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Shower Monte Carlo
what they are
and what they do

Implementations

- COJETS Odorico (1984)
- ISAJET Page+Protopopescu (1986)
- FIELDAJET Field (1986)
- JETSET Sjöstrand (1986)
- PYTHIA Bengtsson+Sjöstrand (1987), Sjöstrand (1994)
- ARIADNE Lönnblad (1991)
- HERWIG Marchesini+Webber (1988), Marchesini+Webber+Abbiendi+Knowles+Seymour+Stanco (1992)
- SHERPA Gleisberg+Hoche+Krauss+Schalicke+Schumann+Winter (2004)

See Les Houches Guidebook hep-ph/0403045 for a complete list.

Capabilities

1. Large library of hard events cross sections (SM and BSM)

2. Dress hard events with QCD radiation

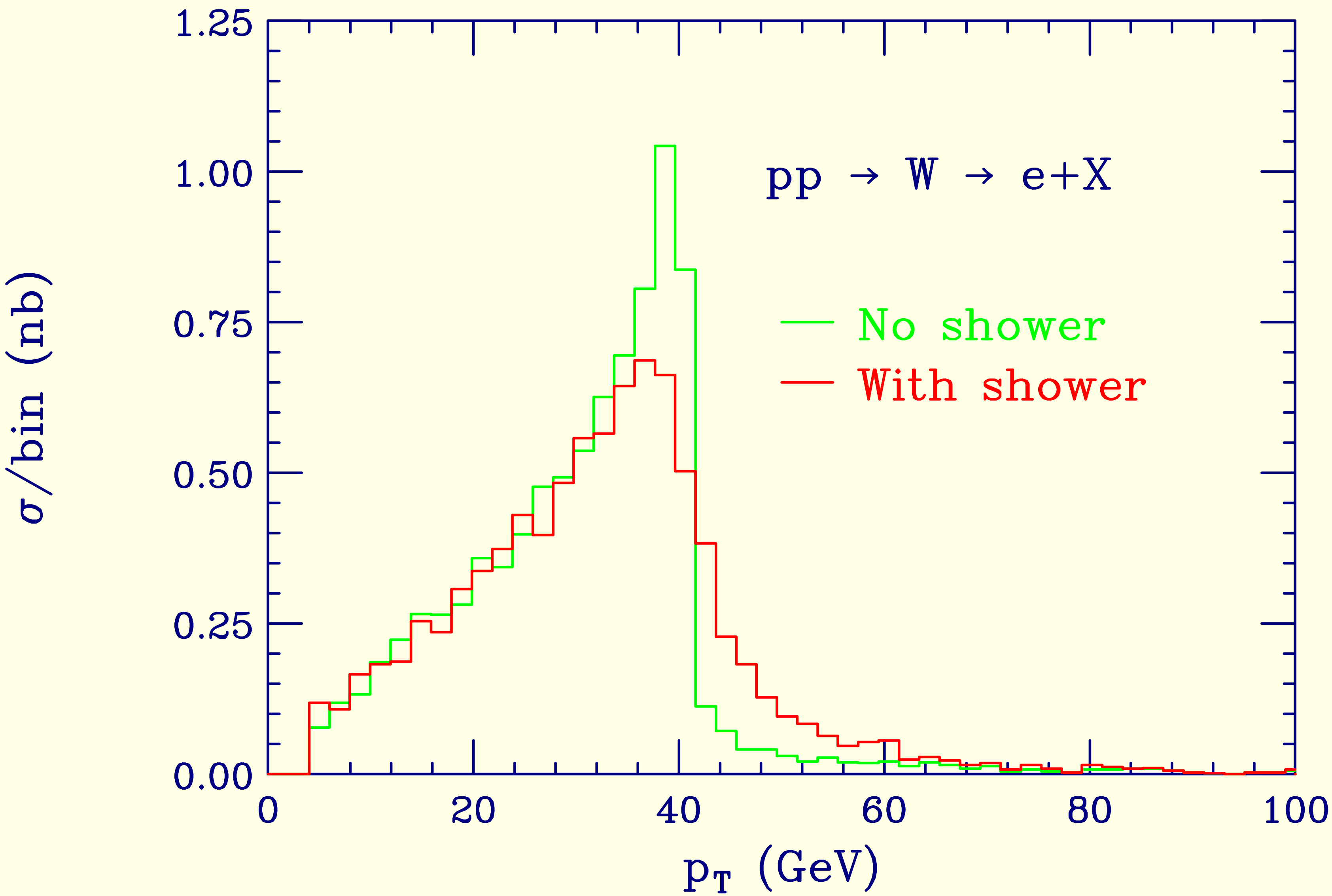
3. Models for hadron formation

4. Models for underlying event, multi-parton collisions, minimum bias

5. Library for (spacetime) decays of unstable particles

The name SHOWER from item 2

Amazing models for hard interactions



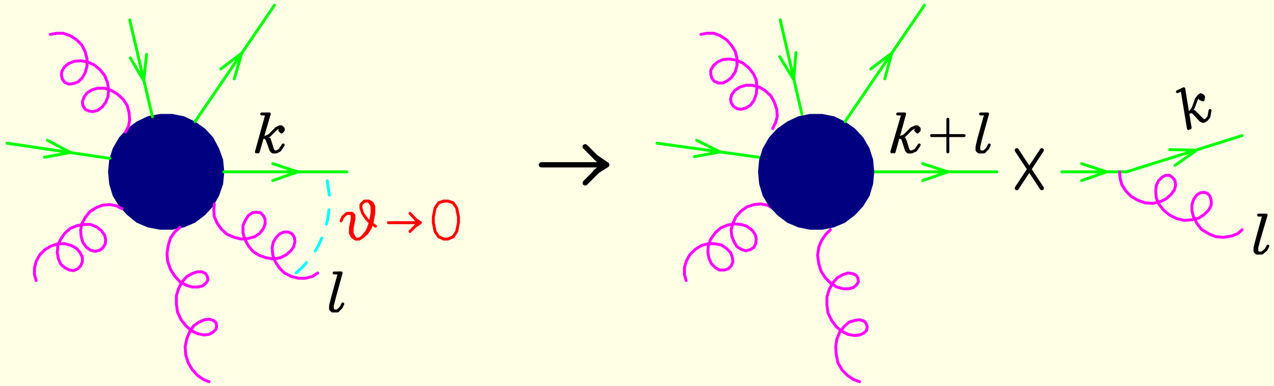
(Half an hour of work)

IHEP	ID	IDPDG	IST	P-X	P-Y	P-Z	ENERGY	M
30	NU_E	12	1	64.30	25.12	-1194.4	1196.4	
31	E+	-11	1	-22.36	6.19	-234.2	235.4	
230	PI0	111	1	0.31	0.38	0.9	1.0	
231	RHO+	213	197	-0.06	0.07	0.1	0.8	
232	P	2212	1	0.40	0.78	1.0	1.6	
233	NBAR	-2112	1	-0.13	-0.35	-0.9	1.3	
234	PI-	-211	1	0.14	0.34	286.9	286.9	
235	PI+	211	1	-0.14	-0.34	624.5	624.5	
236	P	2212	1	-1.23	-0.26	0.9	1.8	
237	DLTABR--	-2224	197	0.94	0.35	1.6	2.2	
238	PI0	111	1	0.74	-0.31	-27.9	27.9	
239	RHO0	113	197	0.73	-0.88	-19.5	19.5	
240	K+	321	1	0.58	0.02	-11.0	11.0	
241	KL_1-	-10323	197	1.23	-1.50	-50.2	50.2	
242	K-	-321	1	0.01	0.22	1.3	1.4	
243	PI0	111	1	0.31	0.38	0.2	0.6	

Shower basics: Collinear factorization

QCD emissions are enhanced near the collinear limit

Cross sections factorize near collinear limit

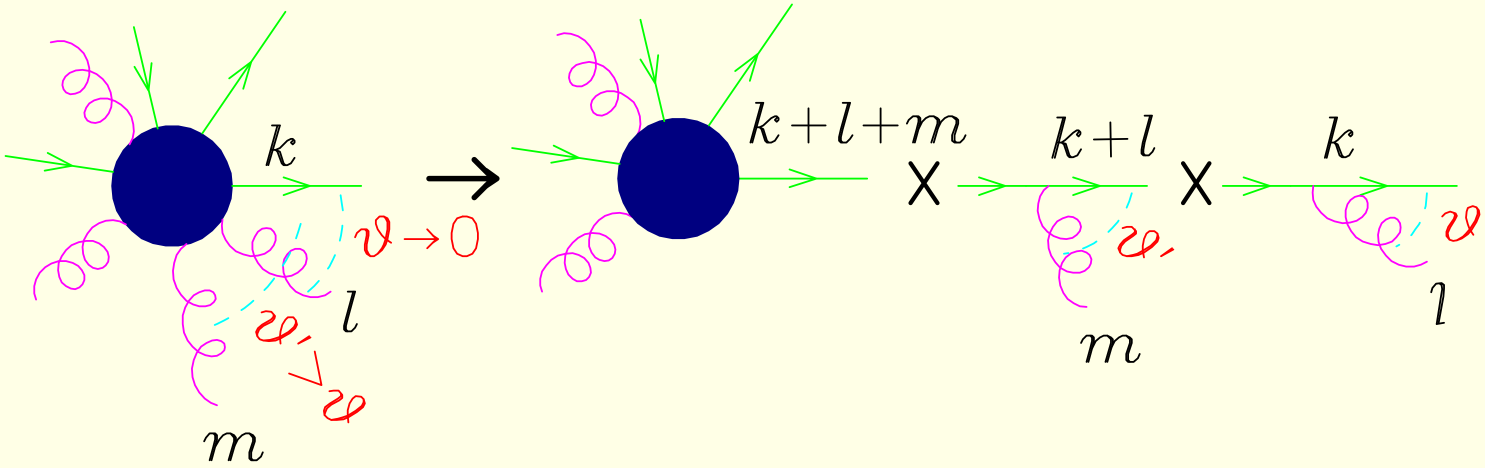


$$|M_{n+1}|^2 d\Phi_{n+1} \Rightarrow |M_n|^2 d\Phi_n \; \frac{\alpha_S}{2\pi} \, \frac{dt}{t} \, P_{qq}(z) \, dz \, \frac{d\phi}{2\pi}$$

t :	virtuality, or p_{T}^2, or $E^2\theta^2$
$z = k^0/(k^0 + l^0)$:	energy (or $p_{\parallel}$, or p^+) fraction of the quark
$P_{qq}(z) = C_F \, \frac{1+z^2}{1-z}$:	Altarelli-Parisi splitting function
	(Ignore $z \to 1$ IR divergence for now)

If another gluon becomes collinear, iterate the previous formula:

$\theta', \theta \to 0$
with $\theta' > \theta$

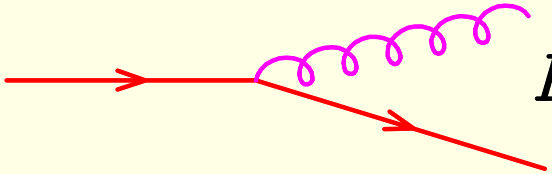


$$|M_{n+1}|^2 d\Phi_{n+1} \Rightarrow \quad |M_{n-1}|^2 d\Phi_{n-1} \quad \times \frac{\alpha_S}{2\pi}\frac{dt'}{t'} P_{qq}(z')\, dz' \frac{d\phi'}{2\pi}$$
$$\times \frac{\alpha_S}{2\pi}\frac{dt}{t} P_{qq}(z)\, dz \frac{d\phi}{2\pi} \;\; \theta(t'-t)$$

Collinear partons can be described by a factorized integral ordered in t.
For m collinear emissions:

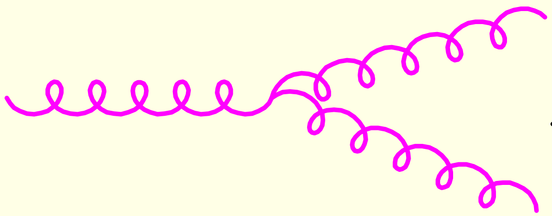
$$\int_{\theta_{\rm min}} d\theta_1 \int_{\theta_1} d\theta_2 \ldots \int_{\theta_{m-1}} d\theta_m \propto \frac{\log^m \frac{1}{\theta^2_{\rm min}}}{m!} \approx \frac{\log^m \frac{Q^2}{\Lambda^2}}{m!}, \quad \Lambda \approx \Lambda_{\rm QCD} \; .$$

Same applies to all splitting processes



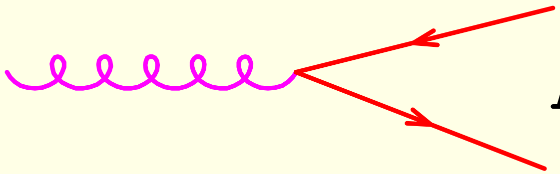
$P_{qq}(z)$

$$P_{qq}(z) = C_F \, \frac{1+z^2}{1-z}$$



$P_{gg}(z)$

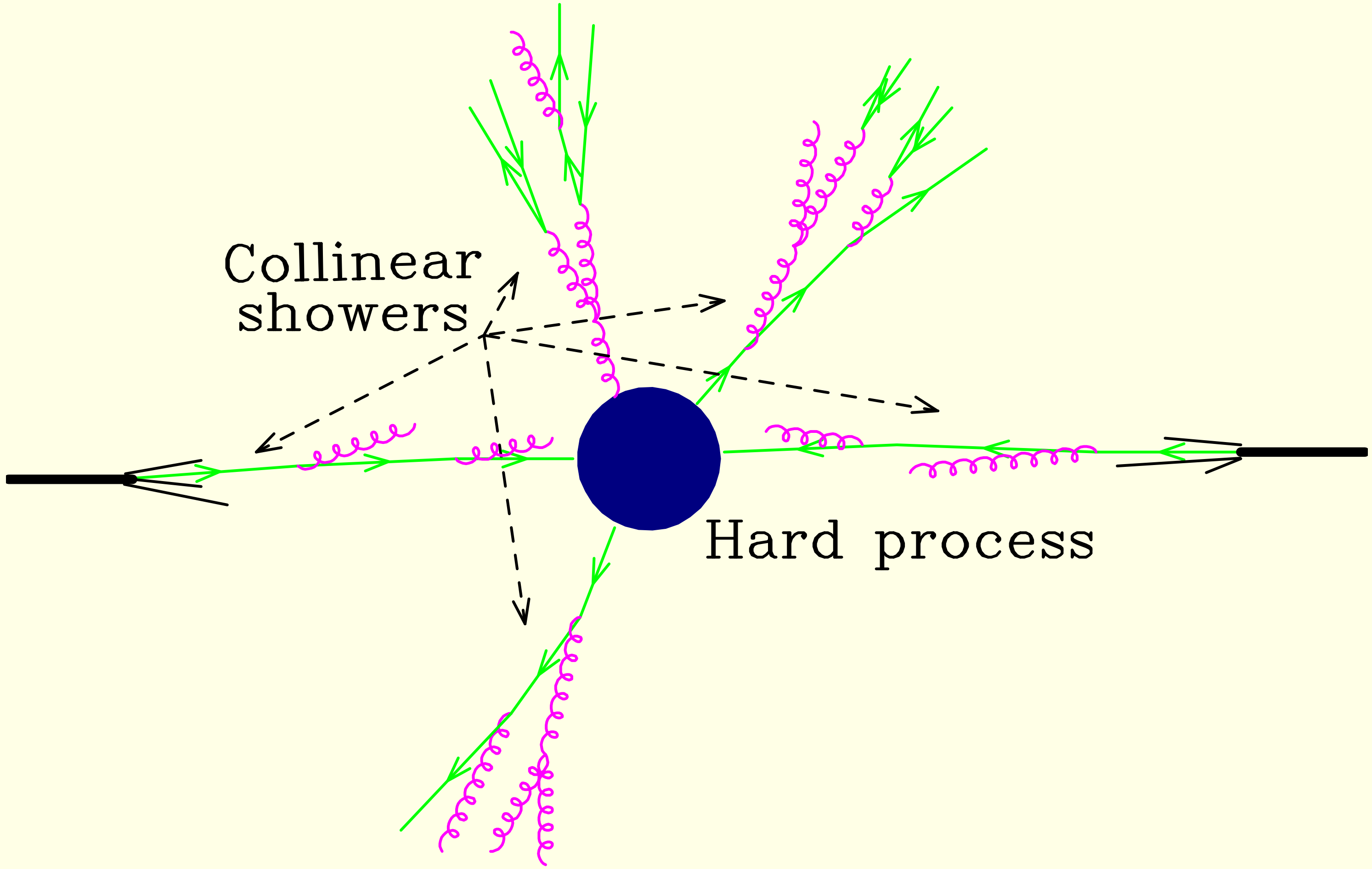
$$P_{gg}(z) = C_A \left(\frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right)$$



$P_{gq}(z)$

$$P_{gq}(z) = T_F \left(z^2 + (1-z)^2 \right)$$

"Leading log" description of hard event



(Initial state radiation treated in similar way)

IR Divergences

Collinear divergences when $t \to 0$

$$|M_n|^2 \int \frac{\alpha_S}{2\pi} \frac{dt}{t} P_{qq}(z)\, dz \frac{d\phi}{2\pi} \propto |M_n|^2 \alpha_S \log\left(\frac{\text{process scale}}{\text{IR cutoff}}\right)$$

According to the KLN (Kinoshita–Lee–Nauenberg) theorem

IR divergences cancel order by order in INCLUSIVE quantities.

So: virtual corrections MUST give

$$-|M_n|^2 d\Phi_n \int \frac{\alpha_S}{2\pi} \frac{dt}{t} P_{qq}(z)\, dz \frac{d\phi}{2\pi} \propto -|M_n|^2 \alpha_S \log\left(\frac{\text{ren. scale}}{\text{IR cutoff}}\right)$$

Must add $n+1$ and n body cross section to get cancellation

Negative divergent correction to n body cross section

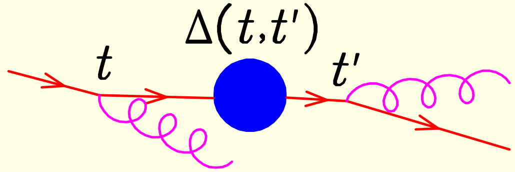
How do we get positive, exclusive, final state cross sections?

Only possible answer: add virtual terms to all orders

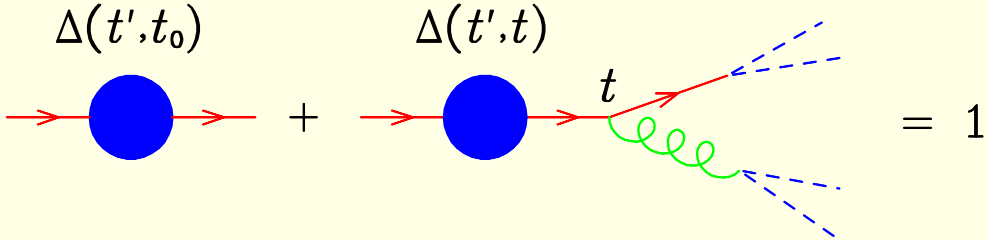
(same answer for textbooks IR problem in QED)

In the shower: UV and IR divergent virtual corrections for internal lines:

- UV renormalization at largest t
- IR cutoff at smallest t



Condition for cancellation:
applies recursively;
(t_0 = IR cutoff $\approx \Lambda^2_{\mathrm{QCD}}$)



$$\Delta(t_{\mathrm{max}}, t_0) + \int_{t_0}^{t_{\mathrm{max}}} \Delta(t_{\mathrm{max}}, t) \frac{\alpha_S}{2\pi} \frac{dt}{t} P_{qq}(z)\, dz \frac{d\phi}{2\pi} = 1$$

$$\Rightarrow \; \Delta(t_{\mathrm{max}}, t) = \exp\left[- \int_{t}^{t_{\mathrm{max}}} \frac{\alpha_S}{2\pi} \frac{dt}{t} P_{qq}(z)\, dz \frac{d\phi}{2\pi} \right]$$

so that:

$$= \int_{t_0}^{t_{\mathrm{max}}} \frac{d\Delta(t_{\mathrm{max}}, t)}{dt} dt = 1 - \Delta(t_{\mathrm{max}}, t_0)$$

Sudakov Form Factor

$\Delta(t_{\mathrm{max}}, t_{\mathrm{max}}) = 1$

$\Delta(t_{\mathrm{max}}, t_0) \rightarrow$ small

i.e. no radiation is suppressed!!

Recipe for shower with virtual corrections:

- Include a factor

$$\frac{dt}{t} dz \, \frac{\alpha_{\mathrm{s}}(t)}{2\pi} \, P_{ij}(z)$$

  at each splitting vertex (use renormalized coupling at scale t!).

- Order the splittings in t: later splittings have smaller t.

- Include the factor $\Delta_i(t_1, t_2)$ on each internal line going from a splitting at the scale t_1 to a splitting at t_2, with

$$\Delta_i(t_1, t_2) = \exp\left[-\sum_j \int_{t_1}^{t_2} \frac{dt'}{t'} \int dz \, \frac{\alpha_{\mathrm{s}}(t')}{2\pi} \, P_{ij}(z) \right]$$

Simple probabilistic interpretation:

$\Delta(t_1, t_2)$ is the probability for having no branching from t_1 to t_2.
The probability to have a branching in the interval $t_2, t_2 + dt_2$ is

$$P(t_1, t_2)dt_2 = [1 - \Delta(t_1, t_2 + dt)] - [1 - \Delta(t_1, t_2)] = \Delta(t_1, t_2)\,\frac{dt_2}{t_2}\frac{\alpha_S(t')}{2\pi}\int P(z)\,dz$$

$\Delta(t_1, t_2)$ has uniform distribution!

Shower algorithm:

- Generate a random number $0 < r < 1$;
- Solve the equation $\Delta(t, t') = r$ for $t' < t$;
- If $t' < t_0$ stop there (unresolvable emission);
- generate a z distributed according to $P(z)$;
- restart for each branch, at an initial value t'.

Elementary example

Simulate a source with a probability p for emission per unit time.
Probability distributions for first emission:

$$P(t)\,dt = \lim_{n \longrightarrow \infty} \left(1 - p\frac{t}{n}\right)^n p\,dt = e^{-pt}\,p\,dt = -d\left(e^{-pt}\right)$$

so $\int P(t)dt$ is distributed uniformly between 0 and 1.
Monte Carlo implementation for emissions between $t = t_0$ and $t = t_f$

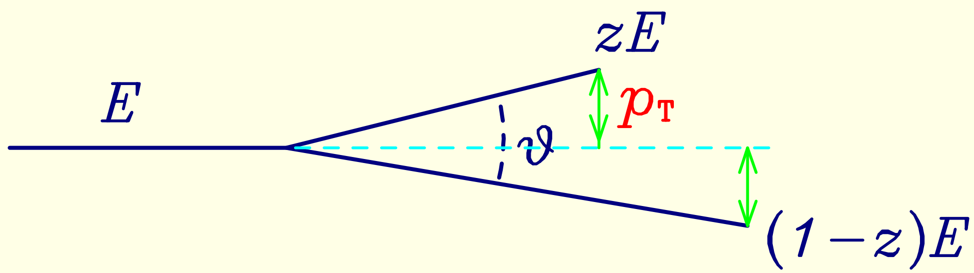
- generate a random number $0 < r < 1$
- solve the equation $e^{-p(t-t_0)} = r$ for t
- if $t > t_f$ stop.
- continue starting from t

Soft divergences: double log region

$z \to 1$ ($z \to 0$) region problematic: for $z \to 1$: $P_{qq}, P_{gg} \propto \frac{1}{1-z}$

Choice of shower variables makes a difference

$$
\begin{aligned}
\text{virtuality:} \quad & t \equiv E^2\, z(1-z)\, \overbrace{\theta^2}^{1-\cos\theta} \\
p_{\mathrm{T}}^2: \quad & t \equiv E^2\, z^2(1-z)^2\, \theta^2 \\
\text{angle:} \quad & t \equiv E^2\, \theta^2
\end{aligned}
$$



$$
\underbrace{\int \frac{dt}{t} \int_{\sqrt{t}/E}^{1-\sqrt{t}/E} \frac{dz}{1-z}}_{\text{virtuality: } z(1-z) > t/E^2} \approx \frac{\log^2 \frac{t}{E^2}}{2}; \underbrace{\int \frac{dt}{t} \int_{t/E^2}^{1-t/E^2} \frac{dz}{1-z}}_{p_{\mathrm{T}}^2\text{: } z^2(1-z)^2 > t/E^2} \approx \log^2 \frac{t}{E}; \underbrace{\int \frac{dt}{t} \int_0^1 \frac{dz}{1-z}}_{\text{angle}} \approx \log t \log \frac{E}{\Lambda}
$$

sizeable difference in double log structure!

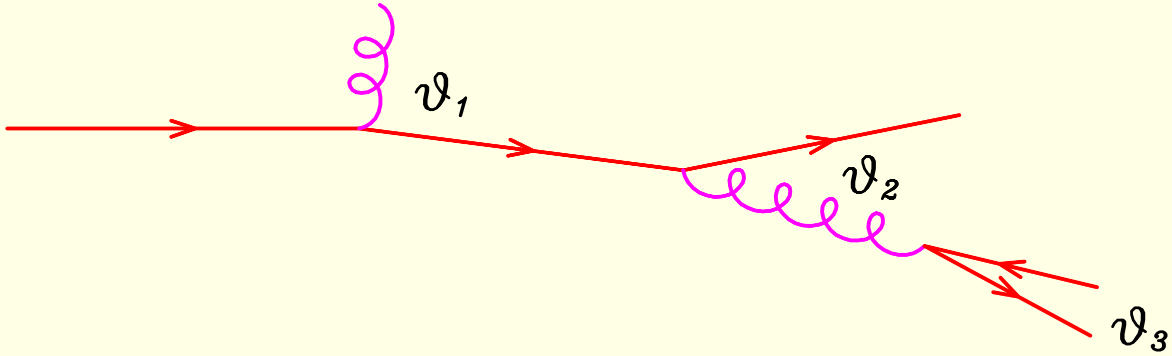
Angular ordering

is the correct choice (Mueller 1981)

$$\frac{d\theta}{\theta}\,\frac{\alpha_{\mathsf{S}}(p_T^2)}{2\pi}\,P(z)\,dz$$

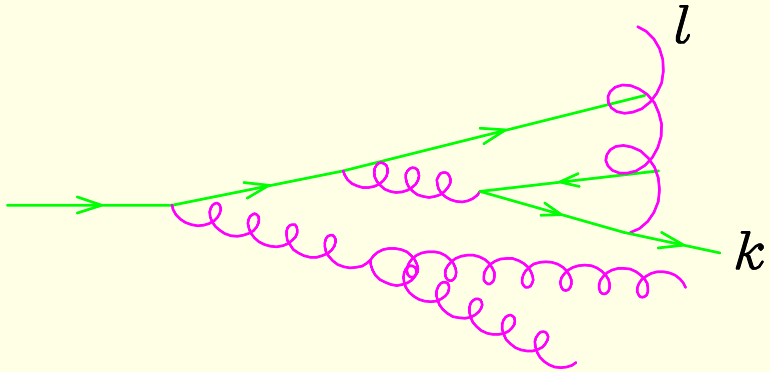
$$\theta_1 > \theta_2 > \theta_3 \ldots$$

$$p_T^2 = E^2\,z^2\,(1-z)^2\,\theta^2$$



$\alpha_{\mathsf{S}}(p_T^2)$ for correct treatment of charge renormalization in soft region.

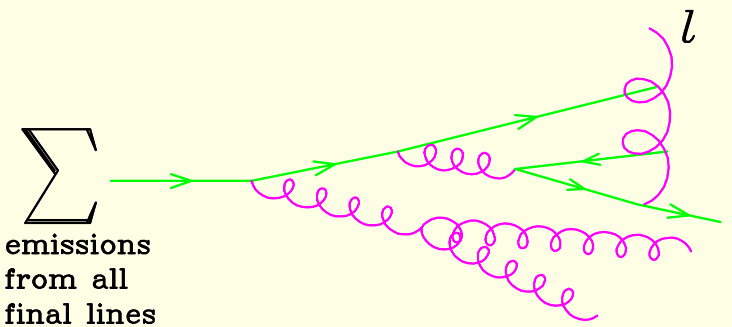
Notice: soft divergence for emission off an off-shell line!



With virtuality ordering:

Soft emissions give small virtuality.
At end of shower, large amount of unrestricted (all angles) soft radiation

But soft gluons emitted at large angles from final state partons add coherently!



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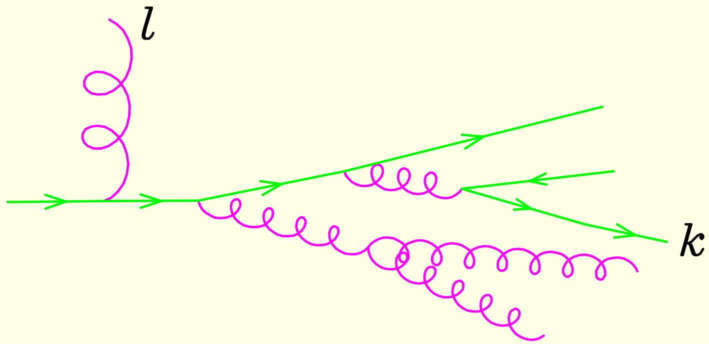


large angle, high energy: already ordered in angle
large angle, small energy: should be reordered by angle

Thus: order in angle

It affects jets distributions in enhanced regions

Most important effect: reduced multiplicity!
(interference is mainly distructive)

- HERWIG: angular ordered
- PYTHIA, SHERPA: virtuality ordered
  interference implemented as fully destructive
  totally vetoing large angle radiation
- ARIADNE implements a different (dipole) approach to soft radiation
  (recent PYTHIA versions borrow this approach)

Vetoing: an algorithm to simply implement cuts (i.e. θ functions) in the elementary emission process: If a branching from t to t' does not satisfy the cut, set $t = t'$ and generate a new $t' < t$.

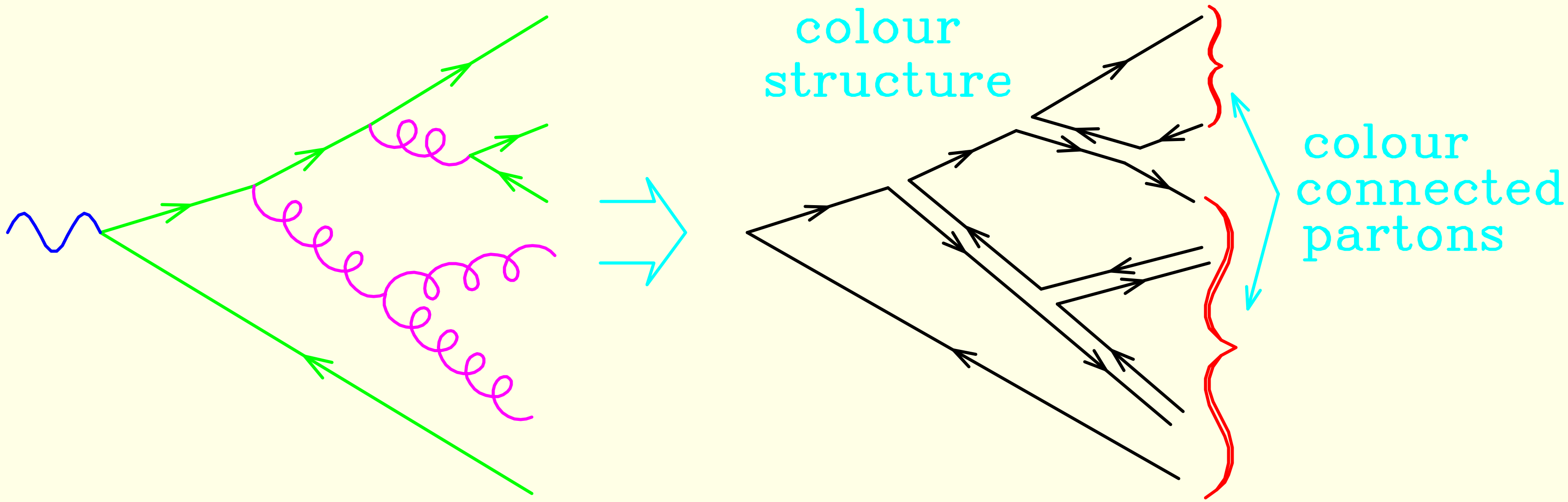
COLOUR

SMC's assign colour labels to partons.

Only colour connections are recorded (as in large N limit).

Initial colour assigned according to cross section for each colour structure.

Showering has colour ambiguities only in $g \to gg$ (exchange of final gluons) (one of the two possibility chosen at random)



Colour assignements are used in the hadronization phase

HADRONIZATION

One very simple model (just to get the feeling): Cluster model

- All gluons are forced to decay into $q\bar{q}$ pairs, so that colour connected partons are just $q\bar{q}$ pairs.
- Compute the mass of the $q\bar{q}$ pair, make it decay into 2 hadrons (or 1 if too light).
- Decay the resonance according to known branchings

Shower dynamics guarantees that clusters have small mass
(Sudakov suppression of large dynamic gaps between final $q\bar{q}$).

Most popular model: Lund String Model.

Conclusions

SMC are amazing models for hard interactions

Include most useful background and signals for LHC

Reliable in leading order for

- Hard interaction
- Jet structure

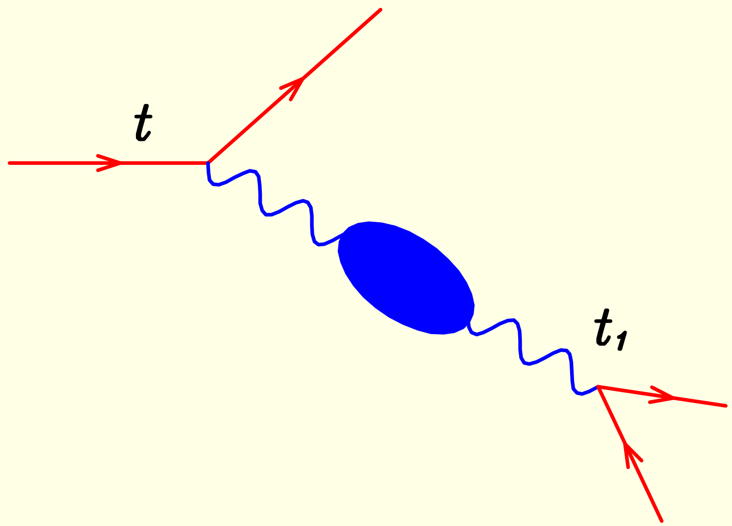
Less reliable (but very realistic) for

- Hadron formation
- Underlying event
- Multiple interactions

BIBLIOGRAPHY
LINKS
EXTRA TOPICS

- Collider Physics, K. Ellis, J. Stirling and B. Webber
- MONTE CARLO SIMULATION OF HARD HADRONIC PROCESSES. By B.R. Webber, Ann.Rev.Nucl.Part.Sci.36:253-286,1986
- PYTHIA homepage: http://www.thep.lu.se/~torbjorn/Pythia.html The PYTHIA manual (pythia6301.pdf): clear and instructive introduction. Tutorial, etc.
- HERWIG page: http://hepwww.rl.ac.uk/theory/seymour/herwig/
- LES HOUCHES GUIDEBOOK TO MONTE CARLO GENERATORS FOR HADRON COLLIDER PHYSICS, hep-ph/0403045

$\Delta(t, t_1)$ example in QED (for Theorists)



In this case, virtual corrections amount to $\alpha \to \alpha(t_1)$ (alpha at scale of photon virtuality). In amplitude squared: $\alpha^2(t_1)$.
According to our recipe, we must supply

$$\alpha(t) \times \alpha(t_1) \times \Delta(t, t_1)$$

that must equal $\alpha^2(t_1)$; so

$$\Delta(t, t_1) = \frac{\alpha(t_1)}{\alpha(t)} = \frac{\log \frac{\Lambda}{t}}{\log \frac{\Lambda}{t_1}}$$

and indeed using

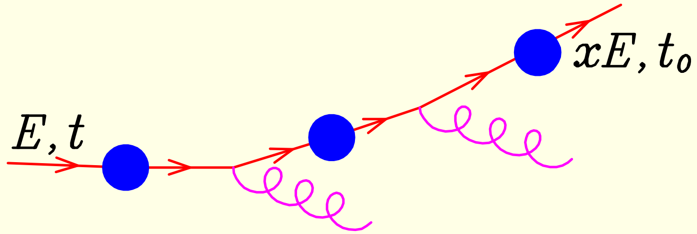
$$\alpha(t) = \frac{1}{b_0 \log \frac{\Lambda}{t}}, \quad b_0 = \frac{4 n_f}{12\pi}$$

we get

$$\Delta(t) = \exp\left[-\int_{t_0}^{t} \frac{dt'}{t'} \frac{\alpha(t')}{2\pi} \overbrace{n_f (z^2 + (1-z)^2)}^{P_{gq} \text{ in QED}} dz \right] = \frac{\log \frac{\Lambda}{t}}{\log \frac{\Lambda}{t_0}}$$

Single Inclusive Distribution

$$D(x,t) = \sum$$



Obeys evolution equation: $$D(x,t) = \Delta(t,t_0)\,\delta(1-x) + \int_{t_0}^{t} \Delta(t,t')\,\frac{dt'}{t'}\,\frac{\alpha_S(t')}{2\pi}\,P(z)\,D(x/z,t')\,\frac{dz}{z}$$

divide by $\Delta(t,t_0)$ $$\frac{D(x,t)}{\Delta(t,t_0)} = \delta(1-x) + \int_{t_0}^{t} \frac{1}{\Delta(t',t_0)}\,\frac{dt'}{t'}\,\frac{\alpha_S(t')}{2\pi}\,P(z)\,D(x/z,t')\,\frac{dz}{z}$$

take $\Delta(t,t_0)\frac{d}{d\log t}$ $$\Delta(t,t_0)\frac{d}{d\log t}\frac{D(x,t)}{\Delta(t,t_0)} =$$

$$\frac{d}{d\log t}D(x,t) - D(x,t)\frac{d}{d\log t}\log\Delta(t) = \frac{\alpha_S(t)}{2\pi}\int P(z)D(x/z,t)\,\frac{dz}{z}$$

The second term on the left hand side regularizes the splitting kernels:

Altarelli-Parisi equation

$$\frac{d}{d\log t}D(x,t) = \frac{\alpha_S(t)}{2\pi}\int P(z)D(x/z,t)\,\frac{dz}{z} - \frac{\alpha_S(t)}{2\pi}\int P(z)D(x,t)\,\frac{dz}{z}$$