SOFT GLUON RESUMMATION
vs
PARTON SHOWER SIMULATIONS

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Introduction: QCD hard scattering

We limit ourselves to considering hard scattering events:
the starting point is the factorization theorem

$$\sigma(p_1,p_2;Q^2)=\sum_{a,b}\int_0^1 dx_1\int_0^1 dx_2\, f_{h_1,a}(x_1,\mu_F^2)\, f_{h_2,b}(x_2,\mu_F^2) \times\hat{\sigma}_{ab}(x_1p_1,x_2p_2,\alpha_S(Q^2),\mu_F^2)$$

⇨ Predictions for the hard cross section can be obtained through:

- Fixed order calculations
- All-order resummed calculations
- Parton Shower simulations

Fixed order calculations

Truncate perturbative expansion at a given order in α_S

$$\hat{\sigma}(p_1,p_2;Q\{Q_1,...\};\mu_F) = \alpha_S^k(\mu_R)\Big(\hat{\sigma}^{(LO)}(p_1,p_2;Q\{Q_1,...\}) + \alpha_S(\mu_R)\,\hat{\sigma}^{(NLO)}(p_1,p_2;Q\{Q_1,...\};\mu_F,\mu_R) + \alpha_S^2(\mu_R)\,\hat{\sigma}^{(NNLO)}(p_1,p_2;Q\{Q_1,...\};\mu_F,\mu_R) + ...\Big)$$

It provides a systematic framework to compute the partonic cross section for an inclusive enough hard scattering process

It is reliable only when all the scales are of the same order:
if $Q_1 \gg Q$ ⇨ large logarithmic contributions of the form

$(\alpha_S L^2)^n$ with $L \equiv \ln Q_1/Q$ arise that may spoil the perturbative expansion

Soft-gluon resummations

In general: even if KLN theorem guarantees the cancellation of IR singularities, soft-gluon effects can still be large when real and virtual contributions are kinematically unbalanced

$\alpha_S L^2$: one soft and one collinear log for each power of α_S

⇨ Look for an improved perturbative expansion when $\alpha_S L \sim 1$

$\alpha_S L^2$	$\alpha_S^2 L^4$	$\cdots\cdots$	$\alpha_S^n L^{2n}$	**Leading Logs (LL)**
$\alpha_S L$	$\alpha_S^2 L^3$	$\cdots\cdots$	$\alpha_S^n L^{2n-1}$	**Next to Leading Logs (NLL)**
				....

Resummation is possible if the observable fulfills the property known as exponentiation

This implies two basic conditions:

- Matrix element factorization
- Phase space factorization

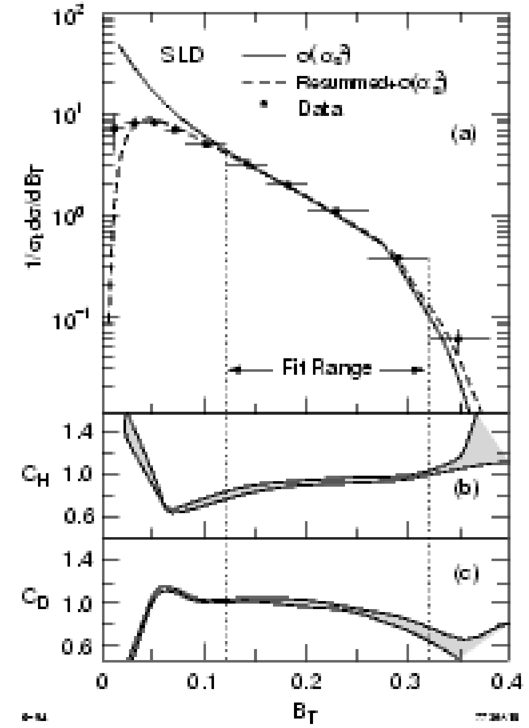
The first is a consequence of gauge invariance and unitarity: in soft and collinear limits the singular structure of QCD matrix elements can be factored out in a (universal) process independent manner

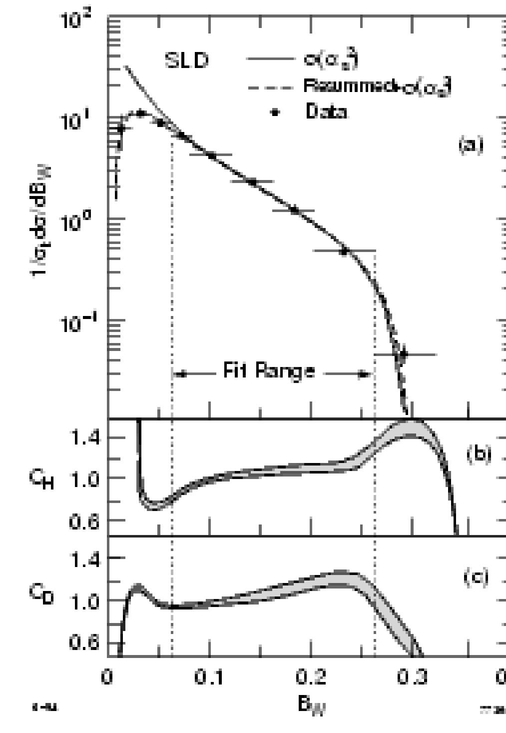
The second condition regards kinematics and depends on the cross section considered
If this condition can be fulfilled (typically working in a conjugate space) resummation is feasible

Well known examples:

- Event shapes
  and jet rates in e^+e^-

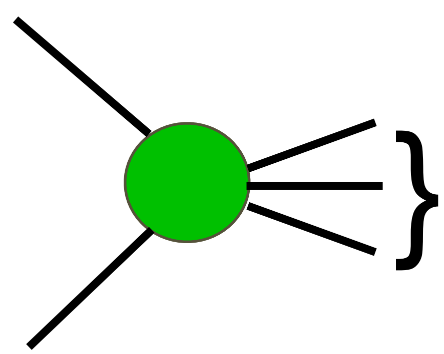
  Typically use Laplace transform





- Resummation of soft gluons near threshold $z = Q^2/\hat{s} \to 1$

$\hat{s}$



Q^2

Work in Mellin N-space

- q_T distributions in hadron collisions or
  Energy-Energy Correlation in e^+e^-

Work in impact parameter b-space

General structure of resummed cross section in conjugate space ν:

$$\hat{\sigma}_\nu^{\mathrm{res.}} \sim \alpha_S^k \, \hat{\sigma}_\nu^{(LO)} \, H(\alpha_S) \exp\{L \, g_1(\alpha_S L_\nu) + g_2(\alpha_S L_\nu) + \alpha_S g_3(\alpha_S L_\nu) + ...\}$$

$$L_\nu = \ln \nu$$

The functions g_1, g_2, g_3 control LL, NLL, NNLL contributions, respectively

$$H(\alpha_S) = \sum_n \alpha_S^n \, H^{(n)}$$

ν-independent hard coefficient

The are defined such that $g_i(\lambda = 0) = 0$

Note: NLL terms formally suppressed by one power of α_S with respect to LL ⇨ The expansion is as systematic as the ordinary perturbative expansion

Note: The terms coming from g_2 are of the same order as those coming from the combined effect of $H^{(1)}$ and g_1

⇨ At NLL we have to include g_1, g_2 but also $H^{(1)}$

How to combine resummed cross section to fixed order ?

→ matching procedure $$\hat{\sigma} = \hat{\sigma}^{\mathrm{res.}} + \hat{\sigma}^{\mathrm{fin.}}$$

Start from resummed contribution $\hat{\sigma}^{\mathrm{res.}}$ which includes all the logarithmically enhanced terms

Define:

$$\hat{\sigma}^{\mathrm{fin.}} = \hat{\sigma}^{\mathrm{f.o.}} - \left[\hat{\sigma}^{\mathrm{res.}}\right]^{\mathrm{f.o.}}$$

↑ standard fixed order result

obtained by subtracting from the fixed order result the truncation of the resummed result at the same order: it does not contain large logarithmic terms

In this way the calculation is everywhere as good as the fixed order result but much better in the region where soft gluon effects are important

Parton showers

Provide an all-order approximation of the partonic cross section in the soft and collinear regions

⇨ somewhat similar to resummed calculations

⊕ Much more flexible, since they can give a fully exclusive description of the final state

⊕ Make possible to include hadronization effects

⊖ Difficult matching with fixed order

⊖ No analytical information

What about logarithmic accuracy ?

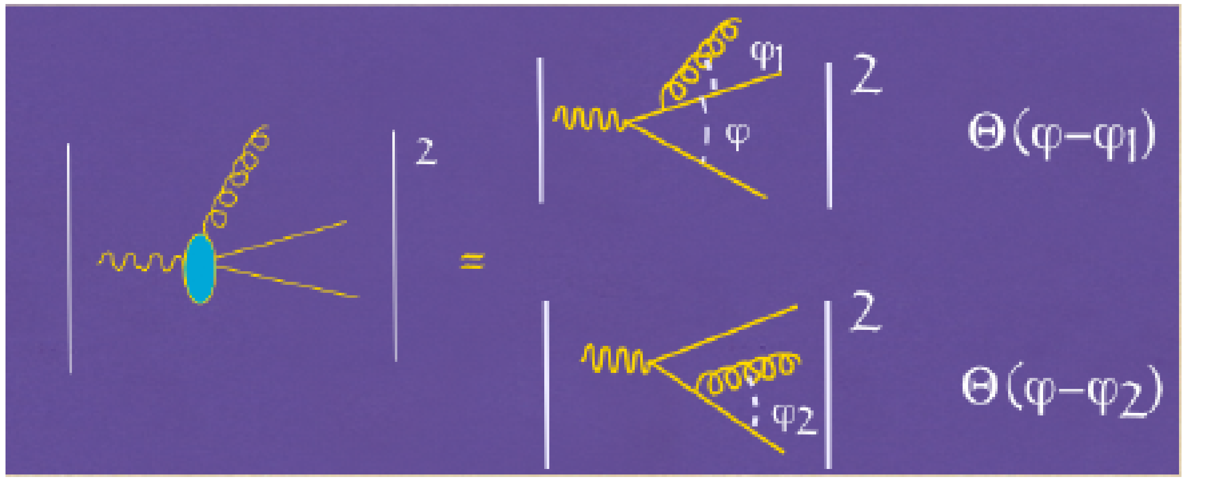
Resummed calculations can be in principle performed to arbitrary logarithmic accuracy

The logarithmic accuracy achievable by parton showers is instead limited by quantum mechanics

Parton showers are essentially probabilistic: quantum interference cannot be taken into account

This problem is overcome by using colour coherence: soft gluon radiated at large angles distructively interfere

The effect of quantum interferences is thus approximated by angular ordering constraint

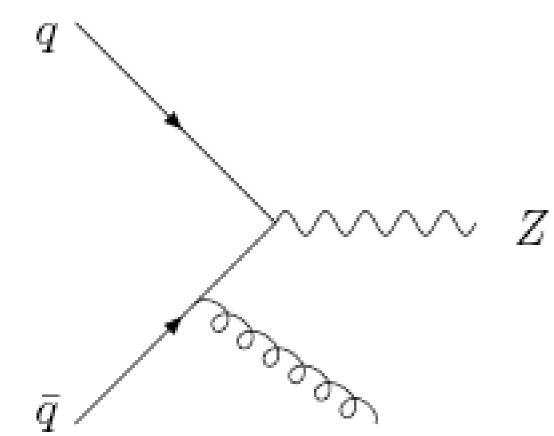


⇨ Angular ordering allows to reach “almost” NLL accuracy (for inclusive enough observables)

The extension to higher logarithmic accuracy is not necessarily feasible

Resummation: an explicit example

Consider the production of a vector boson at small transverse momentum



Two-scale problem: $q_T \ll Q = M_Z$

➡ The recoiling gluon is forced to be either soft or collinear to one of the incoming partons

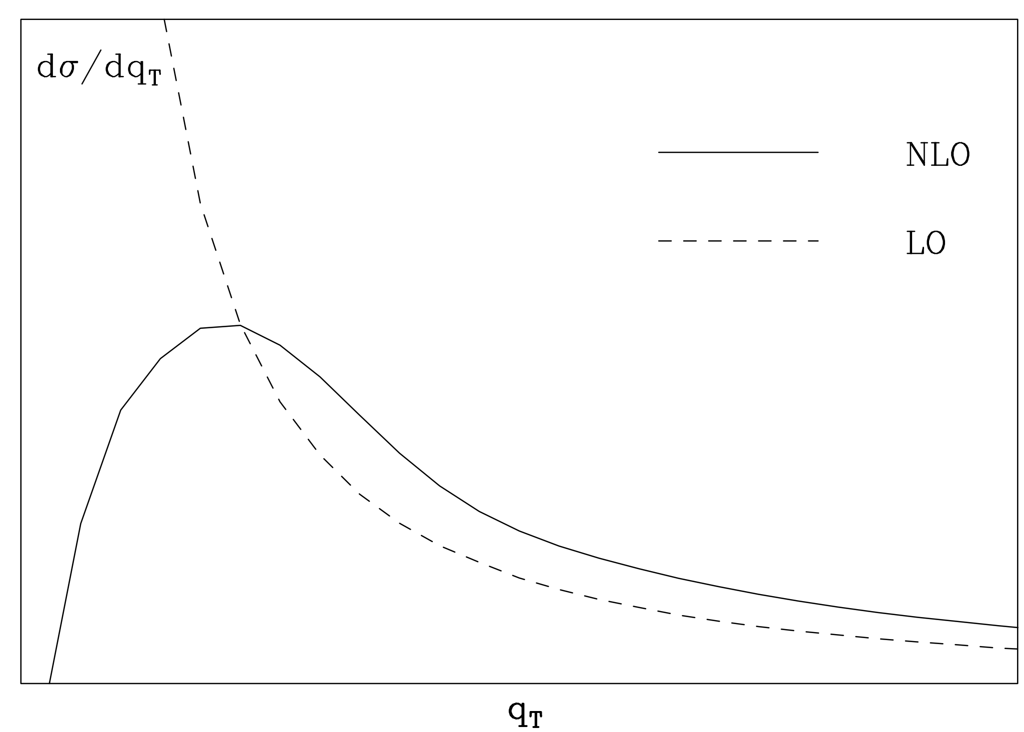
Real radiation strongly inhibited: KLN cancellation still at work but

large logarithmic contributions of the form $\alpha_S^n \log^{2n} Q^2/q_T^2$ appear spoiling the perturbative expansion

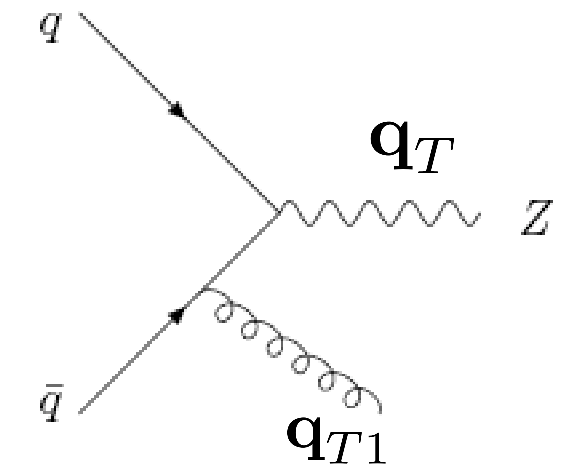
LO: $\frac{d\sigma}{dq_T} \to +\infty$

as $q_T \to 0$

NLO: $\frac{d\sigma}{dq_T} \to -\infty$



Single-gluon contribution in the small $\mathbf{q}_T$ limit



$$\frac{\alpha_S}{2\pi}\int d\omega\,\frac{d\theta^2}{\theta^2}\left(\delta^2(\mathbf{q}_T-\mathbf{q}_{T1})-\delta^2(\mathbf{q}_T)\right)C_F\frac{1+z^2}{1-z}$$

Real — Virtual — qq splitting kernel

$$q_{T1}=\theta E_g$$
$$\omega=E_g/Q$$
$$z=1-\omega$$

$$=\frac{\alpha_S}{2\pi}\int\frac{dq_{T1}^2}{q_{T1}^2}\left(\delta^2(\mathbf{q}_T-\mathbf{q}_{T1})-\delta^2(\mathbf{q_T})\right)C_F\int_0^{1-q_{T1}/Q}dz\left(\frac{2}{1-z}-(1+z)\right)$$

$$=\frac{\alpha_S}{2\pi}\int_0^{Q^2}\frac{dq_{T1}^2}{q_{T1}^2}\left(\delta^2(\mathbf{q}_T-\mathbf{q}_{T1})-\delta^2(\mathbf{q_T})\right)\left(A^{(1)}\log\frac{Q^2}{q_{T1}^2}+B^{(1)}+\ldots\right)$$

$$A^{(1)}=C_F$$
$$B^{(1)}=-\frac{3}{2}C_F$$

Write delta functions in b-space

Neglect $\mathcal{O}(q_{T1}/Q)$ terms

Universal coefficients

$$=\frac{\alpha_S}{2\pi}\int\frac{d^2\mathbf{b}}{(2\pi)^2}e^{i\mathbf{b}\mathbf{q}_T}\int_0^{Q^2}\frac{dq_{T1}^2}{q_{T1}^2}\left(e^{-i\mathbf{b}\mathbf{q}_{T1}}-1\right)\left(A^{(1)}\log\frac{Q^2}{q_{T1}^2}+B^{(1)}\right)$$

Performing irrelevant azimuthal integration ⇒

$$\frac{\alpha_S}{2\pi}\int\frac{d^2\mathbf{b}}{(2\pi)^2}e^{i\mathbf{b}\mathbf{q}_T}\int_0^{Q^2}\frac{dq_{T1}^2}{q_{T1}^2}\left(J_0(q_{T1}b)-1\right)\left(A^{(1)}\log\frac{Q^2}{q_{T1}^2}+B^{(1)}\right)$$

More gluons: in b-space kinematics fulfill exact factorization

$$\delta^2(\mathbf{q}_T - \mathbf{q}_{T1} - \ldots \mathbf{q}_{Tn}) \rightarrow e^{i\mathbf{b}(\mathbf{q}_T - \mathbf{q}_{T1} - \ldots \mathbf{q}_{Tn})} = e^{i\mathbf{b}\mathbf{q}_T} \prod_i e^{-i\mathbf{b}q_{Ti}}$$

Adding a 1/n! symmetry factor the single gluon contribution exponentiates

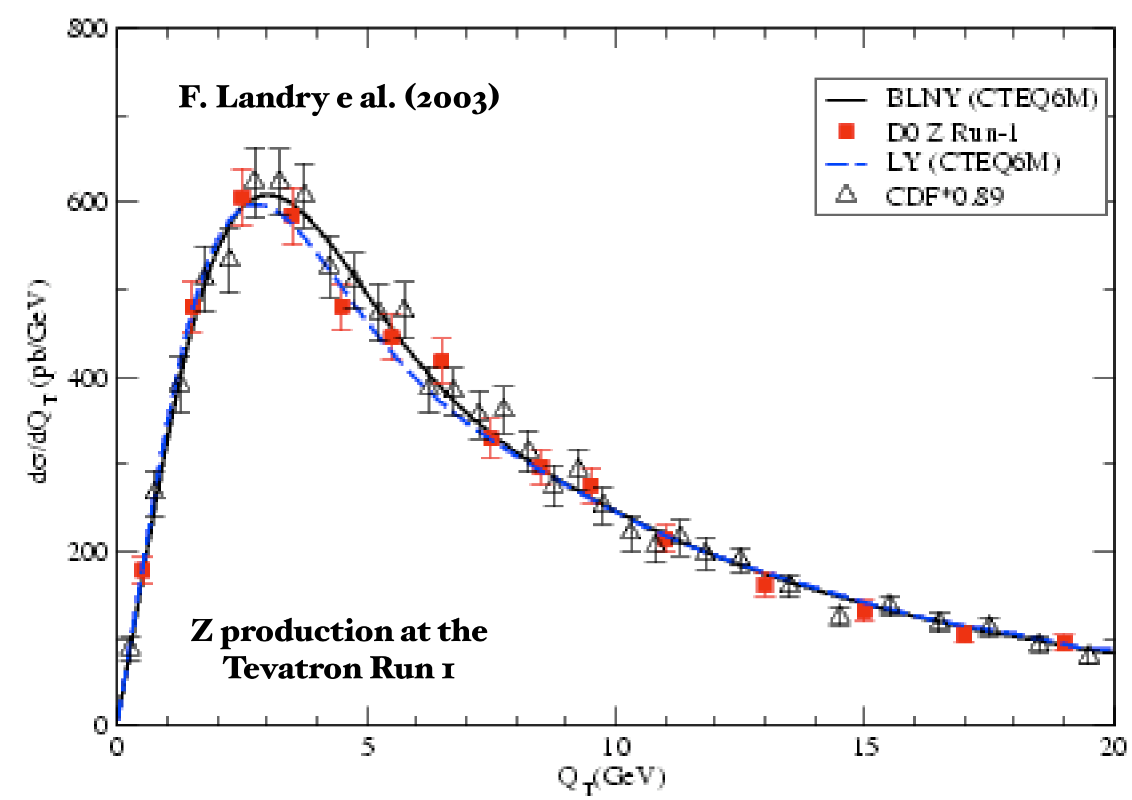
G. Parisi, R. Petronzio (1979)
G. Gurci, M.Greco, Y.Srivastava(1979)
J. Kodaira, L. Trentadue (1982)
J. Collins, D.E. Soper, G. Sterman (1985)

$$\Rightarrow \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b}\mathbf{q}_T} \exp\left\{ \int_0^{Q^2} \frac{dq_{T1}^2}{q_{T1}^2} \left(J_0(bq_{T1}) - 1\right) \frac{\alpha_S}{2\pi} \left(A^{(1)} \log \frac{Q^2}{q_{T1}^2} + B^{(1)} \right) \right\}$$

Replacing $\alpha_S \rightarrow \alpha_S(q_{T1}^2)$ we can control subleading effects

The resummed cross section is now finite as $q_T \rightarrow 0$

This is what we observe in the data !



An improved b-space formalism

We use b-space resummation and introduce some novel features

G. Bozzi, S. Catani, D. de Florian, MG (2005)

Parton distributions are factorized at $\mu_F \sim M$

$$\frac{d\hat{\sigma}_{ac}^{(\mathrm{res.})}}{dq_T^2} = \frac{1}{2}\int_0^\infty db\, b\, J_0(bq_T)\, \mathcal{W}_{ac}(b, M, \hat{s}; \alpha_S(\mu_R^2), \mu_R^2, \mu_F^2)$$

$$\mathcal{W}_N(b, M; \alpha_S(\mu_R^2), \mu_R^2, \mu_F^2) = \mathcal{H}_N\left(\alpha_S(\mu_R^2); M^2/\mu_R^2, M^2/\mu_F^2\right)$$
$$\times \exp\{\mathcal{G}_N(\alpha_S(\mu_R^2), bM; M^2/\mu_R^2, M^2/\mu_F^2)\}$$

where the large logs are organized as:

$$\mathcal{G}_N(\alpha_S, bM; M^2/\mu_R^2, M^2/\mu_F^2) = L\, g^{(1)}(\alpha_S L)$$
$$+ g_N^{(2)}(\alpha_S L; M^2/\mu_R^2) + \alpha_S\, g_N^{(3)}(\alpha_S L; M^2/\mu_R^2, M^2/\mu_F^2) + \dots$$

with $L = \ln M^2 b^2/b_0^2$ ⇒ $\tilde{L} = \ln\left(1 + M^2 b^2/b_0^2\right)$ and $\alpha_S = \alpha_S(\mu_R)$

Unitarity constraint enforces correct total cross section

The qt spectrum of the Higgs

G. Bozzi, S. Catani, D. de Florian, MG (2003,2005)

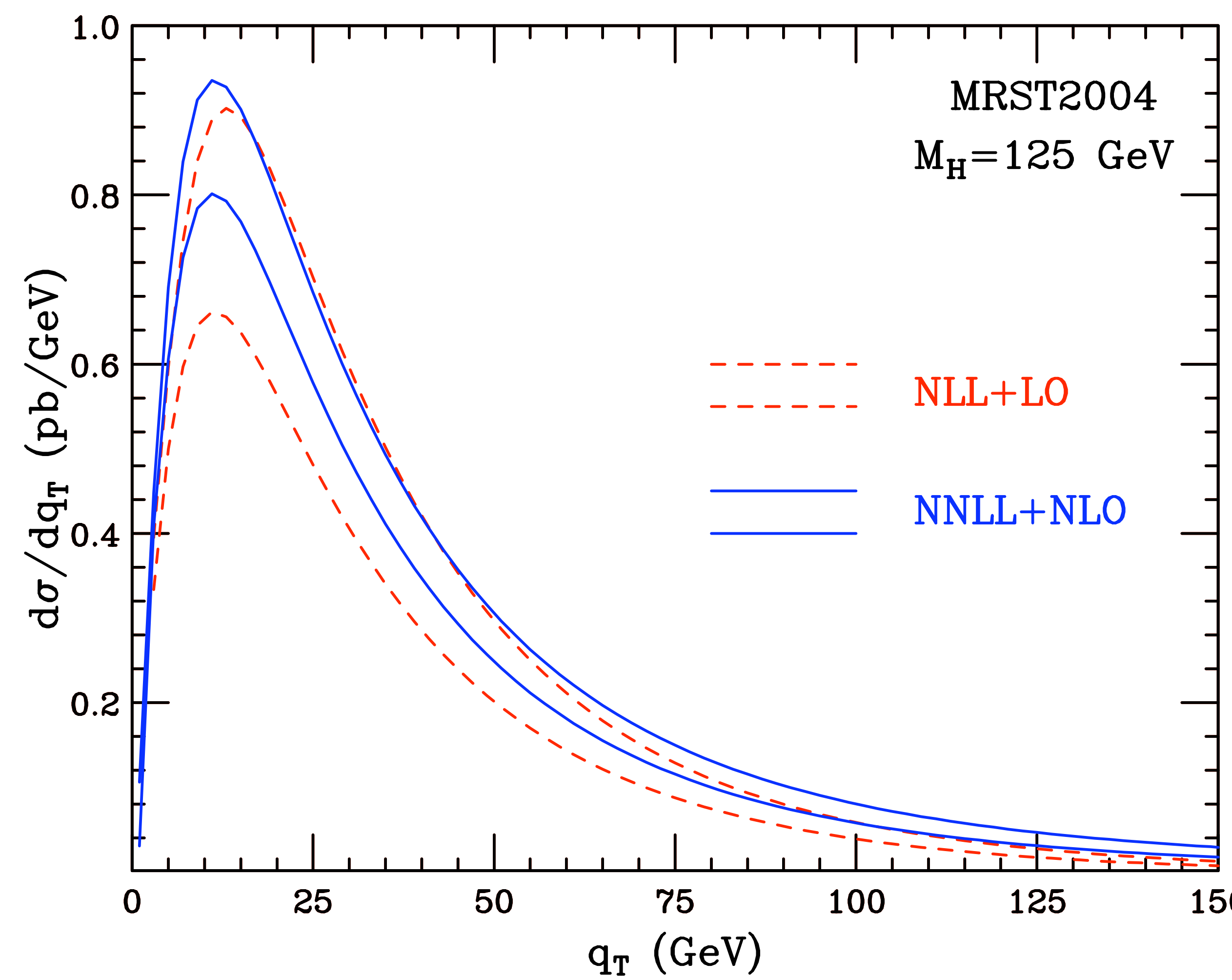
We applied the formalism to compute the Higgs spectrum at the LHC

NLL+LO and NNLL+NLO results with consistent study of theoretical uncertainties and high quality matching to fixed order

Integral of resummed spectra reproduces the correct NLO and NNLO total cross sections

Calculation implemented in the fortran code HqT available at

http://theory.fi.infn.it/grazzini/codes.html

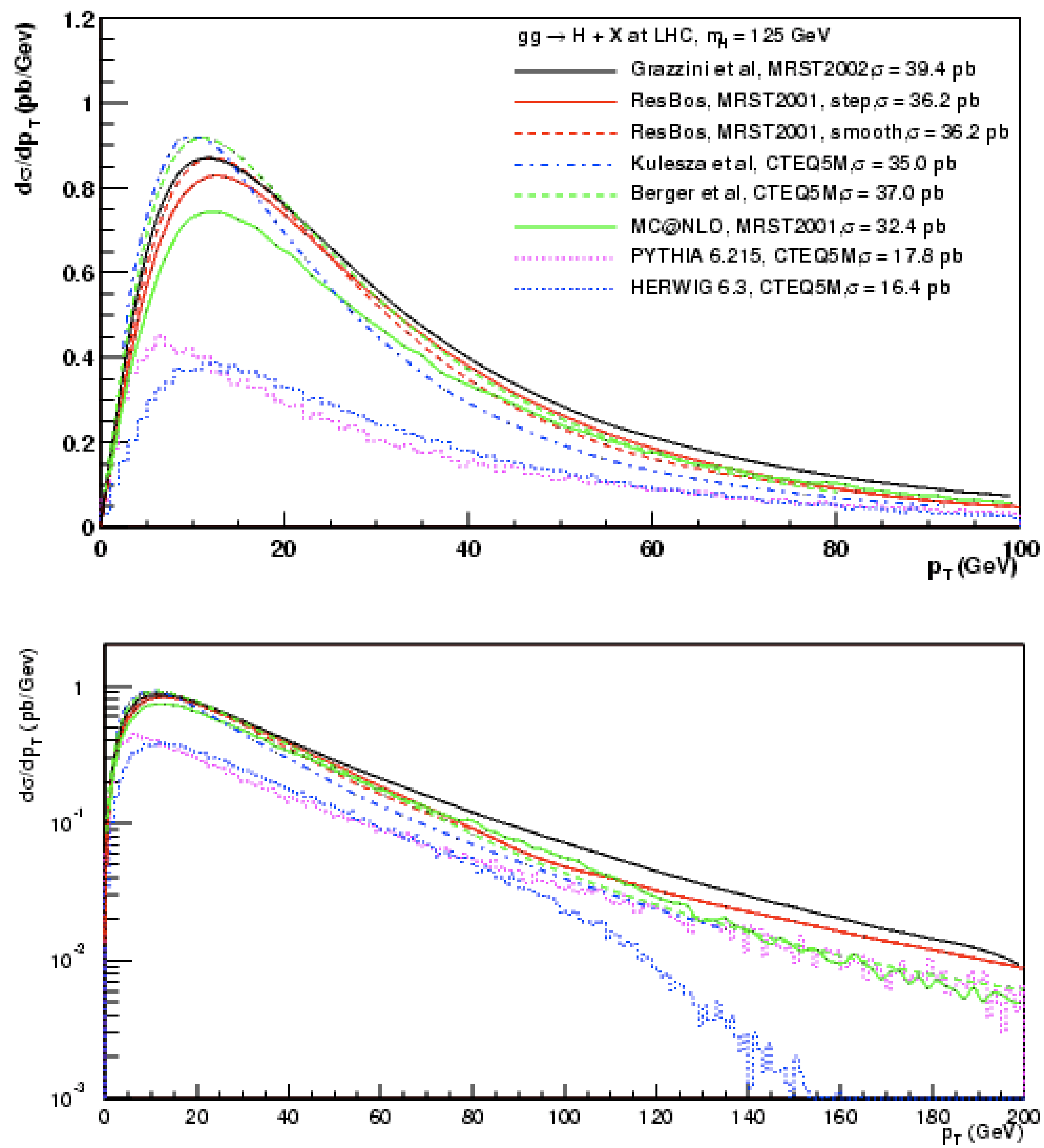


Higgs spectrum: comparison of different approaches

C. Balazs et al., Les Houches 2003

- RESBOS: basically NLL+LO accuracy: NLO at large q_T included though a K-factor
- Berger et al.: basically (N)NLL+LO
- Kulesza et al.: joint resummation of transverse momentum and threshold corrections
- HERWIG 6.3 (no ME correction)
- PYTHIA 6.2 (ME correction included)

Reasonable agreeement in shape but Pythia 6.2 considerably softer !



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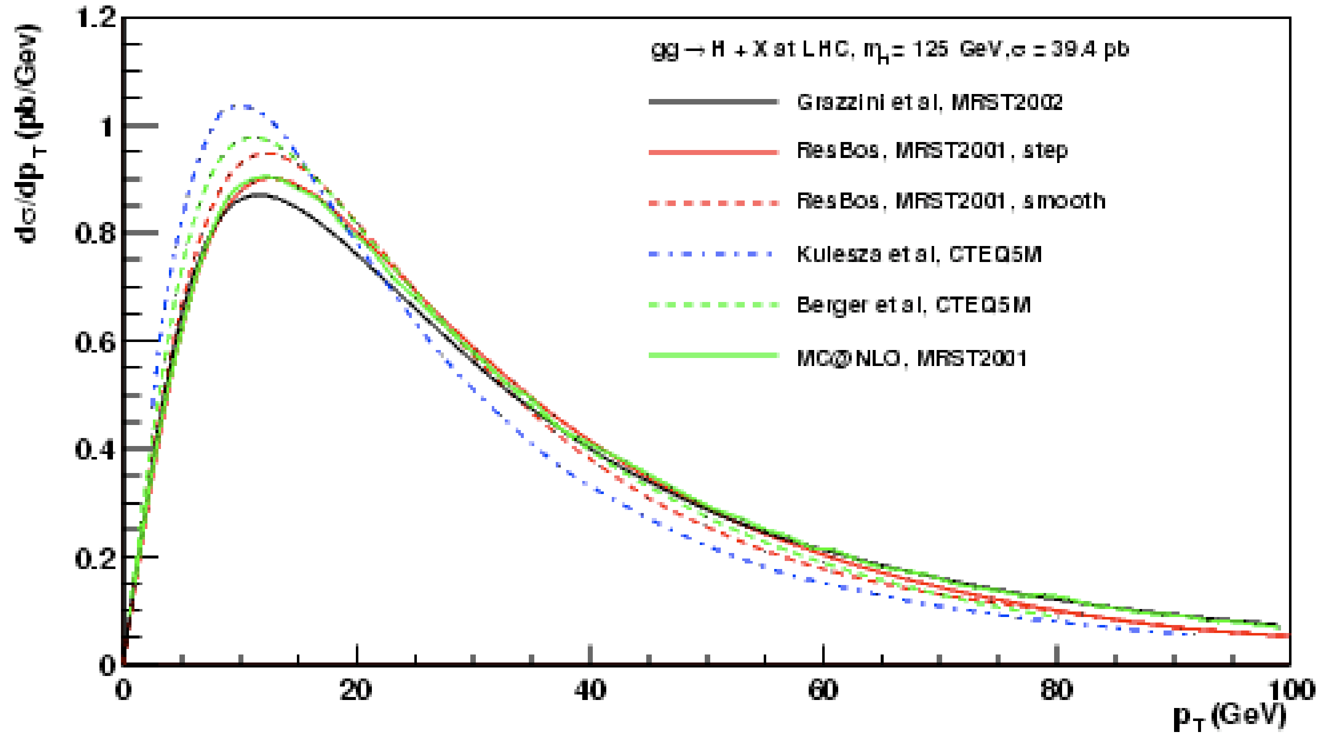
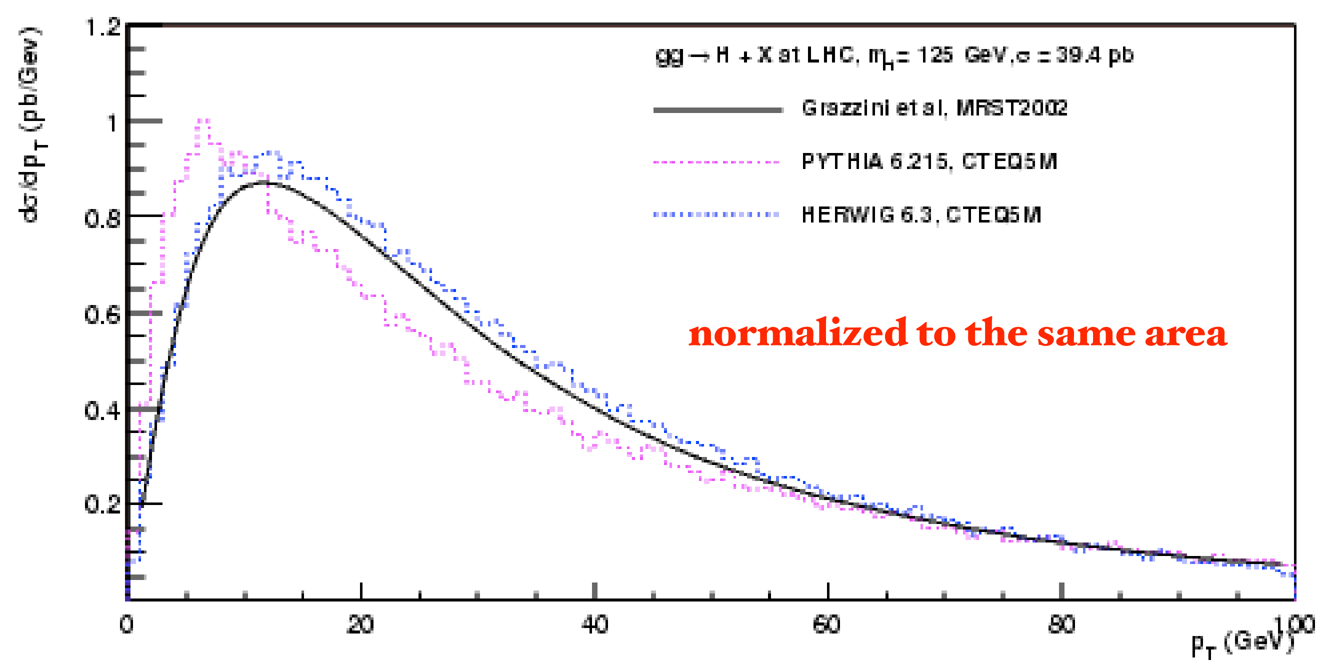
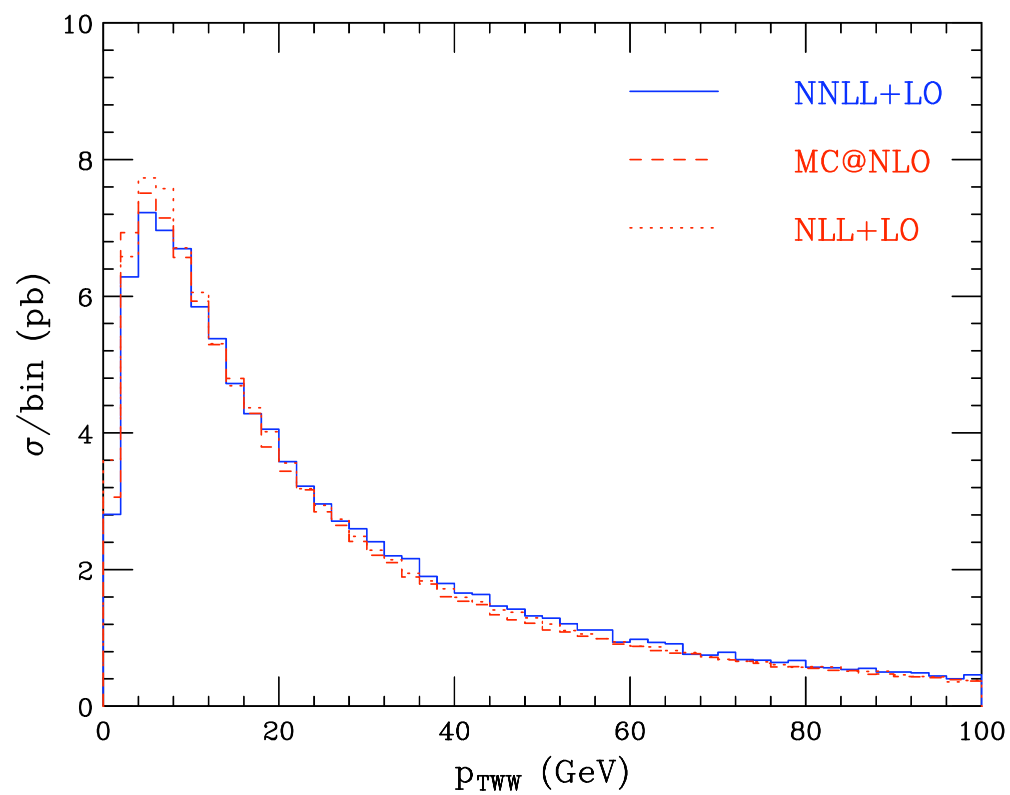
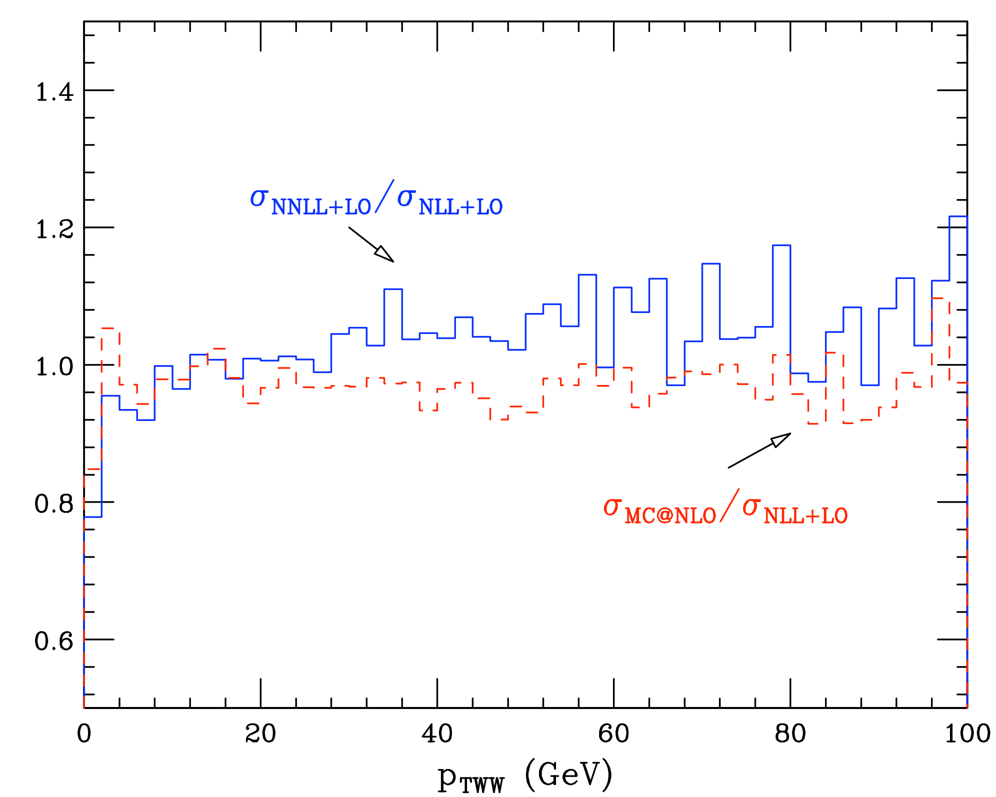




WW : Resummation vs MC@NLO

MG (2005)





NLO result (not shown) diverges to $+\infty$ as $p_T^{WW} \to 0$

Nice agreement with MC@NLO

NNLL effect tends to make the spectrum harder

Resummation effects generally small on leptonic observables

Effects seen only when hard cuts are applied

Recent progress: automated resummation

Fixed order NLO calculations typically implemented in observable-independent parton levels MC codes

Resummed calculations usually worked out (when possible) analytically for each observable ⇨ generally painful

CAESAR

A. Banfi, G. Salam, G. Zanderighi (2003)

Automatic resummation of a large class of event-shape variables

Avoids the need to find the conjugate space in which the observable factorizes

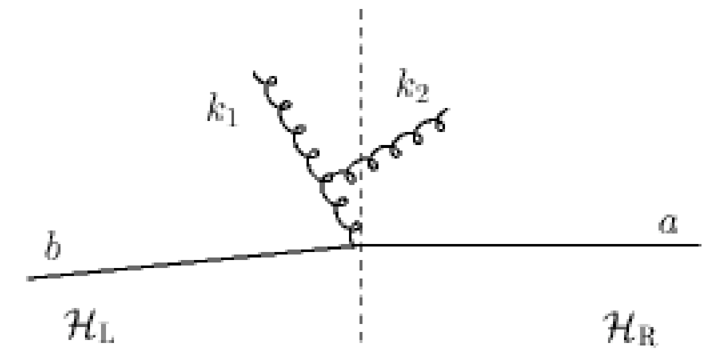
Price to pay: more limited range of applicability and accuracy

To be done: matching with fixed order

Recent progress: non-global logs

Non global observables: sensitive to emissions in only a part of phase space

These observables are affected by previously neglected single logarithmic contributions



M. Dasgupta, G. Salam (2001)

Resummed in closed form only in the large N_c limit

These effects are due to soft-gluon radiation at large angles ⇨ difficult to take them into account in MC parton shower

Recently: even "superleading" terms discovered in gap-between jets cross section at hadron colliders

J.R.Forshaw, A. Kyrieleis, M.H.Seymour (2006)

Summary

Resummed calculations allowed us to push the validity of QCD perturbation theory to the boundary of the available phase space where fixed order predictions are not reliable

Resummed predictions are automatically provided by standard MC:

- (+) Much more flexible, since they can give a fully exclusive description of the final state
- (+) Make possible to include hadronization effects
- (−) Difficult matching with fixed order
- (−) Logarithmic accuracy often unclear
- (−) Difficult to estimate uncertainties

Analytical resummations provide the most advanced theoretical accuracies available at present

- (+) Up to NNLL in some cases (threshold, q_T , EEC)
- (+) Easy matching with fixed order
- (+) Easier to estimate uncertainties
- (−) Have to be worked out for each observable (but progress in automatization is being made)

Bottom line:

MC and analytical resummations are complementary !
Analytical resummed calculation will be particularly helpful in the validation of MC simulation tools

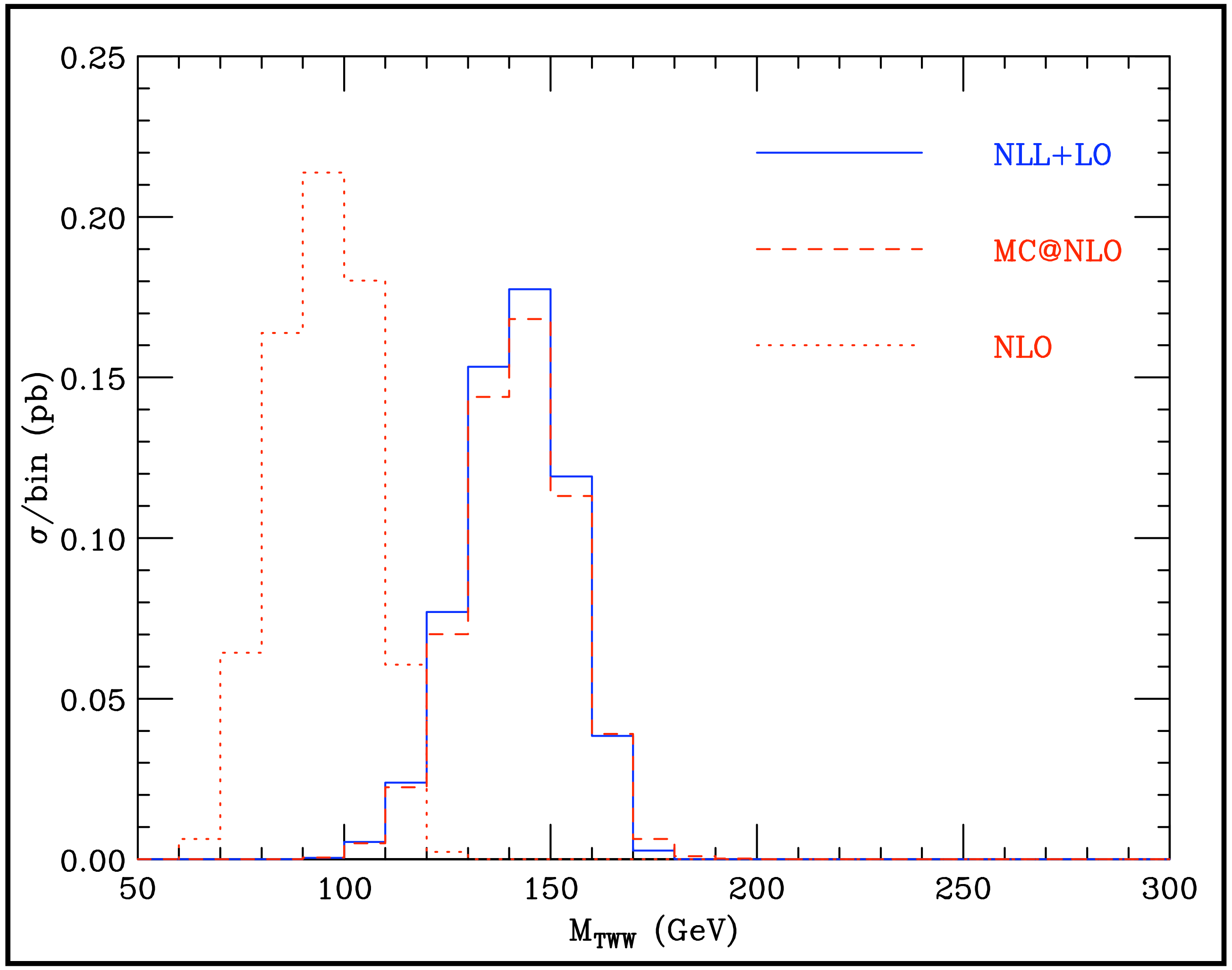
EXTRA SLIDES

But try now with cuts used for Higgs search in the $H \to WW \to l\nu l\nu$ channel

$p^l_{T\,\mathrm{min}} > 25$ GeV $\quad 35$ GeV $< p^l_{T\,\mathrm{max}} < 50$ GeV

$p_T^{\mathrm{miss}} > 20$ GeV $\quad \Delta\phi < 45^o \quad m_{ll} < 35$ GeV

$p_T^{\mathrm{jet}} < 30\,\mathrm{GeV}$



The M_T^{WW} **distribution is completely off at NLO !**
The position of the peak is shifted by about $50\,\mathrm{GeV}$