

Final-state observables in QCD

Resummation vs MC simulations

Andrea Banfi¹

¹Università degli Studi di Milano-Bicocca
Dipartimento di Fisica "G. Occhialini"

24 October 2006 / MCWS Workshop

Resummation and exponentiation

- Any **short distance** cross section can be written as a **power series in $\alpha_s(Q)$**

$$d\sigma = \underbrace{d\sigma_0}_{\text{LO}} + \alpha_s \times \underbrace{d\sigma_1}_{\text{NLO}} + \alpha_s^2 \times \underbrace{d\sigma_2}_{\text{NNLO}} + \dots$$

$Q \gg \Lambda_{QCD} \Rightarrow \alpha_s(Q) \ll 1$, the above expansion is justified

- Coefficients may be large: two physical scales Q and Q_0
Large logarithms $L = \ln Q/Q_0$ from incomplete IRC real-virtual cancellations
- Resummation** = reorganisation of the perturbative expansion

$$d\sigma =$$

Soft and collinear \Rightarrow double logarithms $\alpha_s L^2$

Hard collinear or soft large angle \Rightarrow single logarithms $\alpha_s L$

Finite virtual and exact SC phase space \Rightarrow constant $C(\alpha_s)$

Resummation and exponentiation

- Any **short distance** cross section can be written as a **power series in $\alpha_s(Q)$**

$$d\sigma = \underbrace{d\sigma_0}_{\text{LO}} + \alpha_s \times \underbrace{d\sigma_1}_{\text{NLO}} + \alpha_s^2 \times \underbrace{d\sigma_2}_{\text{NNLO}} + \dots$$

$Q \gg \Lambda_{QCD} \Rightarrow \alpha_s(Q) \ll 1$, the above expansion is justified

- Coefficients may be large: two physical scales Q and Q_0
Large logarithms $L = \ln Q/Q_0$ from incomplete IRC real-virtual cancellations
- Resummation** = reorganisation of the perturbative expansion

$$d\sigma =$$

Soft and collinear \Rightarrow double logarithms $\alpha_s L^2$

Hard collinear or soft large angle \Rightarrow single logarithms $\alpha_s L$

Finite virtual and exact SC phase space \Rightarrow constant $C(\alpha_s)$

Resummation and exponentiation

- Any **short distance** cross section can be written as a **power series in $\alpha_s(Q)$**

$$d\sigma = \underbrace{d\sigma_0}_{\text{LO}} + \alpha_s \times \underbrace{d\sigma_1}_{\text{NLO}} + \alpha_s^2 \times \underbrace{d\sigma_2}_{\text{NNLO}} + \dots$$

$Q \gg \Lambda_{QCD} \Rightarrow \alpha_s(Q) \ll 1$, the above expansion is justified

- Coefficients may be large: two physical scales Q and Q_0
Large logarithms $L = \ln Q/Q_0$ from incomplete IRC real-virtual cancellations
- Resummation** = reorganisation of the perturbative expansion

$$d\sigma = 1 + \alpha_s(L^2 + L + 1) + \alpha_s^2(L^4 + L^3 + L^2 + L + 1) + \dots$$

Soft and collinear \Rightarrow double logarithms $\alpha_s L^2$

Hard collinear or soft large angle \Rightarrow single logarithms $\alpha_s L$

Finite virtual and exact SC phase space \Rightarrow constant $C(\alpha_s)$

Resummation and exponentiation

- Any **short distance** cross section can be written as a **power series in $\alpha_s(Q)$**

$$d\sigma = \underbrace{d\sigma_0}_{\text{LO}} + \alpha_s \times \underbrace{d\sigma_1}_{\text{NLO}} + \alpha_s^2 \times \underbrace{d\sigma_2}_{\text{NNLO}} + \dots$$

$Q \gg \Lambda_{QCD} \Rightarrow \alpha_s(Q) \ll 1$, the above expansion is justified

- Coefficients may be large: two physical scales Q and Q_0
Large logarithms $L = \ln Q/Q_0$ from incomplete IRC real-virtual cancellations
- Resummation** = reorganisation of the perturbative expansion

$$d\sigma = \exp(\underbrace{Lg_1(\alpha_s L)}_{\text{LL}} + \underbrace{g_2(\alpha_s L)}_{\text{NLL}} + \underbrace{\alpha_s g_3(\alpha_s L)}_{\text{NNLL}} + \dots) \times C(\alpha_s) + \text{suppressed terms}$$

Soft and collinear \Rightarrow double logarithms $\alpha_s L^2$

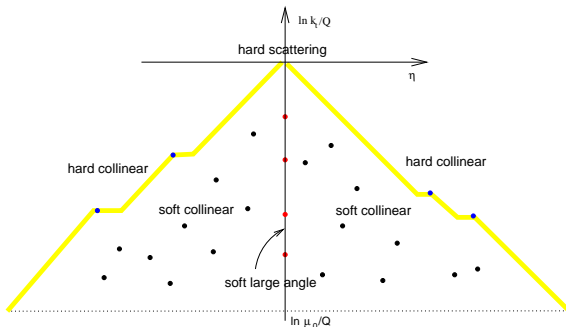
Hard collinear or soft large angle \Rightarrow single logarithms $\alpha_s L$

Finite virtual and exact SC phase space \Rightarrow constant $C(\alpha_s)$

Inclusive and final-state observables

Example: real and virtual corrections to **DY** pair production

[Collins, Soper, Sterman]

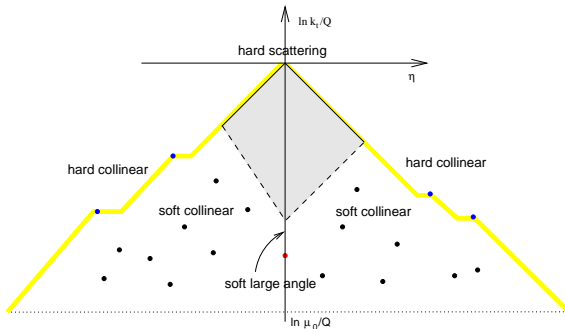


- virtual corrections are universal \Rightarrow exponentiation
- observable \Leftrightarrow veto on real emissions

Inclusive and final-state observables

Example: real and virtual corrections to **DY** pair production

[Collins, Soper, Sterman]

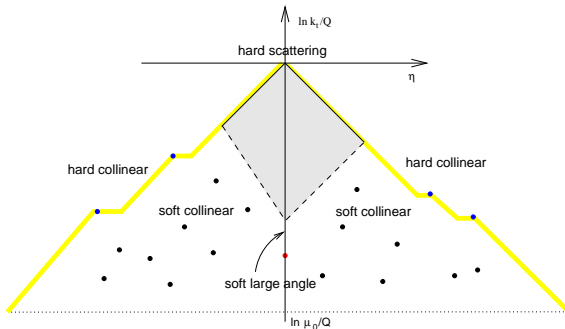


- **virtual corrections** are universal \Rightarrow **exponentiation**
- **observable** \Leftrightarrow veto on **real emissions**

Inclusive and final-state observables

Example: real and virtual corrections to **DY** pair production

[Collins, Soper, Sterman]



- **virtual corrections** are universal \Rightarrow **exponentiation**
- **observable** \Leftrightarrow veto on **real emissions**

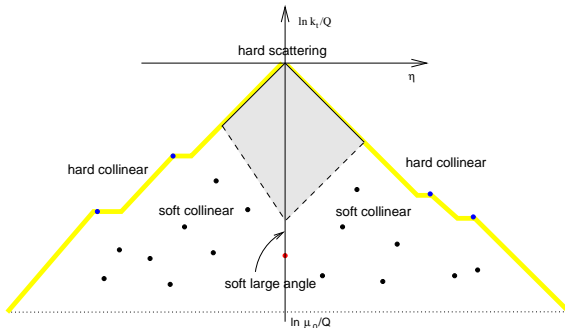
Inclusive observables

- real veto from **kinematics**
- exponentiation in **transform space**

Inclusive and final-state observables

Example: real and virtual corrections to **DY** pair production

[Collins, Soper, Sterman]



- **virtual corrections** are universal \Rightarrow **exponentiation**
- **observable** \Leftrightarrow veto on **real emissions**

Inclusive observables

- real veto from **kinematics**
- exponentiation in **transform space**

Final-state observables

- real veto from **direct measurement**
- need **approximated multi-parton ME**

Event-shape variables

- **Event-shape variables** $V(p_1, \dots, p_n)$ are **continuous measures** of the geometrical properties of hadron energy-momentum flow.
- **Thrust**: longitudinal particle alignment

$$T \equiv \frac{\max_{\vec{n}_T} \sum_i |\vec{p}_i \cdot \vec{n}_T|}{\sum_i |\vec{p}_i|} \quad \underbrace{\Sigma(1-T)}_{\text{th}} = \int_{1-T}^1 dT \underbrace{\frac{1}{\sigma} \frac{d\sigma}{dT}}_{\text{exp}} =$$

Event-shape variables

- **Event-shape variables** $V(p_1, \dots, p_n)$ are **continuous measures** of the geometrical properties of hadron energy-momentum flow.
- **Thrust**: longitudinal particle alignment

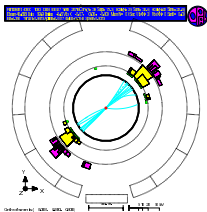
$$T \equiv \frac{\max_{\vec{n}_T} \sum_i |\vec{p}_i \cdot \vec{n}_T|}{\sum_i |\vec{p}_i|} \quad \underbrace{\Sigma(1-T)}_{\text{th}} = \int_{1-T}^1 dT \underbrace{\frac{1}{\sigma} \frac{d\sigma}{dT}}_{\text{exp}} =$$

Event-shape variables

- **Event-shape variables** $V(p_1, \dots, p_n)$ are **continuous measures** of the geometrical properties of hadron energy-momentum flow.
- **Thrust**: longitudinal particle alignment

$$T \equiv \frac{\max_{\vec{n}_T} \sum_i |\vec{p}_i \cdot \vec{n}_T|}{\sum_i |\vec{p}_i|} \quad \underbrace{\Sigma(1-T)}_{\text{th}} = \int_{1-T}^1 dT \underbrace{\frac{1}{\sigma} \frac{d\sigma}{dT}}_{\text{exp}} =$$

Pencil-like event: $1 - T \gtrsim 0$



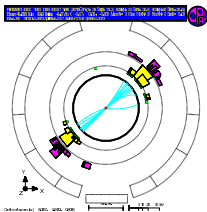
Event-shape variables

- **Event-shape variables** $V(p_1, \dots, p_n)$ are **continuous measures** of the geometrical properties of hadron energy-momentum flow.
- **Thrust**: longitudinal particle alignment

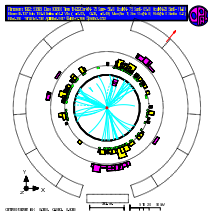
$$T \equiv \frac{\max_{\vec{n}_T} \sum_i |\vec{p}_i \cdot \vec{n}_T|}{\sum_i |\vec{p}_i|}$$

$$\underbrace{\Sigma(1-T)}_{\text{th}} = \int_{1-T}^1 dT \underbrace{\frac{1}{\sigma} \frac{d\sigma}{dT}}_{\text{exp}} =$$

Pencil-like event: $1 - T \gtrsim 0$



Planar event: $1 - T \simeq 1/3$



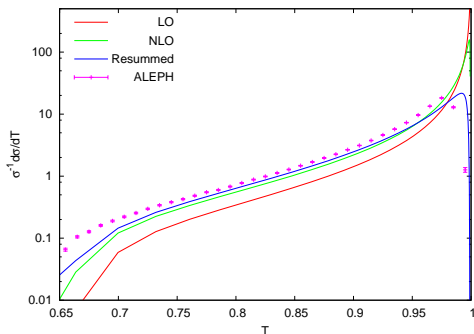
Physics of event shapes

Sensitive to QCD radiation \Rightarrow **measure α_s**

Large **non-perturbative** contributions

- $1/Q$ power corrections $\Rightarrow \alpha_s$ - α_0 fits
[Dokshitzer, Dasgupta, Marchesini, Lucenti, Salam, Webber]
- MC work very well \Rightarrow use **MC hadronisation**

Useful for **discovery?** \Rightarrow No investigation yet



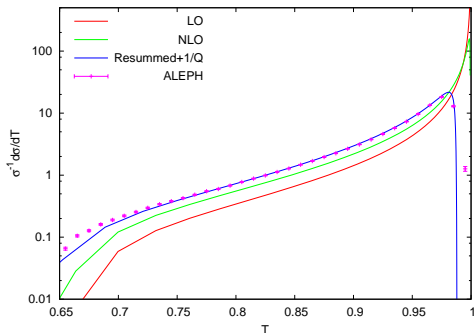
Physics of event shapes

Sensitive to QCD radiation \Rightarrow **measure** α_s

Large **non-perturbative** contributions

- $1/Q$ power corrections $\Rightarrow \alpha_s$ - α_0 fits
[Dokshitzer, Dasgupta, Marchesini, Lucenti, Salam, Webber]
- MC work very well \Rightarrow use **MC hadronisation**

Useful for **discovery**? \Rightarrow No investigation yet



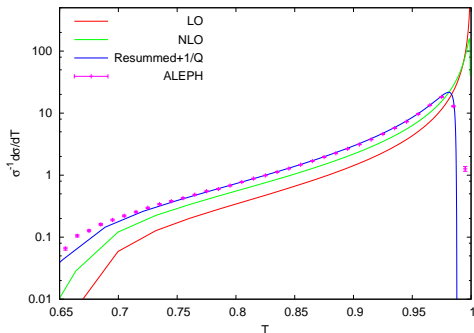
Physics of event shapes

Sensitive to QCD radiation \Rightarrow **measure** α_s

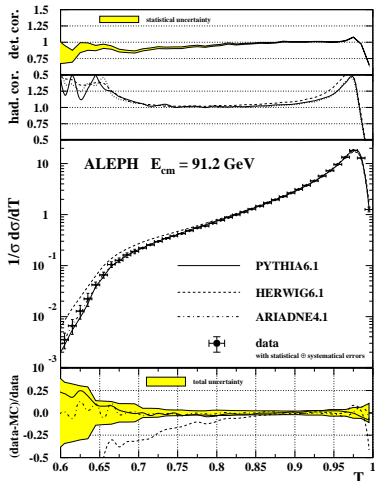
Large **non-perturbative** contributions

- $1/Q$ power corrections $\Rightarrow \alpha_s - \alpha_0$ fits
 [Dokshitzer, Dasgupta, Marchesini, Lucenti, Salam, Webber]
- MC work very well \Rightarrow use **MC hadronisation**

Useful for **discovery**? \Rightarrow No investigation yet



$$\alpha_s(M_Z) = 0.1202 \pm 0.0050$$



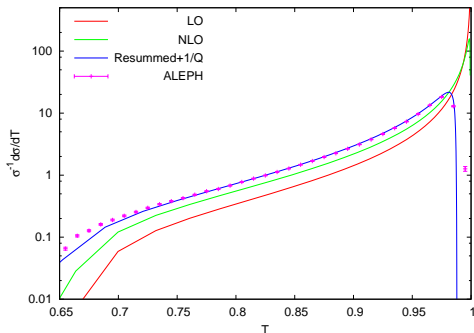
Physics of event shapes

Sensitive to QCD radiation \Rightarrow **measure α_s**

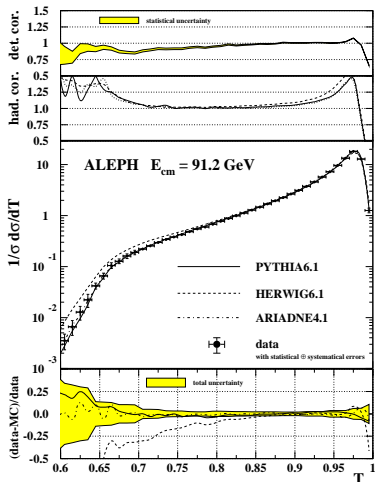
Large **non-perturbative** contributions

- $1/Q$ power corrections $\Rightarrow \alpha_s - \alpha_0$ fits
 [Dokshitzer, Dasgupta, Marchesini, Lucenti, Salam, Webber]
- MC work very well \Rightarrow use **MC hadronisation**

Useful for **discovery**? \Rightarrow No investigation yet



$$\alpha_s(M_Z) = 0.1202 \pm 0.0050$$



Recursive IRC safety

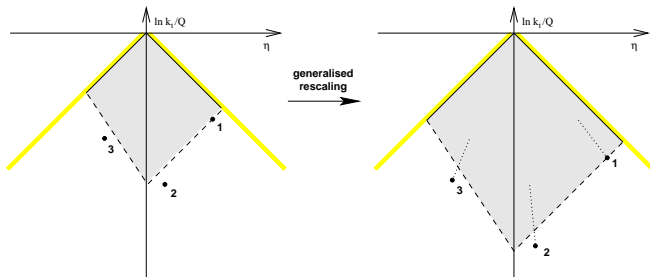
Exponentiation at NLL holds only for variables satisfying **rIRC safety** conditions

Consider $V(k_1, \dots, k_n)$ and $V(k_i) = \zeta_i v \Rightarrow k_i$ is a function of v and ζ_i

Recursive IRC safety

Exponentiation at NLL holds only for variables satisfying **rIRC safety** conditions

Consider $V(k_1, \dots, k_n)$ and $V(k_i) = \zeta_i v \Rightarrow k_i$ is a function of v and ζ_i



- 1 Observable **scaling** properties are the same with any number of emissions

$$\lim_{v \rightarrow 0} \frac{V(k_1, \dots, k_n)}{V(k_1)} = \text{constant} \neq 0$$

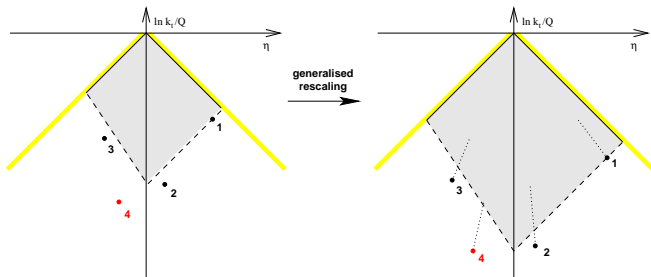
- 2 Addition of a **soft and/or collinear** particle does not modify observable scaling

$$\lim_{\zeta_{n+1} \rightarrow 0} \lim_{v \rightarrow 0} \frac{V(k_1, \dots, k_n, k_{n+1})}{V(k_1)} = \lim_{v \rightarrow 0} \frac{V(k_1, \dots, k_n)}{V(k_1)}$$

Recursive IRC safety

Exponentiation at NLL holds only for variables satisfying **rIRC safety** conditions

Consider $V(k_1, \dots, k_n)$ and $V(k_i) = \zeta_i v \Rightarrow k_i$ is a function of v and ζ_i



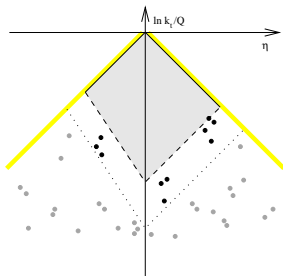
- 1 Observable **scaling** properties are the same with any number of emissions

$$\lim_{v \rightarrow 0} \frac{V(k_1, \dots, k_n)}{V(k_1)} = \text{constant} \neq 0$$

- 2 Addition of a **soft and/or collinear** particle does not modify observable scaling

$$\lim_{\zeta_{n+1} \rightarrow 0} \lim_{v \rightarrow 0} \frac{V(k_1, \dots, k_n, k_{n+1})}{V(k_1)} = \lim_{v \rightarrow 0} \frac{V(k_1, \dots, k_n)}{V(k_1)}$$

NLL resummations vs MC simulations



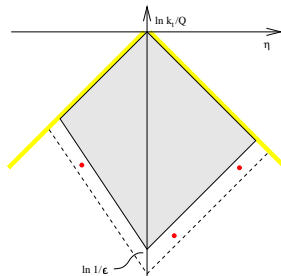
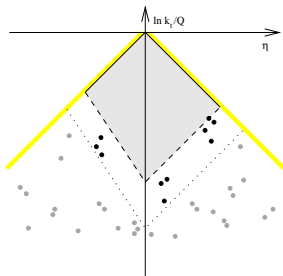
Event generators

- **Angular ordering** from hard legs
 \Rightarrow **LL and NLL**
- **Secondary emissions**
 \Rightarrow more subleading logarithm, but **no control over accuracy**
- Colour exact only in **large N_c** limit
- Missing pure virtual corrections
 \Rightarrow **no Coulomb phases**

Analytical NLL resummations

- Emissions only from hard legs
 \Rightarrow **LL and NLL**
- No secondary emissions and $v \rightarrow 0$
 \Rightarrow **Eliminate all subleading logs**
- Primary emissions only
 \Rightarrow **exact colour structure**
- **Exact virtual corrections** to all orders

NLL resummations vs MC simulations



Event generators

- **Angular ordering** from hard legs
 \Rightarrow LL and NLL
- **Secondary emissions**
 \Rightarrow more subleading logarithm, but **no control over accuracy**
- Colour exact only in **large N_c** limit
- Missing pure virtual corrections
 \Rightarrow **no Coulomb phases**

Analytical NLL resummations

- Emissions only from hard legs
 \Rightarrow LL and NLL
- No secondary emissions and $v \rightarrow 0$
 \Rightarrow **Eliminate all subleading logs**
- Primary emissions only
 \Rightarrow **exact colour structure**
- **Exact virtual corrections** to all orders

General resummation of rIRC safe final-state observables

- **Master Formula** for NLL resummation of $\Sigma(v) = \text{Prob}(V(k_1, \dots, k_n) < v)$

[AB, Salam, Zanderighi]

$$\Sigma(v) = e^{-R(v)} \mathcal{F}(R') \quad R' = -v \frac{dR}{dv}$$

- **Virtual corrections** up to scale v in $R(v) \Rightarrow$ exponentiation of LL (and part of NLL)

$$V(k) \simeq d_\ell \left(\frac{k_t}{Q} \right)^\alpha e^{-b_\ell \eta} g_\ell(\phi) \quad \ell = \text{hard leg}$$

- Multiple **soft and collinear emissions** in a band of **width $\ln 1/\epsilon$** in $\mathcal{F}(R')$

General resummation of rIRC safe final-state observables

- **Master Formula** for NLL resummation of $\Sigma(v) = \text{Prob}(V(k_1, \dots, k_n) < v)$

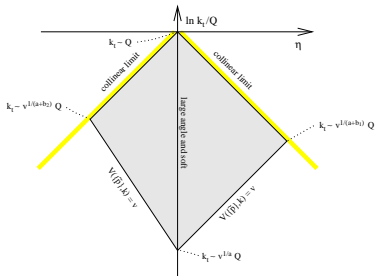
[AB, Salam, Zanderighi]

$$\Sigma(v) = e^{-R(v)} \mathcal{F}(R') \quad R' = -v \frac{dR}{dv}$$

- **Virtual corrections** up to scale v in $R(v) \Rightarrow$ exponentiation of LL (and part of NLL)

$$V(k) \simeq d_\ell \left(\frac{k_t}{Q} \right)^\alpha e^{-b_\ell \eta} g_\ell(\phi) \quad \ell = \text{hard leg}$$

- Multiple **soft and collinear emissions** in a band of width $\ln 1/\epsilon$ in $\mathcal{F}(R')$



General resummation of IRC safe final-state observables

- **Master Formula** for NLL resummation of $\Sigma(v) = \text{Prob}(V(k_1, \dots, k_n) < v)$

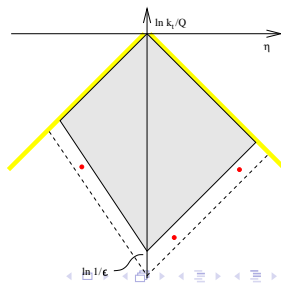
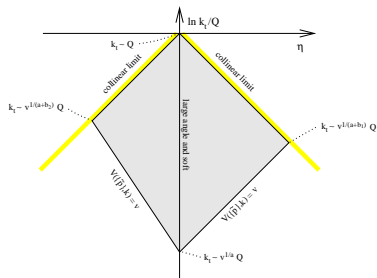
[AB, Salam, Zanderighi]

$$\Sigma(v) = e^{-R(v)} \mathcal{F}(R') \quad R' = -v \frac{dR}{dv}$$

- **Virtual corrections** up to scale v in $R(v) \Rightarrow$ exponentiation of LL (and part of NLL)

$$V(k) \simeq d_\ell \left(\frac{k_t}{Q} \right)^\alpha e^{-b_\ell \eta} g_\ell(\phi) \quad \ell = \text{hard leg}$$

- Multiple **soft and collinear emissions** in a band of **width $\ln 1/\epsilon$** in $\mathcal{F}(R')$



Computer Automated Expert Semi-Analytical Resummer

- 1 For each hard leg ℓ determine single emission parameterisation

$$V(k) \simeq d_\ell \left(\frac{k_t}{Q} \right)^a e^{-b_\ell \eta} g_\ell(\phi) \quad 1 - T = \frac{k_t}{Q} e^{-\eta}$$

- 2 Perform tests of applicability, in particular **rIRC safety**
- 3 Generate emissions ordered in $V(k_i) = v \zeta_i$ with the constraint $V(k_1) = v$

$$dP(k_i(\zeta_i, \eta_i, \ell_i)) = R'_\ell \frac{d\eta_i}{\Delta\eta_i} \frac{d\phi_i}{2\pi} \frac{d\zeta_i}{\zeta_i} \left(\frac{\zeta_i}{\zeta_{i-1}} \right)^{R'_\ell}, \quad \sum_\ell R'_\ell = R'$$

and compute $\mathcal{F}(R')$ as the following mean value

$$\mathcal{F}(R') = \lim_{v \rightarrow 0} \left\langle \left(\frac{V(k_1, \dots, k_n)}{V(k_1)} \right)^{-R'} \right\rangle$$

- 4 Evaluate the **master formula**, and integrate over Born configurations

Important notes:

- No need of integral transform but **NLL accuracy guaranteed**
- The user needs simply to provide a **routine to compute $V(k_1, \dots, k_n)$**

Computer Automated Expert Semi-Analytical Resummer

- 1 For each hard leg ℓ determine single emission parameterisation

$$V(k) \simeq d_\ell \left(\frac{k_t}{Q} \right)^a e^{-b_\ell \eta} g_\ell(\phi) \quad 1 - T = \frac{k_t}{Q} e^{-\eta}$$

- 2 Perform tests of applicability, in particular **rIRC safety**
- 3 Generate emissions ordered in $V(k_i) = v \zeta_i$ with the constraint $V(k_1) = v$

$$dP(k_i(\zeta_i, \eta_i, \ell_i)) = R'_\ell \frac{d\eta_i}{\Delta\eta_i} \frac{d\phi_i}{2\pi} \frac{d\zeta_i}{\zeta_i} \left(\frac{\zeta_i}{\zeta_{i-1}} \right)^{R'_\ell}, \quad \sum_\ell R'_\ell = R'$$

and compute $\mathcal{F}(R')$ as the following mean value

$$\mathcal{F}(R') = \lim_{v \rightarrow 0} \left\langle \left(\frac{V(k_1, \dots, k_n)}{V(k_1)} \right)^{-R'} \right\rangle$$

- 4 Evaluate the **master formula**, and integrate over Born configurations

Important notes:

- No need of integral transform but **NLL accuracy guaranteed**
- The user needs simply to provide a **routine to compute $V(k_1, \dots, k_n)$**

Computer Automated Expert Semi-Analytical Resummer

- 1 For each hard leg ℓ determine single emission parameterisation

$$V(k) \simeq d_\ell \left(\frac{k_t}{Q} \right)^\alpha e^{-b_\ell \eta} g_\ell(\phi) \quad 1 - T = \frac{k_t}{Q} e^{-\eta}$$

- 2 Perform tests of applicability, in particular **rIRC safety**
- 3 Generate emissions ordered in $V(k_i) = v \zeta_i$ with the constraint $V(k_1) = v$

$$dP(k_i(\zeta_i, \eta_i, \ell_i)) = R'_\ell \frac{d\eta_i}{\Delta\eta_i} \frac{d\phi_i}{2\pi} \frac{d\zeta_i}{\zeta_i} \left(\frac{\zeta_i}{\zeta_{i-1}} \right)^{R'_\ell}, \quad \sum_\ell R'_\ell = R'$$

and compute $\mathcal{F}(R')$ as the following mean value

$$\mathcal{F}(R') = \lim_{v \rightarrow 0} \left\langle \left(\frac{V(k_1, \dots, k_n)}{V(k_1)} \right)^{-R'} \right\rangle$$

- 4 Evaluate the **master formula**, and integrate over Born configurations

Important notes:

- No need of integral transform but **NLL accuracy guaranteed**
- The user needs simply to provide a **routine to compute $V(k_1, \dots, k_n)$**

Computer Automated Expert Semi-Analytical Resummer

- 1 For each hard leg ℓ determine single emission parameterisation

$$V(k) \simeq d_\ell \left(\frac{k_t}{Q} \right)^\alpha e^{-b_\ell \eta} g_\ell(\phi) \quad 1 - T = \frac{k_t}{Q} e^{-\eta}$$

- 2 Perform tests of applicability, in particular **rIRC safety**
- 3 Generate emissions ordered in $V(k_i) = v \zeta_i$ with the constraint $V(k_1) = v$

$$dP(k_i(\zeta_i, \eta_i, \ell_i)) = R'_\ell \frac{d\eta_i}{\Delta\eta_i} \frac{d\phi_i}{2\pi} \frac{d\zeta_i}{\zeta_i} \left(\frac{\zeta_i}{\zeta_{i-1}} \right)^{R'_\ell}, \quad \sum_\ell R'_\ell = R'$$

and compute $\mathcal{F}(R')$ as the following mean value

$$\mathcal{F}(R') = \lim_{v \rightarrow 0} \left\langle \left(\frac{V(k_1, \dots, k_n)}{V(k_1)} \right)^{-R'} \right\rangle$$

- 4 Evaluate the **master formula**, and integrate over Born configurations

Important notes:

- No need of integral transform but **NLL accuracy guaranteed**
- The user needs simply to provide a **routine to compute $V(k_1, \dots, k_n)$**

Computer Automated Expert Semi-Analytical Resummer

- 1 For each hard leg ℓ determine single emission parameterisation

$$V(k) \simeq d_\ell \left(\frac{k_t}{Q} \right)^\alpha e^{-b_\ell \eta} g_\ell(\phi) \quad 1 - T = \frac{k_t}{Q} e^{-\eta}$$

- 2 Perform tests of applicability, in particular **rIRC safety**
- 3 Generate emissions ordered in $V(k_i) = v \zeta_i$ with the constraint $V(k_1) = v$

$$dP(k_i(\zeta_i, \eta_i, \ell_i)) = R'_\ell \frac{d\eta_i}{\Delta\eta_i} \frac{d\phi_i}{2\pi} \frac{d\zeta_i}{\zeta_i} \left(\frac{\zeta_i}{\zeta_{i-1}} \right)^{R'_\ell}, \quad \sum_\ell R'_\ell = R'$$

and compute $\mathcal{F}(R')$ as the following mean value

$$\mathcal{F}(R') = \lim_{v \rightarrow 0} \left\langle \left(\frac{V(k_1, \dots, k_n)}{V(k_1)} \right)^{-R'} \right\rangle$$

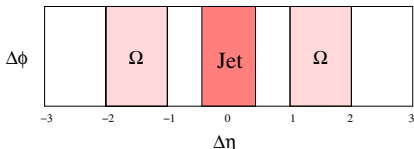
- 4 Evaluate the **master formula**, and integrate over Born configurations

Important notes:

- No need of integral transform but **NLL accuracy guaranteed**
- The user needs simply to provide a **routine to compute $V(k_1, \dots, k_n)$**

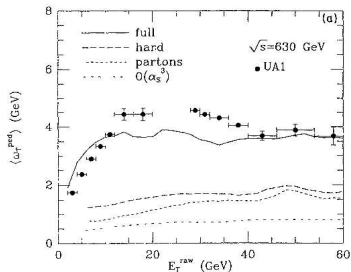
Away-from-jet E_t flow

The energy flow **away** from the jet region has been a subject of study for many years



$$E_t = \sum_{i \in \Omega} E_{ti} \quad i \in \text{hadrons}$$

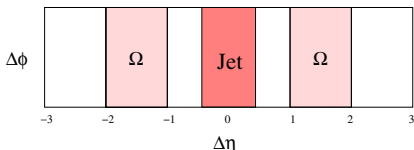
$$\Sigma(Q_\Omega) = \int_0^{Q_\Omega} dE_t \frac{1}{\sigma} \frac{d\sigma}{dE_t}$$



- Soft underlying event in hadronic collisions
- Dijet rapidity gaps in photoproduction
- Current-hemisphere jet-shapes in DIS

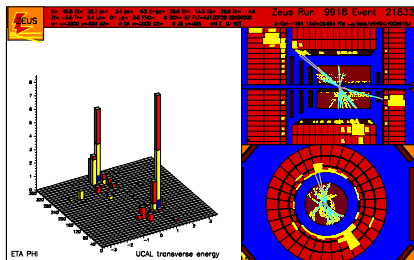
Away-from-jet E_t flow

The energy flow **away** from the jet region has been a subject of study for many years



$$E_t = \sum_{i \in \Omega} E_{ti} \quad i \in \text{hadrons}$$

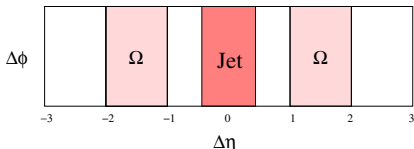
$$\Sigma(Q_\Omega) = \int_0^{Q_\Omega} dE_t \frac{1}{\sigma} \frac{d\sigma}{dE_t}$$



- Soft underlying event in hadronic collisions
- Dijet rapidity gaps in photoproduction
- Current-hemisphere jet-shapes in DIS

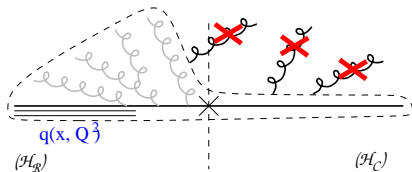
Away-from-jet E_t flow

The energy flow **away** from the jet region has been a subject of study for many years



$$E_t = \sum_{i \in \Omega} E_{ti} \quad i \in \text{hadrons}$$

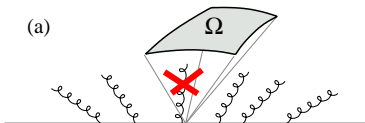
$$\Sigma(Q_\Omega) = \int_0^{Q_\Omega} dE_t \frac{1}{\sigma} \frac{d\sigma}{dE_t}$$



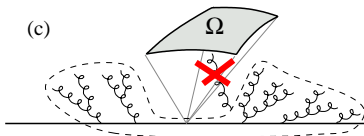
- Soft underlying event in hadronic collisions
- Dijet rapidity gaps in photoproduction
- Current-hemisphere jet-shapes in DIS

Non-global logarithms

The E_t flow is a non-global variable, measures particles in a **restricted region Ω**



Global variable \Rightarrow **Independent** emission from **primary** partons!



Non-global variable \Rightarrow **Coherent** emission from **all** partons!

- Evolution variable $t \sim \alpha_s \ln \frac{Q}{Q_\Omega}$

$$\Sigma_\Omega(t) = \Sigma_P(t) \times S(t)$$

Note: LL are single logarithms

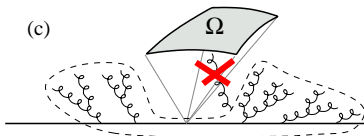
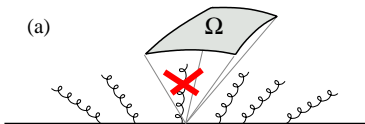
- Primary emission

$$\Sigma_P(t) = \exp\{-4 \times \text{Area}_\Omega \times t\}$$

- Non-global logarithms in $S(t)$

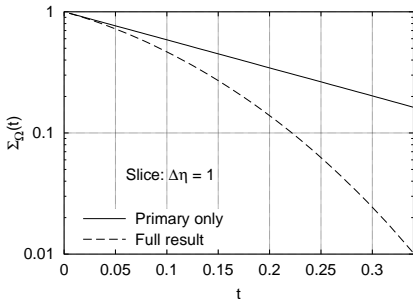
Non-global logarithms

The E_t flow is a non-global variable, measures particles in a **restricted region Ω**



Global variable \Rightarrow **Independent** emission from **primary** partons!

Non-global variable \Rightarrow **Coherent** emission from **all** partons!



- Evolution variable $t \sim \alpha_s \ln \frac{Q}{Q_\Omega}$

$$\Sigma_\Omega(t) = \Sigma_P(t) \times S(t)$$

Note: LL are single logarithms

- Primary emission

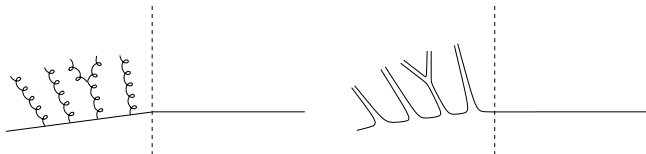
$$\Sigma_P(t) = \exp\{-4 \times \text{Area}_\Omega \times t\}$$

- Non-global logarithms in $S(t)$

MC resummation of non-global logarithms

- Resummation of NG logs is performed in the **large N_c limit** via a dipole MC

[Dasgupta,Salam]



- Emissions are ordered in $t \sim \alpha_s \ln Q/k_t$, and a gluon k_i is emitted from a dipole j in the **dipole back-to-back frame** with a probability

$$dP(k_i(t_i, \eta_i, \phi_i)) = 2C_A d\eta_i \Theta\left(\frac{\Delta\eta_j}{2} - |\eta_i|\right) \frac{d\phi_i}{2\pi} dt_i \left(\frac{t_i}{t_{i-1}}\right)^{2C_A \Delta\eta_{\text{tot}}}$$

$\Delta\eta_j$ is a collinear cutoff and $\Delta\eta_{\text{tot}} = \sum_j \Delta\eta_j$

- After boost in the lab frame, if $k_i \in \Omega$, the bin $t = t_i$ is filled, and a new configuration is generated
- When the MC stops we have reconstructed the distribution $d\Sigma/dt$ at LL accuracy

Non-global logarithms: resummation vs HERWIG

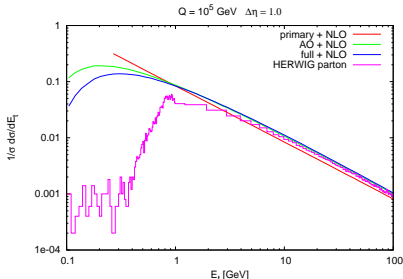
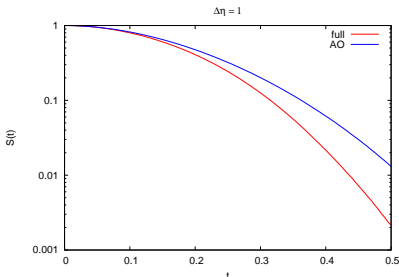
- Modify emission probability of gluon k with **angular ordering**

[AB, Corcella, Dasgupta]

$$\frac{1 - \cos \theta_{ab}}{(1 - \cos \theta_{ak})(1 - \cos \theta_{kb})} \rightarrow \left[\frac{\Theta(\cos \theta_{ak} - \cos \theta_{ab})}{1 - \cos \theta_{ak}} + \frac{\Theta(\cos \theta_{kb} - \cos \theta_{ab})}{1 - \cos \theta_{kb}} \right]$$

The resummation MC remains the same, different Lorentz transform to the lab

- AO is close to full distribution and not to $\Sigma_P(t) \Rightarrow$ **HERWIG describes well $d\sigma/dE_t$**



- Radiation suppression is **single-logarithmic** \Rightarrow sensitivity to **shower cutoff**

Super-leading logarithms

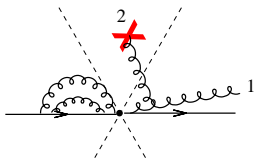
- Are leading logarithms for E_t flow are always single logarithms?
- The answer is NO: in hadron-hadron collisions **super-leading logarithms**
[Forshaw, Kyrieleis, Seymour]
- k_1 incoming and collinear + k_2 in a gap Y + non-cancelling Coulomb phases

Super-leading logarithms

- Are leading logarithms for E_t flow are always single logarithms?
- The answer is NO: in hadron-hadron collisions **super-leading logarithms**
[Forshaw, Kyrieleis, Seymour]
- k_1 incoming and collinear + k_2 in a gap Y + non-cancelling Coulomb phases

Super-leading logarithms

- Are leading logarithms for E_t flow are always single logarithms?
- The answer is NO: in hadron-hadron collisions **super-leading logarithms**
 [Forshaw, Kyrieleis, Seymour]
- k_1 incoming and collinear + k_2 in a gap Y + non-cancelling Coulomb phases

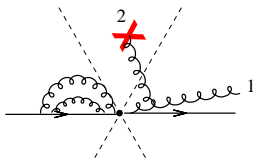


The diagram shows a quark line (solid line) with a gluon loop (curly line) and a gluon emission (curly line). A red cross is placed over the loop. The diagram is labeled with '2' and '1'.

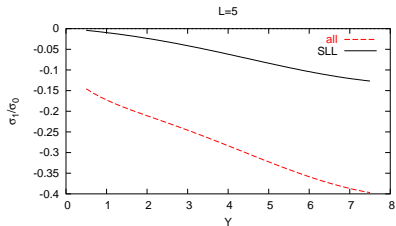
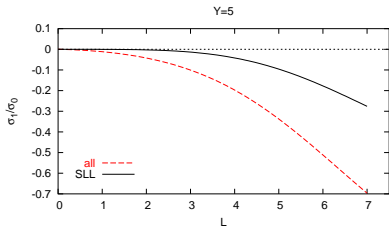
$$= \alpha_s^4 \pi^2 L^5 Y$$

Super-leading logarithms

- Are leading logarithms for E_t flow are always single logarithms?
- The answer is NO: in hadron-hadron collisions **super-leading logarithms**
 [Forshaw, Kyrieleis, Seymour]
- k_1 incoming and collinear + k_2 in a gap Y + non-cancelling Coulomb phases



$$= \alpha_s^4 \pi^2 L^5 Y$$



Conclusions and outlook

Conclusions

- **rIRC safety** is the condition that determines resumability
[AB, Salam, Zanderighi]
- **CAESAR** is now a powerful tool to resum rIRC safe observables
- A lot of work is in progress to understand NG observables
 - Connection with **small-x physics** in saturation regime
[AB, Marchesini, Smye]
[Marchesini, Mueller]
[Marchesini, Onofri]
 - Impact of **k_t algorithm** on non-global logs \Rightarrow fundamental for jet cross sections
[Appleby, Seymour]
[Appleby, AB, Dasgupta, Delenda]
 - **Superleading logarithms** in hadronic collisions?
[Forshaw, Kyrieleis, Seymour]

For coming year . . .

- NLL+NLO analysis of **event shapes in hadron-hadron collisions** (CAESAR v1.0)
- Resummation of **dijet cross sections** ($\Delta\phi_{JJ}$ near π)