COMBINING PARTON SHOWERS (PS) WITH TREE-LEVEL MATRIX ELEMENTS (ME)

(or, better,

"Introduction to CKKW*")

MC WS
LNF, Frascati
Feb. 2006

Stefano Catani
INFN, Firenze

- A basic introduction to:
  (and very very elementary/simplified; no technicalities)

  i) the topic / issue

  ii) a method

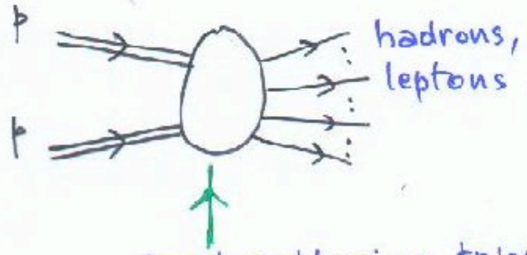
[Overall phylosophy (in our approach):
start from PS ("complex object")
and improve it by implementing
info from ME ("simpler object")]

* F. Krauss, R. Kuhn, B.R. Webber + S.C.
JHEP 0111 (2001) 063,
hep-ph/0109231

THE TOPIC and THE ISSUE

Parton-shower Monte Carlo :

- very useful (essential) tool
  for physics at high-energy colliders :
  complete simulation of real physical events
  produced by hard scattering
  ↳ hard scale
  $Q \gg M_{hadron} \sim 1\,\text{GeV}$



hard-scattering trigger
(high-mass or high-$p_\perp$ system)

- QCD basis of PS MC :

perturbative ⊗ non-perturbative

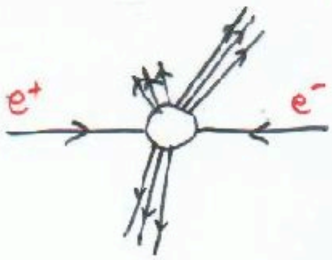
multiple production of quarks and gluons	hadronization, underlying event, multiple scattering, ...
⬇	⬇
"good approximation" of underlying (<u>true</u>) QCD dynamics	based on QCD "inspired" <u>models</u> and, anyhow, tuned to available data

↳ aimed at
description of
<u>bulk</u> of hard-scattering events
("most probable" events)

- bulk of events / most probable events :

examples

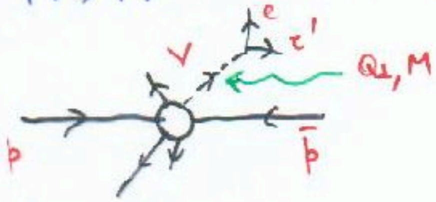
hadronic final state in high-energy e^+e^- collisions (PETRA, LEP, ...)

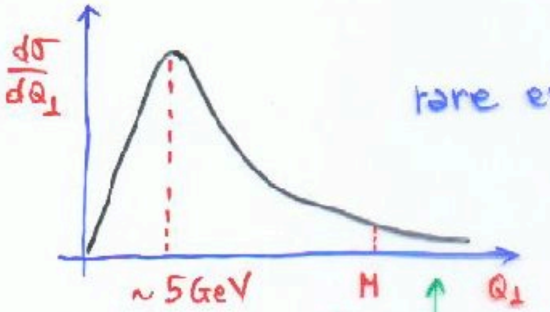


mostly 2-jet events,
some 3-jet events,
few 4-jet events,
⋮

rare events

vector boson ($V = W^{\pm}, Z$) production in $p\bar{p}$, pp collisions





rare events

mostly low-$Q_\perp$ vector boson (with no recoiling high-$p_\perp$ jet)

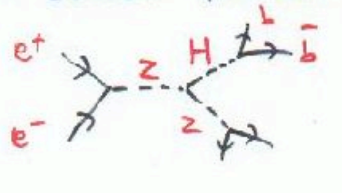
some high-$Q_\perp$ vector boson with 1 recoiling (balancing) high-$p_\perp$ ($p_\perp \sim Q_\perp$) jet

few events with accompanying 2, 3, ... high-$p_\perp$ jets

- multijet hadronic final states:

  rare events but, nonetheless, essential for study of new physics at high-energy colliders ← both background and signal

e.g. — search for SM Higgs boson at LEP



vs. 4-jet QCD background

— SUSY particles at the LHC

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- topic: Monte Carlo simulation of multijet (hadronic) final states

  ↑ beyond the original goal (motivation for) Parton-shower MC

- issue: improve physics of PS by implementing additional info on QCD dynamics

  ↗ as given by exact (tree-level) QCD Matrix Elements
  (indeed, physics of multijet events is mainly perturbative physics, and thus well approximated by perturbative matrix elements)

(Differences from) OTHER RECENT TOPICS IN PHYSICS OF PS MC's

- Hard Matrix Element corrections to PS

- Matching PS ⊕ NLO

- Higher logarithmic accuracy in PS

  e.g.
  - $\alpha_s|_{\overline{MS}} \longrightarrow \alpha_s|_{\text{Bremsstrahlung scheme}}$
  - shower from heavy quarks (where $M_{\text{Quark}} \ll Q$ ← hard scale)
  - improved kinematical variables for shower evolution ({energy; angle} vs. ↪ {longitudinal momentum-fraction, invariant $k_\perp$})

- Non-perturbative / higher-twist effects

  e.g.
  - better hadronization models
  - particle decays
  - multiple interactions, underlying event

- Plan of presentation:

  from exact ME to PS

  and back to

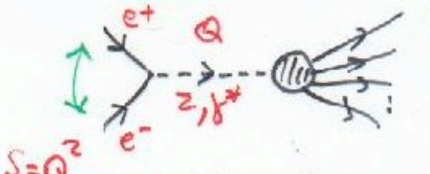
  combining PS ⊕ ME

- use $e^+e^- \to$ hadrons
  as simplest (and best understood)
  process / illustrative example

- comments on hadron collisions
  (depending on remaining
  available time)

e^+e^- ANNIHILATION : HADRONIC FINAL STATES IN PERTURBATIVE QCD

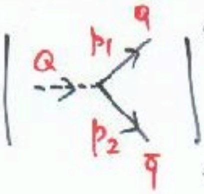
$e^+e^- \to$ hadrons
(PETRA, SLC, LEP, ...)



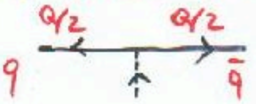
$S = Q^2$ ← hard scale

- 2 parton final state (lowest perturbative order: tree level)

$$\left|M^{(tree)}_{q\bar q}\right|^2 \sim \left|\quad\right|^2$$



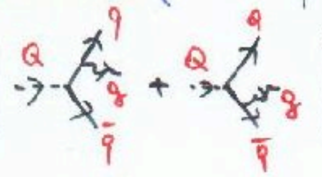
final state fully specified by kinematics
$(2p_1\cdot p_2 = 2p_i\cdot Q = Q^2)$:



$q\bar q$ are back-to-back in e^+e^- c.m. frame

- 3 parton final state (lowest perturbative order: tree level)

$$\left|M^{(tree)}_{q\bar q g}\right|^2 \sim \left|\quad + \quad\right|^2 \qquad \left(\propto \alpha_s = g_s^2/4\pi\right)$$



- relevant feature : soft/coll. limit

Recall : gluon phase space
$$\frac{d^3p_g}{2\omega_g} \sim d\omega_g^2\, d\theta^2$$

$$\left|M^{(tree)}_{q\bar q g}\right|^2 \underset{\uparrow}{\sim} \left|M^{(tree)}_{q\bar q}\right|^2 \cdot 4 g_s^2 C_F \frac{1}{\omega_g^2}\frac{1}{\theta^2_{gq(\bar q)}}$$

$\omega_g \to 0$
$\theta_{gq(\bar q)} \to 0$

singular (not integrable) behaviour in soft/coll. region

- interpretation:

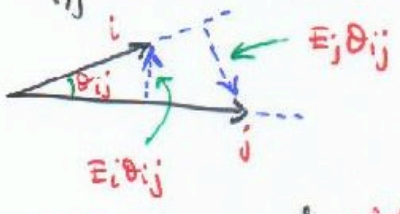
3 parton configuration $\xrightarrow[\text{soft/coll. region}]{}$ indistinguishable from 2 jet event

– some Key points / lessons:

i) introduce resolution variable (lower limit on energy/angle) to separate 3 jet vs. 2 jet events

simpler ("best") resolution variable: minimum relative q_t

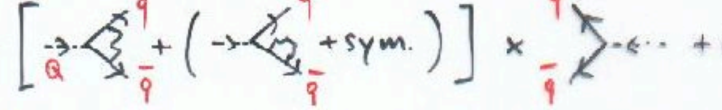
$$q \sim \min_{i,j} \{ E_i \theta_{ij}, E_j \theta_{ij} \}$$



if $q > q_0$ (fixed):
⇒ 3 jet event

ii) if $q < q_0$ ⇒ 2 jet event:

combine $q\bar q g$ at the tree level (REAL EMISSION)
with $q\bar q$ at 1-loop level (VIRTUAL CORRECTION)

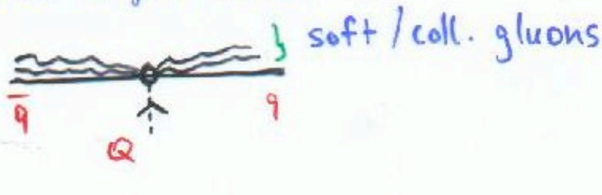
$|M_{q\bar q}^{(1\text{-loop})}|^2 \sim$  $+\ c.c.$

⇒ soft/coll. divergences cancel to get a finite (and q_0-dependent) 2 jet rate at $O(\alpha_s)$

iii) despite cancellation of soft/coll. divergences, (logarithmic) enhancement of tree-level ME in soft/coll. region

↓ implies

jet-like structure of final state: most probable events are 2-jet like



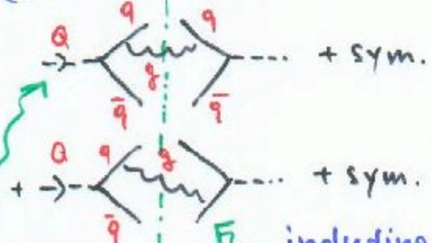
— dominant structure at higher perturbative orders:

- **Tree level**

$$\left| ME^{(tree)}_{n\text{-partons}} \right|^2 \simeq \left| ME^{(tree)}_{n\text{-partons}} \right|^2_{app.}$$

($\simeq$: construct suitable approximation — soft/coll. approximation)

$\left| ME^{(tree)}_{n\text{-partons}} \right|^2$: exact tree-level ME (positive definite); all Feynman graphs ($|A+B+\dots|^2$)



positive definite

including interferences (sign NOT definite!)

$\left| ME^{(tree)}_{n\text{-partons}} \right|^2_{app.}$: logarithmic approximation

$$\alpha_S^n \left(C_{2n} L^{2n} + C_{2n-1} L^{2n-1} + \dots \right)$$

- C_{2n}: exact
- L^{2n}: LL, leading logs
- C_{2n-1}: exact in e^+e^-
- L^{2n-1}: NLL, next-to-leading logs
- $\dots$: neglect/incorrect

$$L \sim \ln \frac{Q^2}{Q^2_{jet}} \gg 1$$

$Q \gg Q_{jet}$; Q: hard scale; Q_{jet}: small resolution scale for jets (e.g.: jet resolution, jet invariant mass, jet cone size, ..)

- introduce cut-off Q_0: minimum parton resolution (avoid IR divergent region of phase space)

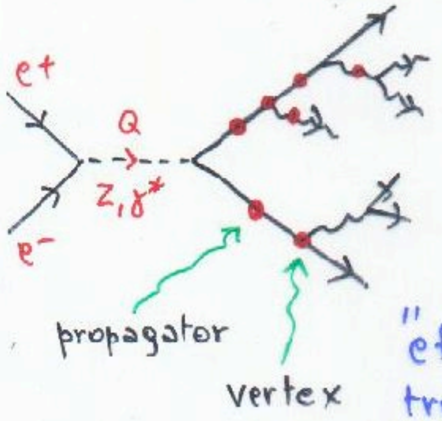
$$\min_{ij} q_{\perp ij} > Q_0$$

- include all-order virtual corrections and combine with real emission below Q_0 to cancel (logarithmic) dependence on cut-off to the same log accuracy as at the tree level

– logarithmic approximation and PS : branching

$$\left| ME^{(\text{tree+loops})} \right|^2 \simeq \left| ME \right|^2_{\text{app.}, Q_0}$$

$$\left| ME \right|^2_{\text{app.}, Q_0} = \sum_{\substack{\text{tree} \\ \text{diagrams}}}$$



"effective" tree-like (branching) diagrams (NOT FEYNMAN GRAPHS!)

⇓

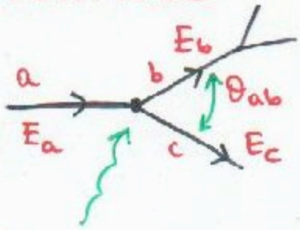
"quasi-local" tree-like structure implementable / implemented in Parton Shower

each vertex / propagator depends only on local branching variables and on previous branching

Note:
logarithmic accuracy :
- NLL for e^+e^-
- almost NLL for hadron collisions

– rules / ingredients of branching / PS diagrams

•) VERTICES



$P_{a\to b}(z)$: Altarelli–Parisi splitting function

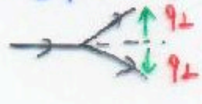
e.g. $P_{q\to q}(z) = C_F \, \frac{1+z^2}{1-z}$

$$V_{a\to bc} = \alpha_s(q)\ \frac{1}{2\pi}\ \frac{1}{\theta_{ab}^2}\ P_{a\to b}\left(\frac{E_b}{E_a}\right)\ \Theta(q > Q_0)$$

($z = E_b/E_a$; $\Theta(q > Q_0)$: lower cut-off for resolvable branching)

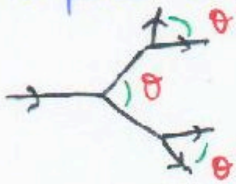
scale of α_s: transverse momentum in the splitting process

$$q \sim \min\left(E_a\theta_{ab}, E_b\theta_{ab}\right) \sim q_\perp$$



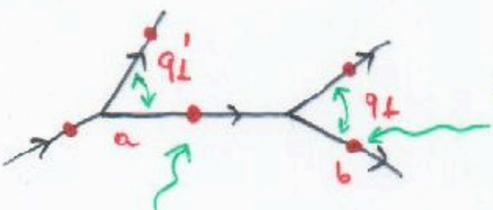
•) ANGULAR ORDERING (due to QUANTUM INTERFERENCES)

impose ordering of branching angles



$\theta > \theta', \theta''$ and so forth

•) PROPAGATORS (virtual corrections + integrated real emission below resolution scale Q_0)



$$\frac{\Delta_a(q_\perp', Q_0)}{\Delta_a(q_\perp, Q_0)}$$

external line: $\Delta_b(q_\perp, Q_0) \simeq \exp\left\{-\int_{Q_0}^{q_\perp} \frac{dq}{q}\, V\right\}$ (V: vertex)

Δ_b: Sudakov form factor: probability to evolve from scale $q_\perp$ to scale Q_0 without any branching (resolvable at scale Q_0)

parton shower : weak points w.r.t. hard multijet region

this region is correctly described only to leading (and, possibly, next-to-leading) logarithmic accuracy

in particular:

- physical kinematical region outside angular-ordered region is empty ("dead zones")
- hard emission inside angular-ordered region is dynamically approximated (by soft/coll. limit) already at the tree-level (lowest order in α_s)

↓

CKKW procedure:

A method to combine PS ⊕ exact tree-level ME, by avoiding "major" problems of

(i) missing phase space regions

(ii) double counting of perturbative terms

(iii) strong dependence on unphysical scales (cut-off parameters)

CKKW PROCEDURE

Main features of the method:

$Q \gg Q_1 > Q_0$ (~ hadronization)

hard scale → Q

Q_0 ← lower resolution scale for PS

Q_1 ← introduce "arbitrary" scale for hard multijet production (ideally, $Q_1 \to 0$, or, better, $Q_1 \to Q_0$)

(i) at scales above Q_1:
multiparton cross sections are given according to exact tree-level ME times approximated (to NLL accuracy) higher-order real (e.g. scale of α_s) and virtual (Sudakov form factors) corrections

← n partons, with $n \leq N$ fixed though, in principle, arbitrary

(ii) at scales below Q_1:
multiparton distribution are given by parton shower subjected to a "veto" procedure

Note: ← in the e^+e^- case, it can be (has been) proven analytically

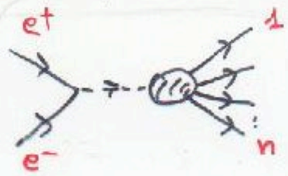
- dependence on the "unphysical" scale Q_1 cancel up to NLL accuracy ($\alpha_s^n \ell^{2n}$, $\alpha_s^n \ell^{2n-1}$: $\ell = \ln Q^2/Q_1^2$)

  [any (unavoidable) residual dependence on Q_1 could be exploited for tuning less singular terms to obtain optimal agreement with data]

- accuracy of original PS recovered up to NLL order ($L = \ln Q^2/Q_0^2$)

(i) scales above Q_1:

- generate kinematics of n parton configuration



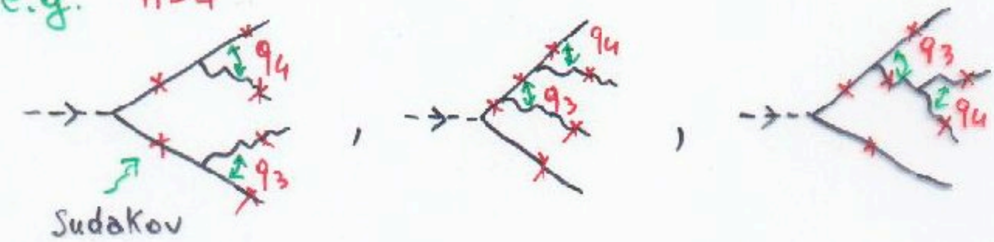
$n = 2, 3, \ldots \max(n) = N$

- compute iteratively $\min q_{1ij}$ to determine the (transverse-momentum) resolution scales

$$Q_1 < q_n < q_{n-1} \cdots < q_3 < q_2 = Q$$

at which $n, n-1, \ldots 3, 2$ jets are resolved; reconstruct accordingly branching diagram ("shower history")

e.g. $n = 4$



- weigh the configuration by

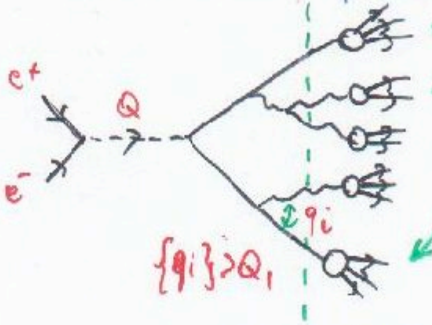
$$\frac{1}{(n-2)!} \; \frac{\alpha_s(q_n)}{\alpha_s} \cdots \frac{\alpha_s(q_3)}{\alpha_s} \; \left| ME_n^{(tree)} \right|^2 \; \prod_i (\text{Sudakov factors})$$

- $\frac{1}{(n-2)!}$: phase space factor for identical partons
- $\frac{\alpha_s(q_n)}{\alpha_s} \cdots \frac{\alpha_s(q_3)}{\alpha_s}$: higher-order corrections according to PS
- $\left| ME_n^{(tree)} \right|^2$: **exact** — NO PS approximation at the tree level (i.e. branching vertices times angular ordering)
- $\prod_i$ (Sudakov factors): Sudakov factor of branching diagram

[Note: depends only on $\{q_n, \ldots, q_2\}$ and NOT on actual branching diagram]

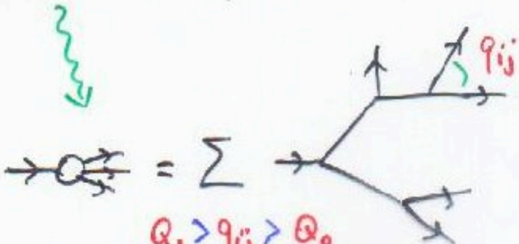
(ii) scales below Q_1 : ↙ above Q_1

supplement n-parton "hard" configuration with subsequent shower down to scale Q_0



n partons as in (i)

vetoed shower



$Q_1 > q_{ij} > Q_0$

customary angular-ordered PS with additional veto on any branching at scale $q > Q_1$

[to avoid double counting of tree level terms]

- important point (recall: original PS is angular ordered and not $q_\perp$ ordered)

starting scale for vetoed shower

E : energy of vetoed shower

θ_{in} : maximum angle of vetoed shower

angle at the "primary branching" in the branching history (branching with hardest parton in the branch)

[to reproduce angular-ordered emission of softest partons at scale Q_0

e.g.

θ'' can be larger than θ' if $E'' < E'$]

CKKW in e^+e^-

Recall: "Standard" PS MCs already include exact (tree-level) 3-parton ME ($q\bar{q}g$)

HERWIG
PYTHIA
ARIADNE

— pilot HERWIG study:

- include exact 4-parton ME
- dependence on hard multijet scale Q_1 ⇝ Fig.

— approximate* version of CKKW procedure implemented in APACIC++ (Krauss, Kuhn, Soff, Ivanyi)

[* Mainly because APACIC++ uses virtuality (invariant mass) as evolution variable in PS, and then angular ordering is enforced (similar to PYTHIA)]

- use exact tree-level ME up to $n=5$
- multijet scale $Q_1 \sim Q/20$ (i.e. $Q_1 \sim 5\,\text{GeV}$ at Z-pole) [recall: $Q_0 \sim 1 \div 2\,\text{GeV}$] (very weak dependence on Q_1)
- illustrative result on event shapes ⇝ Fig.

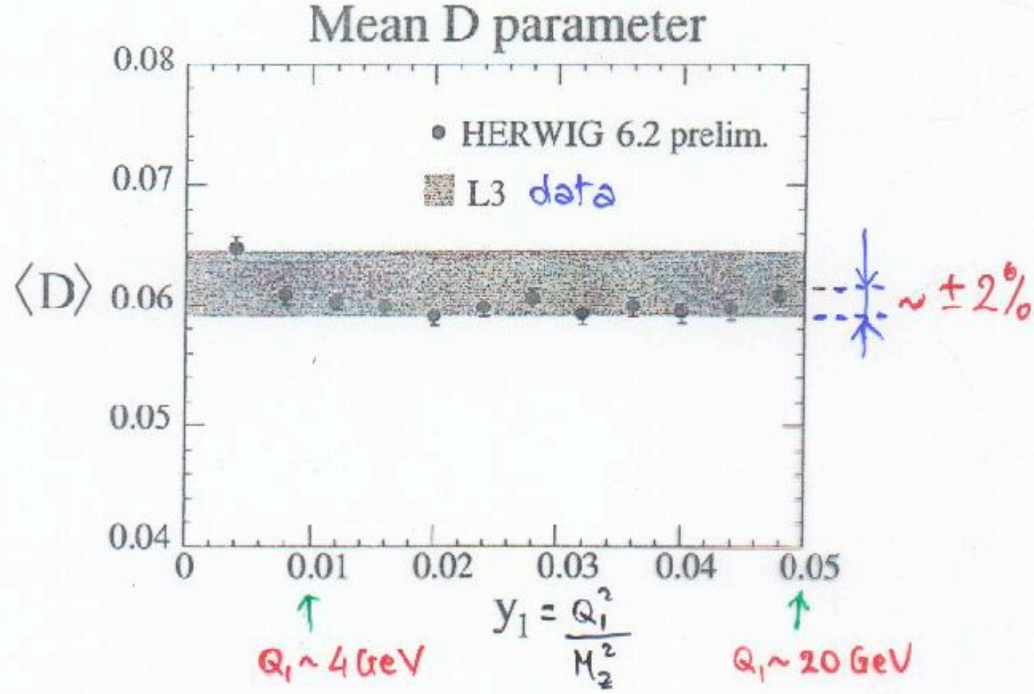
⇓

some improvement in multijet region

[although difficult to improve upon "Standard" MCs:
- because of their extreme tuning to data
- because they include exact 3-parton ME]

Example of four-jet event shape at LEP

pilot HERWIG study



linearized momentum tensor

$$T^{\alpha\beta} = \sum_i \frac{p_i^\alpha p_i^\beta}{|\vec{p}_i|} \Big/ \sum_j |\vec{p}_j|$$

c.m. frame: $\vec{p}_i \equiv (p_i^x, p_i^y, p_i^z)$

$\lambda_1, \lambda_2, \lambda_3$: corresponding eigenvalues $(\lambda_1 + \lambda_2 + \lambda_3 = 1)$

D-parameter

$$D \equiv 27\, \lambda_1 \lambda_2 \lambda_3$$

$D \neq 0$ only if event is not planar (at least 4 partons)

– Some event shapes at LEP1

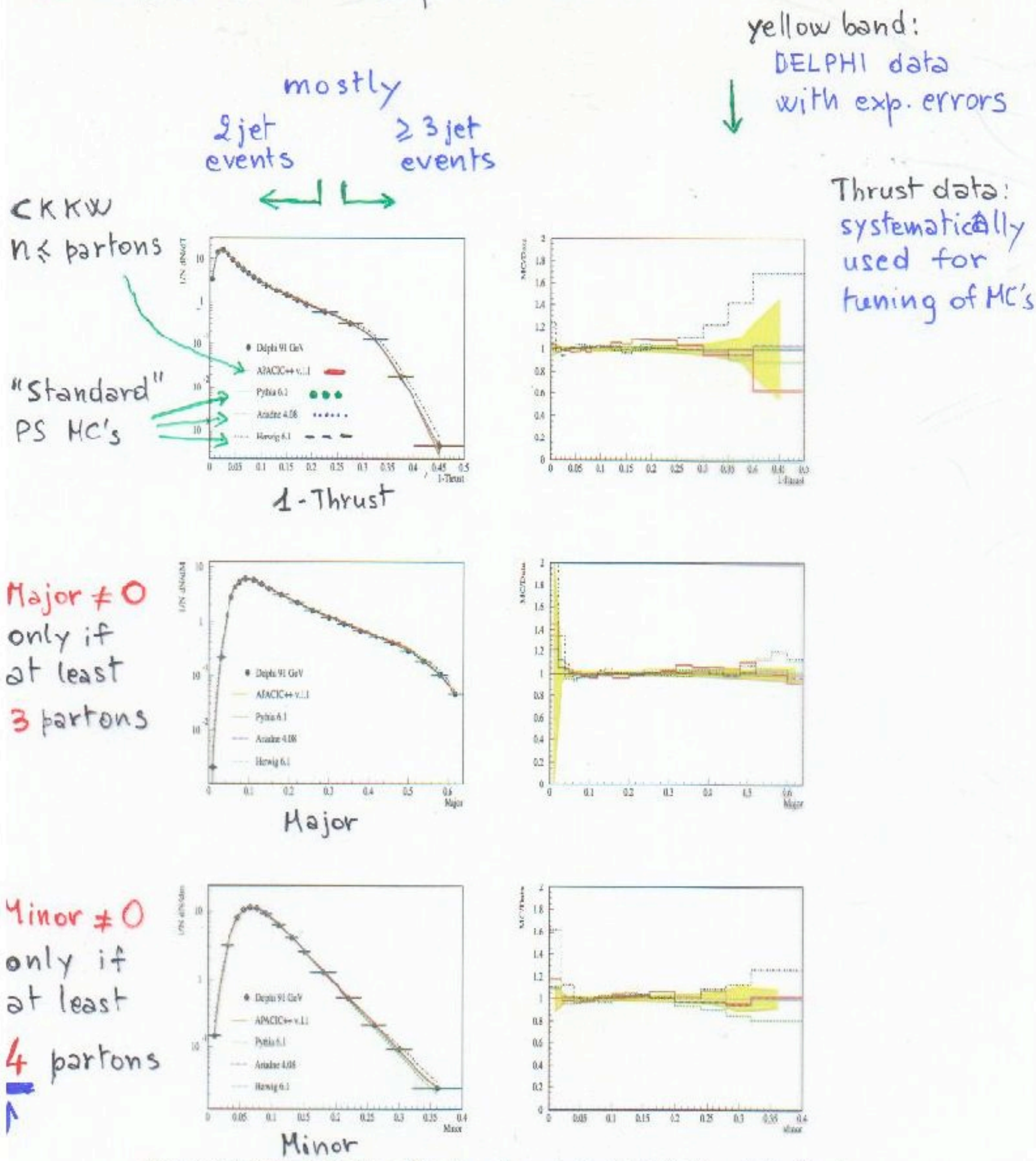


Figure 10: Some event shape (thrust, major and minor) distributions at the Z–pole.

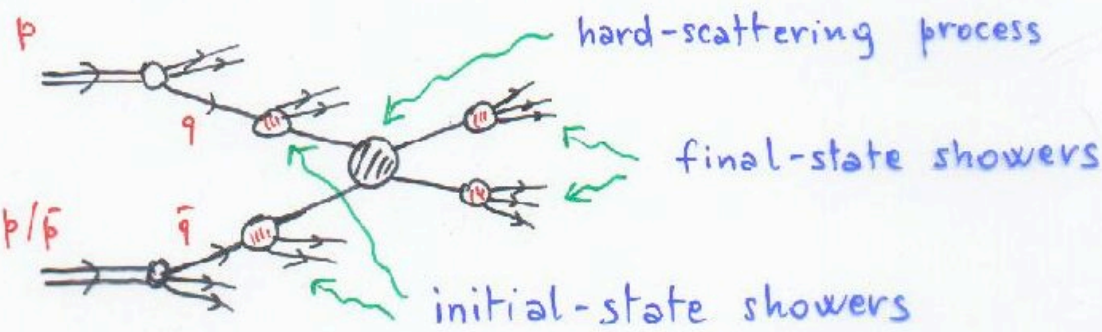
HADRON - HADRON COLLISIONS

Overall picture of PS (and ME) analogous to e^+e^-, but some <u>conceptual</u> and <u>practical</u> differences

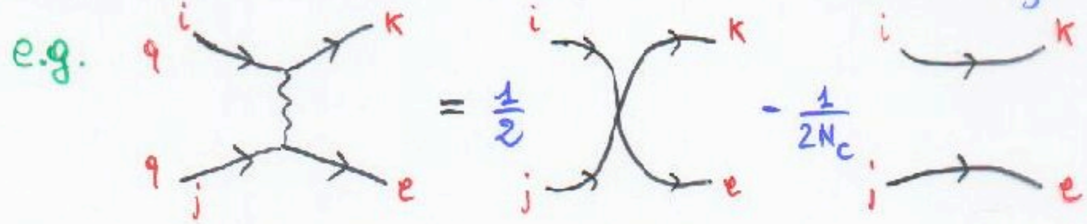
- trivial correspondence between kinematical variables :

e^+e^- c.m.s.		$pp/p\bar{p}$ c.m.s.
$E_i\ ;\ \theta_{ij}$	$\longrightarrow$	$E_{iT}\ ;\ R_{ij} = \sqrt{(\phi_i-\phi_j)^2+(y_i-y_j)^2}$

- QCD radiation from the initial state (absent in e^+e^-):

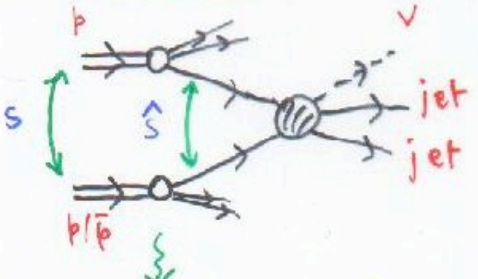


- non-trivial colour flow in hard scattering :



$\Rightarrow$ angular ordering <u>does not control</u> NLL terms at $\mathcal{O}(1/N_c^2)$

- <u>multiple</u> scales in multiparton hard scattering



hard scales : vector boson mass; jet E_T; dijet invariant mass; ⋮

hadronic (partonic) c.m. energy S ($\hat{S}$) does not set overall hard scale (as in e^+e^-)

- different PS MC generators implement, in practice, different PS algorithms
  probably, quite similar at LL accuracy;
  definitely different at NLL order

[e.g. even in HERWIG:
initial-state shower uses (for practical reasons) backward evolution, and then it implements $q_\perp$-ordering rather than angular-ordering]

⬇

owing to more differences (w.r.t. e+e-) in PS algorithms,
more difficulties (freedom/ambiguities/uncertainties) in combining PS ⊕ ME

- 3 practical implementations investigated so far

- CKKW (F. Krauss et al.; SHERPA)
  (pretty similar to original CKKW for e+e-)

- Lönnblad
  (~ modification of CKKW implementable in "dipole splitting model" / ARIADNE;
  ↳ iteration of $2 \to 3$ splitting, rather than $1 \to 2$ splitting)
  ($q\bar{q} \to q g \bar{q}$)

- M.L. Mangano (encoded in ALPGEN)
  (more different from original CKKW;
  start from ME and then showering:
  Sudakov reweighting + vetoed shower replaced by rejection procedure)

- illustrative comparison:

  W^+ + multijets at the Tevatron and the LHC

  - PS ⊕ exact tree-level ME :

    $2 \to W + n\text{-partons} \quad , \quad n \leq 4$

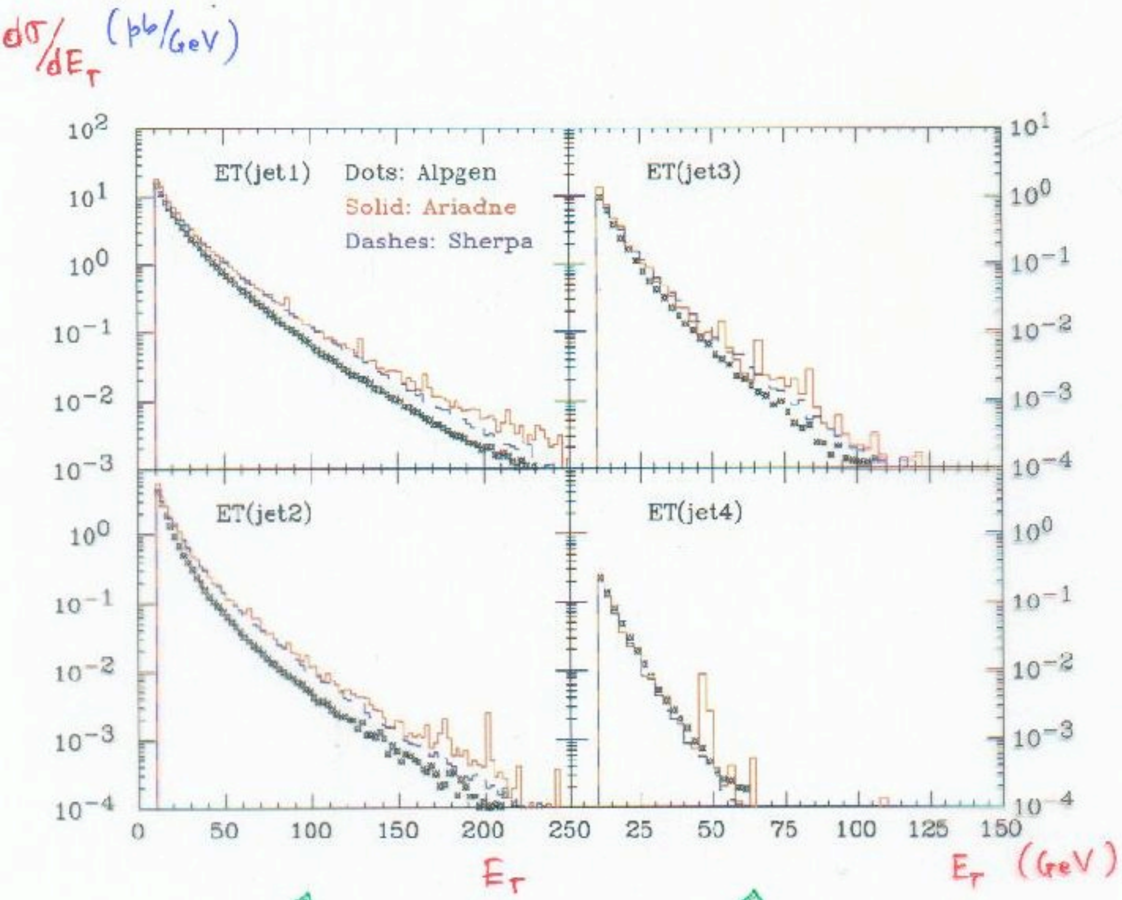
  - hard multijet scale $Q_1 \sim 10$ GeV

  - inclusive E_T spectra of the 4 leading (higher-E_T) jets ⇝ Figs.

⇓

overall consistency of the three results, although differences/uncertainties appear (→ further studies, investigations, data, ...)

- Inclusive E_T spectra of the 4 leading jets in W^+ + jets production at the Tevatron Run II ($\sqrt{s} = 1.96$ TeV)

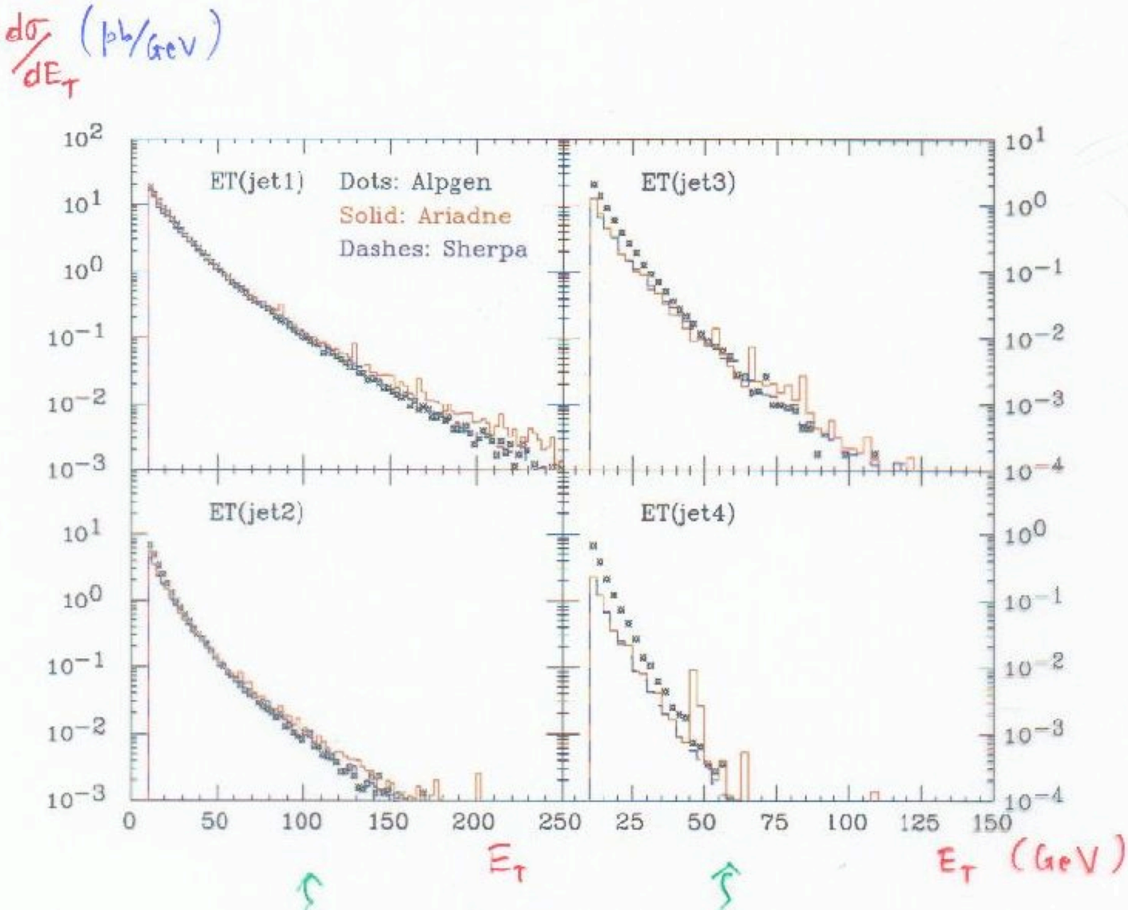


ALPGEN spectra are typically softer

good agreement

⟹ But ... →

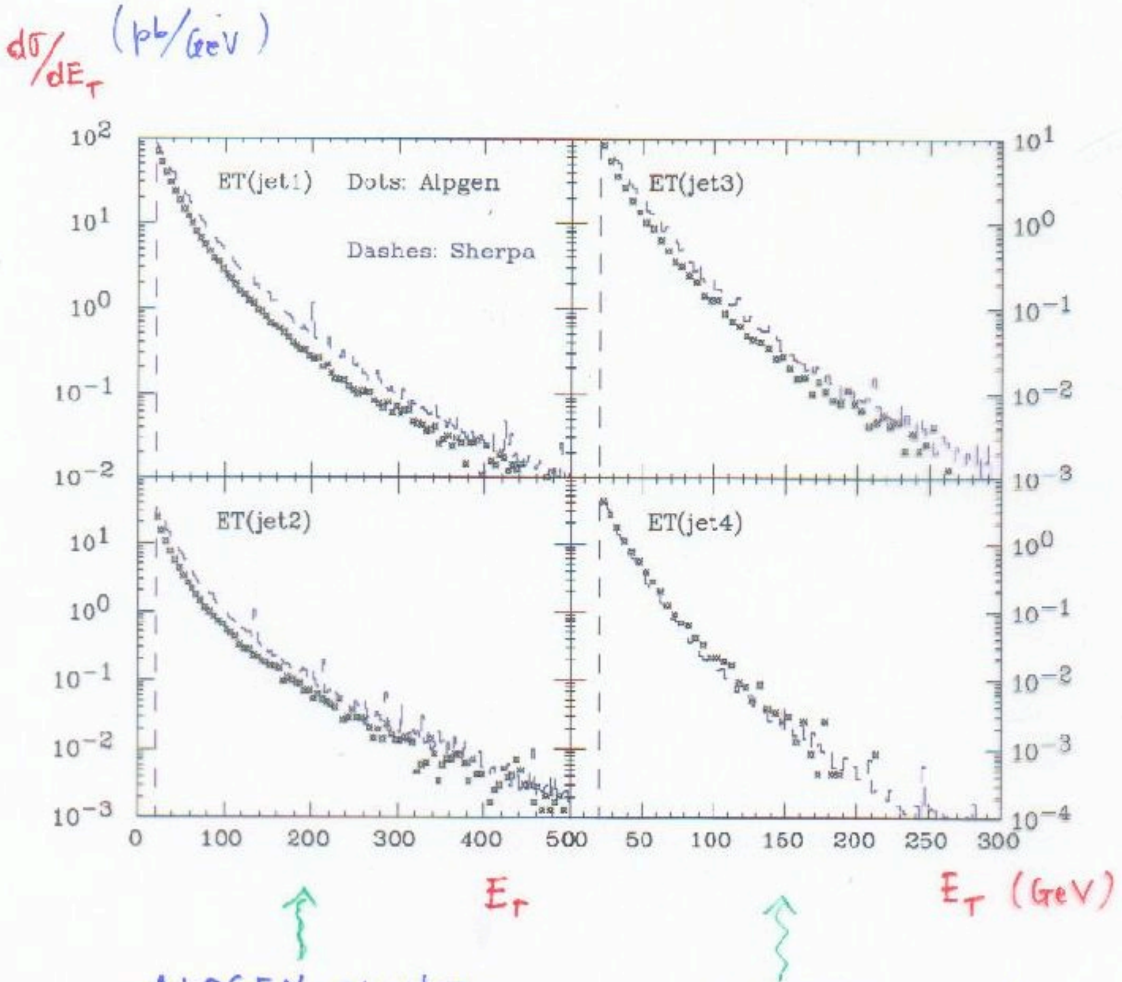
..... ⟹ rescaling $\alpha_s(q) \to \alpha_s(\frac{1}{2}q)$ in ALPGEN



good agreement

ALPGEN spectra are typically harder

- Inclusive E_T spectra of the 4 leading jets in W^++jets production at the LHC



ALPGEN spectra are typically softer

good agreement

→ same qualitative behaviour as at the Tevatron, but differences are smaller at the LHC

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CONCLUSIONS / OUTLOOK

- combining PS ⊕ ME

  [by avoiding :
  - missing phase space regions
  - double counting of perturbative terms
  - strong dependence on (fine tuning of) unphysical scales (cut-off parameters)]

  is FEASIBLE

- independently of the actual procedure,
  - introduction / definition of hard multiparton scale
  - reweighting (by $\alpha_s(q)$ and Sudakov factors) of $|ME^{(tree)}|^2$

    +
  - vetoed shower

  are (likely to be) necessary features of any consistent procedure

- plenty of space (especially in the case of hadron collisions) for ideas, interesting work

  .... and hard work