frac{1}{t} d\Phi_r \right\}$$

$$\Delta_t = \exp \left[ - \int_t \frac{\alpha}{2\pi} P(z) \frac{1}{t'} d\Phi_{r'} \right]$$

All order emission probability in POWHEG:

$$\langle O \rangle = \int \bar{B}(\Phi_n) d\Phi_n \left\{ O(\Phi_n) \Delta_{t_{\text{min}}} + \int O(\Phi_n, \Phi_r) \Delta_{t_r} \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} d\Phi_r \right\}$$

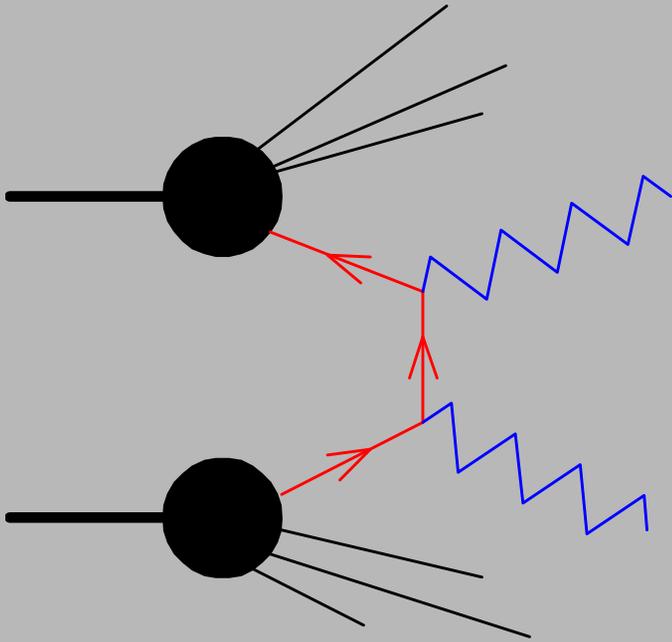
$$\Delta_t = \exp \left[ - \int \theta(t_r - t) \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} d\Phi_r \right]$$

with  $t_r = k_T(\Phi_n, \Phi_r)$ , the transverse momentum for the radiation.

Positive if  $\bar{B}$  is positive (i.e.  $\text{NL} < \text{LO}$ ).

# First example: $ZZ$ production in hadron collisions

(Ridolfi, P.N.)



- NLO known  
(Mele, Ridolfi, P.N.)
- Intermediate complexity
- Hadrons in initial state
- Similar to  $WZ$ ,  $WW$ ,  $Q\bar{Q}$

# $\bar{\Phi}$ and $\Phi_r$ variables

$\bar{\Phi}$  variables: choose  $M_{ZZ}$ ,  $Y_{ZZ}$  and  $\theta$ , where

- $M_{ZZ}$ : invariant mass of the  $ZZ$  pair
- $Y_{ZZ}$ : rapidity of  $ZZ$  pair
- $\theta$ : go in the (longitudinally) boosted frame where  $Y_{ZZ} = 0$ .  
go to the  $ZZ$  rest frame with a transverse boost  
In this frame  $\theta$  is the angle of a  $Z$  to the longitudinal direction.

$\Phi_r$  variables:

- $x = M_{ZZ}/s$ , ( $s$  is the invariant mass of the incoming parton system)  
 $x \rightarrow 1$  is the soft limit
- $y$ : cosine of the angle of the radiated parton to the beam direction in the partonic CM frame.
- $\phi$ : radiation azimuth.

## Few tricks to do it

$$\bar{B}(\Phi) = B(\Phi) + V(\Phi) + \int d\Phi_r [R(\Phi, \Phi_r) - C(\Phi, \Phi_r)]$$

Seems to need one  $\Phi_r$  integrations to get weight of each  $\Phi$  point.

In fact, write

$$\tilde{B}(\Phi, \Phi_r) = N[B(\Phi) + V(\Phi)] + R(\Phi, \Phi_r) - C(\Phi, \Phi_r), \quad N = \frac{1}{\int d\Phi_r}.$$

so that

$$\bar{B}(\Phi) = \int \tilde{B}(\Phi, \Phi_r) d\Phi_r.$$

Use standard procedures ([SPRING-BASES, Kawabata](#)) to generate unweighted events for  $\tilde{B}(\bar{\Phi}, \Phi_r) d\Phi_r d\bar{\Phi}$ .  
**discard**  $\Phi_r$  (same as integrating over it!).

$$\Delta(\Phi, p_T) = \exp \left[ - \int \frac{R(\Phi, \Phi_r)}{B(\Phi)} \theta(k_T(\Phi, \Phi_r) - p_T) d\Phi_r \right],$$

Look for an upper bounding function;

$$\frac{R(\Phi, \Phi_r)}{B(\Phi)} \leq U(\Phi) = N \frac{\alpha_S(k_T)}{(1-x)(1-y^2)}$$

Generate  $x, y$  according to

$$\exp \left[ - \int U(\Phi) \theta(k_T(\Phi, \Phi_r) - p_T) d\Phi_r \right]$$

accept the event with a probability

$$\frac{R(\Phi, \Phi_r)}{B(\Phi)U(\Phi)}.$$

If the event is rejected generate a new one for smaller  $p_T$ , and so on  
(This procedure reconstructs the exact emission probability).

In the  $ZZ$  case, an event is generated with about six calls to  $R(\Phi, \Phi_r)$ .

# Interfacing to SMC's

For a  $p_T$  ordered SMC, nothing else needs to be done.

Use the standard Les Houches Interface for User's Processes (LHI):

put partonic event generated by POWHEG on the LHI;

Run the SMC in the LHI mode.

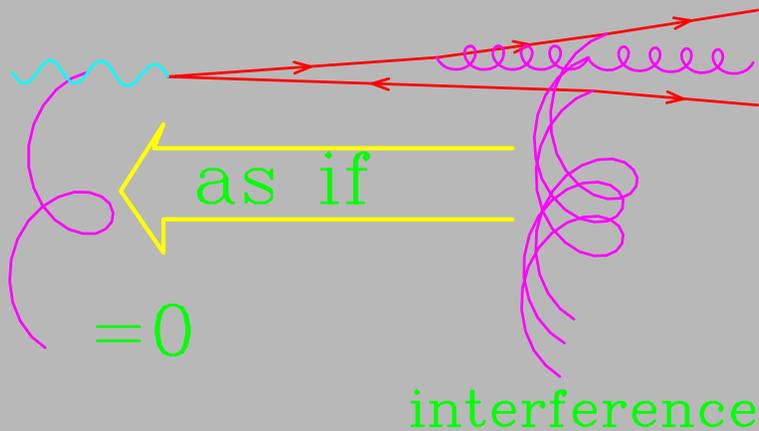
The LHI provides a facility to pass the  $p_T$  of the event to the SMC (SCALUP).

As far as the hardest emission is concerned, POWHEG guarantees:

- NLO accuracy of (integrated) shape variables
- Collinear, double-log, soft (large  $N_c$ ) accuracy of the Sudakov FF.  
(In fact, corrections that exponentiates are obviously OK)

As far as subsequent (less hard) emissions, the output has the accuracy of the SMC one is using.

For angular ordered SMC's (i.e. HERWIG):



Angular ordering accounts for soft gluon interference.

Intensity for photon jets = 0

Intensity for gluon jets =  $C_A$

instead of  $2C_F + C_A$

Consistent with a boosted jet pair, in the case of a photon jet.

In angular ordered SMC large angle soft emission is generated first.

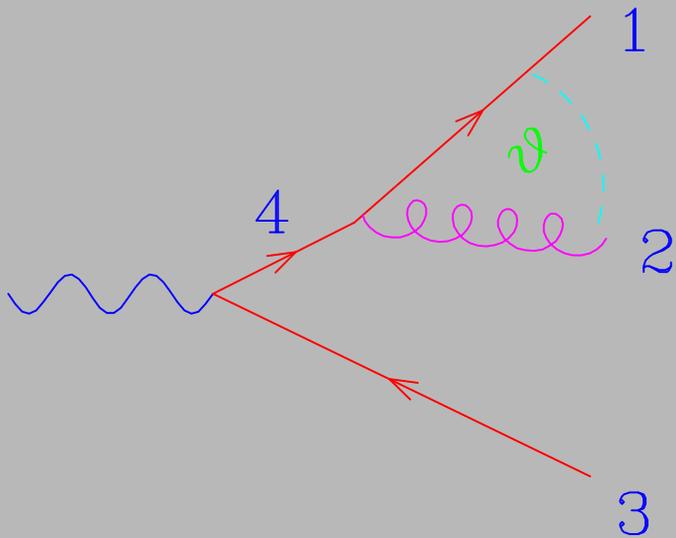
Hardest emission (i.e. highest  $p_T$ ) happens later.

Difficult to correct it explicitly.

# Recipe for angular ordered showers

- Generate event with hardest emission
- Generate all subsequent emissions with a  $p_T$  veto equal to the hardest emission  $p_T$
- Pair up the partons that are nearest in  $p_T$
- Generate an angular ordered shower associated with the paired parton, stopping at the angle of the paired partons (**truncated shower**)
- Generate all subsequent (vetoed) showers

## Example of truncated shower: $e^+e^-$



Nearby partons: 1,2

Truncated shower: 1,2 pair,  
from maximum angle to  $\theta$

1 and 2 shower from  $\theta$  to cutoff

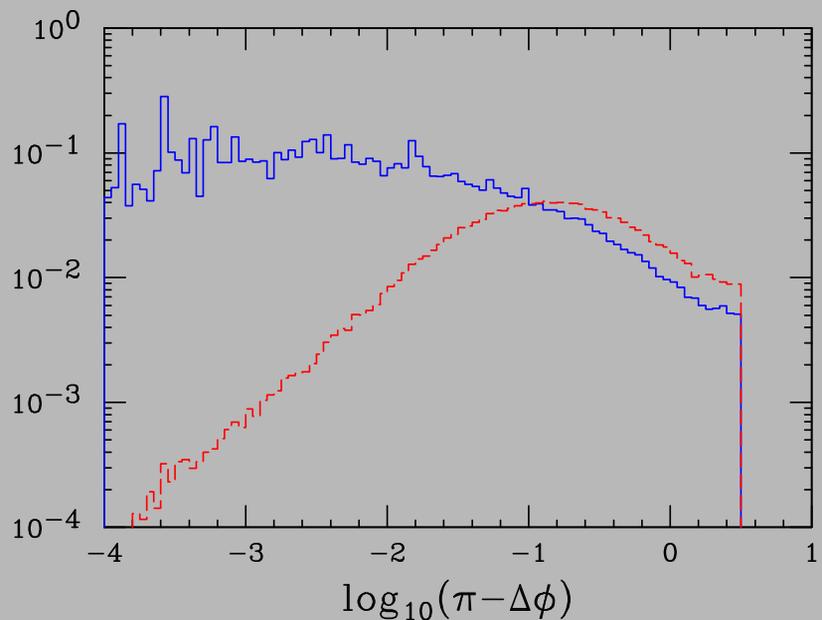
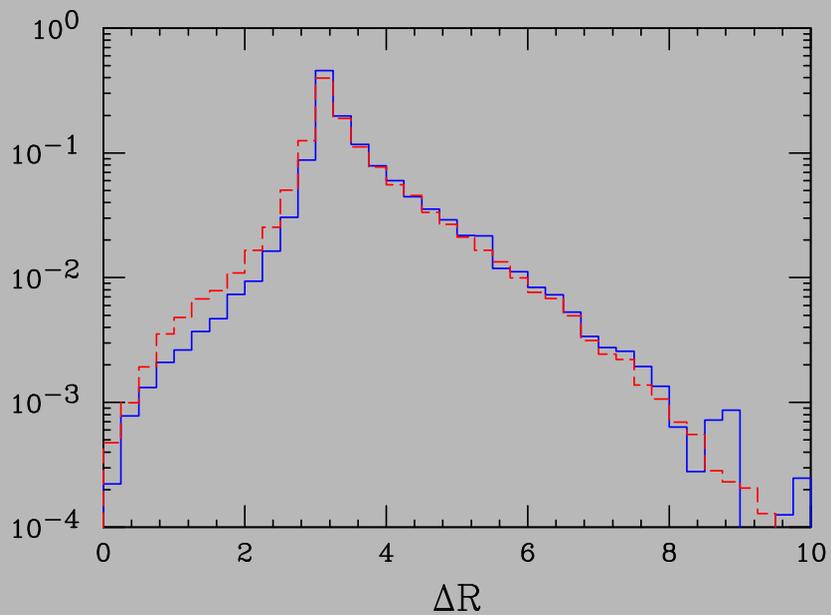
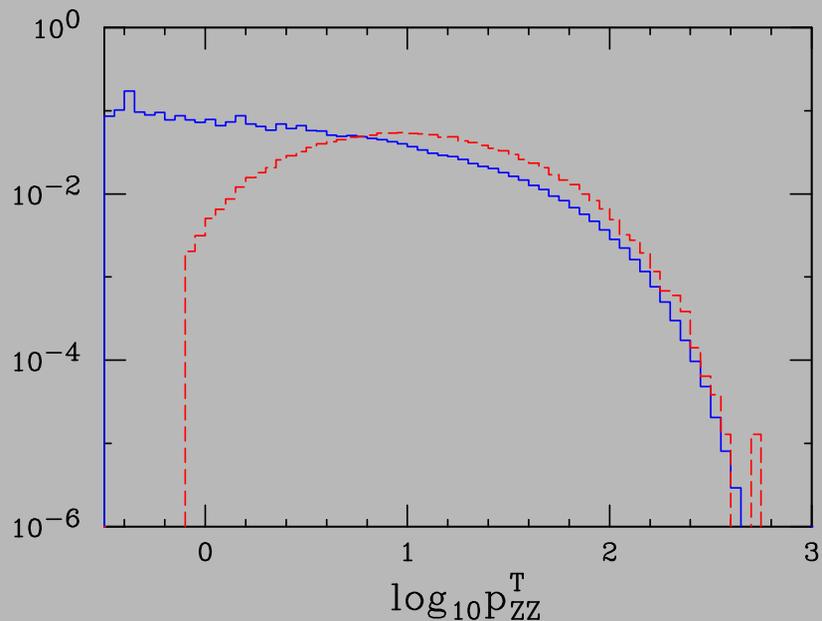
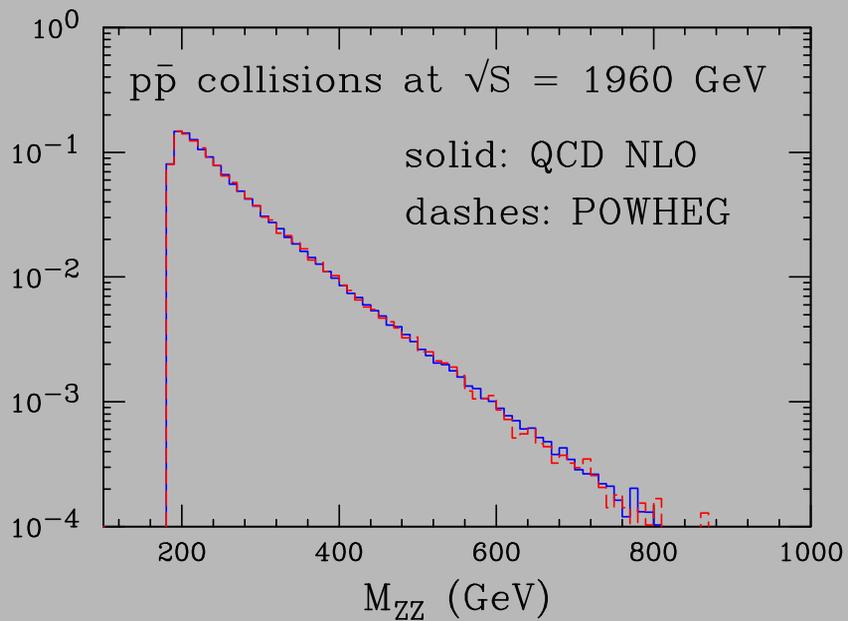
3 showers from maximum to cutoff

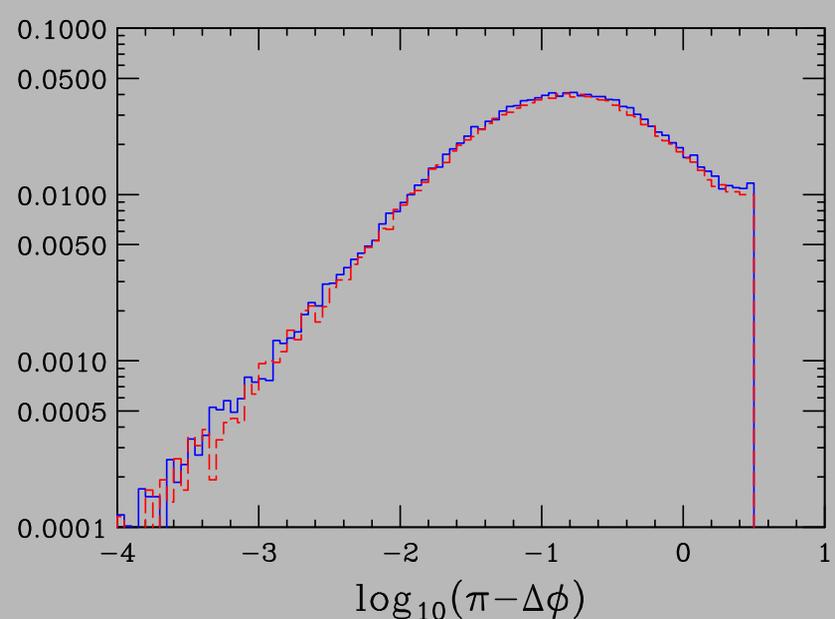
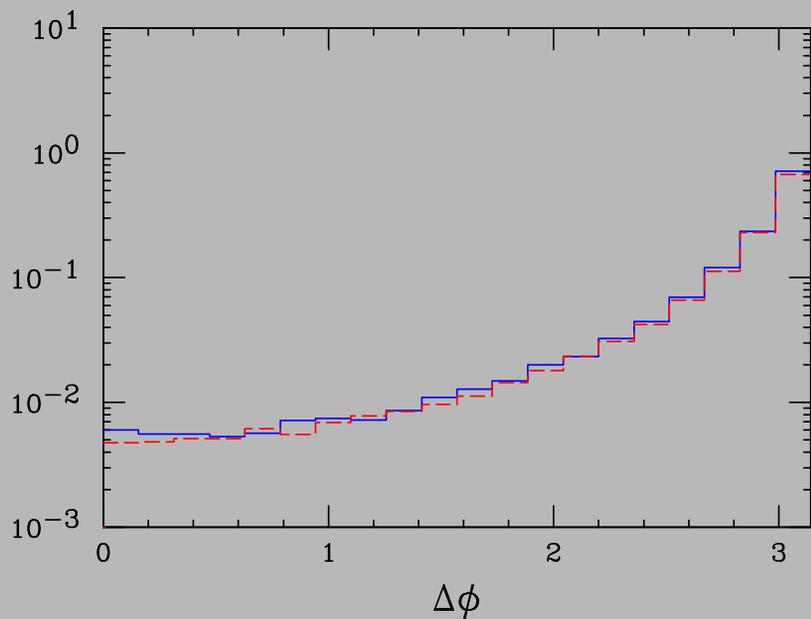
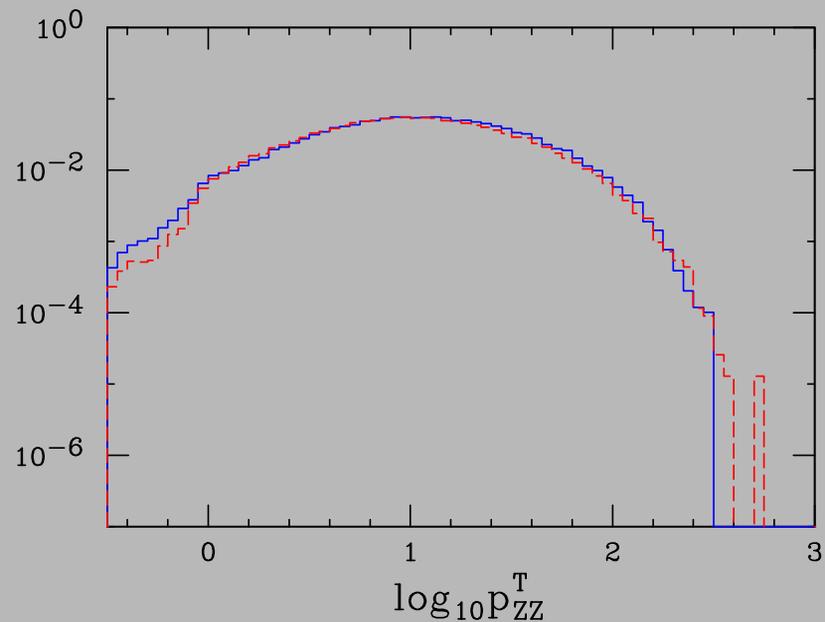
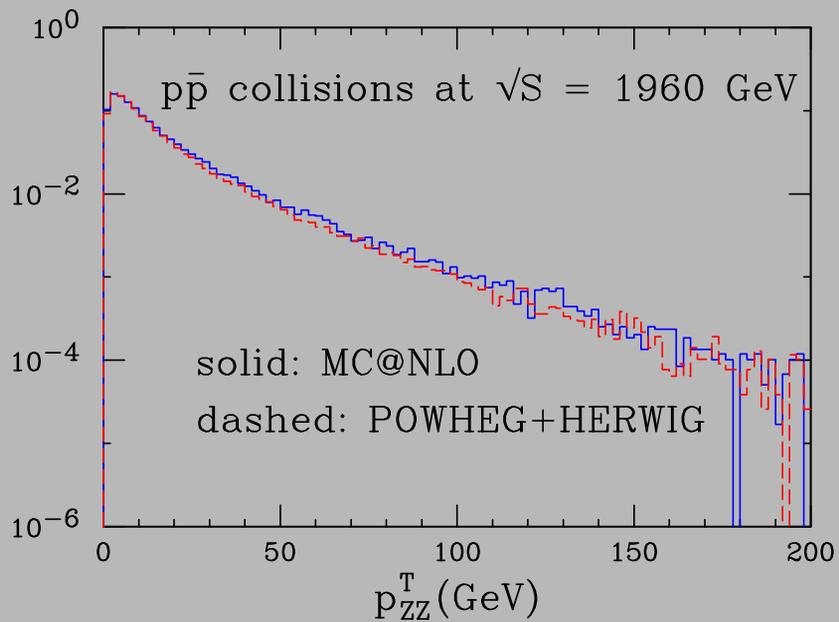
The truncated shower reintroduces coherent soft radiation from 1,2 at angles larger than  $\theta$  (Angular ordered SMC's generate those earlier).

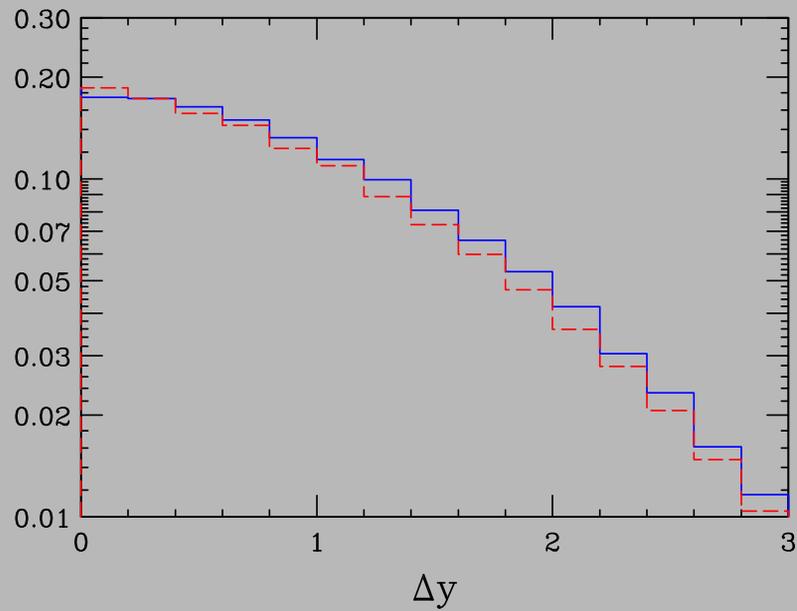
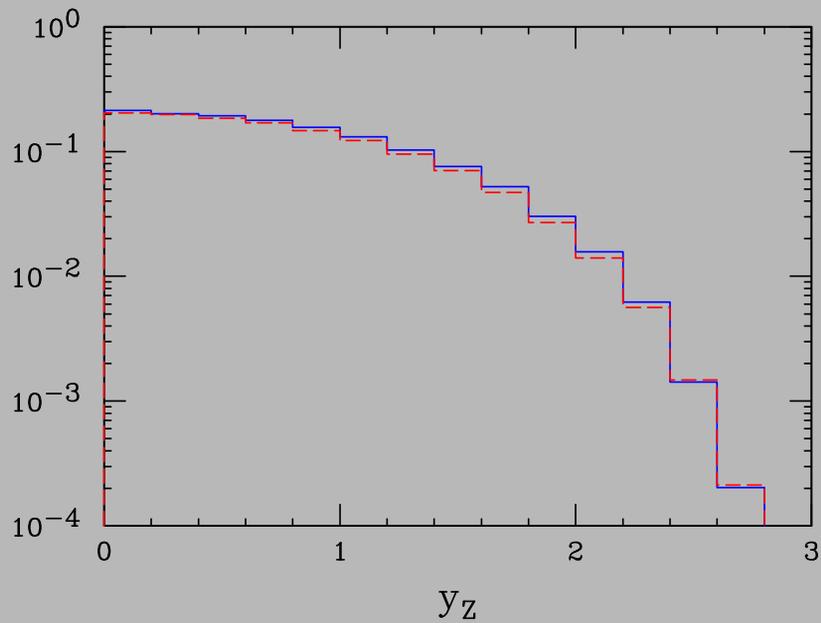
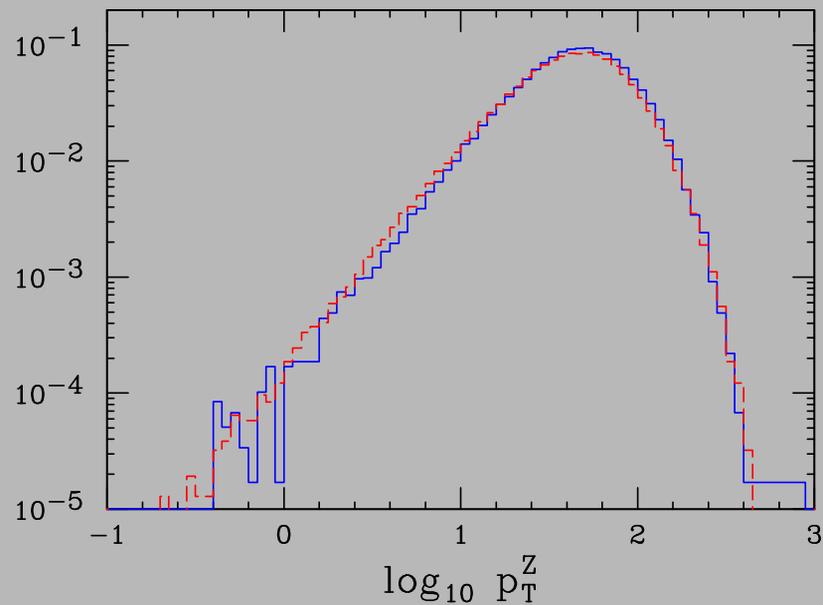
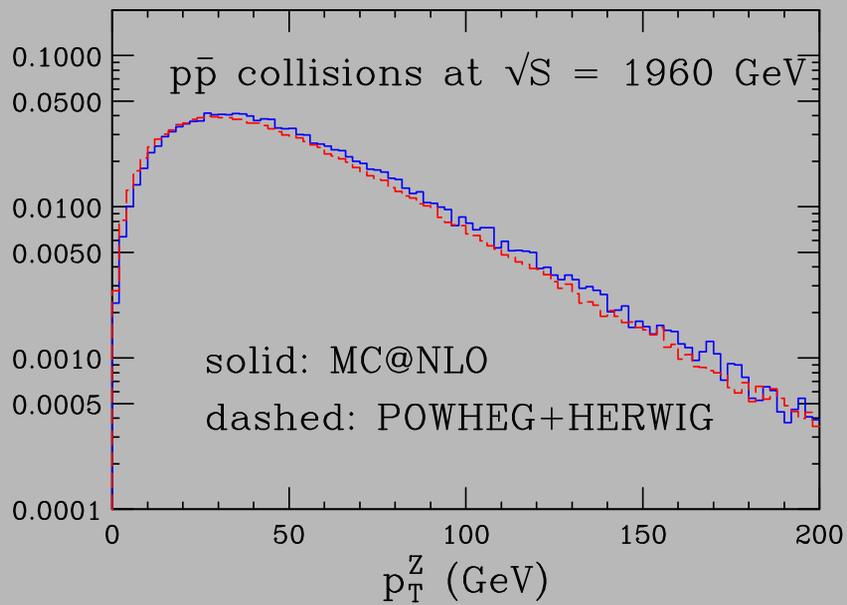
Truncated showers not yet implemented; work in progress with [S. Frixione](#).  
(No evidence of effects from their absence up to now)

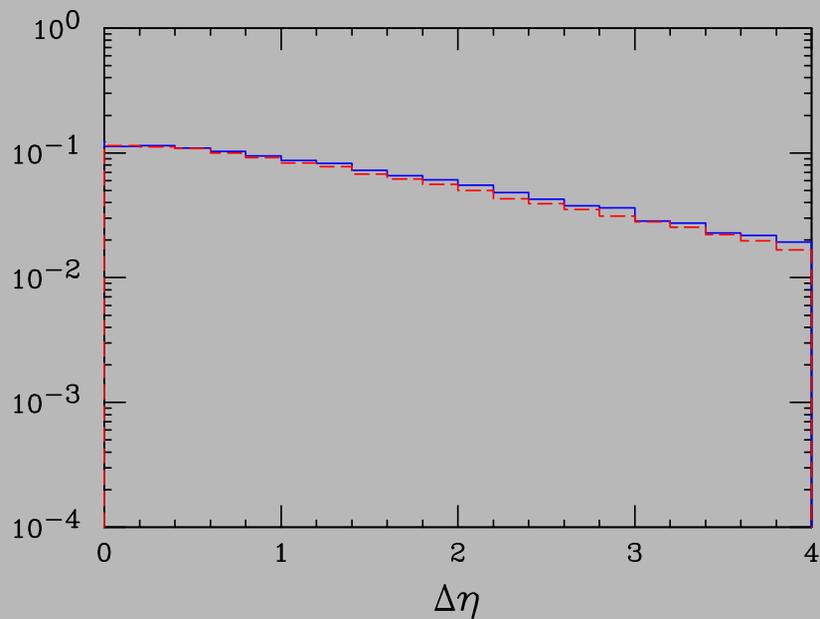
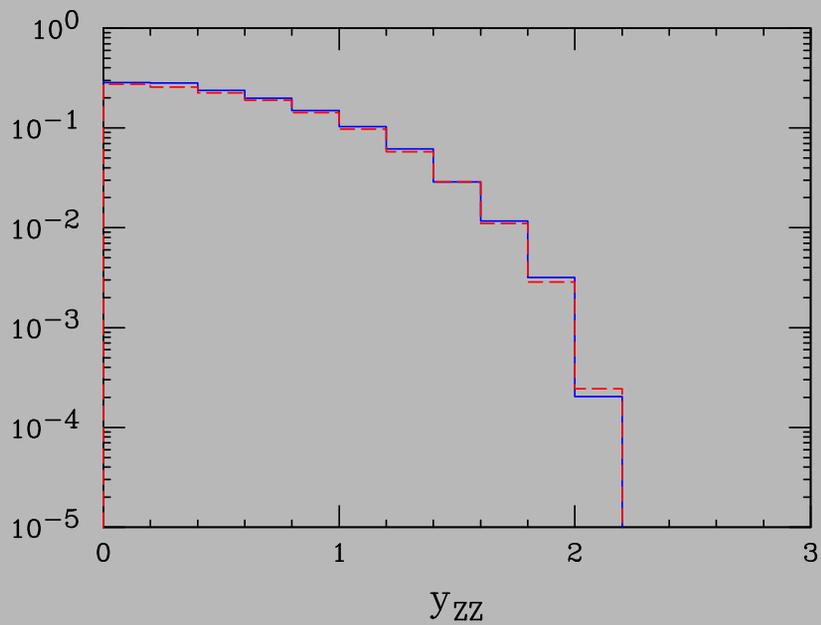
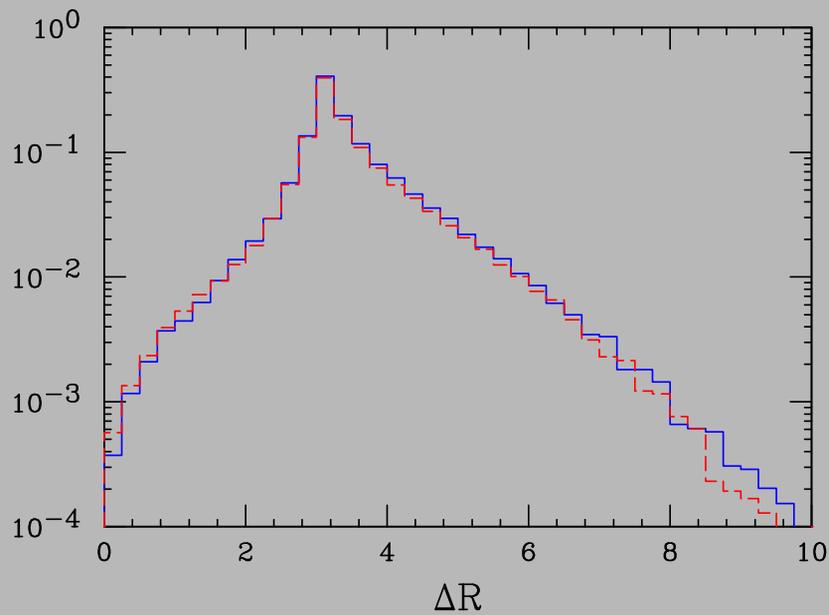
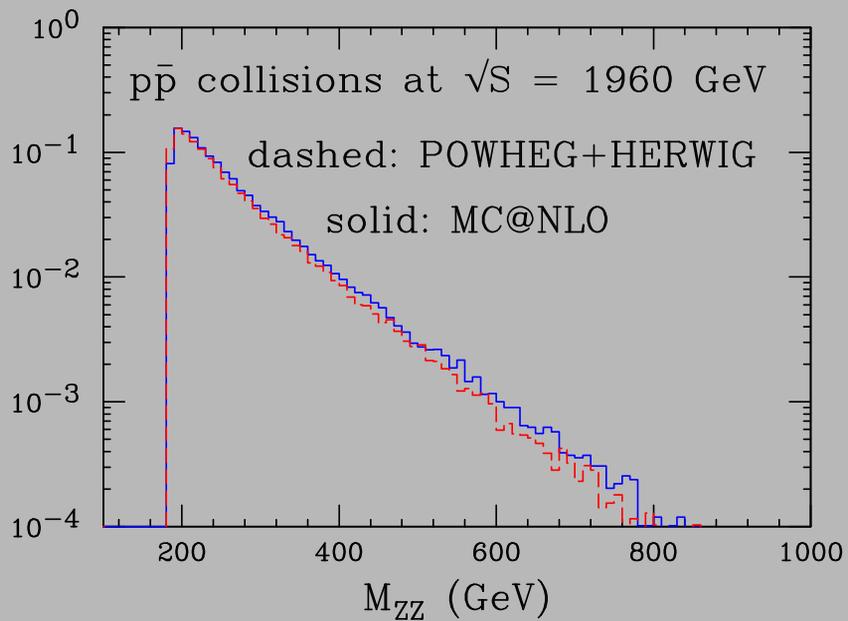
## Results

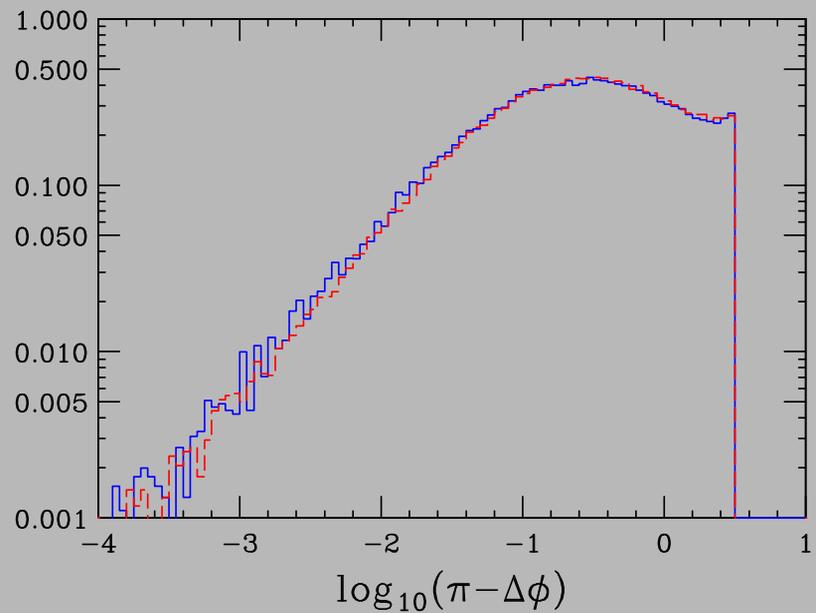
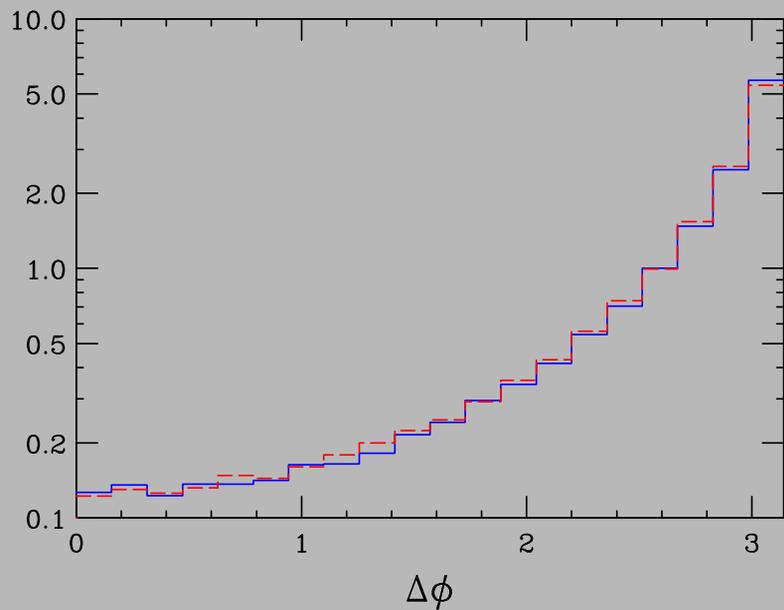
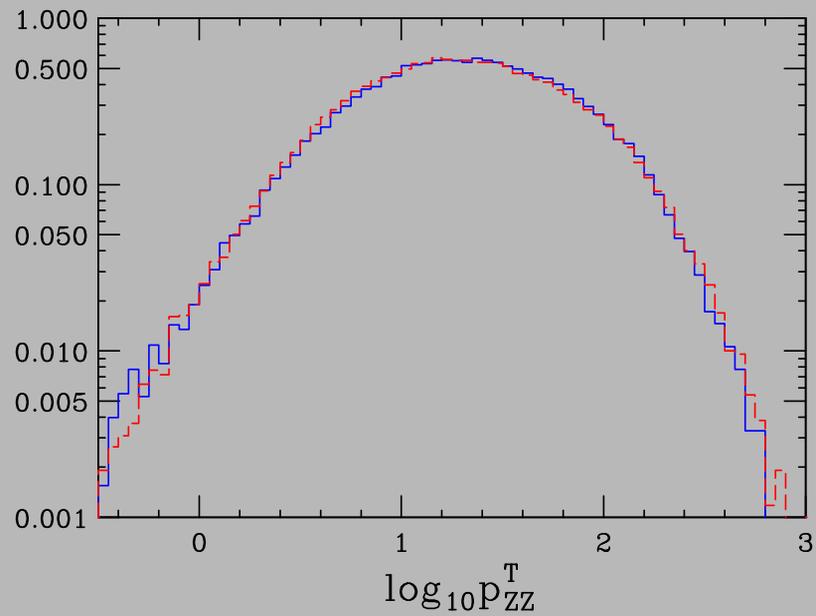
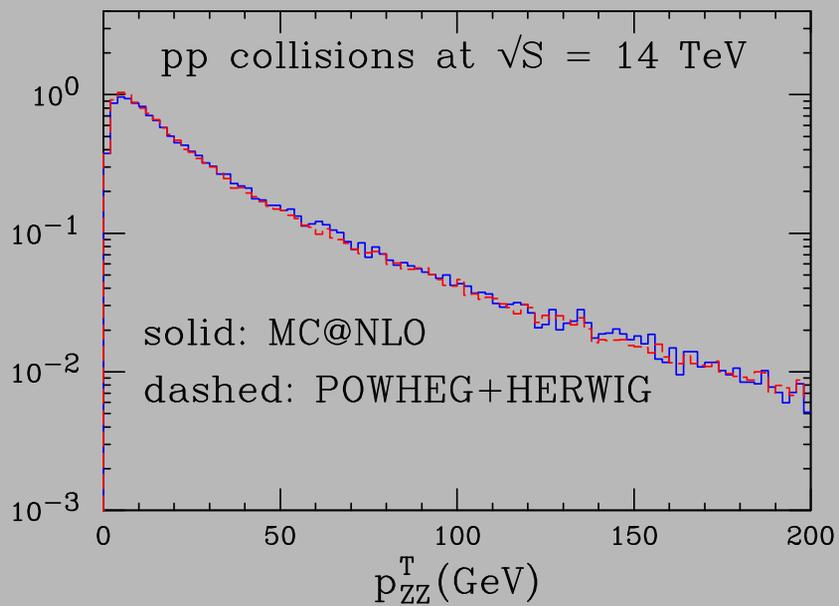
- We studied distributions for LHC and the TEVATRON
- We interfaced the output to both HERWIG and PYTHIA
- No truncated showers for now
- Haven't found yet significant differences from from MC@NLO

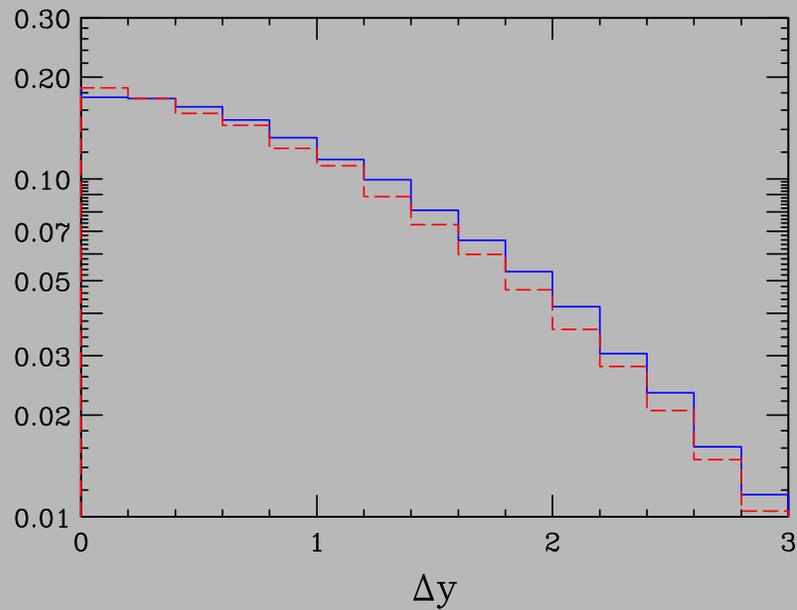
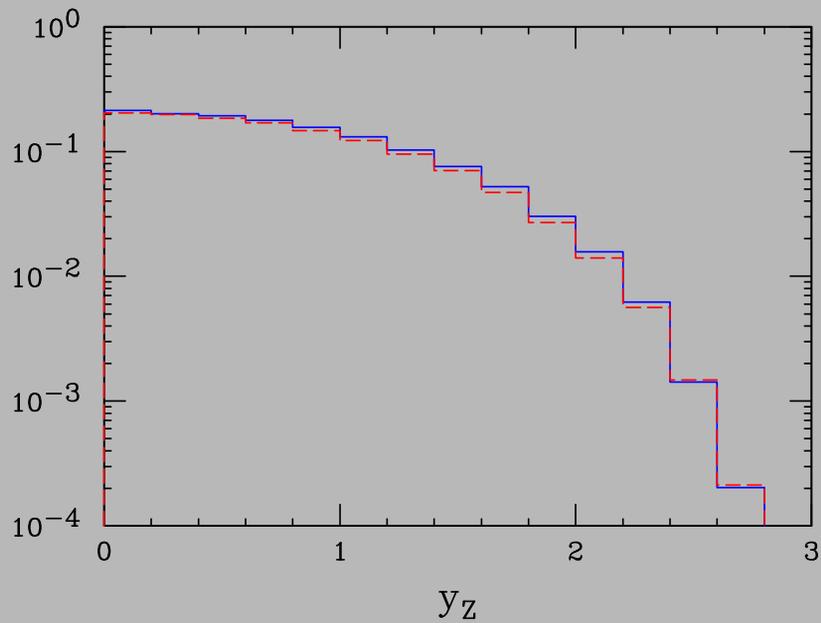
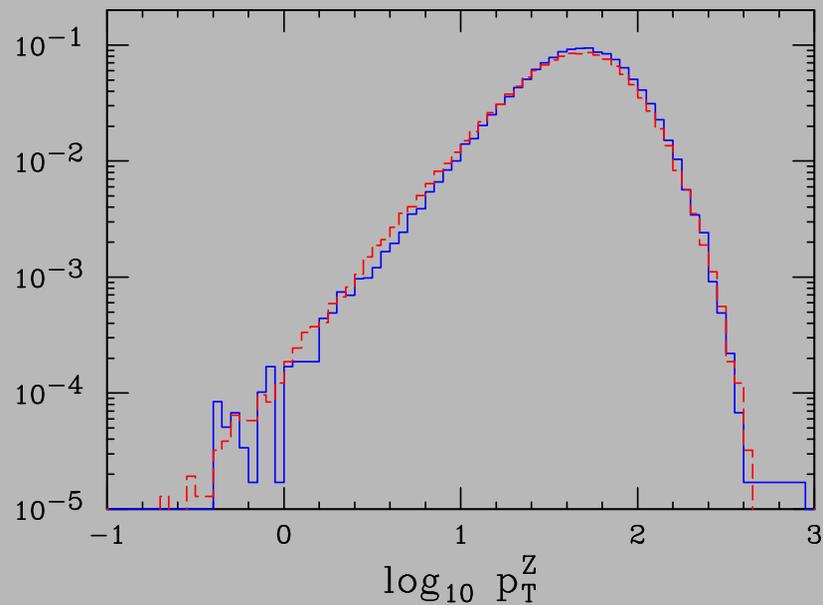
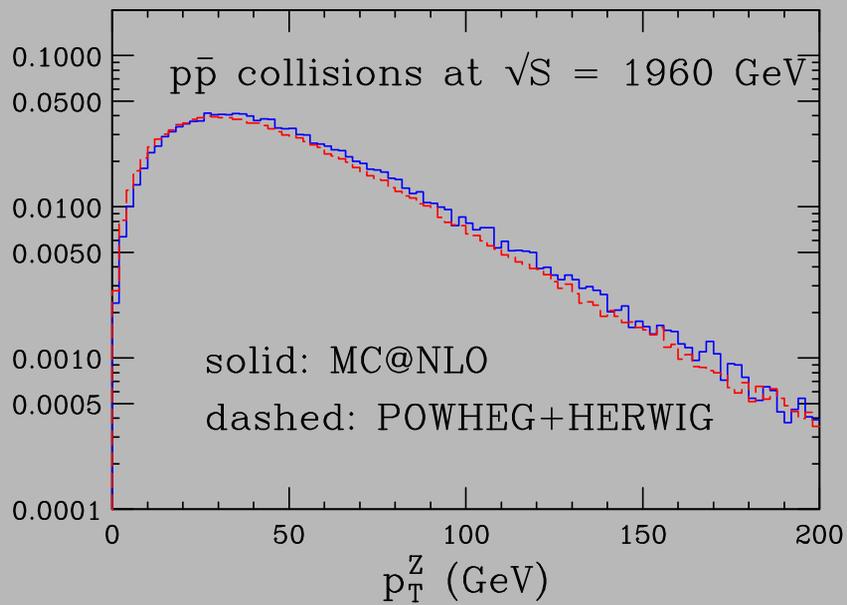


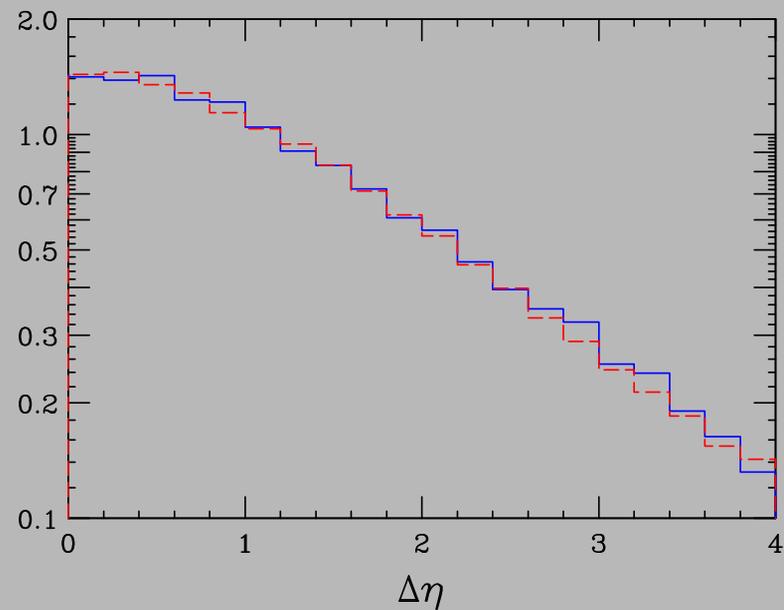
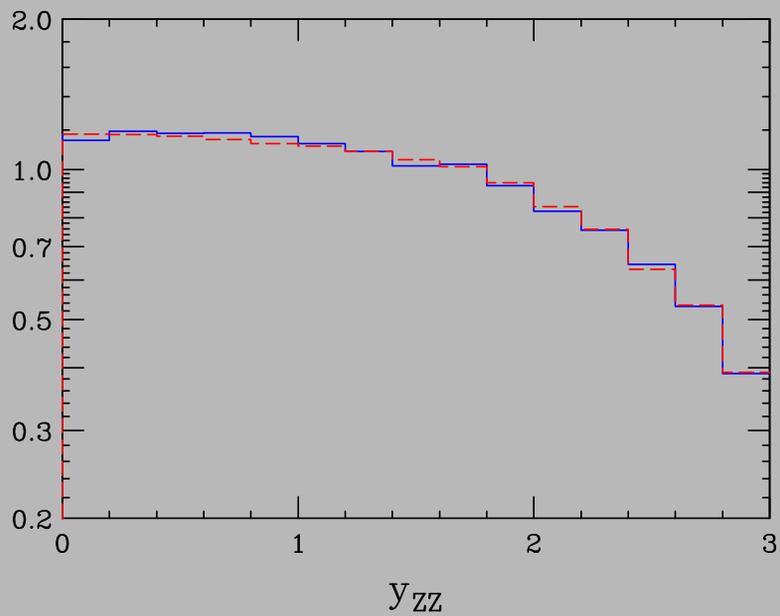
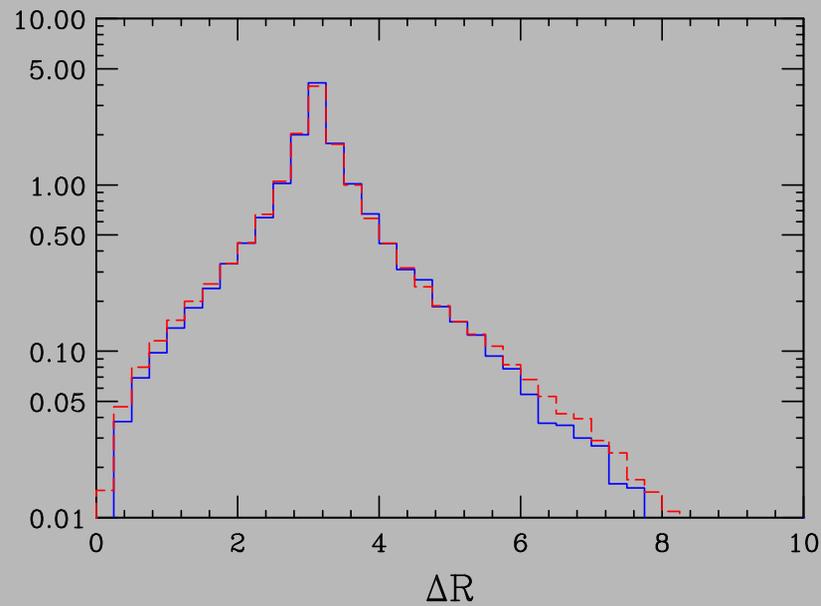
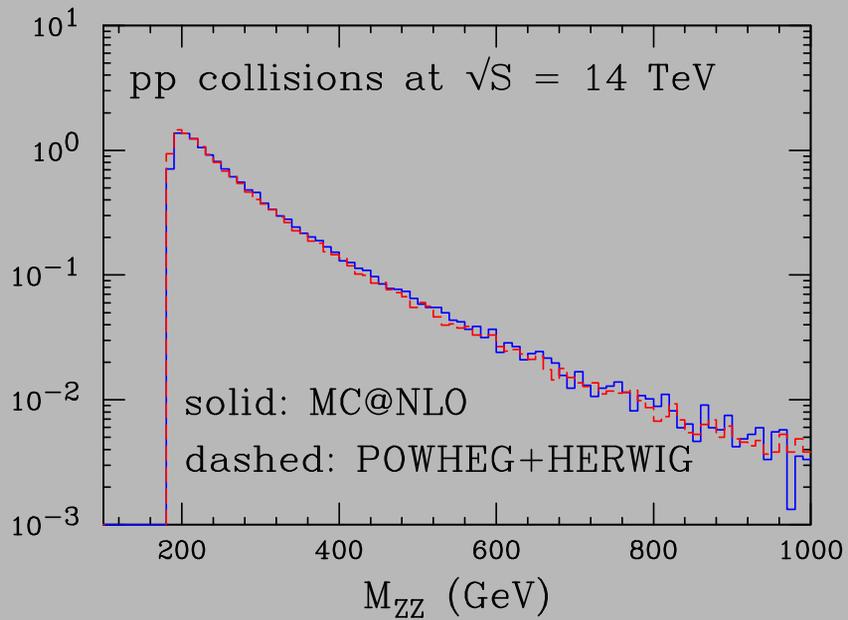


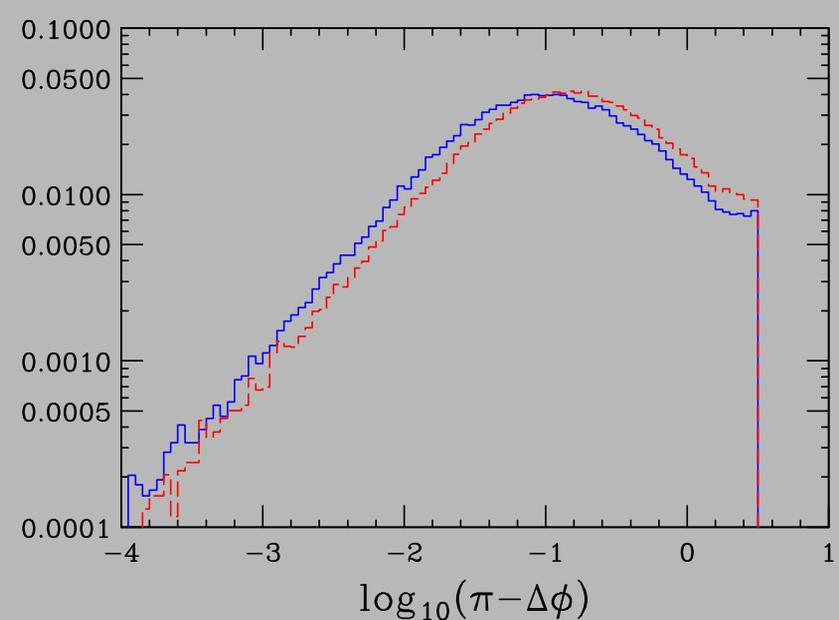
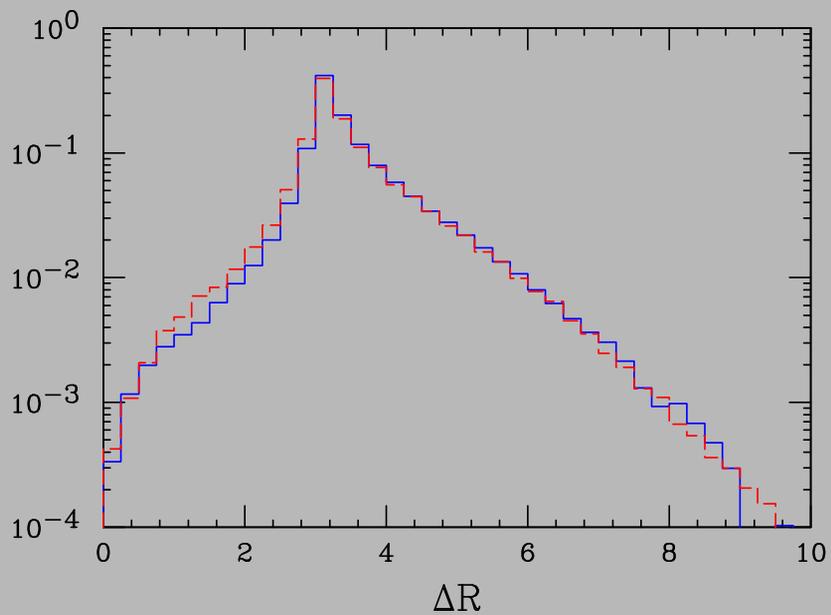
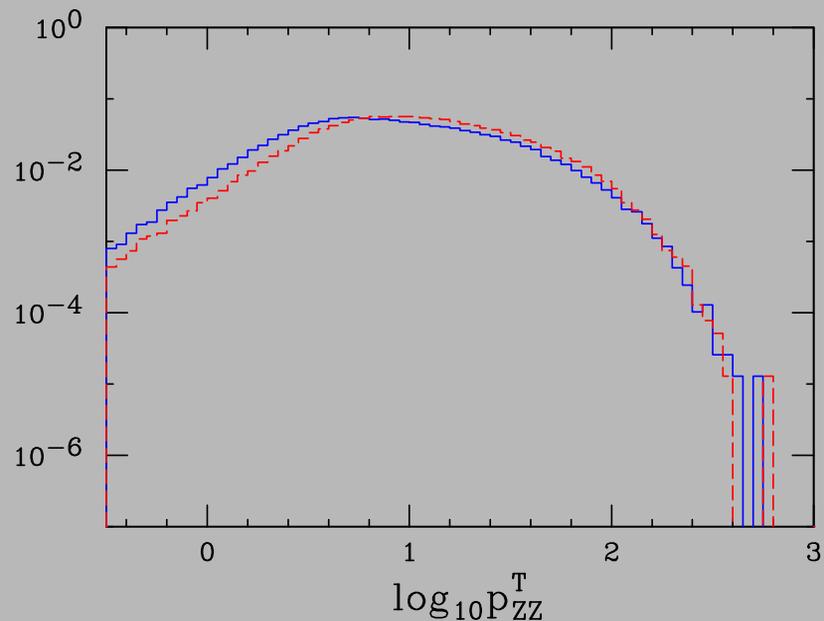
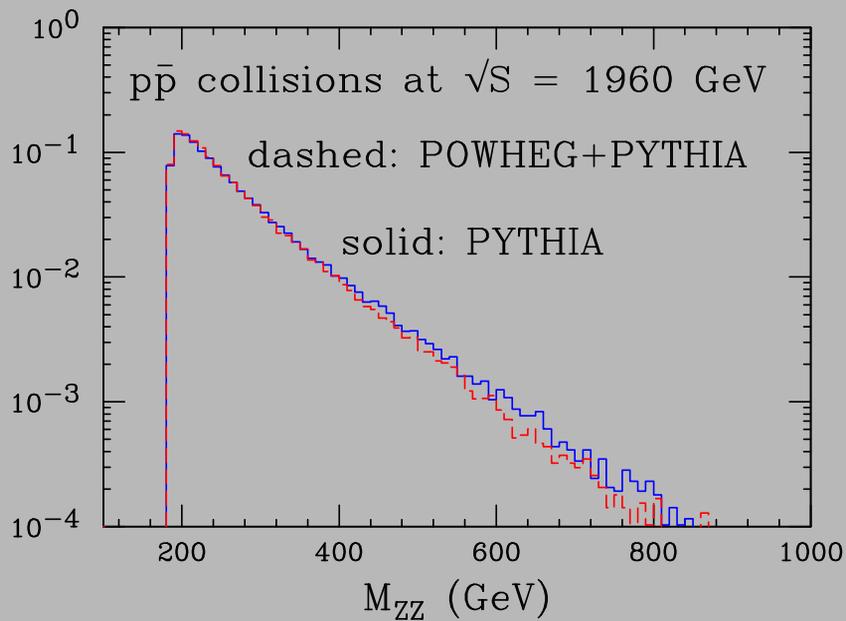


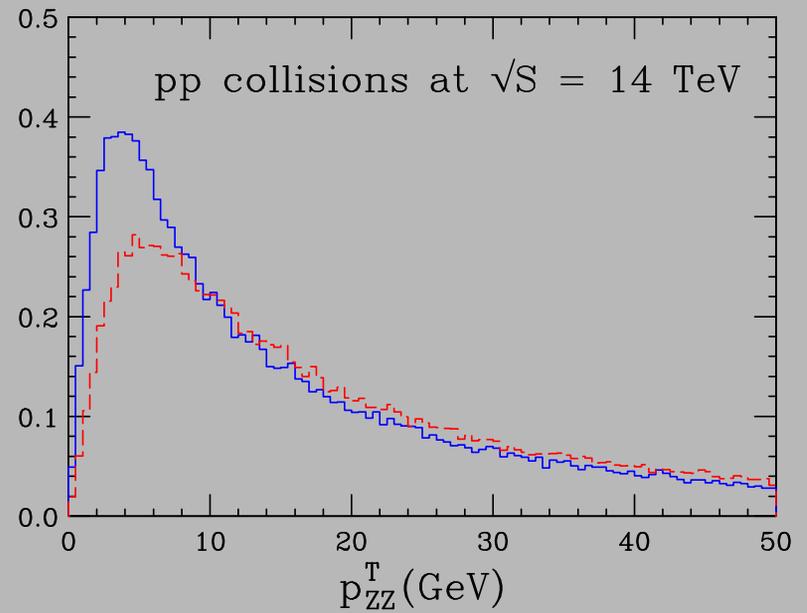
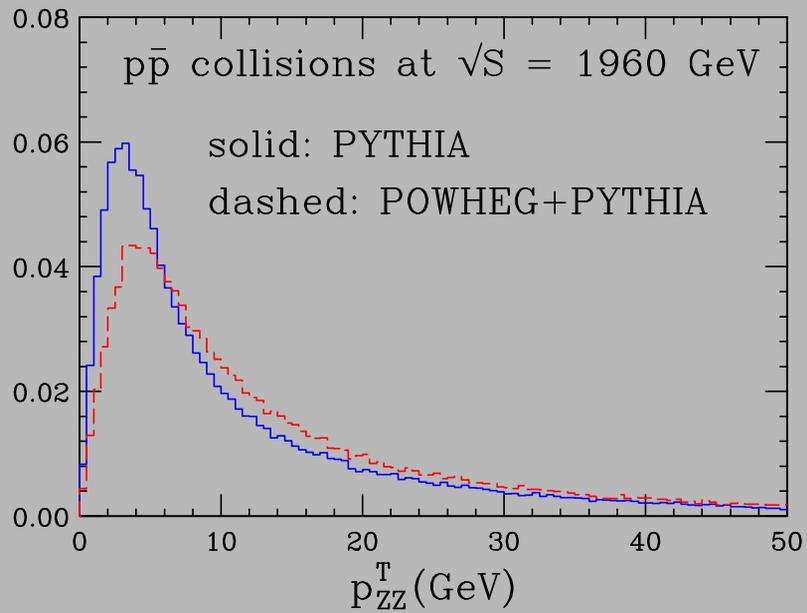












# Accuracy of the Sudakov Form Factor

POWHEG's Sudakov FF has the form (with  $c \approx 1$ )

$$\Delta_t = \exp \left[ - \int_t^{Q^2} \frac{dk_T^2}{k_T^2} \frac{\alpha_S(c k_T^2)}{\pi} \left\{ A \log \frac{M^2}{k_T^2} + B \right\} \right]$$

We know that the NLL Sudakov form factor has the form

$$\Delta_t^{\text{NLL}} = \exp \left[ - \int_t^{Q^2} \frac{dk_T^2}{k_T^2} \frac{\alpha_S(k_T^2)}{\pi} \left\{ \left( A_1 + A_2 \frac{\alpha_S(k_T^2)}{\pi} \right) \log \frac{M^2}{k_T^2} + B \right\} \right]$$

provided the colour structure of the process is sufficiently simple ( $\leq 3$  coloured legs). Can use this to fix  $c$  in POWHEG's Sudakov FF. (Suggested in (Catani, Webber, Marchesini, 1991) for HERWIG)  
 $\geq 4$  coloured legs: exponentiation only holds in LL, or large  $N_c$

Summarizing:

POWHEG Sudakov is: always LL accurate,

NLL accurate for  $\leq 3$  coloured legs, NLL accurate in leading  $N_c$  in all cases.

# Some topics on general formulation of POWHEG

Frixione, Oleari, P.N., work in progress

Extension to the general case only a matter of bookkeeping;  
POWHEG is fully general, can be applied in any subtraction framework.

We look in details at POWHEG in

- the FKS (Frixione, Kunszt, Signer)
- the CS (Catani, Seymour) subtraction frameworks.

## Flavour separation

There are several allowed flavour structures in the  $n$  body process. A flavour structure is a flavour assignment to the incoming and outgoing partons. The  $B$  and  $V$  contributions are labelled by the flavour structure index  $f_b$ .

There are several allowed flavour structures in the  $n + 1$  body process. Thus  $R$  is labelled by a flavour structure index  $f_r$ . Each component  $R_{f_r}$  has several singularity regions. We thus write

$$R = \sum_{\alpha_r} R^{\alpha_r}$$

where each  $R^{\alpha_r}$  has a specific flavour structure, and is singular in only one singular region. This partition of  $R$  is trivial to perform:

- FKS provides specific kinematic functions  $S_{\alpha_r}$ , with  $\sum_{\alpha_r} S_{\alpha_r} = 1$  that suppress all but one singular regions.
- in CS one can use instead  $S_{\alpha_r} = C_{\alpha_r} / (\sum_{\alpha_r} C_{\alpha_r})$  where  $C_{\alpha_r}$  are the dipole subtraction terms.

$\bar{B}$  carries an  $f_b$  index;

Sudakov FF also carries an  $f_b$  index:

$$\Delta^{f_b}(\Phi_n, p_T) = \exp \left\{ - \sum_{\alpha_r \in \{\alpha_r | f_b\}} \int \frac{[d\Phi_r R(\Phi_n, \Phi_r) \theta(k_T - p_T)]_{\alpha_r}}{B^{f_b}(\Phi_n)} \right\}$$

or

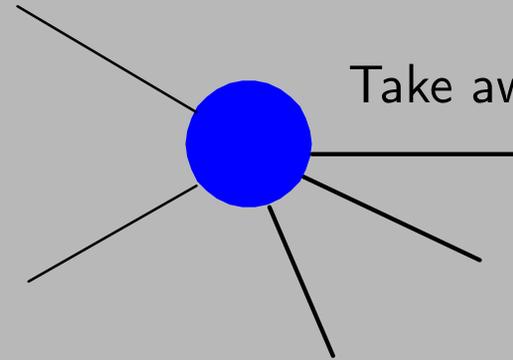
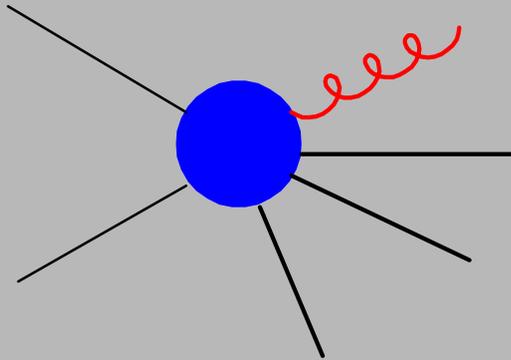
$$\Delta^{f_b}(\Phi_n, p_T) = \prod_{\alpha_r \in \{\alpha_r | f_b\}} \exp \left\{ - \sum \int \frac{[d\Phi_r R(\Phi_n, \Phi_r) \theta(k_T - p_T)]_{\alpha_r}}{B^{f_b}(\Phi_n)} \right\}$$

where

- $\{\alpha_r | f_b\}$  is the set of all singular regions having the underlying Born configuration with flavour structure  $f_b$ .
- $[\dots]_{\alpha_r}$  means that everything inside is relative to the  $\alpha_r$  singular term: thus  $R$  is  $R_{\alpha_r}$ , the parametrization  $(\Phi_n, \Phi_r)$  is the one appropriate to the  $\alpha_r$  singular region

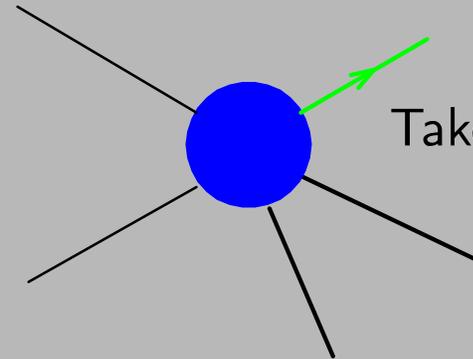
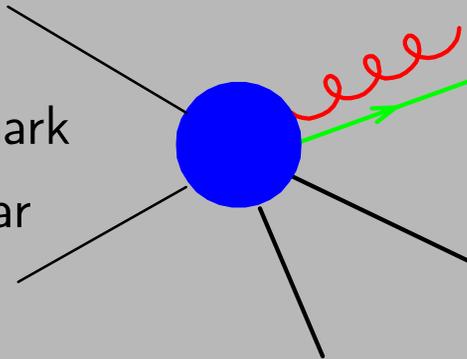
The last expression is closer to typical SMC's, with each emission considered independently.

Soft:



Take away gluon

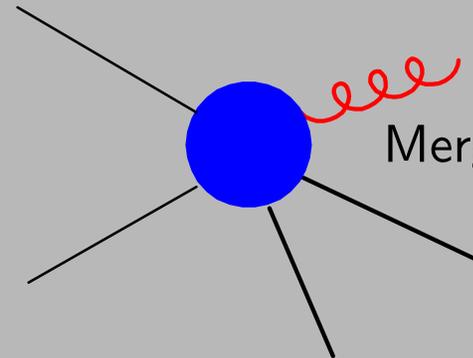
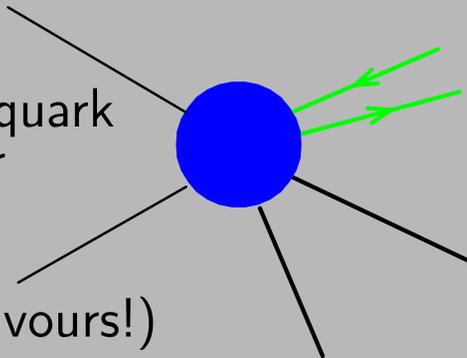
Gluon quark  
collinear



Take away gluon

Quark Antiquark  
collinear

(opposite flavours!)



Merge into gluon

# Work in progress: Heavy Flavour Production

with S. Frixione, M. Mangano and G. Ridolfi

NLO calculation much more complex than in  $ZZ$  production.

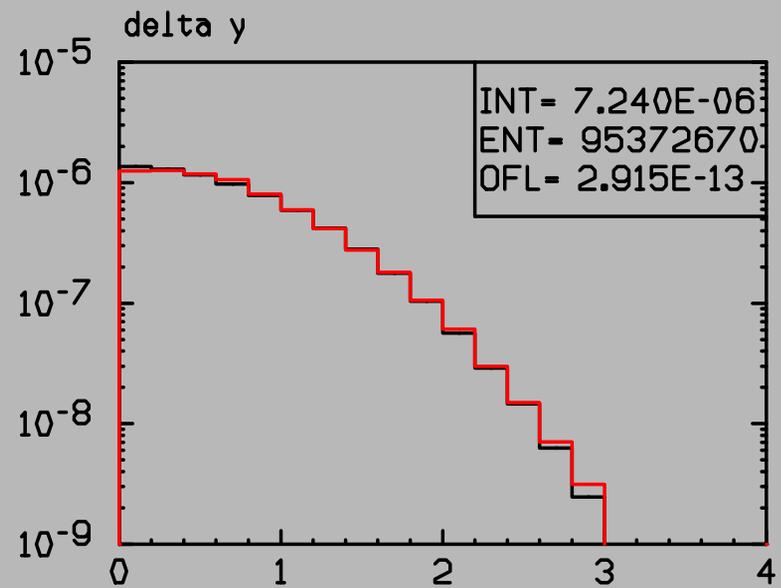
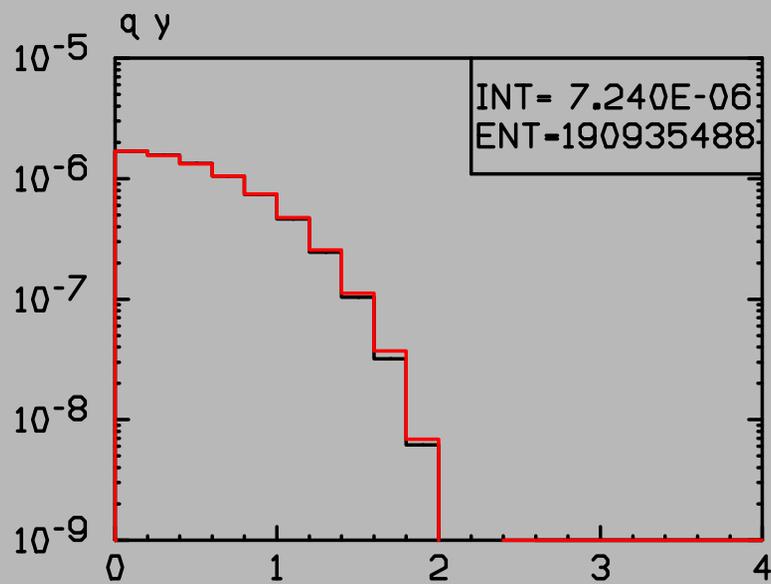
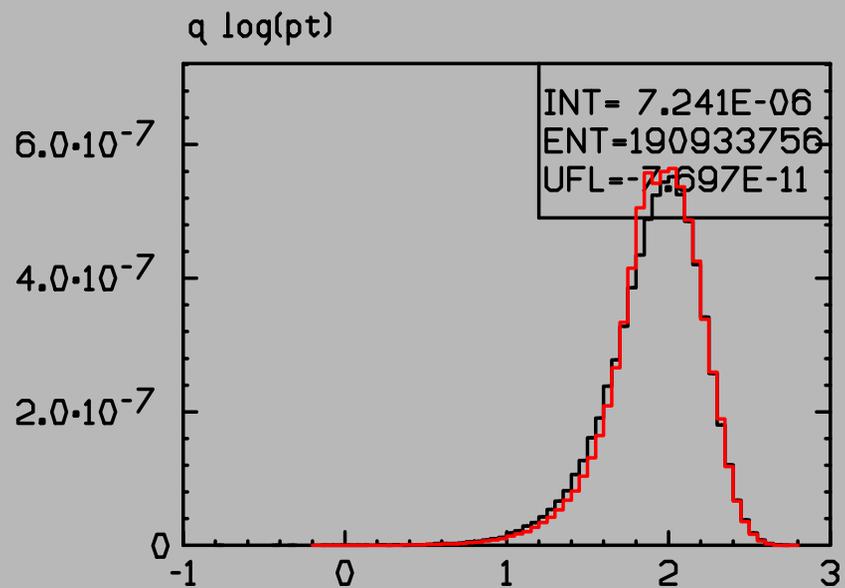
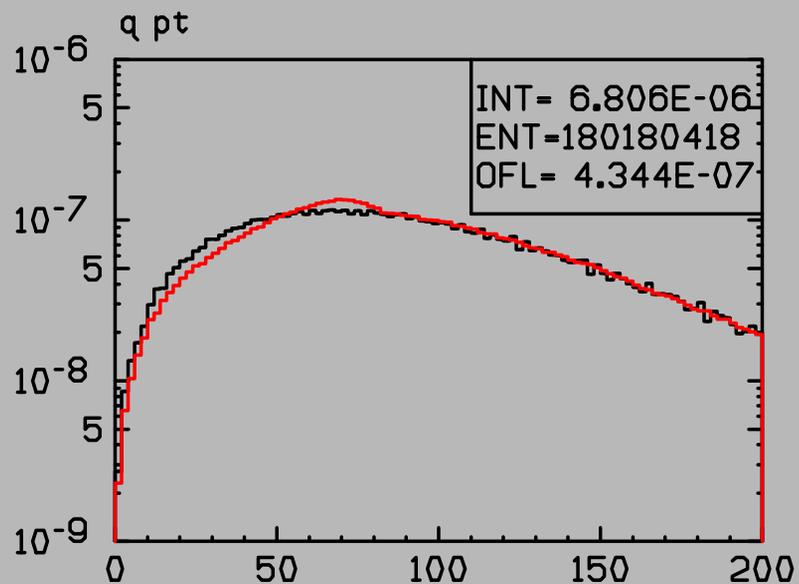
However, POWHEG implementation is **as easy**.

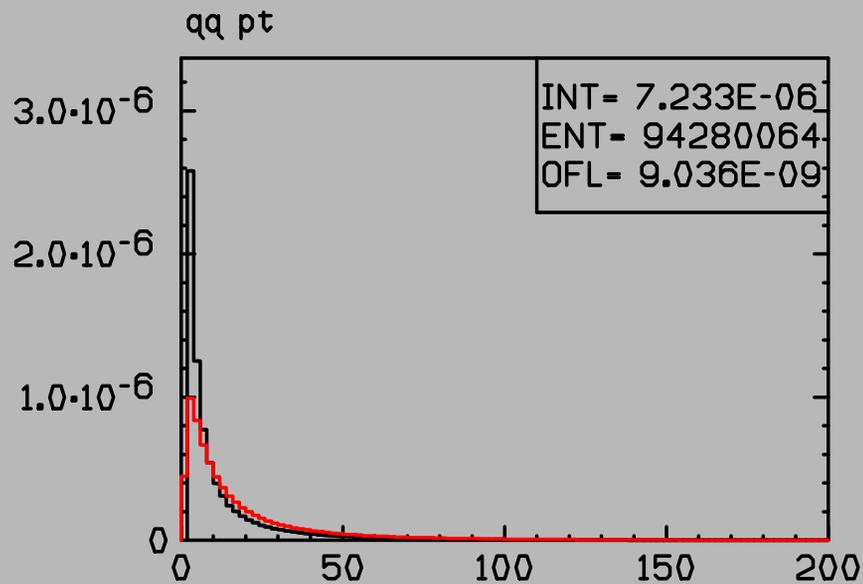
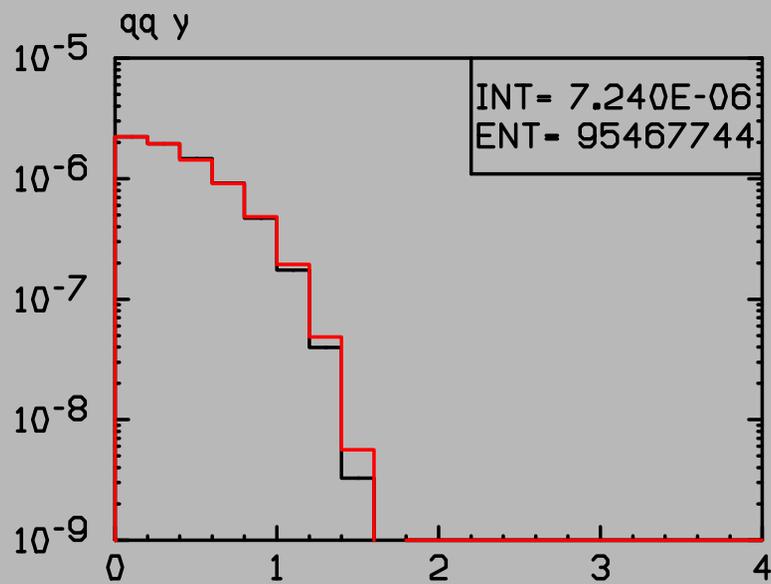
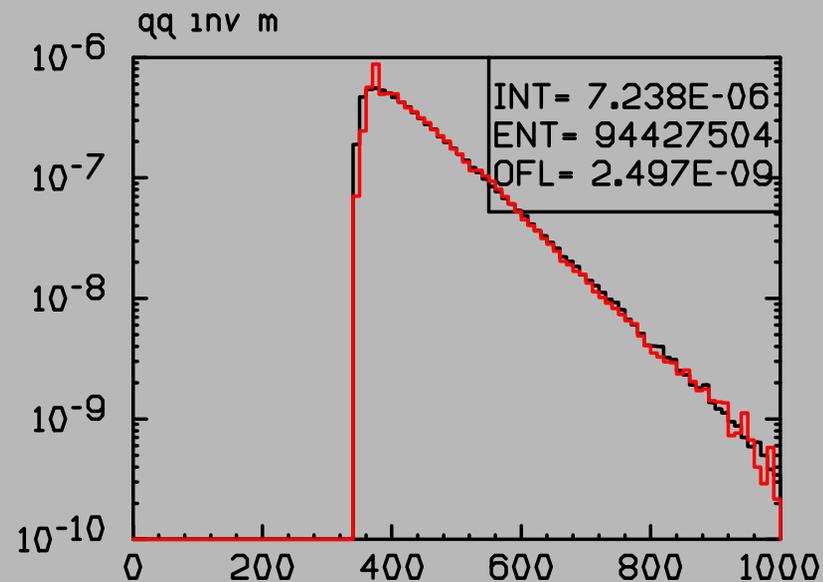
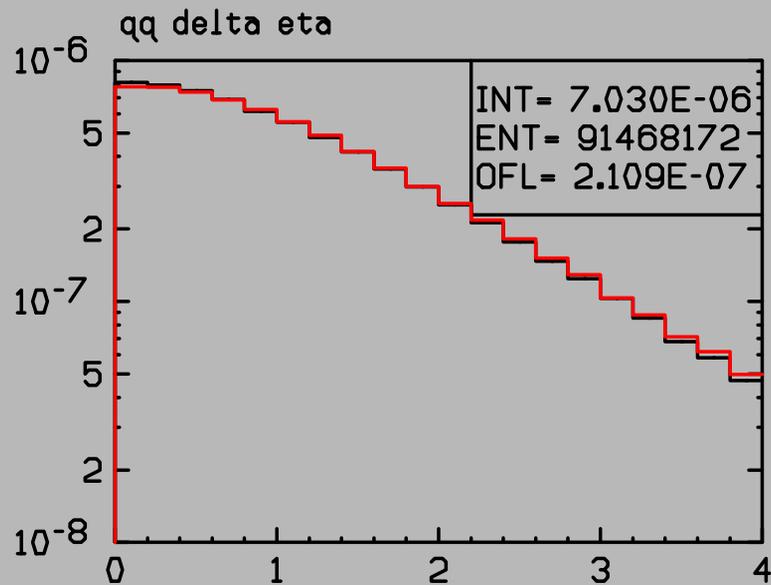
Detailed soft structure of HVQ production hidden in the ratio  $R/B$ .

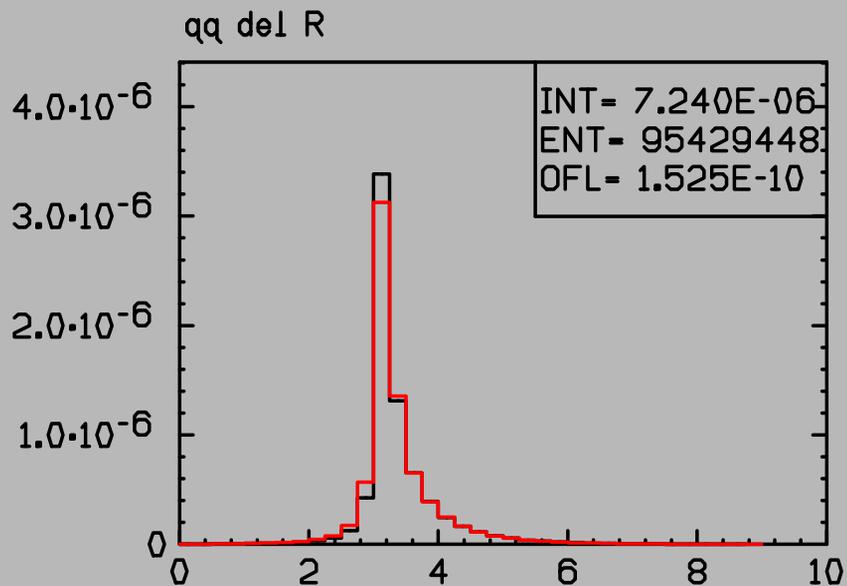
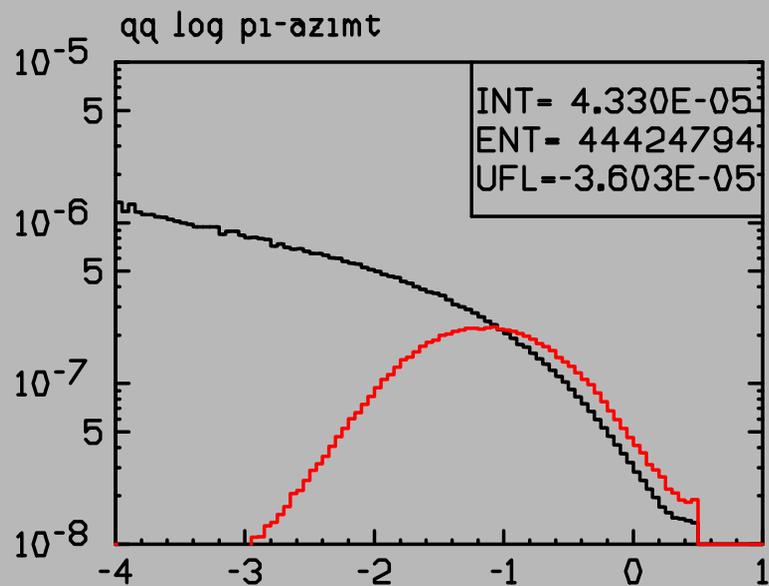
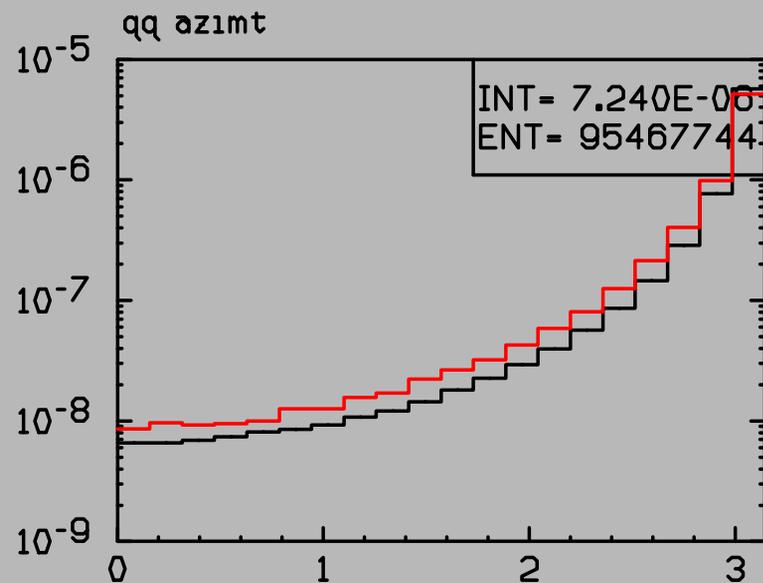
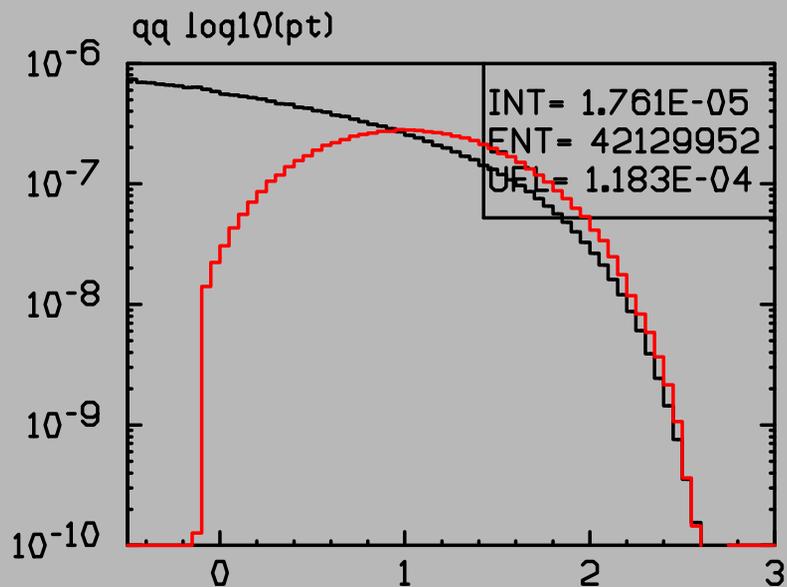
Status:

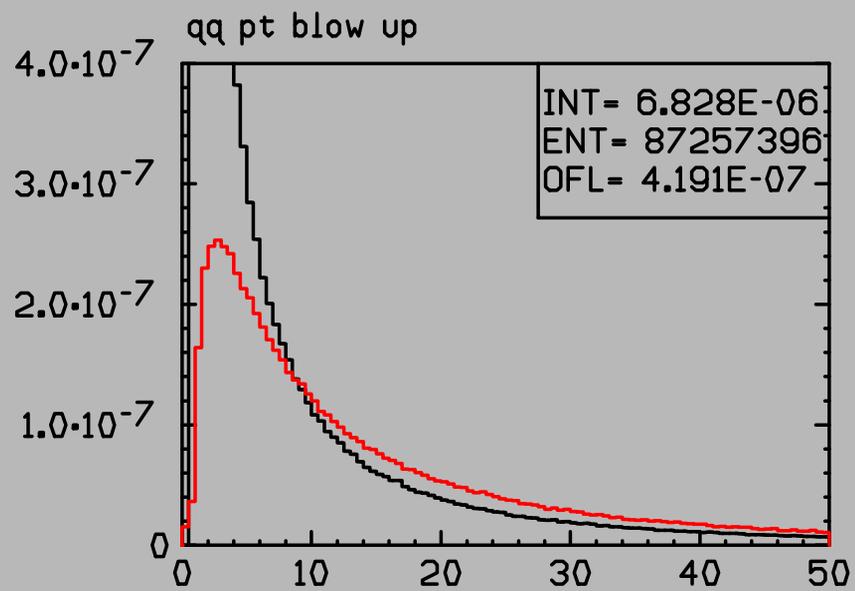
- POWHEG Generator ready and working
- LHI interface under work

Some raw plots follow ...









# Conclusions and Perspective

- POWHEG is a viable method for interfacing NLO and SMC
- It is easy to implement, does not require new NLO computations
- Does not require commitment to specific SMC implementations
- Its output is closer to traditional SMC's: positive weighted events
- To get it going, we will implement a number of processes: vector bosons and boson pairs, Higgs, Heavy Flavour, etc.
- We collect and publish material to make it easy for others to implement POWHEG with their NLO calculation