Higgs plus dijet production with semi-numerical methods

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Theory Division, CERN

Monte Carlo, la Fisica e le Simulazioni a LHC
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→ At the LHC *everything involves QCD*! QCD provides interactions (the beam), the background, the challenge

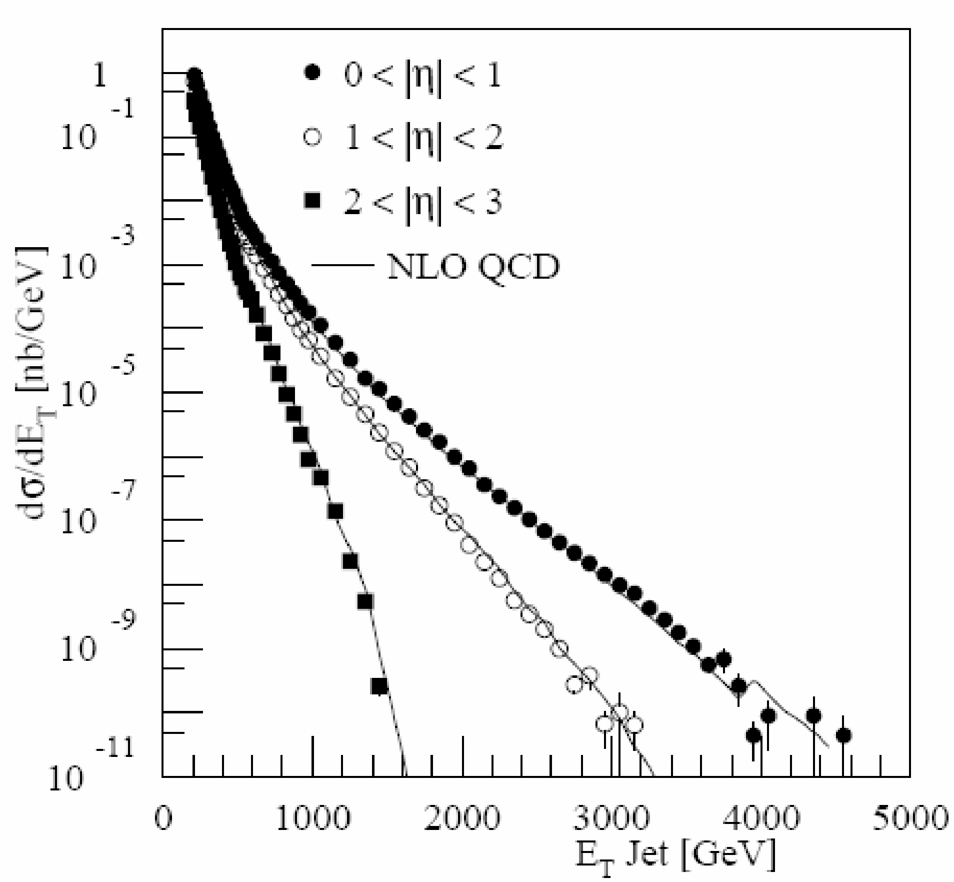
NLO at the LHC

QCD Studies

A. De Roeck
La Thuille '06

E.g. Jet Physics

Huge cross sections:
Eg for 1 fb^{-1} ~ 10000 events with E_T> 1 TeV
100 events with E_T> 2 TeV



- PDFs
- Jet shape
- α_s
- New physics?

Understanding QCD at 14 TeV will be one of the first topics at LHC
Then: precise measurements of W,Z, tt, Drell-Yan production
Then: W,Z+1 jet; W,Z+2 jets etc
⇒ Use to tune Monte Carlos

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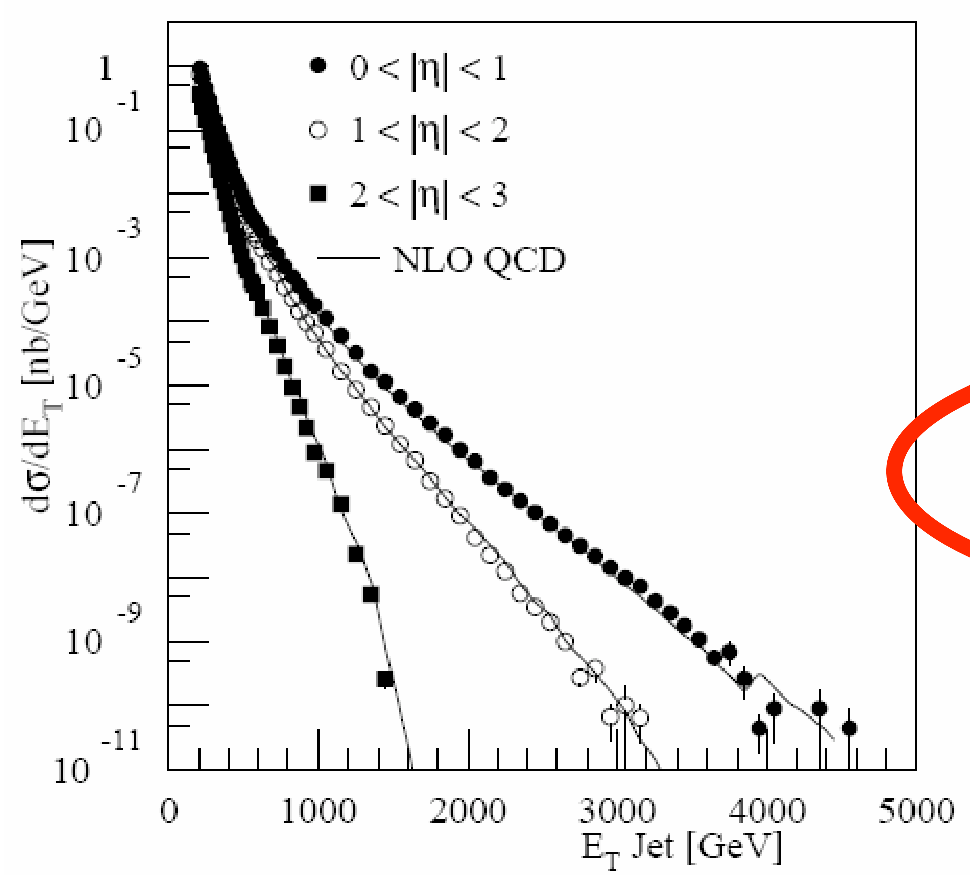

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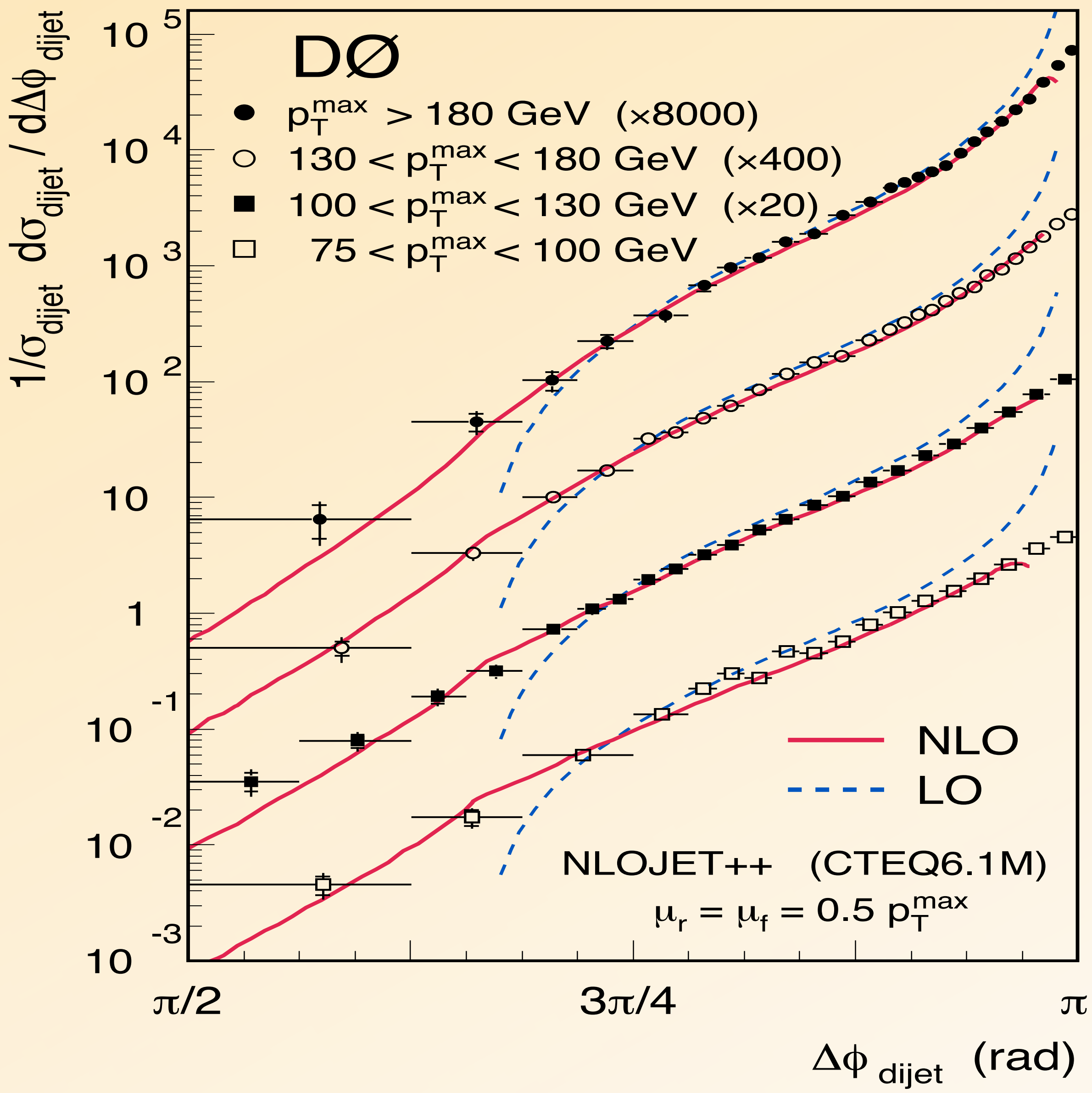

D0 '04

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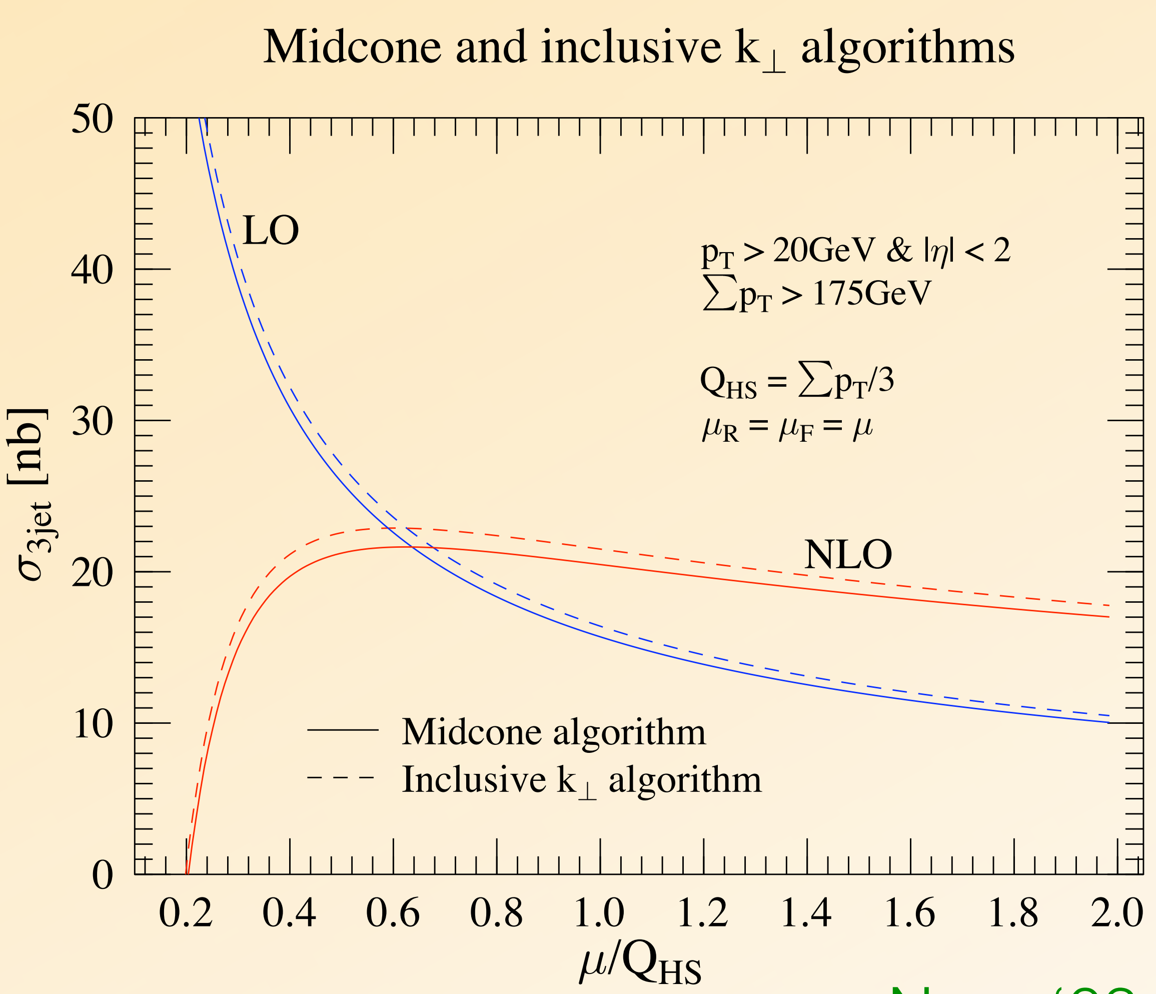
- establish normalization and shape of cross-sections
- reduce unphysical scale dependence



Nagy '03

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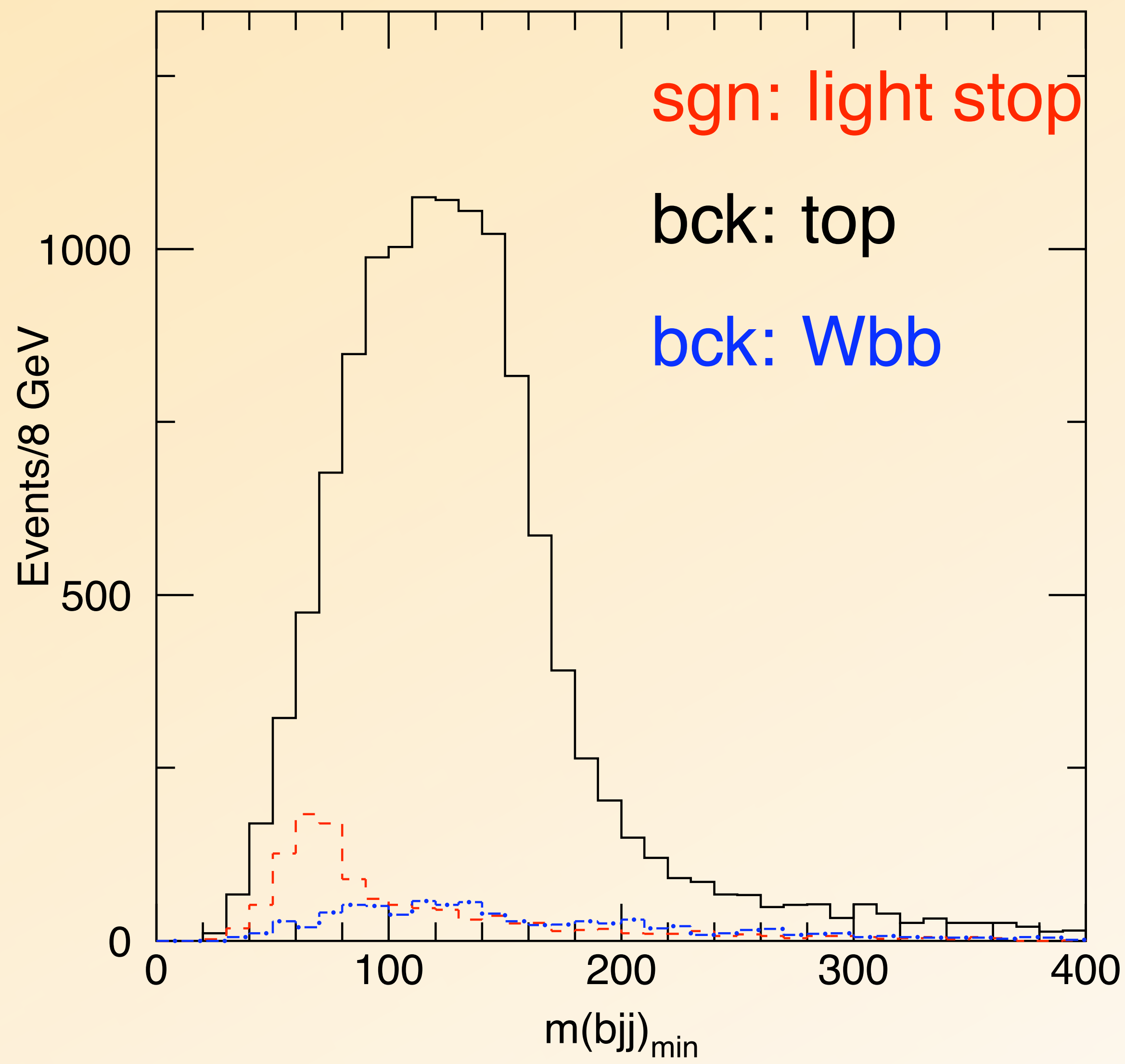

Les Houches BSM summary ‘06

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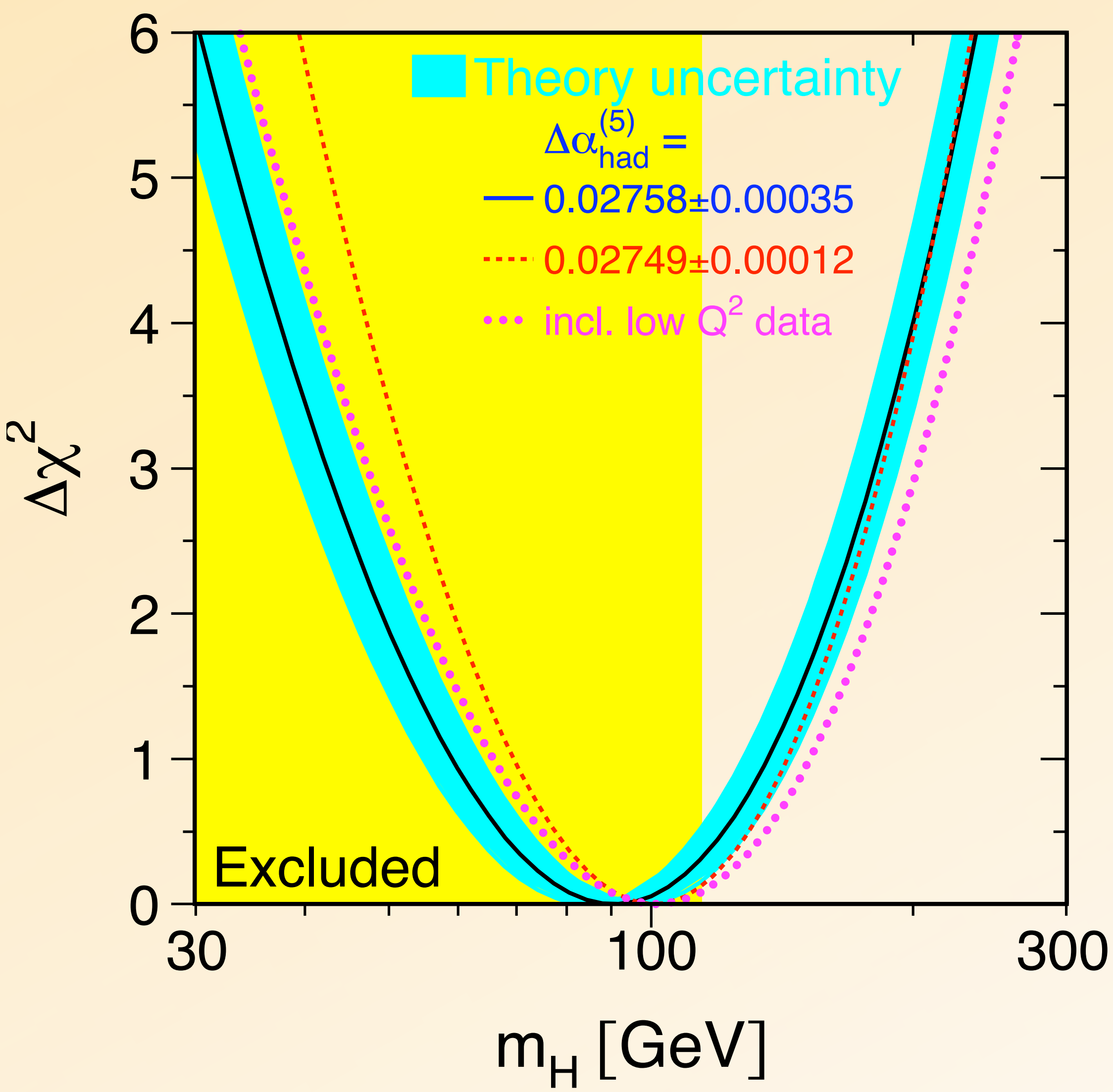
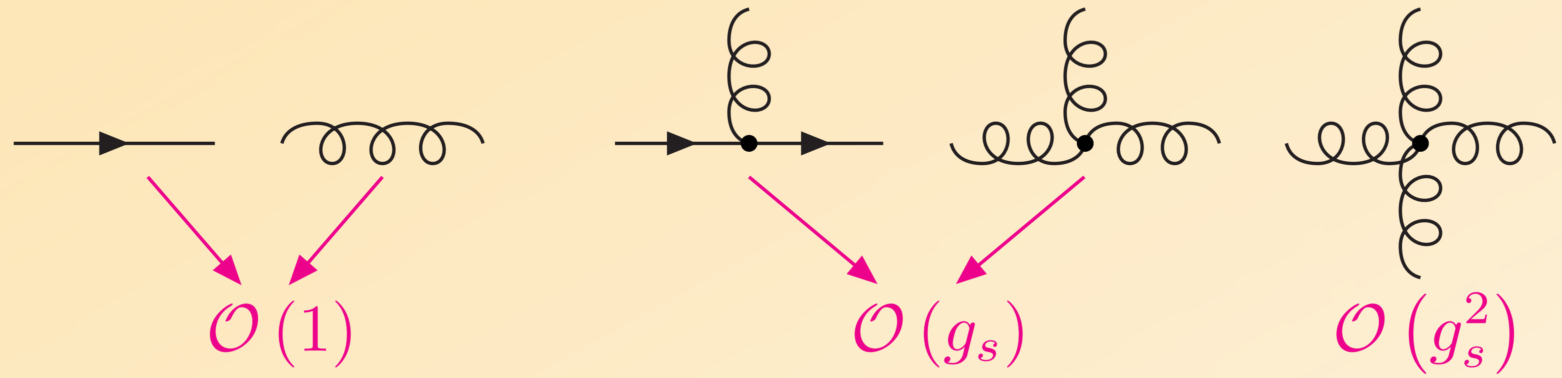
- establish normalization and shape of cross-sections
- reduce unphysical scale dependence
- new physics searches requires good knowledge of signals and backgrounds
- get indirect informations about sectors not directly accessible



Higgs working group '05

Feynman diagrams

QCD building blocks



Feynman diagrams

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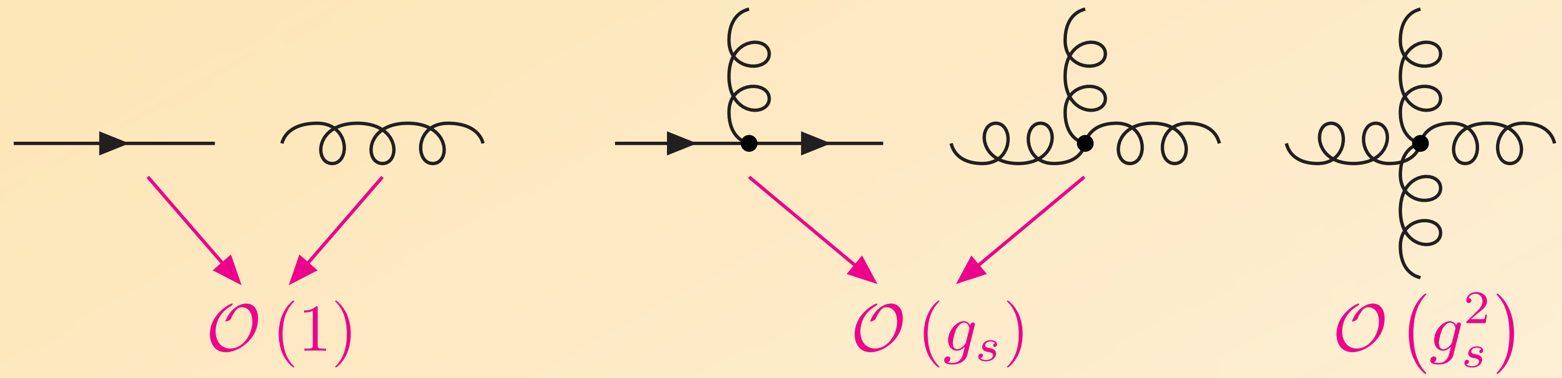

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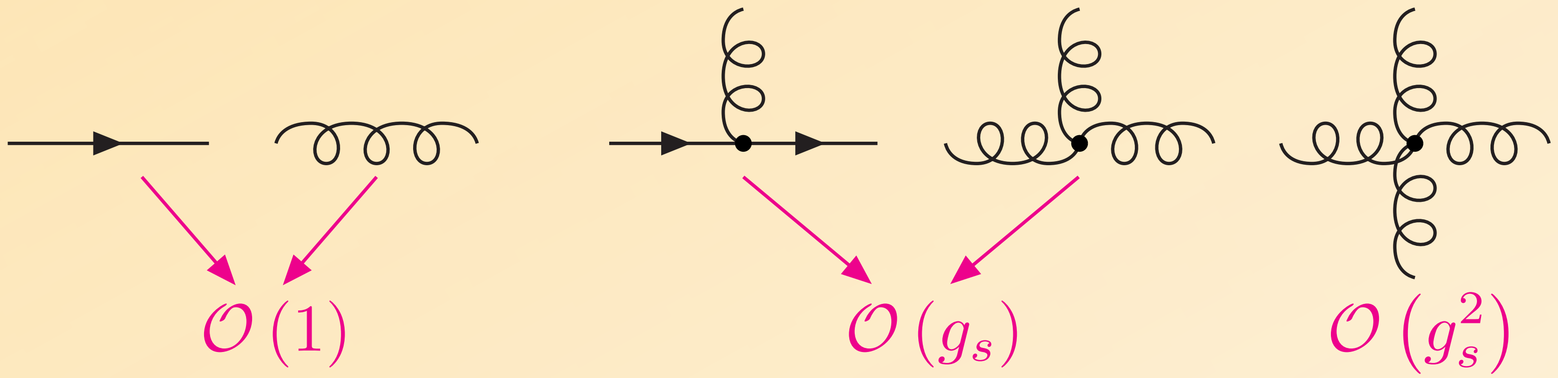

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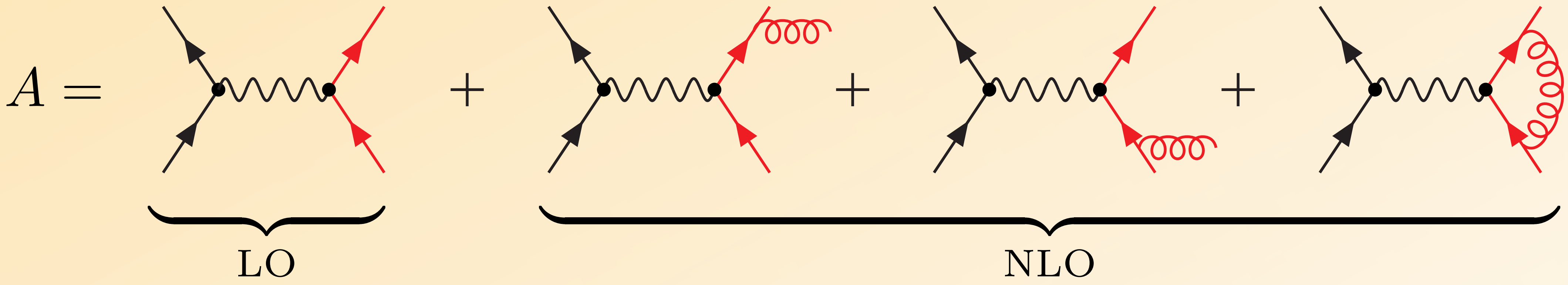
Example: for ee->qq @ NLO compute



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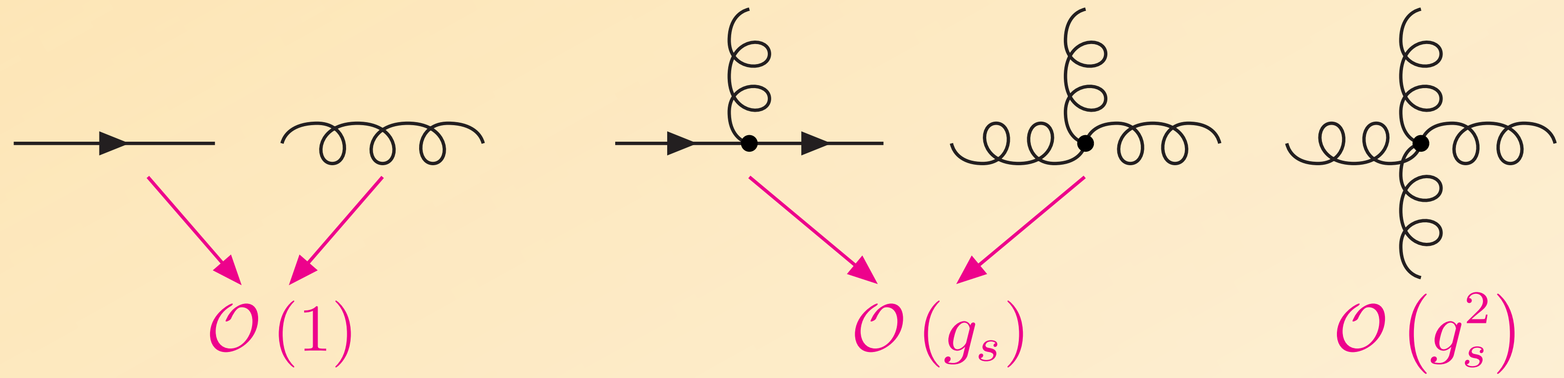

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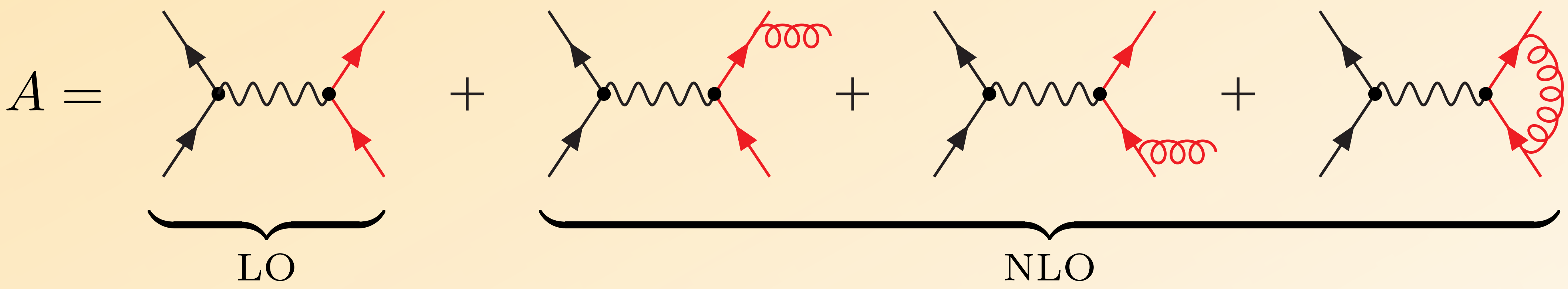

given the amplitude compute physical cross-sections

$$\frac{d\sigma}{dv} = \frac{1}{F}\int [d\Phi] |A|^2 \delta(v - V(\Phi))$$

F:flux factor; Φ :phase space; V: measurement

NLO HEPcode for LHC

- NLOJET++ *[Nagy]* pp ⇒ 3j
- AYLEN/Emilia *[Dixon,De Florian,Kunszt,Signer]* pp ⇒ WW,WZ, ZZ, Wg, Zg
- PHOX *[Aurenche,Binoth,Fontannaz,Guillet,Heinrich, Pilon,Werlen]* pp ⇒ g+1j, gg
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- heavy quark production *[Mangano, Nason, Ridolfi]* pp ⇒ QQ
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Complete list available at http://www.cedar.ac.uk/hepcode

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Problem for the LHC where most processes/ backgrounds involve high multiplicity final states

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NB: the merging is *essential* for a proper comparison with data

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➡ Here I will mainly address the calculation of virtual corrections

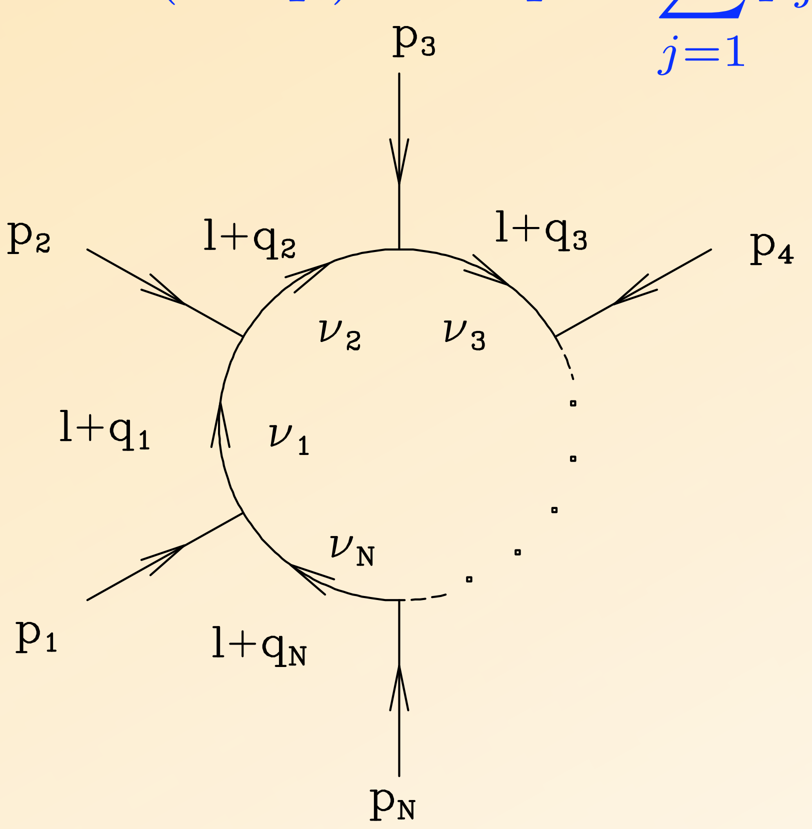
[of course in the final predictions I will show all three points will have been addressed]

Notation

The generic M-tensor N-point integral:

$$I^{\mu_1\cdots\mu_M}(D;\nu_1,\ldots,\nu_N) \equiv \int \frac{d^Dl}{i\pi^{D/2}}\,\frac{l^{\mu_1}\ldots l^{\mu_M}}{d_1^{\nu_1}d_2^{\nu_2}\cdots d_N^{\nu_N}}$$

$$D = 4-2\epsilon \qquad d_i \equiv (l+q_i)^2 \qquad q_i \equiv \sum_{j=1}^{i} p_j \qquad \sigma \equiv \sum_{i=1}^{N} \nu_i$$



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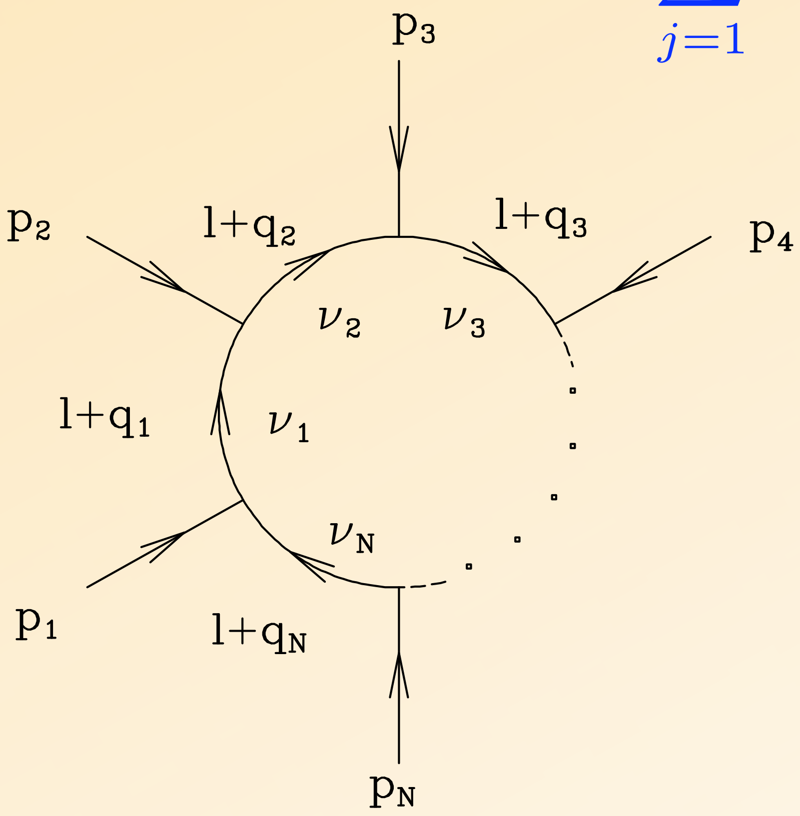

NB: we consider here only massless propagators, although the method is more general (see later)

Our seminumerical method

- Use Qgraf/FeynArts to generate the NLO amplitude for a specific process (provide propagators/interaction vertexes)

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- use Davydychev's reduction of tensor integrals

$$\begin{aligned} I_{\mu_1 \ldots \mu_M}(D; \{\nu_l\}) &= \sum_{\substack{\lambda, \kappa_1, \kappa_2, \ldots, \kappa_N \geq 0 \\ 2\lambda + \sum_i \kappa_i = M}} \left(-\tfrac{1}{2}\right)^\lambda \{[g]^\lambda [q_1]^{\kappa_1} \ldots [q_N]^{\kappa_N}\}_{\mu_1 \ldots \mu_M} \\ &\times \quad (\nu_1)_{\kappa_1} \ldots (\nu_N)_{\kappa_N} \; I(D + 2(M - \lambda); \{\nu_l + \kappa_l\}) \end{aligned}$$

Semi-numerical method (II)

Use standard integration by part techniques

$$\int \frac{d^D l}{i\pi^{D/2}} \frac{\partial}{\partial l^\mu} \left(\frac{\left(\sum_{i=1}^N y_i\right) l^\mu + \left(\sum_{i=1}^N y_i q_i^\mu\right)}{d_1^{\nu_1} d_2^{\nu_2} \cdots d_N^{\nu_N}} \right) = 0 \qquad \forall \{y_i\}_{i=1}^N$$

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to derive the following identities

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Notation: $S_{ij} = (q_i - q_j)^2\,;\quad b_i \equiv \sum_{j=1}^N S_{ij}^{-1};\quad B \equiv \sum_{j=1}^N b_i = \sum_{i,j=1}^N S_{ij}^{-1}$

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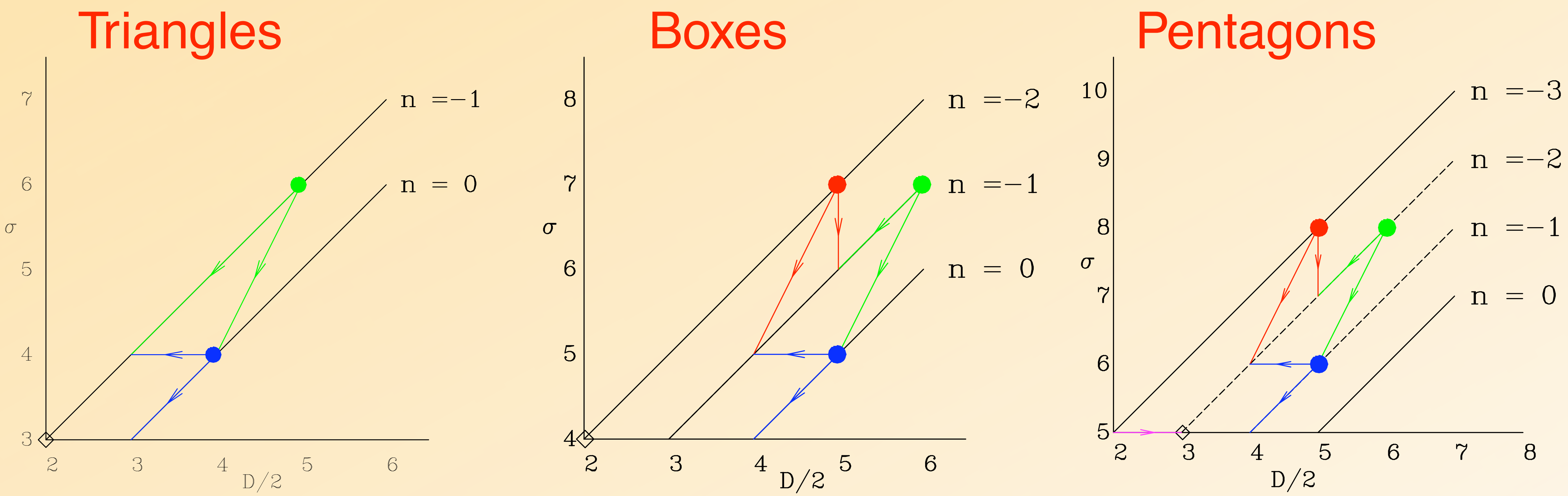
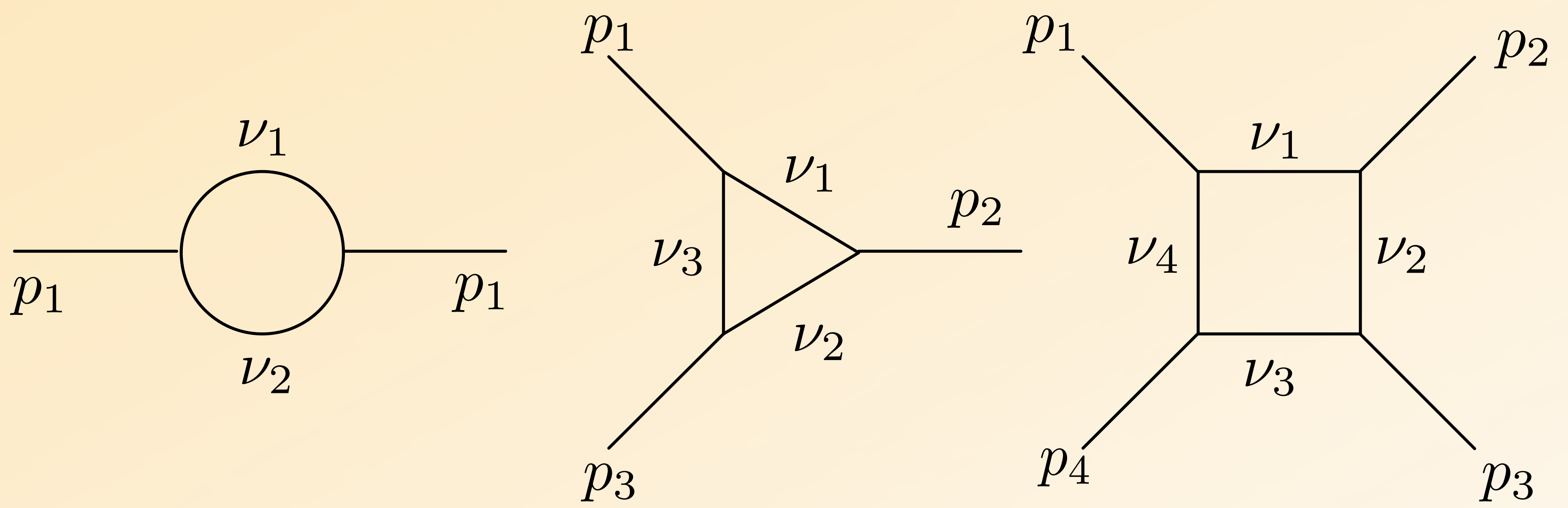


These few identities allow one to reduce *any* scalar integral to a set of *analytically known basis integrals*

Basis integrals

- two-point functions in any D
- three-point functions with one off-shell leg in any D
- three-point functions with three off-shell legs in D=4
- four-point functions in D=4
- six dimensional pentagon



All integrals known analytically!

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  - practical implication: *the method does not work!*
  - this is the *main problem of numerical methods*

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<u>and get</u>

$$I(D-2;\{\nu_l\}) = \sum_{i=1}^{N} \underbrace{b_i I(D-2;\{\nu_l - \delta_{li}\})}_{\text{"big terms": } \mathcal{O}(1)\text{, "simpler" integrals}} + \underbrace{(D-1-\sigma)\; B\, I(D;\{\nu_l\})}_{\text{"small term": } \mathcal{O}(B)\text{, "more difficult" integral}}$$

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➪ Recursive application of *the same relation* allows one to compute integrals with *arbitrary accuracy*

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Does this work?

A method without application is like a closed bottle of red wine: no matter how big the name on it, you don't know if it's good till you open it

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- new results
  - ☑ Higgs plus di-jet production via gluon fusion in the large mt-limit at NLO
  - ☑ six-gluon amplitudes
  - ☐ ongoing...

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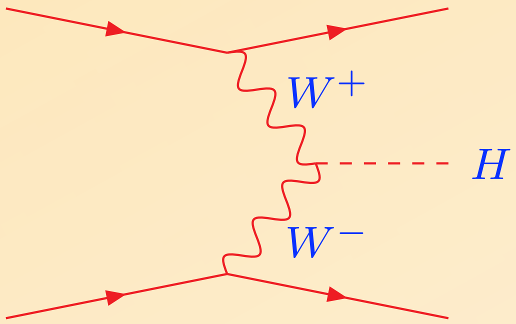
Vector Boson Fusion



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-clean: small QCD effects
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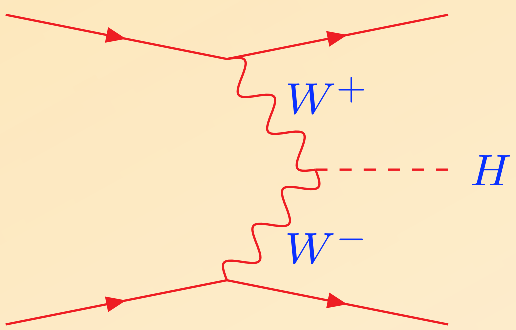
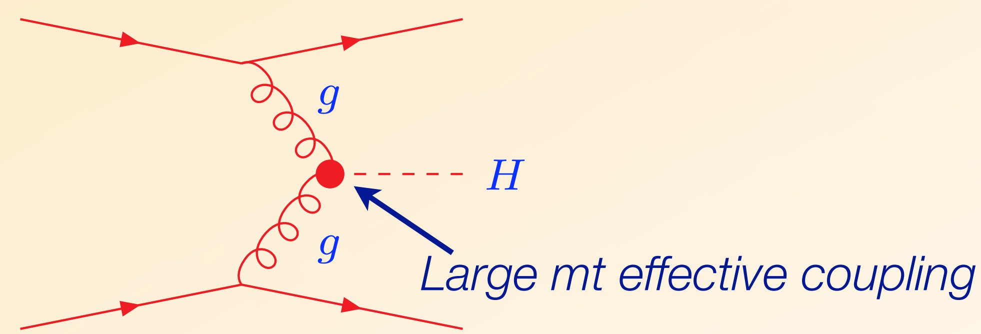


Gluon gluon fusion



$\mathcal{O}\left(\alpha_{ew}^2\right)$ ⟵ $\sim$ ⟶ $\mathcal{O}\left(\alpha_s^4\right)$

-clean: small QCD effects
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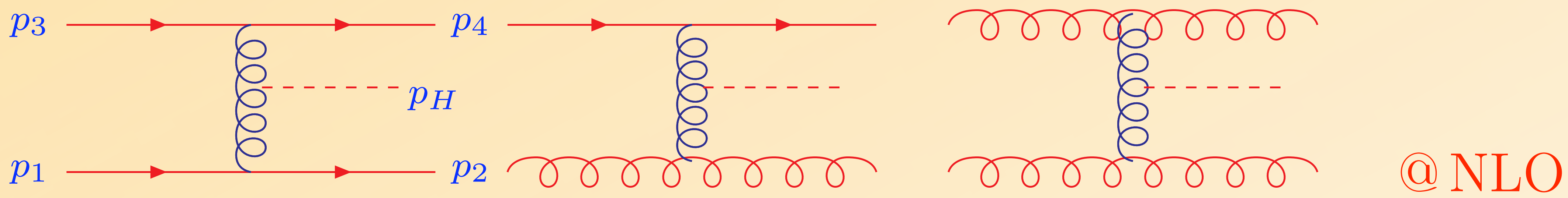
-large QCD corrections
-dominant background to VBF-Higgs production
-important to know it at NLO

Virtual Higgs plus 4 parton processes



Diagrams 30/60 191 739

[Ellis,Giele,GZ '06]

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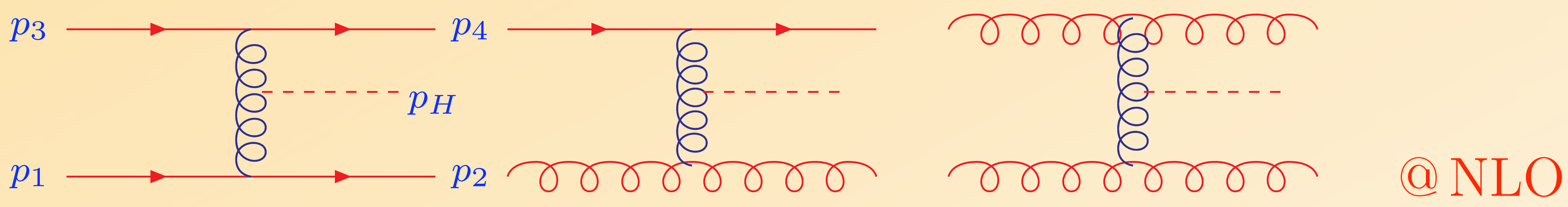

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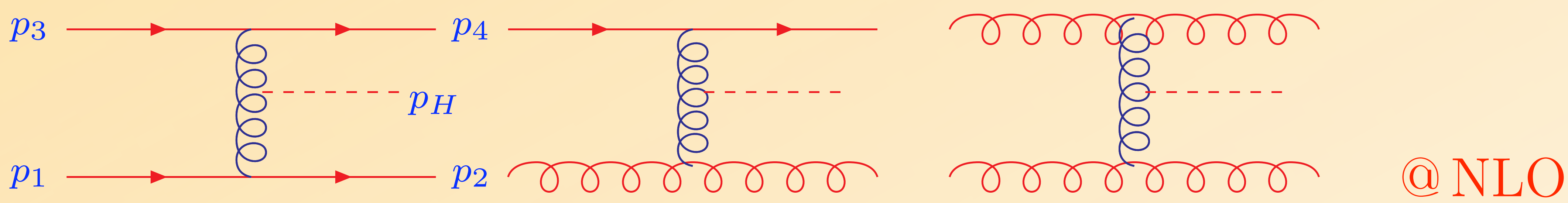

Diagrams	30/60	191	739

- checks:
  - ✓ for the simpler H+4q done analytical calculations too
  - ✓ Ward identities, identities between color amplitudes (decoupling identities....), symmetries, collinear limits, poles
- relative accuracy:
  - ✓ for non-exceptional points: $\mathcal{O}(10^{-13})$
  - ✓ for exceptional points (predefined): $\mathcal{O}(10^{-6})$

[Ellis,Giele,GZ '06]

Selection cuts

Signal most interesting in mass range $115GeV \lesssim M_H \lesssim 160GeV$

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↪ *cuts designed to suppress ggf compared to VBF signal*

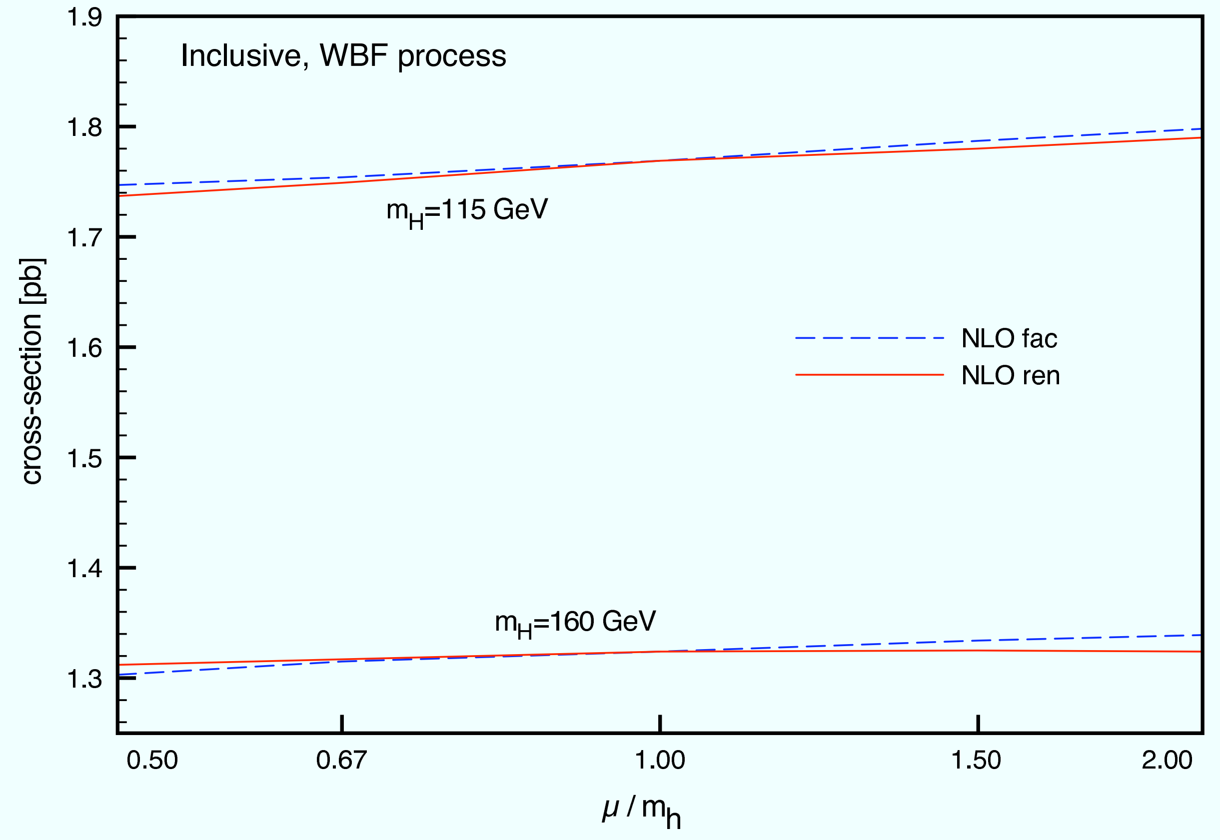
Total cross section

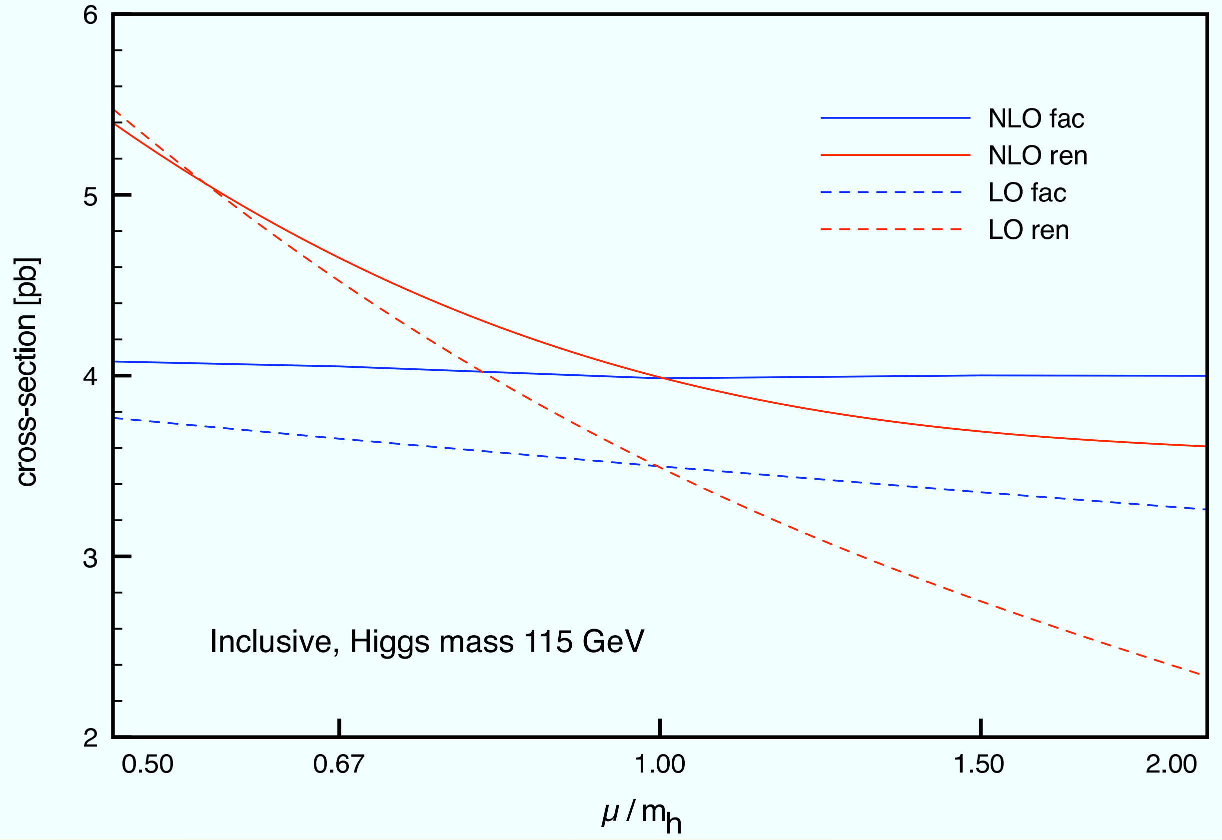
	inclusive		WBF-cuts		
m_H	115 GeV	160 GeV	m_H	115 GeV	160 GeV
σ_{LO} [pb]	3.50	2.19	σ_{LO} [pb]	$.271$	$.172$
σ_{NLO} [pb]	4.03	2.76	σ_{NLO} [pb]	$.346$ (± 5)	$.236$ (± 3)
σ_{WBF} [pb]	1.77	1.32	σ_{WBF} [pb]	$.911$	$.731$

- NLO moderate:
  $\sim 15(25)\%$ for $m_H = 115(160) GeV$
- background dominates:
  $\sigma_{\text{VBF}} \sim 1/2\, \sigma_{\text{ggF}}$
- NLO more important:
  $\sim 30(40)\%$ for $m_H = 115(160) GeV$
- signal dominates:
  $\sigma_{\text{VBF}} \sim 5/2\, \sigma_{\text{ggF}}$

[Campbell, Giele,GZ '06]

Scale uncertainties





VBF signal:

- very moderate dependence:
  $\sim 1.5\%$ for $1/2 < \mu/M_H <= 2$

ggf background:

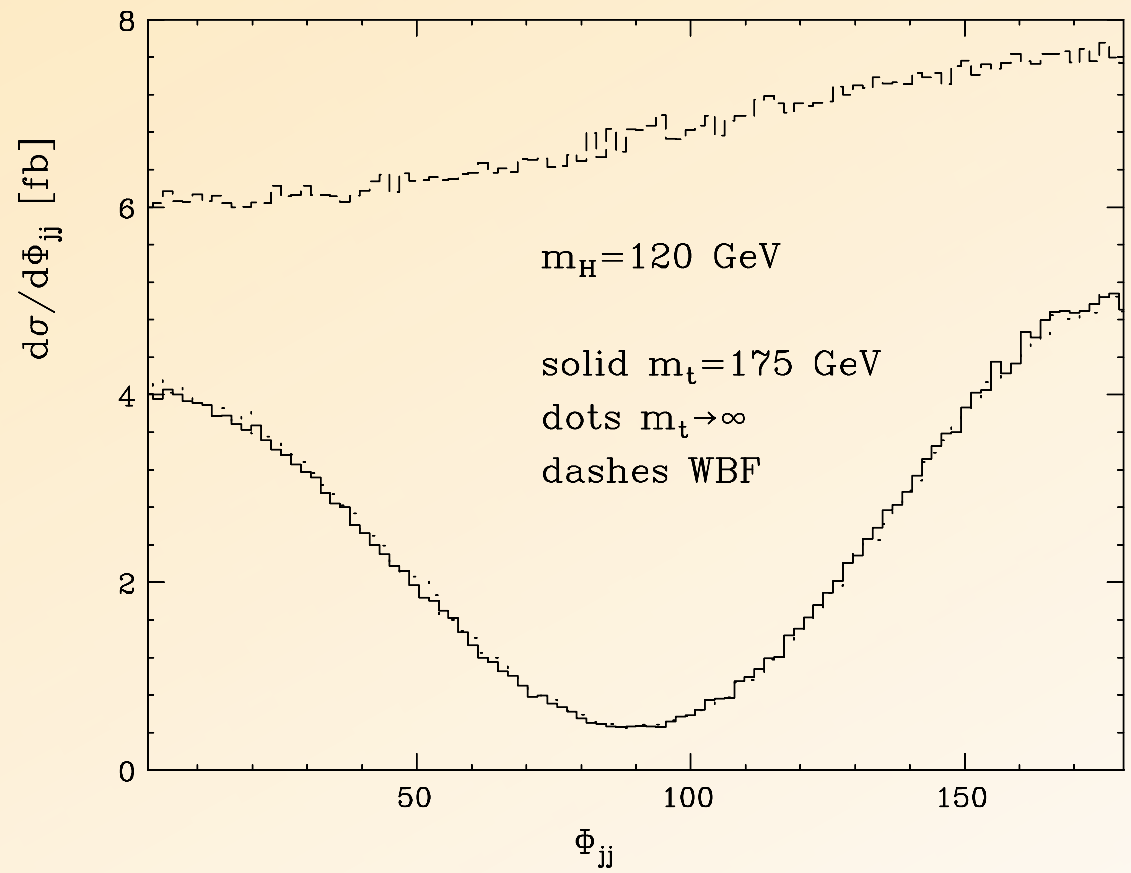
- reduced dependence at NLO,
  $\sim 30(40)\%$ for $m_H = 115(160) GeV$
  but still important (similar for VBF-cuts)

[Campbell, Giele,GZ '06]

Example: azimuthal angle between jets

Because of CP even nature of SM Higgs, azimuthal distribution of jets is peaked at $\phi_{jj} = 0, \pi$, for the ggF processes, while it's almost flat for VBF

[Del Duca et al. '01]



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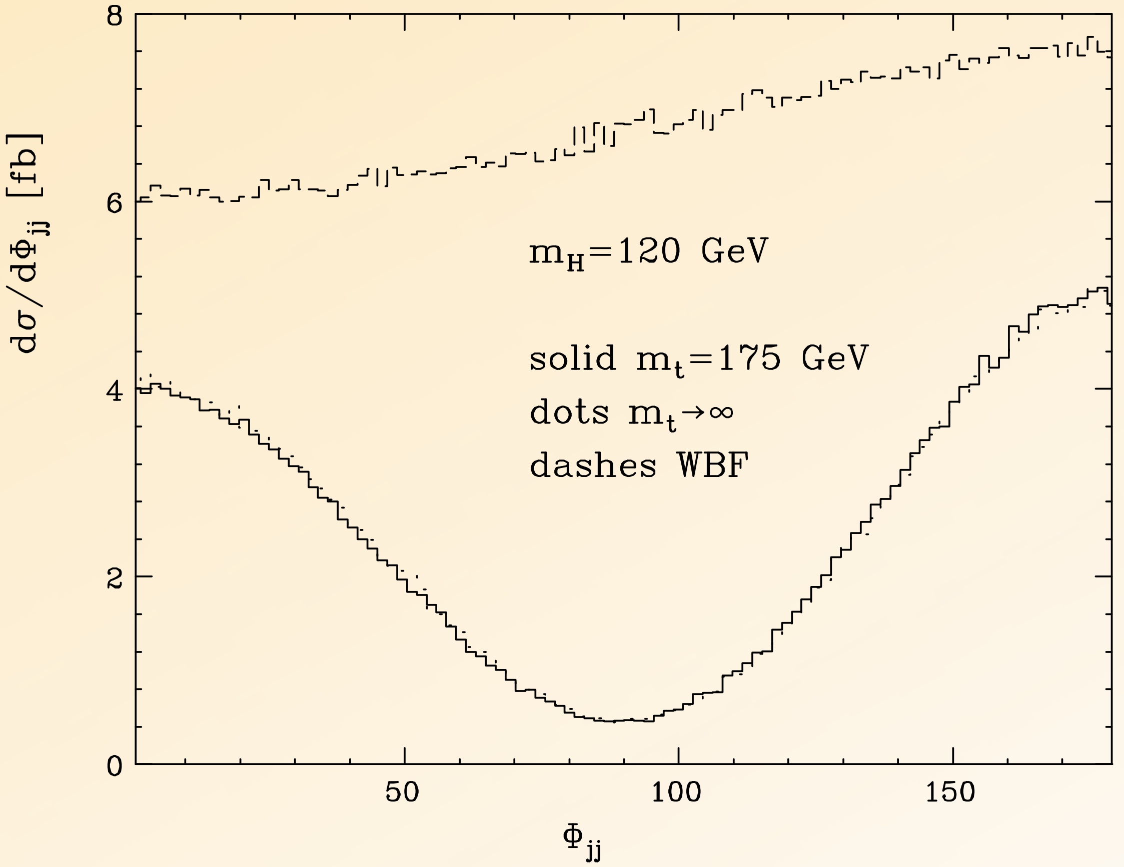

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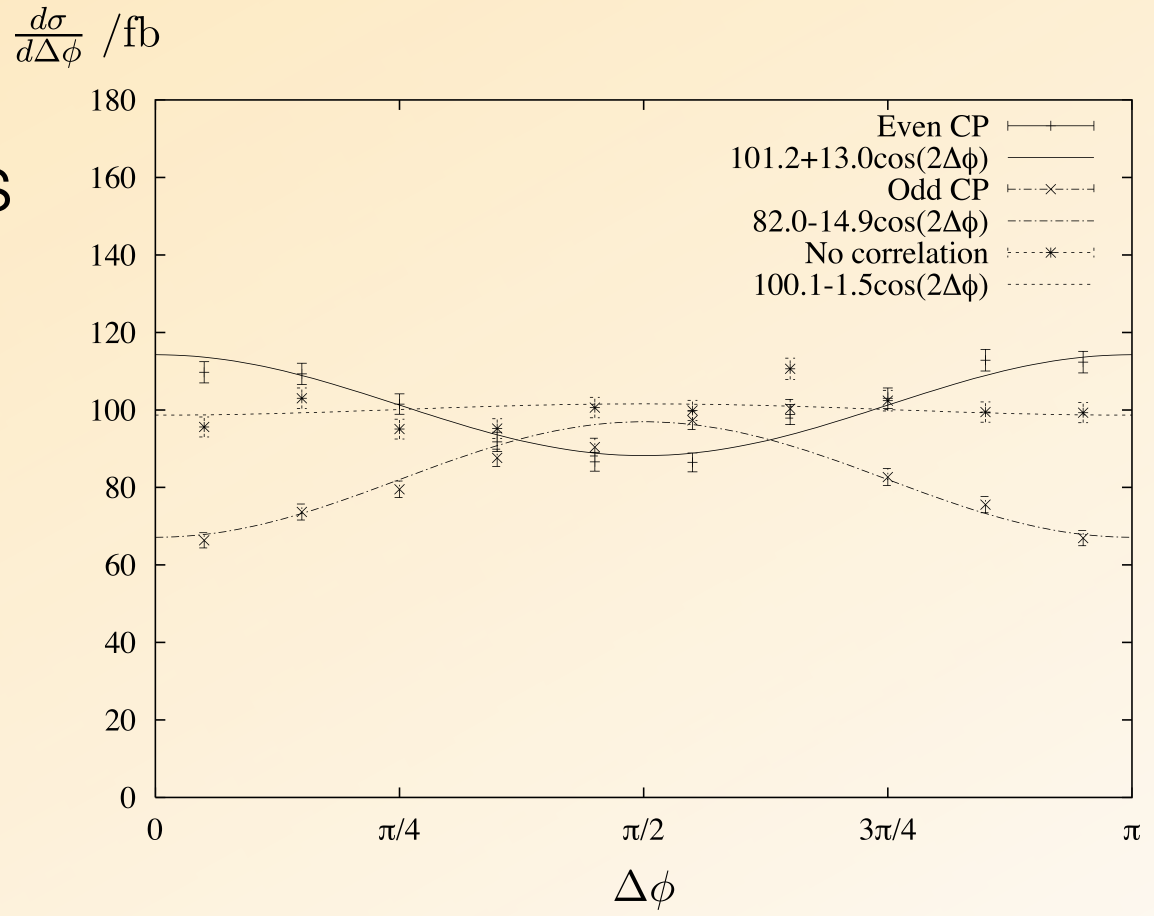

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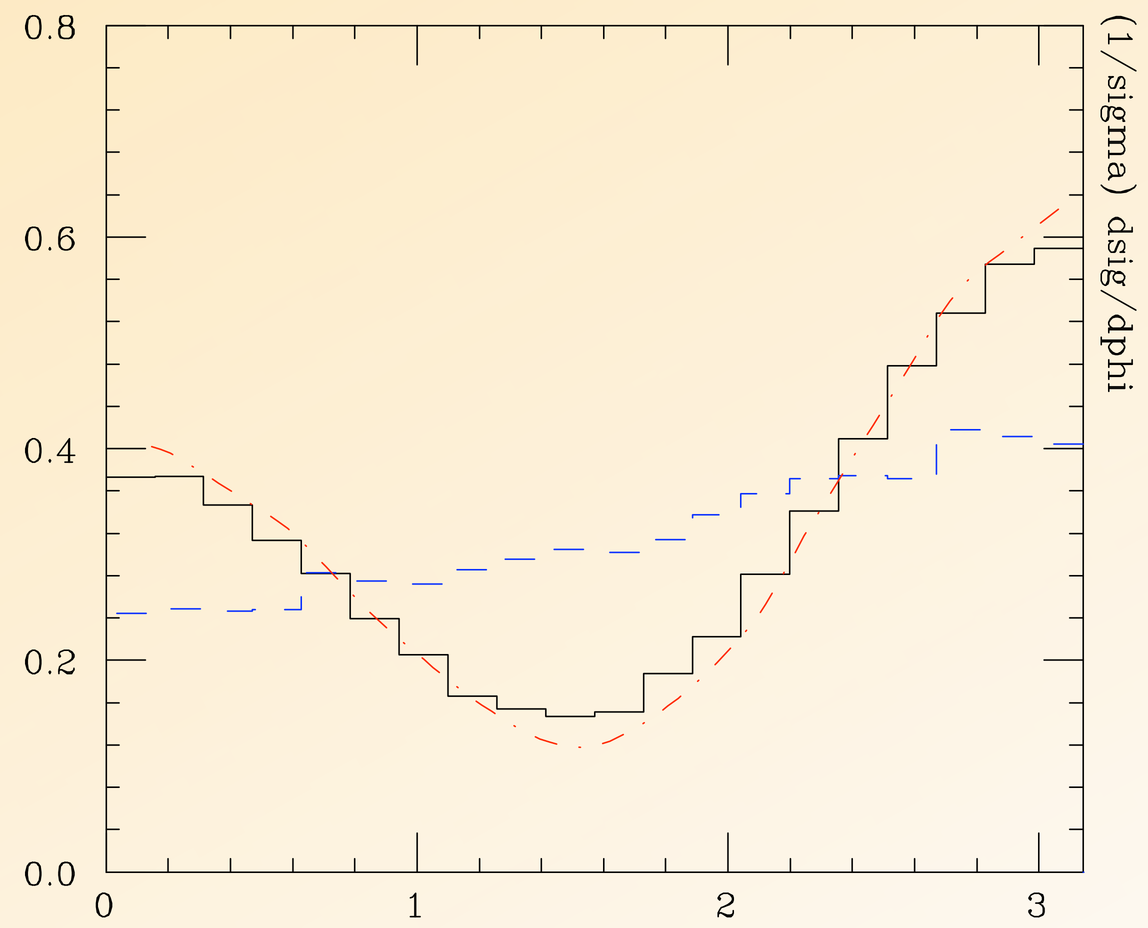

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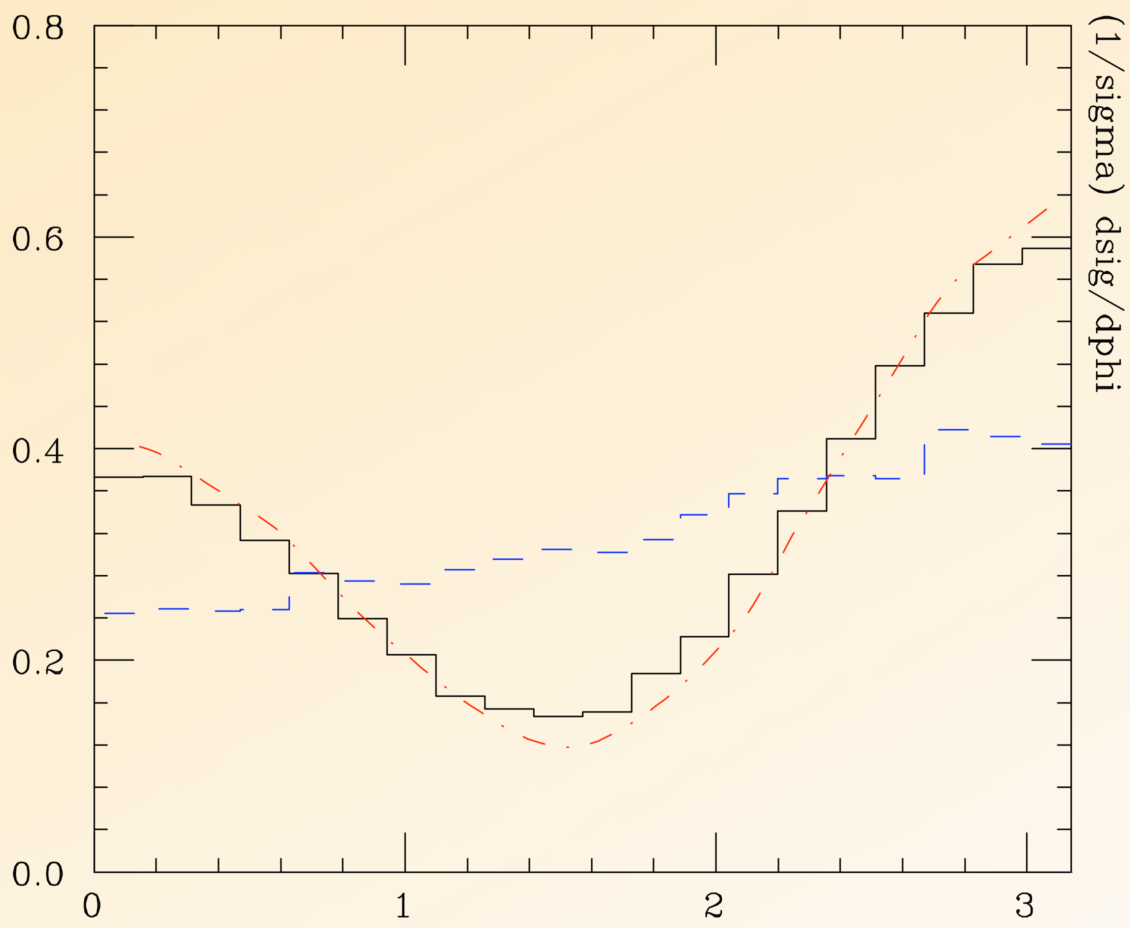
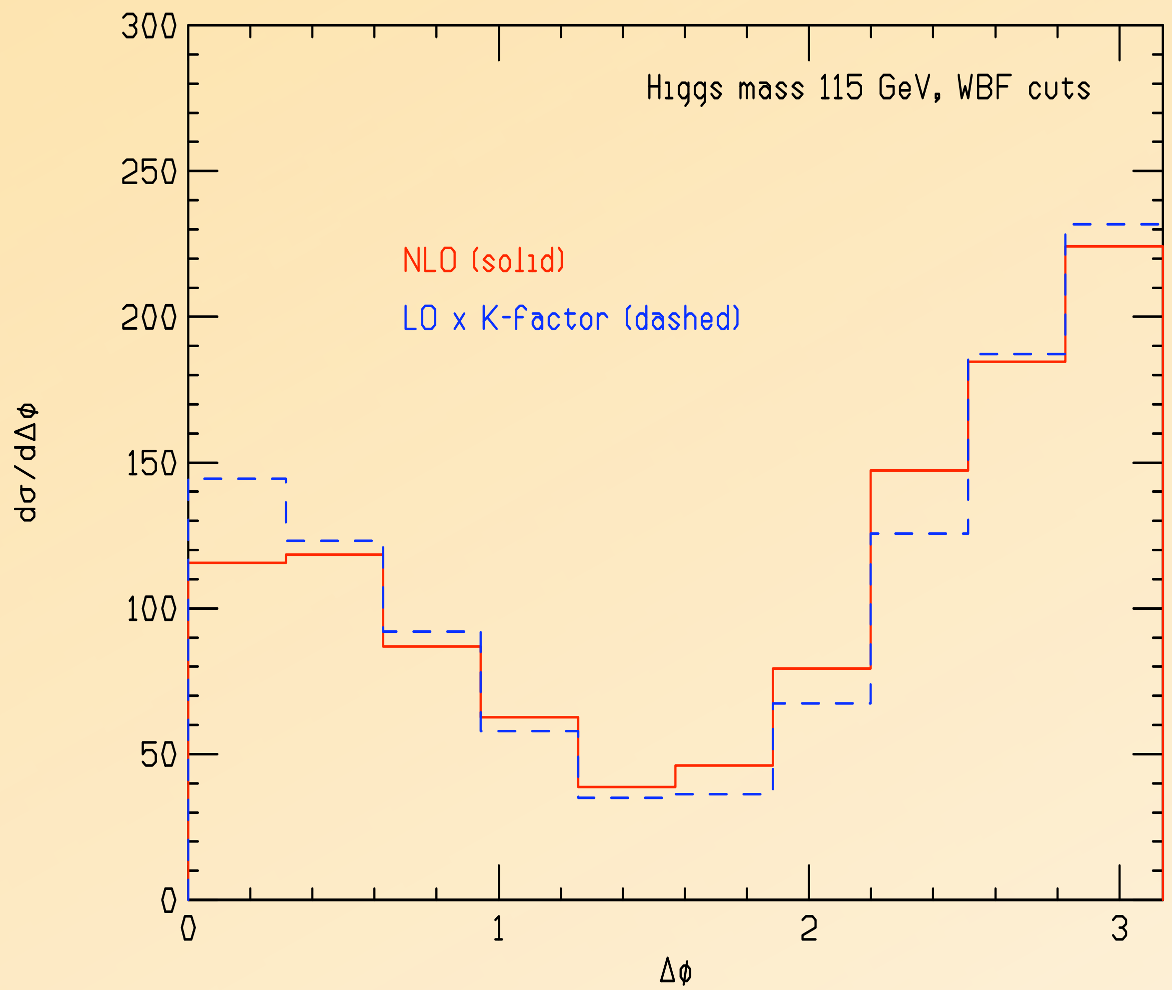
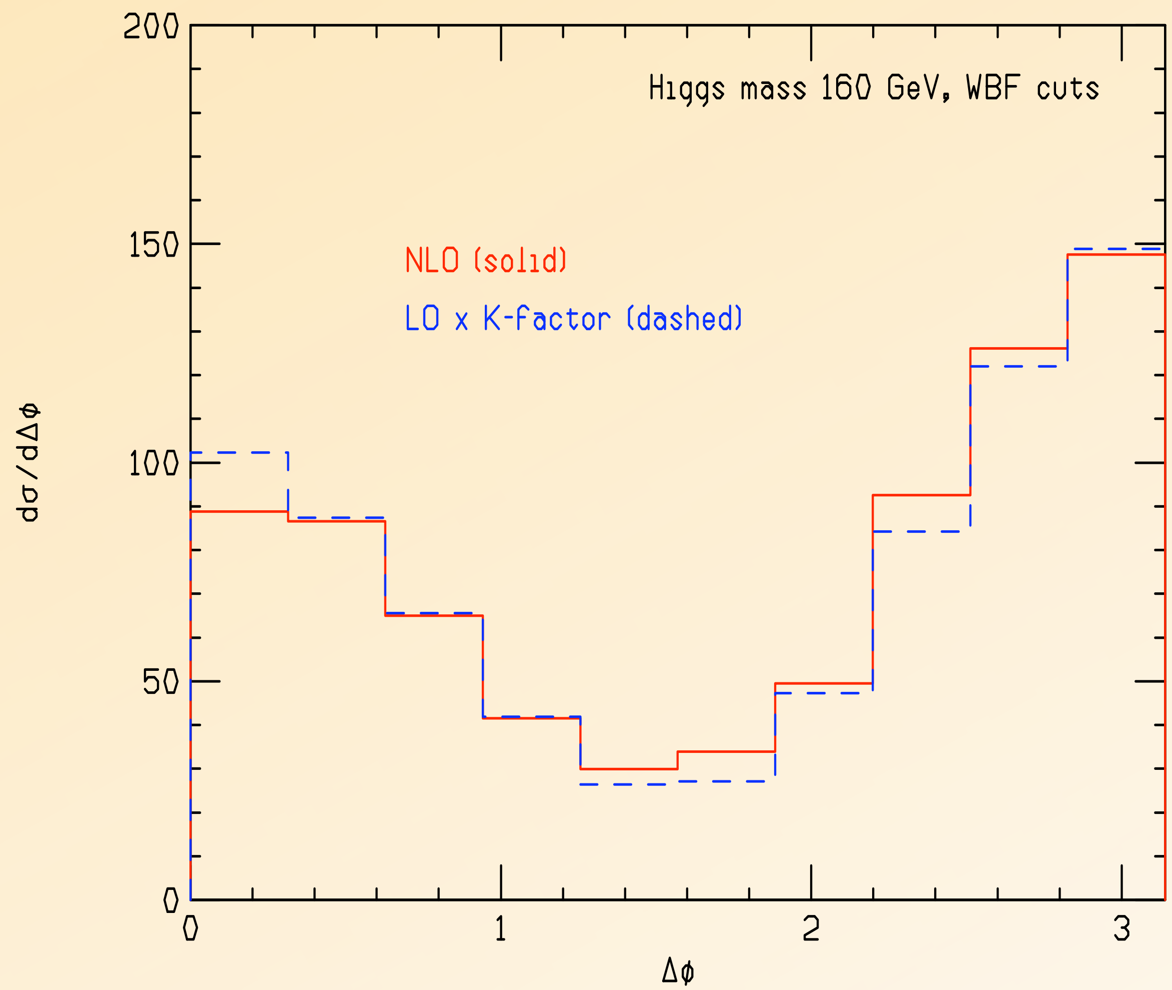


⇒ interesting to see what happens at NLO

Angular distribution





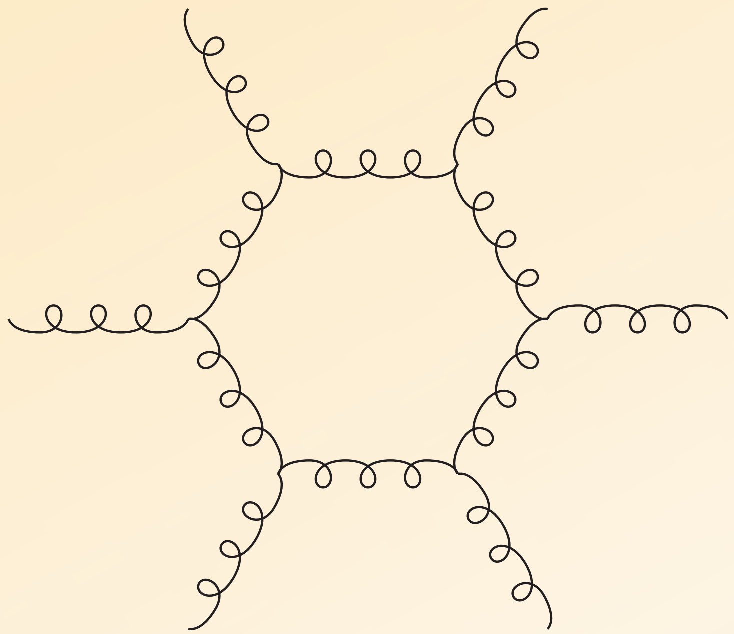
→ no appreciable change of shape, correlation survives at NLO

[Ellis, Giele,GZ '06]

The one-loop six gluon amplitude (I)

Motivation:

- essential ingredient for NLO four-jet production at LHC
- most complicated six-leg calculation in QCD
- tests applicability of the method to six-leg processes



$+ \sim 12000$ diagrams

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Elements of the calculation:

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[Ellis, Giele,GZ '06]

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- number of diagrams involved:12000, the most complicated involve up to rank six six-point integrals

[Ellis, Giele,GZ '06]

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For N=6 it's still a long way to go from amplitudes to cross-section. For the moment we decided not to purse this further.

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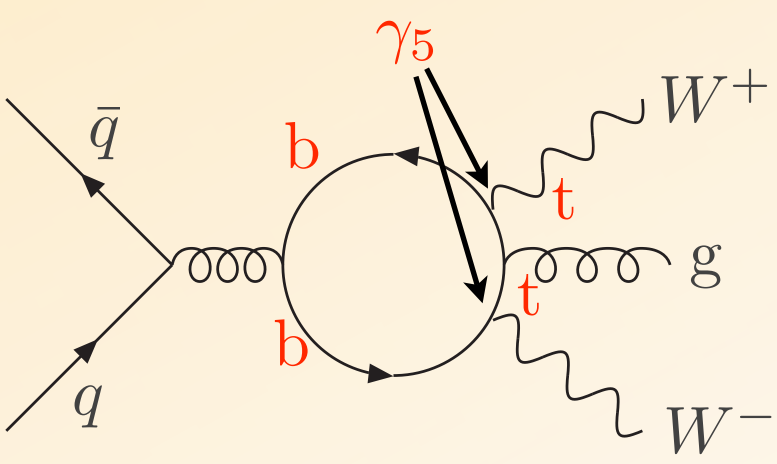

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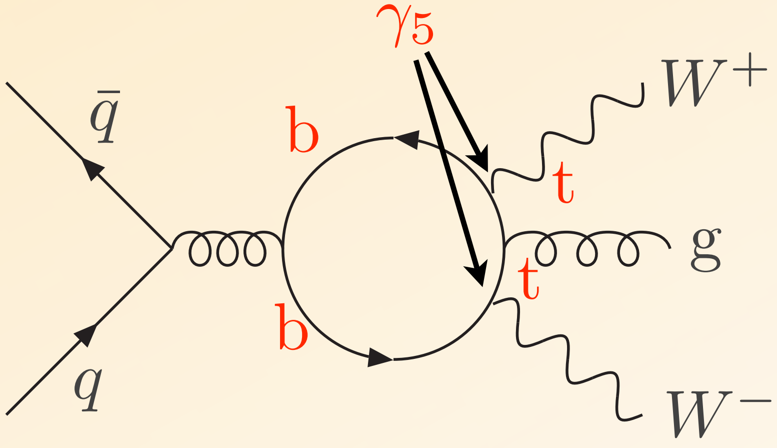

- higher order $gg \to WWg$ (enhanced by gluons PDFs)
  - virtual calculation done

[Campbell,Ellis,GZ in progress]

Conclusions

- We developed a method to evaluate one-loop corrections to N-leg (N<7) processes and implemented it in a numerical program
- Applications:
  - Higgs plus dijet via gluon fusion at the LHC
  - six gluon amplitudes
  - in progress: WW+jet, ZZ+j,WWW, ZZW
  - Next: WW+2 jets