NLO AND NNLO: STATUS AND PROGRESSES

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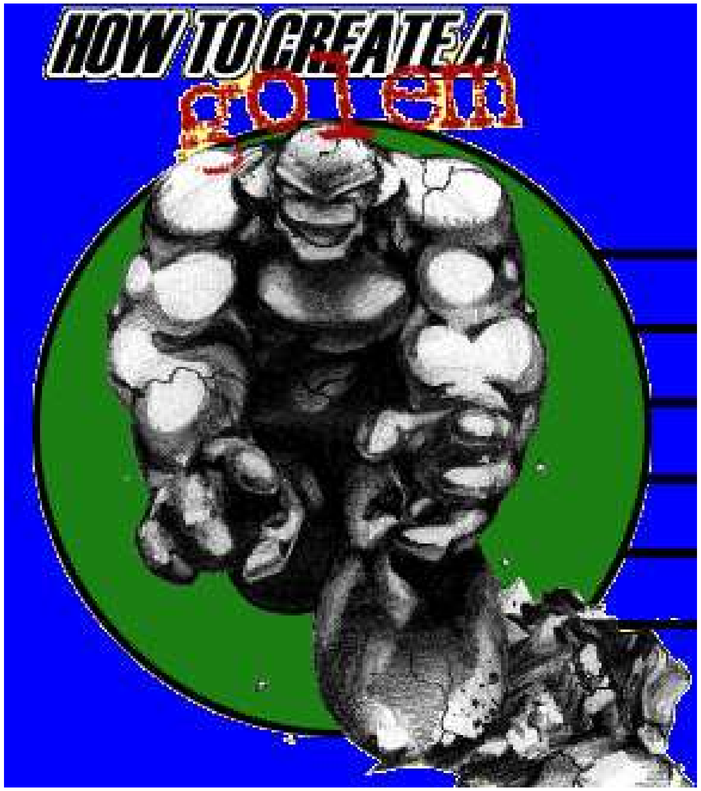
MCWS, Frascati, 27 February 2006

- NLO: where we stand
- NNLO: where we are going
- Ahead of us

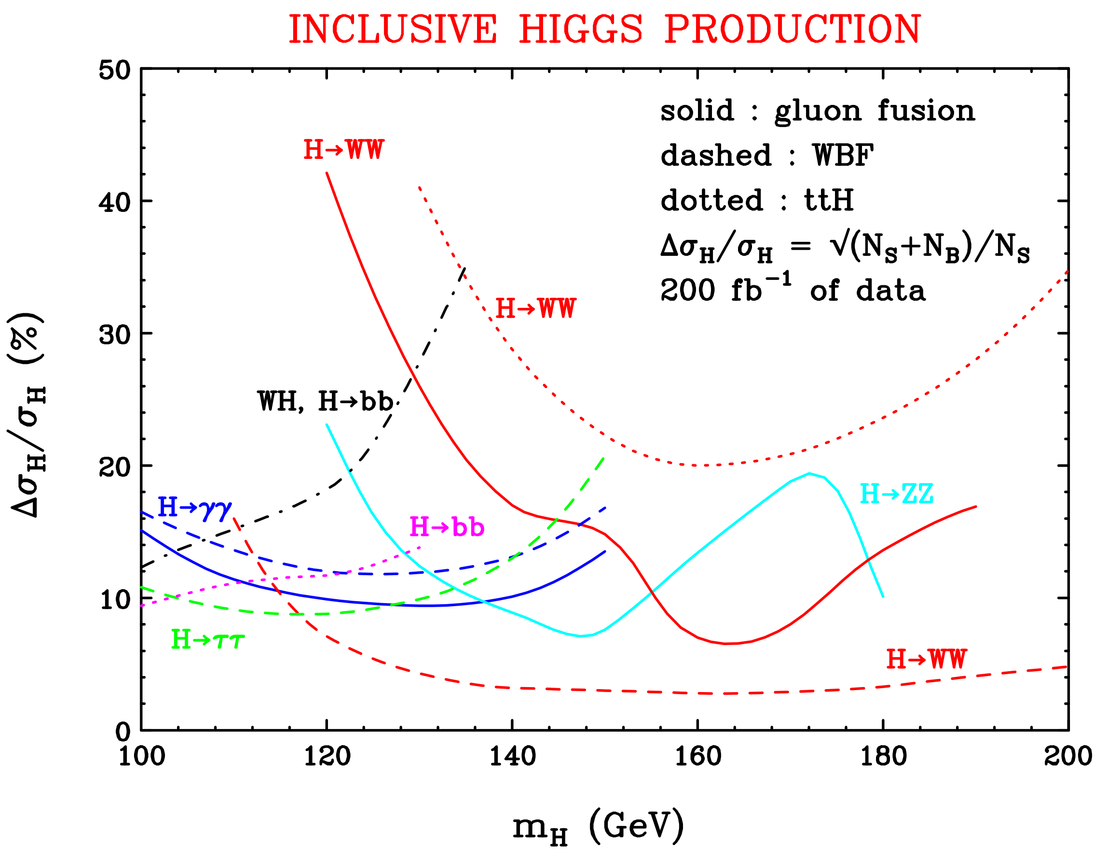


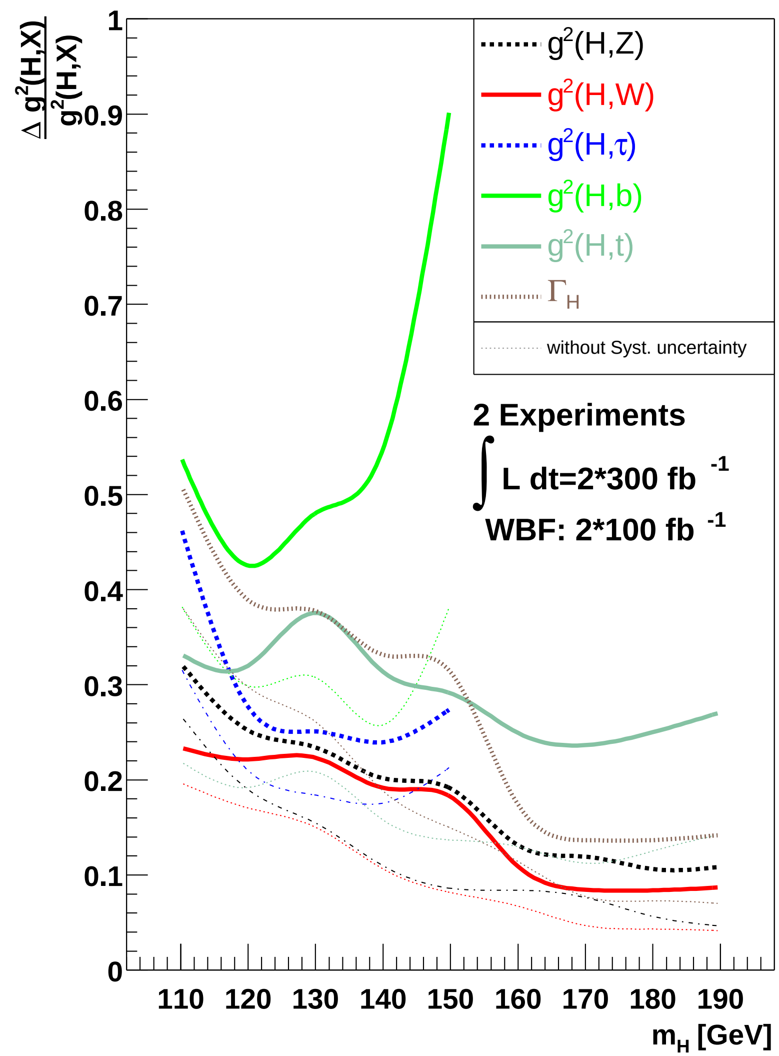
NLO? Yes, thank you!

- NLO calculations are essential to extract reliable estimates for total and differential production rates. This is true not only for the signal, but for the backgrounds too.
- But NLO calculations do not produce events (negative weights).
- It is highly non-trivial to establish an accurate connection between what is computed (partonic quantities) and what is measured (hadronic quantities).
- QCD physics at LEP and Tevatron has taught us that the concept of infrared (soft and collinear) safety is essential to justify the use of fixed-order perturbative calculations . . .
- . . . but does NOT guarantee the accuracy of such calculations. In fact:
  - power corrections effects
  - large logarithms (that need to be resummed to all order)

  can invalidate a fixed-order calculation.
- In addition, showering and hadronization effects need to be understood at a deeper level. This is the reason why we are all here today!

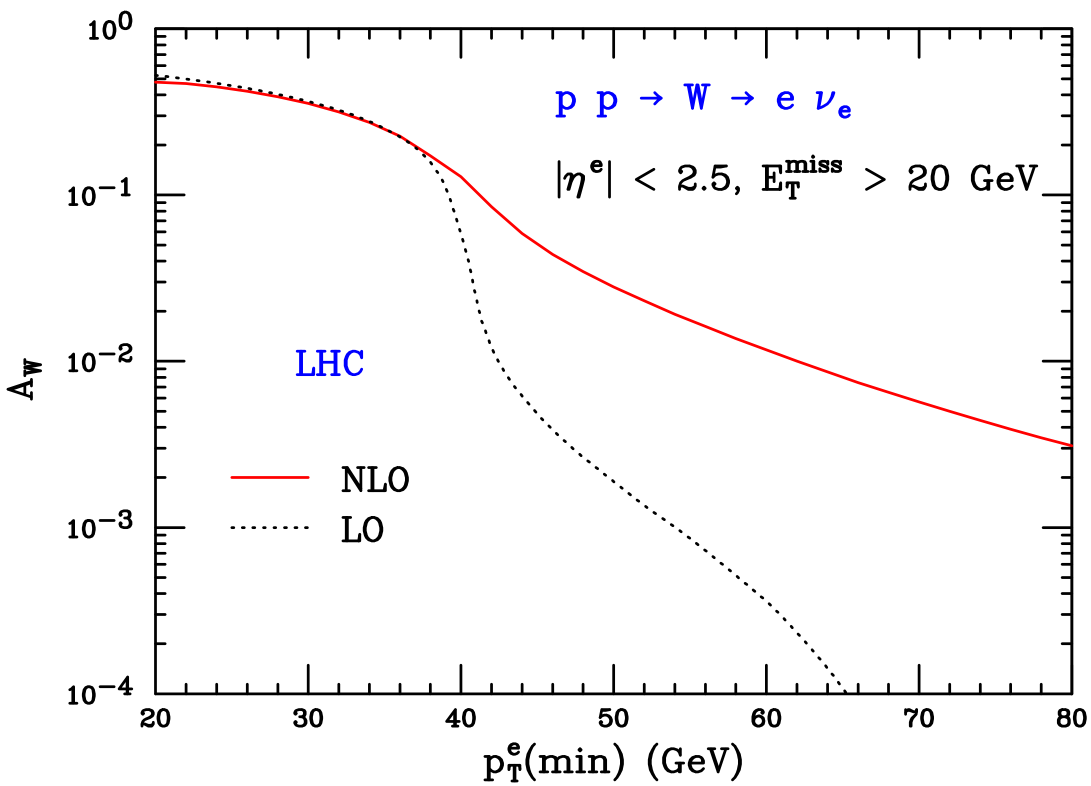
Higgs boson couplings





✓ Extraction of Higgs-boson couplings possible at the level of 10–30% $\Longrightarrow$ NLO corrections needed

W production



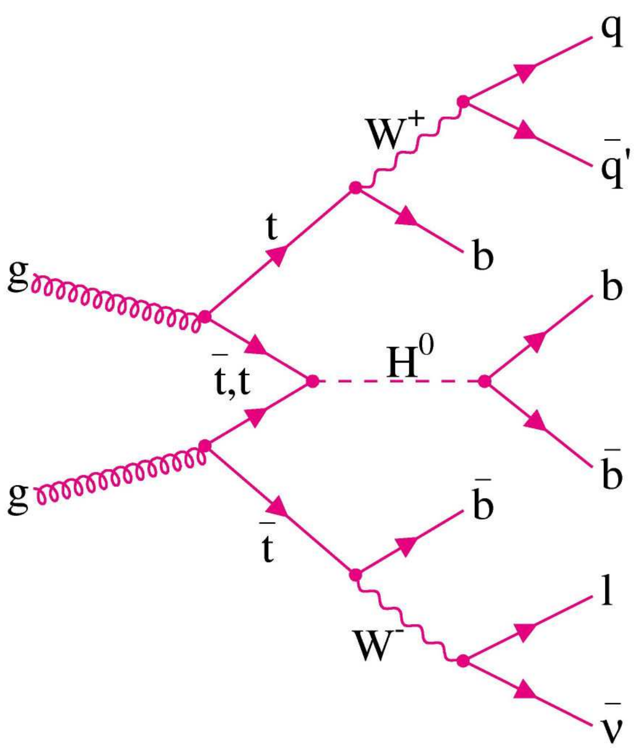
✓ $A_W = \dfrac{1}{\sigma^{(tot)}} \displaystyle\int_{p_T^e(\min)}^{\sqrt{S}/2} dp_T^e \frac{d\sigma}{dp_T^e}(\text{cuts})$

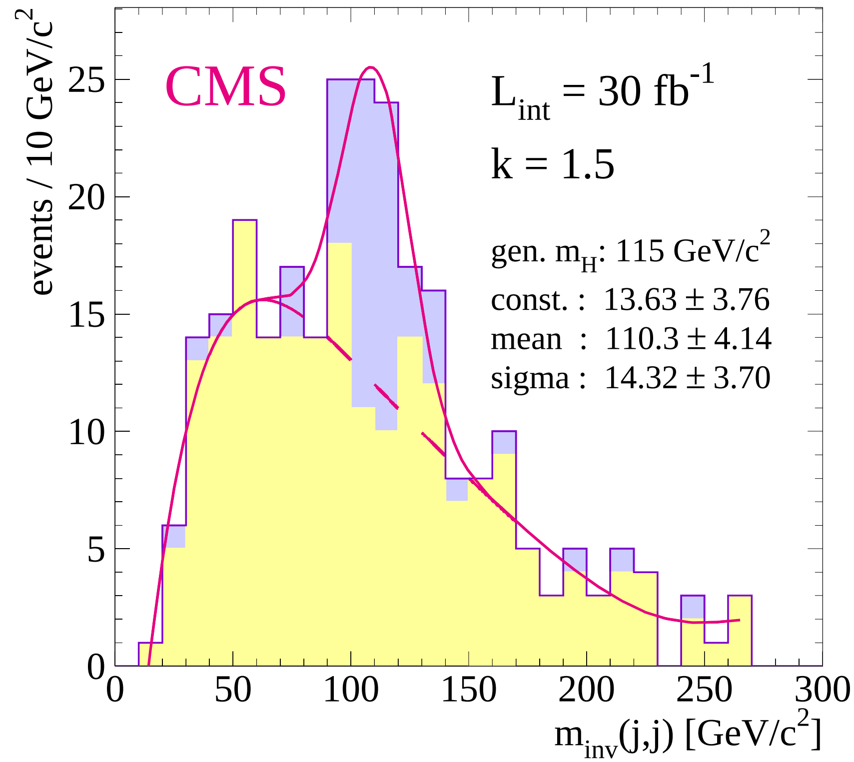
✓ $K(x) = \dfrac{d\sigma_{NLO}/dx}{d\sigma_{LO}/dx}$

K factors STRONGLY phase-space dependent.

Lepton spin correlations have to be taken account correctly!

$t\bar{t}H \longrightarrow t\bar{t}b\bar{b}$





- ✓ important channel for early discover (difficult measurement: combinatorics, *b*-tagging, invariant-mass resolution, good knowledge of detector needed)
- ✓ $h_t = t\bar{t}H$ Yukawa coupling $\Longrightarrow$ measure h_t^2 BR$(H \rightarrow b\bar{b})$
- ✗ must know the background normalization precisely (NLO level).

Status of NLO programs at hadron colliders

- NLOJET++ [Nagy] $pp \to$ (2,3) jets
- AYLEN/EMILIA [de Florian, Dixon, Kunszt, Signer] $pp \to (W, Z) + (W, Z, \gamma)$
- DIPHOX/EPHOX [Aurenche, Binoth, Fontannaz, Guillet, Heinrich, Pilon, Werlen] $pp \to \gamma$ + 1 jet, $pp \to \gamma\gamma$, $\gamma^* p \to \gamma$ + 1 jet
- MCFM [Campbell, Ellis] $pp \to (W, Z)$ + (0,1,2) jets, $pp \to (W, Z) + b\bar{b}, \ldots$
- heavy-quark production [Mangano, Nason, Ridolfi] $pp \to Q\bar{Q}$
- single-top production [Harris, Laenen, Phaf, Sullivan, Weinzierl] $pp \to Q\bar{q}$
- associated Higgs production with $t\bar{t}$ [Dawson, Jackson, Orr, Reina, Wackeroth, Beenakker, Dittmaier, Kramer, Plumper, Spira, Zerwas] $pp \to HQ\bar{Q}$
- VBFNLO [Figy, Zeppenfeld, C.O.] $pp \to (W, Z, H, WW, ZZ, WZ) + 2$ jets, QCD corrections to electroweak production, when typical vector-boson fusion cuts are applied
- di-photon production [del Duca, Maltoni, Nagy, Trocsanyi] $pp \to \gamma\gamma$ + 1 jet

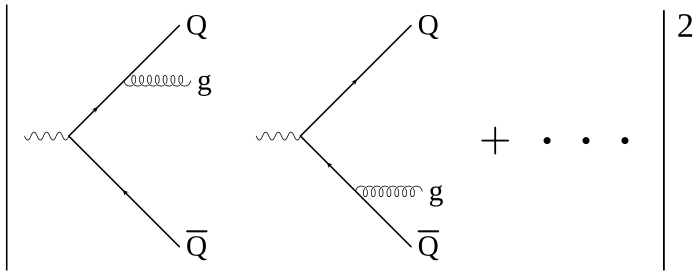
For a more complete list, and the corresponding web pages, see:

`http://www.cedar.ac.uk/hepcode`

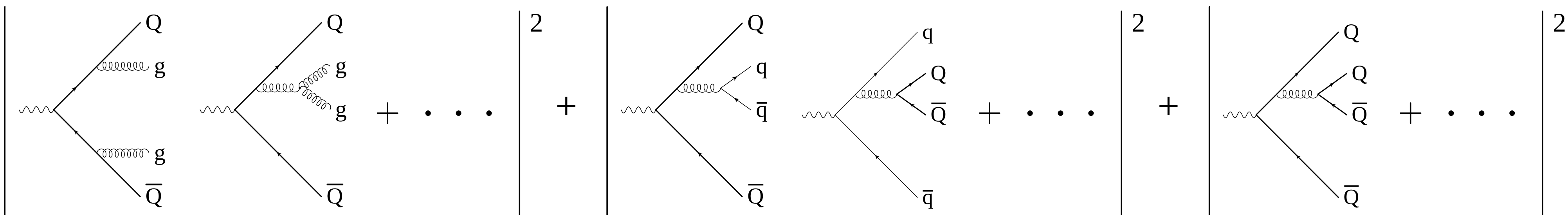
NLO ingredients

$Z/\gamma \longrightarrow$ 3 jets

Born term: order α_s



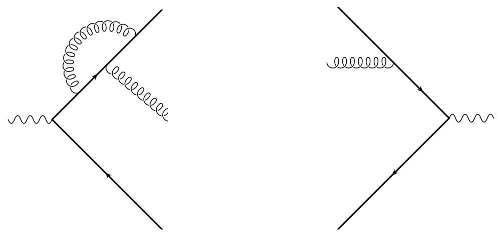
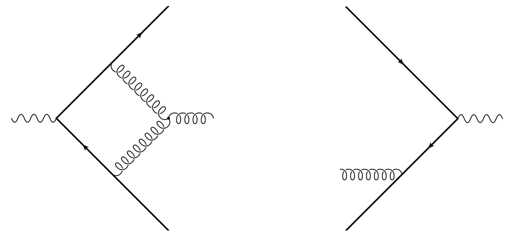
Real terms: order α_s^2



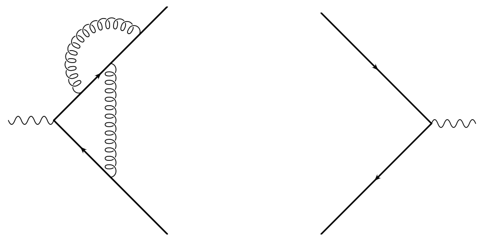
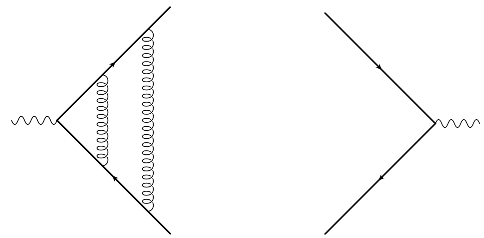
NLO ingredients, cont'd

$$Z/\gamma \rightarrow \text{ 3 jets}$$

Virtual terms: order α_s^2

 + cc +  + cc + . . .

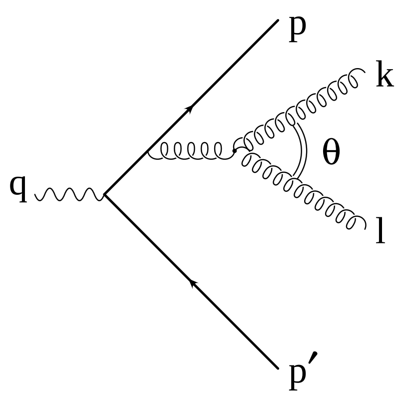
Two-loop terms: order α_s^2

 + cc +  + cc + . . .

Divergences!

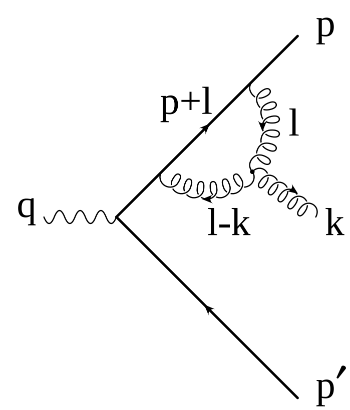
- UV divergences $\Longrightarrow$ renormalization
- IR divergences: SOFT and COLLINEAR

Real terms: divergences come from integration in particular regions of phase space



$$\frac{1}{(k+l)^2} = \frac{1}{2\,k\cdot l} = \frac{1}{2\,E_k E_l\,(1-\cos\theta)}$$

Virtual terms: divergences come from loop integration



$$\int d^4l\,\frac{1}{l^2(l-k)^2(p+l)^2} \overset{l_E\to 0}{\sim} \int \frac{dl_E^2}{l_E^2}\,\frac{d\cos\theta}{1-\cos\theta}$$

Dimensional Regularization: $d = 4-2\epsilon$.

Divergences appear as poles: $1/\epsilon$ and $1/\epsilon^2$, and they cancel for sufficiently (infra-red safe) inclusive observables.

The recipe: the subtraction method

Consider an n-parton final state

$$\sigma^{NLO} \equiv \int d\sigma^{NLO} = \int_{n+1} d\sigma^{R} \quad \Longleftarrow \text{divergences from collinear and soft regions}$$

$$+ \int_{n} d\sigma^{V} \quad \Longleftarrow \text{divergences from loop integration}$$

separately divergent (poles in ϵ), although their sum is finite.

The general idea of the subtraction method is to use the identity

$$d\sigma^{NLO} = \left[d\sigma^{R} - d\sigma^{A} \right] + d\sigma^{A} + d\sigma^{V}$$

where, in the singular regions, in d dimensions

$$\frac{d\sigma^{A}}{d\sigma^{R}} \sim 1$$

$d\sigma^{A}$ acts as a local counterterm for $d\sigma^{R}$

$$\sigma^{NLO} = \underbrace{\int_{n+1} \left[d\sigma^{R} - d\sigma^{A} \right]}_{\text{finite by construction}} + \int_{n+1} d\sigma^{A} + \int_{n} d\sigma^{V} = \underbrace{\int_{n+1} \left[\left(d\sigma^{R} \right)_{\epsilon=0} - \left(d\sigma^{A} \right)_{\epsilon=0} \right]}_{\text{done numerically in 4 dimensions}} + \int_{n} \underbrace{\left[d\sigma^{V} + \int_{1} d\sigma^{A} \right]_{\epsilon=0}}_{\text{done analytically}}$$

Bottlenecks of NLO

✓ The construction of the counterterm $d\sigma^A$ can be done in an automated and simple way.

✓ The integrations over the singular phase-space regions of $d\sigma^A$ are done once and for all. They are universal and process-independent functions. [Catani & Seymour, Frixione, Kunszt & Signer; ...].

✗ The analytic calculation of scalar loop integrals is complicated and process-specific.

✗ The tensor-reduction procedure of virtual integrals is challenging.

Scalar integrals

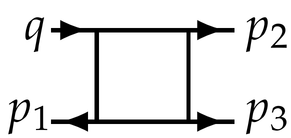
- Mellin-Barnes [Smirnov, Veretin & Tausk] to tackle loop integrals in the Feynman parametrization

$$(A+B)^{-\nu} = \frac{1}{2\pi i}\frac{1}{\Gamma(\nu)\,B^{\nu}}\int_{c-i\infty}^{c+i\infty} dz \left(\frac{A}{B}\right)^{z} \Gamma(-z)\,\Gamma(\nu+z)$$

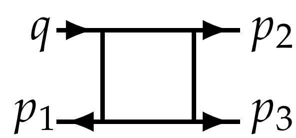
and use the residue theorem to compute the integral

- differential equations [Gehrmann & Remiddi]

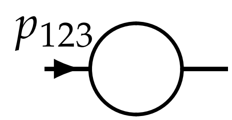
$$s_{23}\frac{\partial}{\partial s_{23}}$$



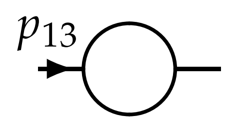
$$= \frac{d-6}{2}$$



$$-\frac{2(d-3)}{s_{12}+s_{23}}\left[\frac{1}{s_{123}}\right.$$



$$\left. - \frac{1}{s_{13}}\right.$$

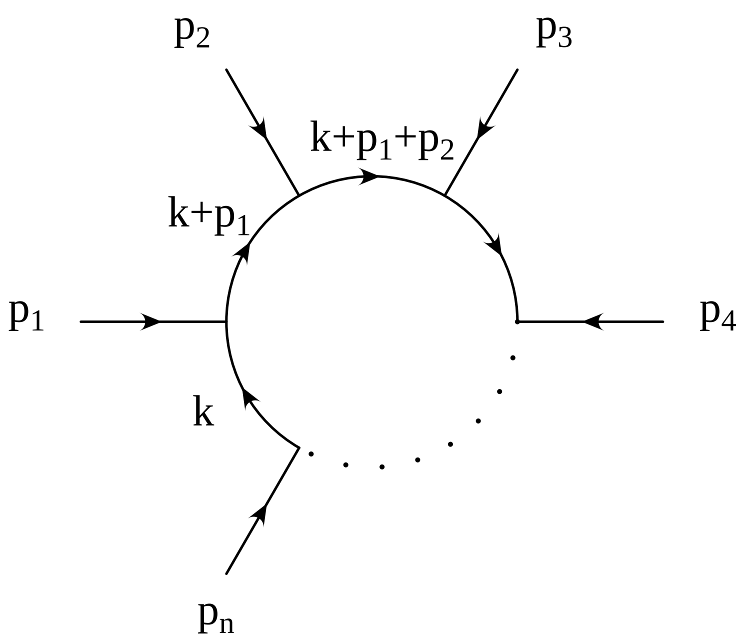


$$\left.\right]$$

plus initial conditions (very easy to obtain).

Same tricks used for two-loop integrals!

Tensor integrals



When the loop momentum appears in the numerator (tensor integrals), things get more involved

$$\int d^d k \frac{k^\alpha k^\beta \ldots}{k^2 (k+p_1)^2 (k+p_1+p_2)^2 (k+p_1+p_2+p_3)^2 \ldots}$$

- analytical tensor-reduction procedures are available but…
- …is it possible to go numerical? If yes, "when" one goes numerical? Before or after some reductions?

Only very recently some of these issues have been addressed.

Tensor reduction: problems

$$\int d^d k \, \frac{\overbrace{k^{\mu_1} k^{\mu_2} \ldots k^{\mu_m}}^{\text{rank } m}}{D_1^{\nu_1} D_2^{\nu_2} \ldots D_n^{\nu_n}} = \sum_{i,j,\ldots} C_{i,j,\ldots}^{d,m} (p_r \cdot p_s) \{ p_i^{\mu_1} p_j^{\mu_2} \ldots, \; g^{\mu_1 \mu_2} \ldots \}$$

$$D_i = \left(k + \sum p_j\right)^2$$

Passarino and Veltman derived recursion relations that connect the tensor coefficients $C_{i,j,\ldots}^{d,m}$ with $C_{i,j,\ldots}^{d,m-1}$, when $\nu_i = 1$, down to scalar integrals ($m = 0$) in d dimensions ($d = 4$ or $4 - 2\epsilon$ in dimensional regularization).

- ✗ But these relations suffer from the presence of quantities in the denominator (Gram determinants) that can approach zero, in particular phase-space regions. They give rise to spurious singularities $\Longrightarrow$ numerical instabilities!

- ✗ The iterative procedure gives rise to large intermediate expressions, difficult to handle and factorize.

Tensor reduction: solutions

- ✓ Close to the "critical regions", expand the tensor coefficients around limits of vanishing Gram determinants, or other kinematics determinants, and then reduce all tensor coefficients to the usual scalar integrals [Denner, Dittmaier et al.]

- ✓ Relate the tensor integrals to other integrals in higher dimension d and higher powers of the propagators [Davydychev]. Then, separate explicitly the infrared and ultraviolet divergences analytically from the finite one-loop contributions, which can then be evaluated numerically using recursion relations [Giele, Glover, Binoth et al.]

None of these methods is completely numeric. They are a mixture of analytic reductions and numeric calculations.

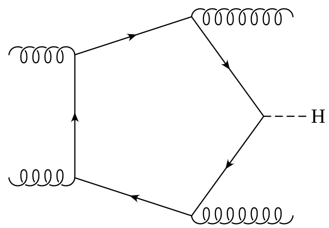
- ✗ The only totally-numerical evaluation of a tensor integral, using Mellin-Barnes technique, is in its infancy [Anastasiou & Daleo (hep-ph/0511176)]: it takes two hours to evaluate a single phase-space point in a rank-6 hexagon!!

Phenomenological applications

- analytic reduction of pentagon integrals [Bern, Dixon & Kosower (hep-ph/9306240)].

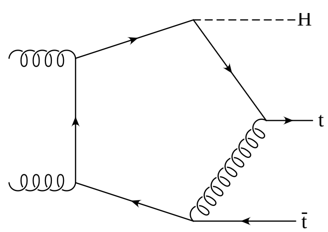
$$\mathrm{PENT}(d=4-2\epsilon) = \underbrace{\sum^{5} \mathrm{BOX}(d=4-2\epsilon)}_{\text{IR divergences}} + (d-4)\times \underbrace{\mathrm{PENT}(d+2=6-2\epsilon)}_{\text{finite}}$$

$pp \longrightarrow H + 2\,\text{jets}$



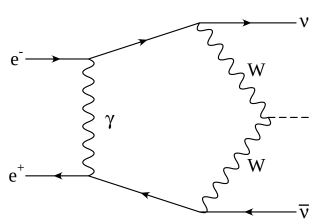
[Del Duca, Kilgore, Schmidt, Zeppenfeld & C.O. (hep-ph/0108030)]

$pp \longrightarrow t\bar{t}H$



[Beenakker, Dittmaier, Krämer, Plümper, Spira & Zerwas, (hep-ph/0211352); Dawson, Jackson, Orr, Reina & Wackeroth (hep-ph/0305087)]

$e^+e^- \longrightarrow \nu\bar{\nu}H$



[Belanger, Boudjema, Fujimoto, Ishikawa, Kaneko, Kato & Shimizu (hep-ph/0211268); Jegerlehner & Tarasov (hep-ph/0212004); Denner, Dittmaier, Roth & Weber (hep-ph/0302198)]

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Kurihara, Y. and Kaneko, T., "Numerical contour integration for loop integrals", arXiv:hep-ph/0503003.

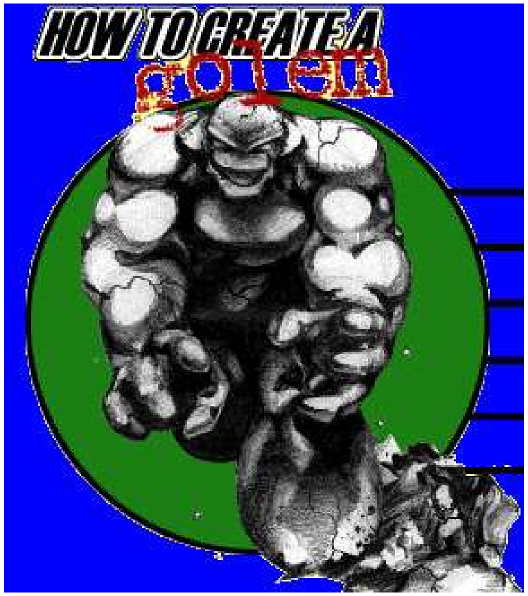
+ Unitary-cut techniques [Bern, Dixon, Kosower et al.]

+ Maximum-Helicity-Violating techniques [Britto, Cachazo, Feng, Witten et al.] + . . .

Final goal

The goal is to build a program that computes scattering processes at NLO in a completely automated way.

We will obtain the GOLEM = General One-Loop Evaluation of Matrix elements!



NNLO?

Do we need NNLO jet cross sections at hadron colliders?

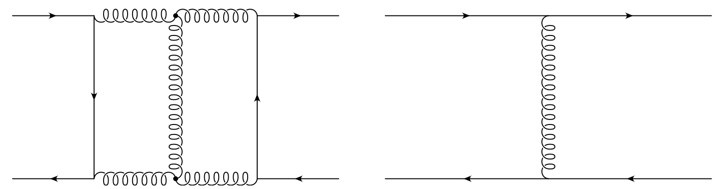
- jets are very complicated objects
- steep E_T-dependence magnifies energy-scale and luminosity uncertainties
- underlying events are surely a problem

YES. At least it helps to focus more attention on

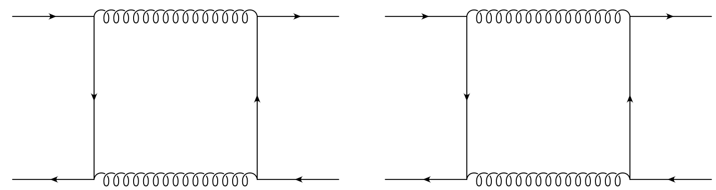
- reduction of renormalization- and factorization-scale dependence of the cross sections
- less worries (hopefully!) about matching theoretical and experimental jet algorithms, and reducing dependence from artificially-introduced parameters (R_{sep})
- more complicated transverse-momentum final states, due to double initial-state radiation (no need of intrinsic k_T)
- reduced dependence on power-correction effects.

Ingredients for NNLO n-jet final state

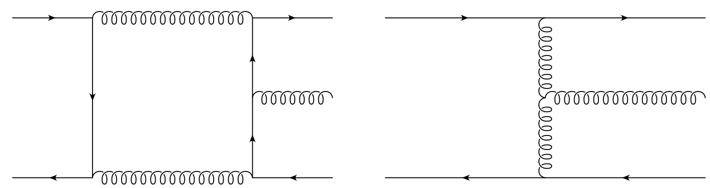
- Two-loop $2 \to 2$ matrix elements
  $\mathcal{M}_{\text{2-loop}}(n) \times \mathcal{M}_{\text{0-loop}}(n) + cc$



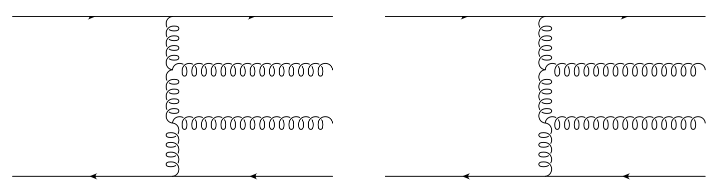
- One-loop $2 \to 2$ matrix elements
  $|\mathcal{M}_{\text{1-loop}}(n)|^2$



- One-loop $2 \to 3$ matrix elements
  $\mathcal{M}_{\text{1-loop}}(n+1) \times \mathcal{M}_{\text{0-loop}}(n+1) + cc$



- Tree-level $2 \to 4$ matrix elements
  $|\mathcal{M}_{\text{0-loop}}(n+2)|^2$



Technical breakthroughs

- algorithms (in `FORM, Maple, Mathematica`) to reduce recursively or by Gauss elimination, large systems of linear equations (10^4–10^6) to 10–30 master integrals, the building blocks of the computation.
  - Integration-by-Parts [Chetyrkin & Tkachov] to build recursive relations

$$\int d^d k \, \frac{\partial}{\partial k^\mu} f(k, p_i) = 0 \qquad p_i = \text{ external momenta}$$

  - Lorentz invariance [Gehrmann & Remiddi]

$$\int d^d k \, f(k, p_i) = F(p_i \cdot p_j)$$

  - implementation of efficient computer-algebra algorithms

Technical breakthroughs, cont'd

– sector decomposition: an automated procedure to break an integration domain into various singular regions, disentangling the overlapping singularities.

$$
\begin{aligned}
I &= \int_0^1 dx\, dy\, x^{-1-\epsilon} y^{-1-\epsilon} (x+y)^{-\epsilon} = \ldots \\
&= \int_0^1 dx\, dy\, x^{-1-3\epsilon} y^{-1-\epsilon} (1+y)^{-\epsilon} + \int_0^1 dx\, dy\, x^{-1-\epsilon} y^{-1-3\epsilon} (1+x)^{-\epsilon}
\end{aligned}
$$

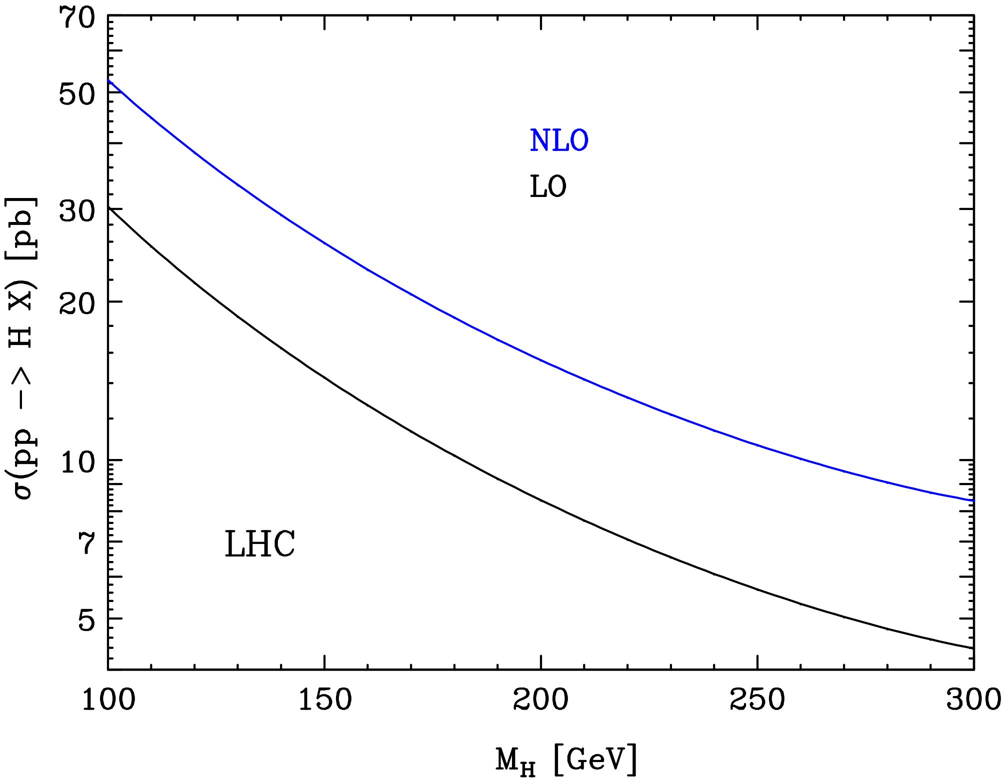
It has been used

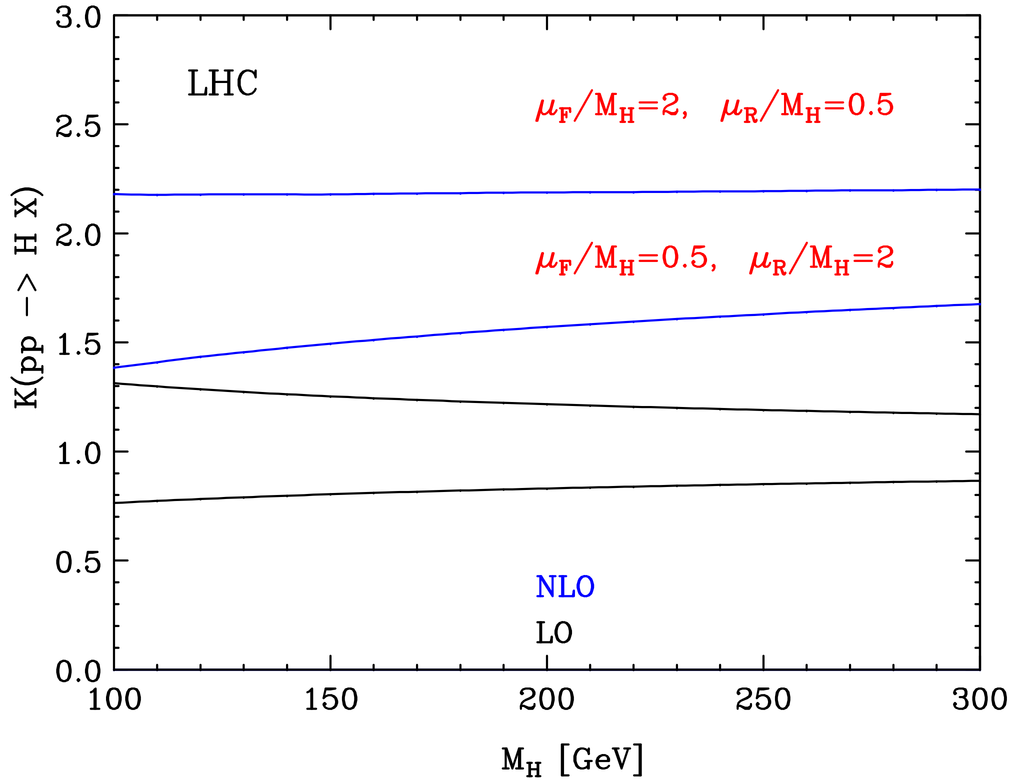
- ∗ in the numerical evaluation of hexagon integrals [Binoth, Heinrich & Kauer]
- ∗ to express the $1 \to 4$ phase-space volume, in a way suitable for numerical integration (all divergences extracted) [Anastasiou, Melnikov & Petriello (hep-ph/0311311)].

– harmonic (nested) sums [Moch, Uwer & Weinzierl]

$$
S(n; m_1, ..., m_k; x_1, ..., x_k) = \sum_{n \geq i_1 \geq i_2 \geq ... \geq i_k \geq 1} \frac{x_1^{i_1}}{{i_1}^{m_1}} \cdots \frac{x_k^{i_k}}{{i_k}^{m_k}}
$$

Totally inclusive: Higgs production at LHC

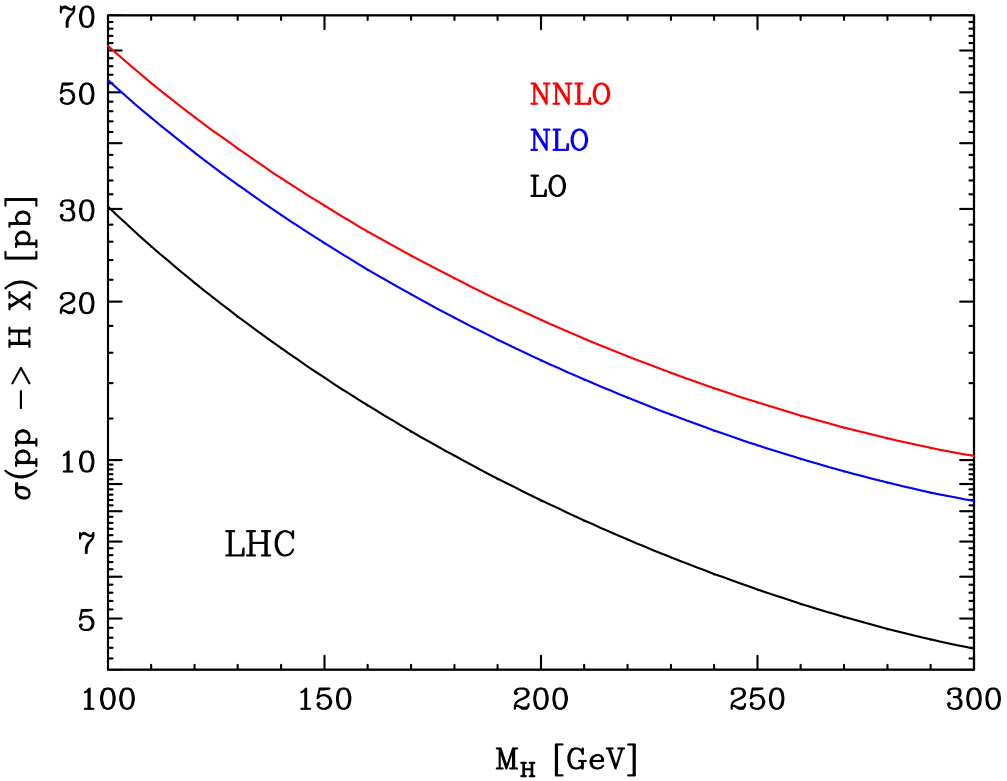


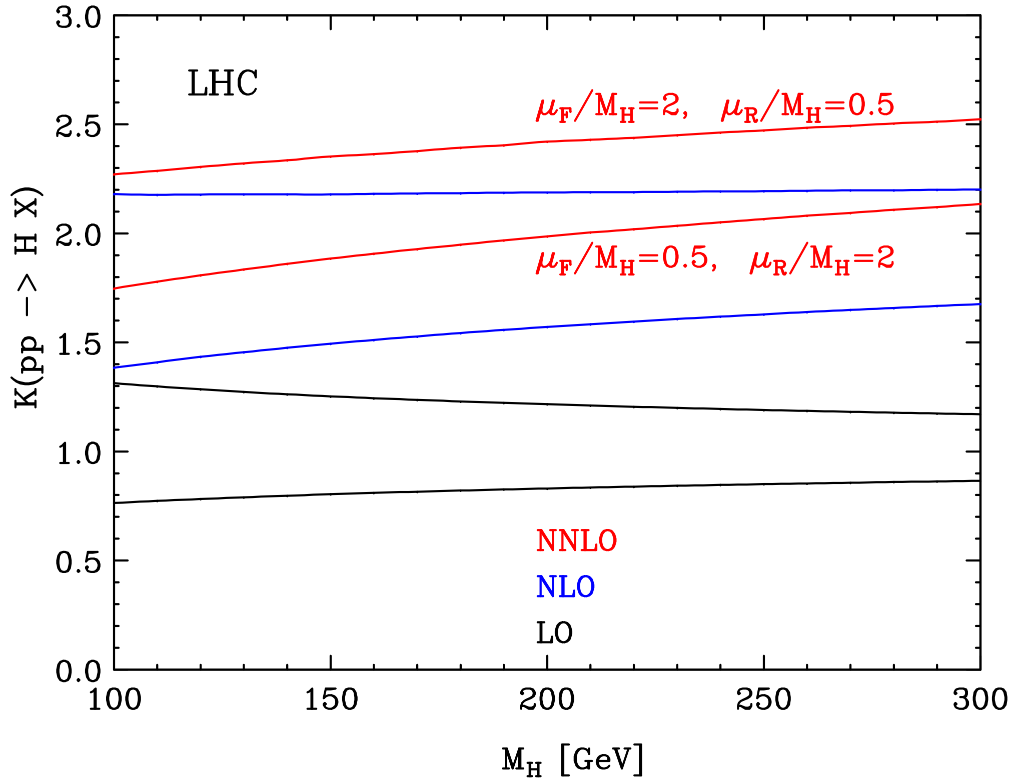


NLO corrections are 80% of the LO!

Is the series well behaved?

Totally inclusive: Higgs production at LHC





Is the series well behaved? $\Longrightarrow$ YES NNLO 15%

- using "conventional" techniques & series expansions [Harlander & Kilgore (hep-ph/0201206)]
  Result cross-checked without approximation [Smith, Ravindran & van Neerven (hep-ph/0302135)]
- confirmed using a new technique [Anastasiou & Melnikov (hep-ph/0207004)]

New technique

- Convert phase-space integrals into loop integrals $i \to f$ (n particles)

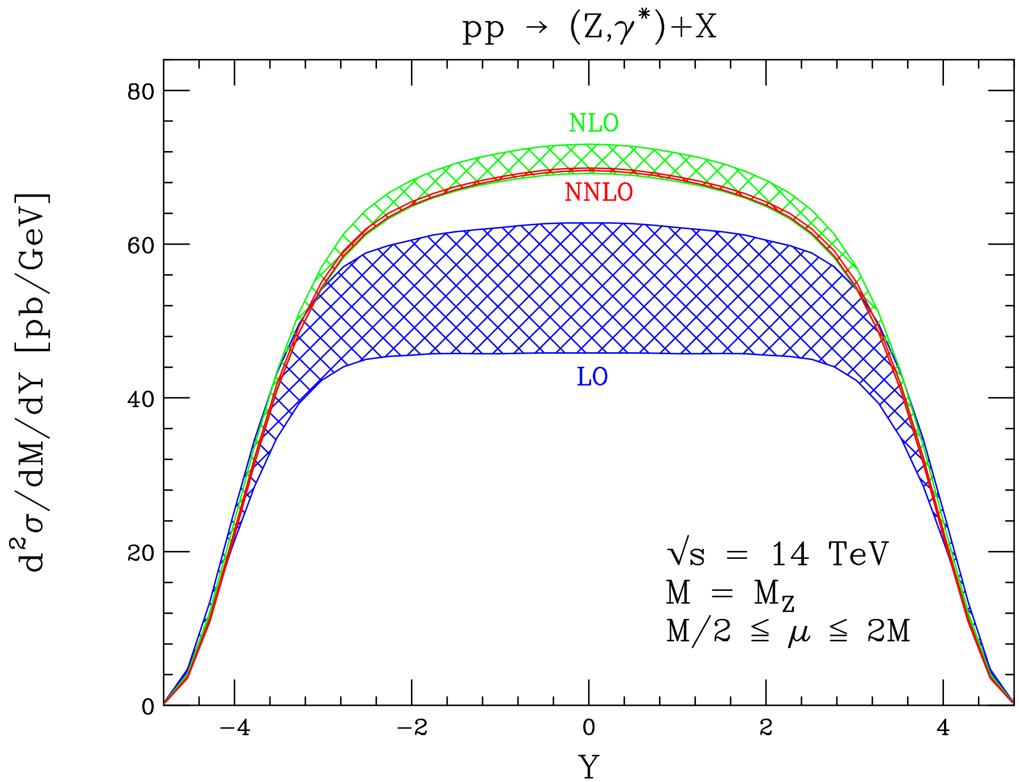
$$\int |\mathcal{M}_{i\to f}|^2 \, d\text{LIPS}(n-1) \underbrace{\frac{d^{d-1}\vec{p}}{2E}}_{E^2=\vec{p}^2+m^2} = \int |\mathcal{M}_{i\to f}|^2 \, d\text{LIPS}(n-1) \, d^d p \, \delta(p^2-m^2) \, \theta(E)$$

$$\delta(x) = \frac{1}{2\pi i}\left(\frac{1}{x-i0} - \frac{1}{x+i0}\right)$$

$$= \int |\mathcal{M}_{i\to f}|^2 \, d\text{LIPS}(n-1) \, \theta(E) d^d p \left[\frac{1}{p^2-m^2-i0} - \frac{1}{p^2-m^2+i0}\right]\frac{1}{2\pi i}$$

Use the formalism developed for the loop reduction to deal with integration over the phase space of final-state particles.

Partially inclusive: rapidity distribution at NNLO



- ✓ Semi-inclusive quantities have been computed: rapidity-distribution for W production [Anastasiou, Dixon, Melnikov & Petriello (hep-ph/0312266)]
- ✓ Remarkable stability to QCD corrections.
- ✓ Use W and Z production to monitor proton-proton luminosity and constrain PDFs at LHC.
- ✗ But spin-correlations effects can be more important than NNLO effects [Frixione & Mangano, (hep-ph/0405130)], when cuts are applied to final-state lepton. Same for electroweak corrections.

First totally exclusive results

- $e^+e^- \to 2$ jets

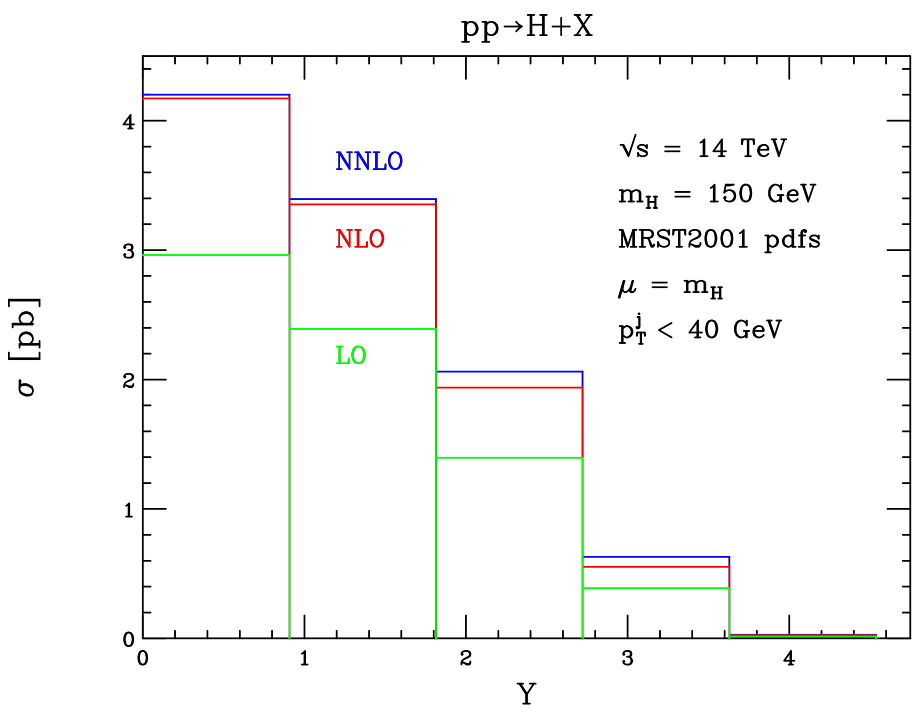
  [Anastasiou, Melnikov & Petriello (hep-ph/0402280)].

  Infrared structure studied also in Gehrmann-De Ridder, Gehrmann & Glover (hep-ph/0403057).

Higgs production in gluon fusion: $pp \to H$

- [Anastasiou, Melnikov & Petriello (hep-ph/0409088)]

  $m_H = 150$ GeV $\quad p_T^j < 40$ GeV



Ahead of us!

- $e^+e^- \longrightarrow 3$ jets at NNLO
- $pp(\bar{p}) \longrightarrow 2$ jets at NNLO
- ...