Lattice QCD with light quarks compares to chiral perturbation theory

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CERN - Theory Group

In Collaboration with L. Del Debbio (Edinburgh), M. Lüscher (CERN), R. Petronzio and N. Tantalo (Tor Vergata)

Outline

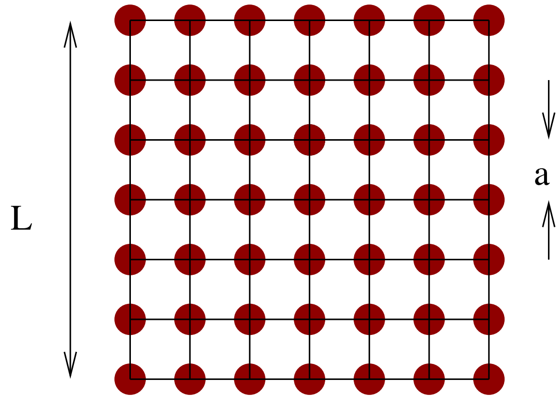
- QCD with two flavors with Wilson fermions
- ChPT with two flavors

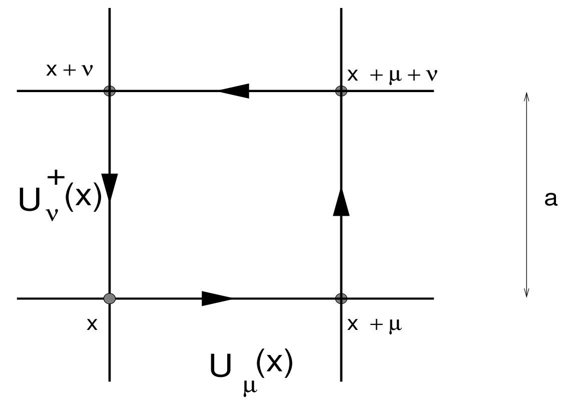
- A new algorithm for full QCD simulations: SAP
- Simulation parameters and costs

- Results for meson masses and decay constants
- Scaling of M_P^2 and F_P with the lattice spacing
- Lattice results confront ChPT

- Conclusions

QCD with two degenerate flavors with the Wilson action





- The Wilson action for the $SU(3)$ Yang–Mills theory is $\left(\beta = 6/g^2\right)$

$$S_{\rm YM} = \beta \sum_{x,\mu<\nu} \left\{ 1 - \frac{1}{6} {\rm Tr} \left[U_{\mu\nu}(x) + U^\dagger_{\mu\nu}(x) \right] \right\}$$

$$U_{\mu\nu}(x) = U_\mu(x) U_\nu(x+\mu) U^\dagger_\mu(x+\nu) U^\dagger_\nu(x)$$

- Periodic boundary conditions for gauge fields

QCD with two degenerate flavors with the Wilson action

- The fermion Wilson action we use is

$$S_{\rm F} = \sum_{i=1}^{2} \sum_{x,y} \bar\psi_i(x)\, D_m(x,y)\, \psi_i(y) \qquad\qquad \psi \equiv \{\psi_1, \psi_2\}$$

$$D_m = \frac{1}{2}\Big\{\gamma_\mu(\nabla^*_\mu + \nabla_\mu) - a\nabla^*_\mu\nabla_\mu\Big\} + m_0$$

where $am_0 = \big(1/k - 8\big)/2$ and

$$\begin{aligned} \nabla_\mu\psi_i(x) &= \frac{1}{a}\Big[U_\mu(x)\psi_i(x+a\hat\mu) - \psi_i(x)\Big] \\ \nabla^*_\mu\psi_i(x) &= \frac{1}{a}\Big[\psi_i(x) - U^\dagger_\mu(x-a\hat\mu)\psi_i(x-a\hat\mu)\Big] \end{aligned}$$

- Fermion fields with periodic boundary conditions in space and anti-periodic in time

Renormalized axial Ward identities for Wilson fermions Bochicchio et al. 85

- It is possible to define renormalized operators

$$\hat{A}^a_\mu(x) = Z_A A^a_\mu(x) \qquad A^a_\mu(x) = \bar{\psi}(x)\gamma_\mu\gamma_5\frac{\sigma^a}{2}\psi(x)$$

$$\hat{P}^a(x) = Z_P P^a(x) \qquad P^a(x) = \bar{\psi}(x)\gamma_5\frac{\sigma^a}{2}\psi(x)$$

that satisfy renormalized axial Ward identities of the form

$$\partial^*_\mu\langle\hat{A}^a_\mu(x)\hat{P}^a(0)\rangle = 2\,\hat{m}\,\langle\hat{P}^a(x)\hat{P}^a(0)\rangle + \mathcal{O}(a) \qquad x \neq 0$$

- The "on-shell" non-perturbative definition of the quark mass is

$$m = \frac{1}{2}\frac{\partial^*_\mu\langle A^a_\mu(x)P^a(0)\rangle}{\langle P^a(x)P^a(0)\rangle} \qquad \hat{m} = \frac{Z_A}{Z_P}\,m$$

Non-linear sigma model with two degenerate flavors

- The fundamental fields

$$U \equiv \exp\left\{\frac{2i}{F}\Phi\right\}\,, \qquad\qquad \Phi = \sum_a \phi^a\,\frac{\sigma^a}{2}$$

transforms under chiral symmetry as

$$U \to V_R U V_L^\dagger\,, \qquad\qquad U^\dagger \to V_L U^\dagger V_R^\dagger$$

with $V_L V_L^\dagger = I$ and $V_R V_R^\dagger = I$

- The $\mathcal{O}(p^2)$ Euclidean action which encodes the SSB is

$$\mathcal{S}^{(2)} = \int d^4x \frac{F^2}{4}\left\{\mathrm{Tr}\left[\partial_\mu U^\dagger \partial_\mu U\right] - M^2\,\mathrm{Tr}\left[U^\dagger + U\right]\right\}$$

where $M^2 = 2B\hat{m}$

Meson mass and decay constant at NLO

- The $\mathcal{O}(p^4)$ Euclidean Action is given by

$$\mathcal{S}^{(4)} = \int d^4x \Big\{ \frac{M^4(\hat{l}_4 - \hat{l}_3)}{16} \mathrm{Tr}\big[U^\dagger + U\big] \mathrm{Tr}\big[U^\dagger + U\big] + \frac{M^2 \hat{l}_4}{8} \mathrm{Tr}\big[\partial_\mu U^\dagger \partial_\mu U\big] \mathrm{Tr}\big[U^\dagger + U\big] + \text{four deriv. terms} \Big\}$$

- The meson mass and decay constant at $\mathcal{O}(p^4)$ are given by

$$M_P^2 = M^2 \Big\{ 1 + \frac{M^2}{32\pi^2 F^2} \log\big(\frac{M^2}{\mu^2}\big) + \frac{2M^2}{F^2} \hat{l}_3(\mu) \Big\}$$

$$F_P = F \Big\{ 1 - \frac{M^2}{16\pi^2 F^2} \log\big(\frac{M^2}{\mu^2}\big) + \frac{M^2}{F^2} \hat{l}_4(\mu) \Big\}$$

Matching a non-linear sigma model with the experiment: M_P^2 Gasser Leutwyler 84

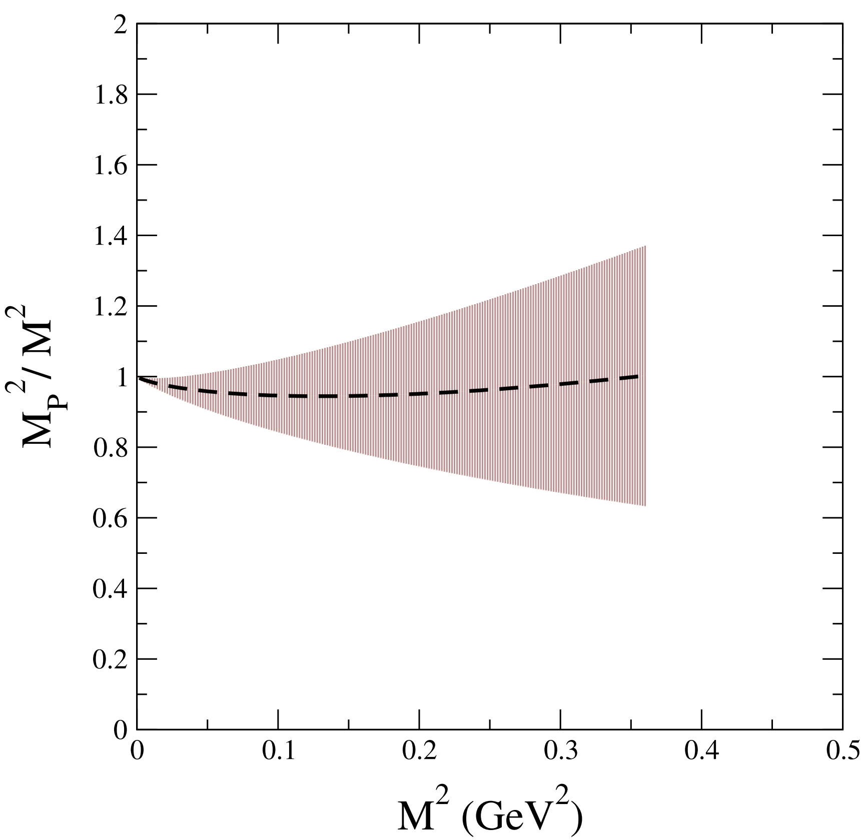
- If we define

$$\hat{l}_3(\mu) = \frac{-1}{64\pi^2}\Big(\bar{l}_3 + \log\big(\frac{M^2}{\mu^2}\big)\Big)\Big|_{M=139.6\mathrm{MeV}}$$

$$\bar{l}_3 = \log\big(\frac{\Lambda_\pi^2}{M^2}\big)\Big|_{M=139.6\mathrm{MeV}}$$

then

$$M_P^2 = M^2\left\{1 + \frac{M^2}{32\pi^2 F^2}\log\big(\frac{M^2}{\Lambda_\pi^2}\big)\right\}$$



- A crude estimate from experimental values of meson masses gives

$$\bar{l}_3 = 2.9 \pm 2.4$$

Matching a non-linear sigma model with the experiment: F_P Gasser Leutwyler 84, Colangelo Gasser Leutwyler 01

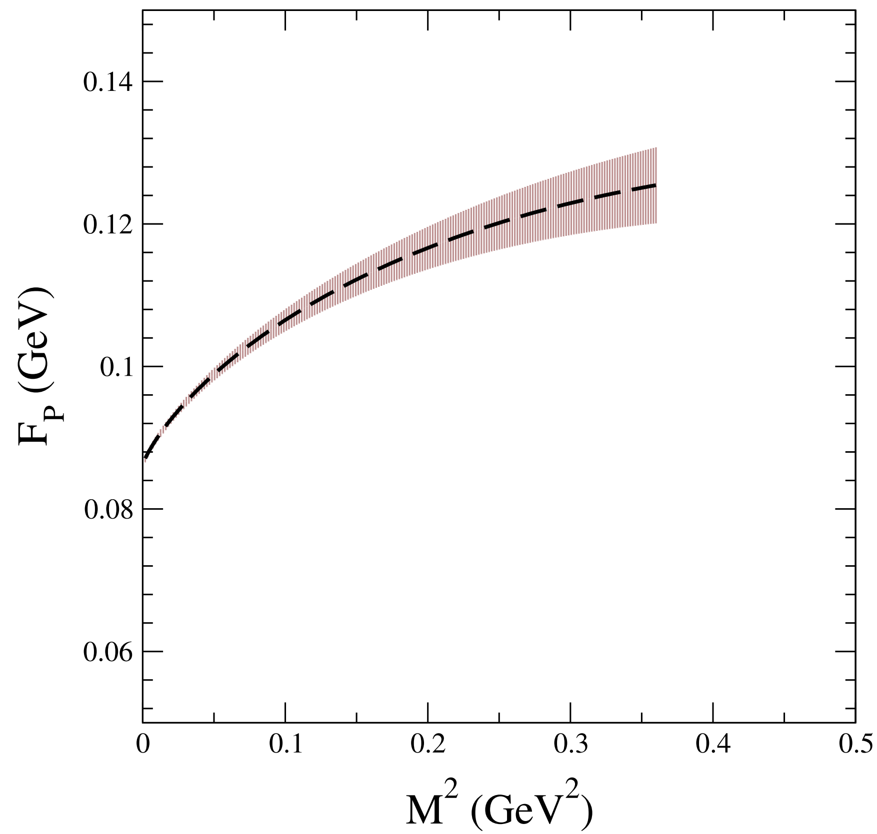
- If we define

$$\hat{l}_4(\mu) = \frac{1}{16\pi^2}\Big(\bar{l}_4 + \log\big(\frac{M^2}{\mu^2}\big)\Big)\Big|_{M=139.6\mathrm{MeV}}$$

$$\bar{l}_4 = \log\big(\frac{\Lambda_F^2}{M^2}\big)\Big|_{M=139.6\mathrm{MeV}}$$

then

$$F_P = F\left\{1 - \frac{M^2}{16\pi^2 F^2}\log\big(\frac{M^2}{\Lambda_F^2}\big)\right\}$$

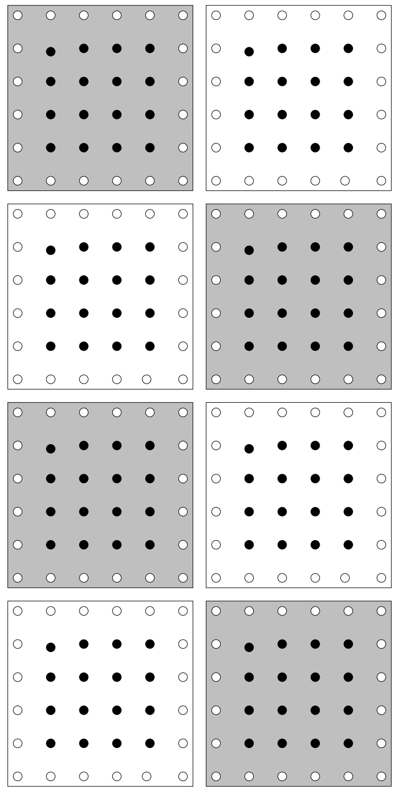


- An estimate from the scalar radius of the pion gives

$$\bar{l}_4 = 4.4 \pm 0.2$$

Schwarz-preconditioned Hybrid Monte Carlo (SAP) Lüscher 03 04

- Decomposition of the lattice into blocks with Dirichlet b.c. with $q \geq \pi/L > 1$ GeV



- Asymptotic freedom: quarks are weakly interacting in the blocks $\Longrightarrow$ QCD easy (*cheaper*) to simulate

- Block interactions are weak and are taken into account exactly

$$S(x,y) \sim \frac{1}{|x-y|^3}$$

Block decomposition of the Dirac operator

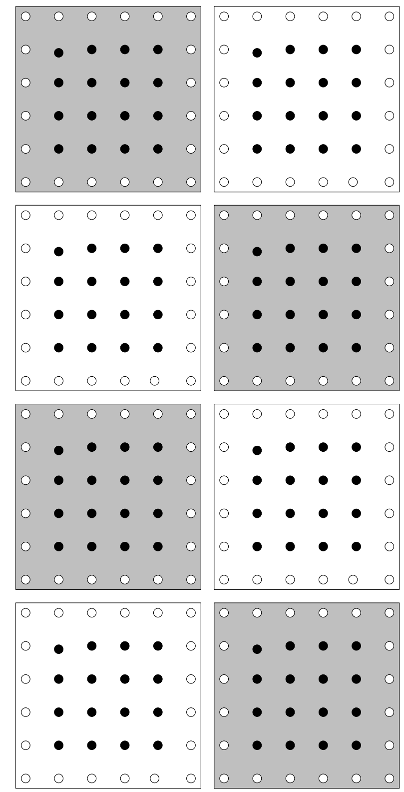
- The Wilson–Dirac operator

$$D_m = \frac{1}{2}\left\{\gamma_\mu(\nabla_\mu^* + \nabla_\mu) - \nabla_\mu^*\nabla_\mu\right\} + m_0$$

can be decomposed as

$$D = D_{\Omega^*} + D_\Omega + D_{\partial\Omega^*} + D_{\partial\Omega}$$

where



$$D_{\Omega^*} = \sum_{\text{white } \Lambda} D_\Lambda \qquad D_\Omega = \sum_{\text{black } \Lambda} D_\Lambda$$

Ω^*, Ω are white and black blocks, $\partial\Omega$, $\partial\Omega^*$ are exterior boundaries

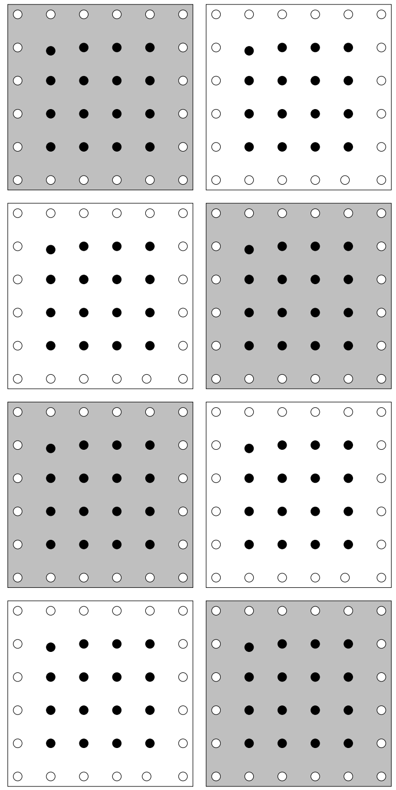
Factorization of the determinant

- The determinant of the Dirac operator written as

$$\det D_W = \prod_{\text{all}\Lambda} \det \hat{D}_\Lambda \ \det R$$

with the block interaction

$$R = 1 - P_{\partial\Omega^*} D_\Omega^{-1} D_{\partial\Omega} D_{\Omega^*}^{-1} D_{\partial\Omega^*}$$



- For two flavors can be written as integral over scalar fields

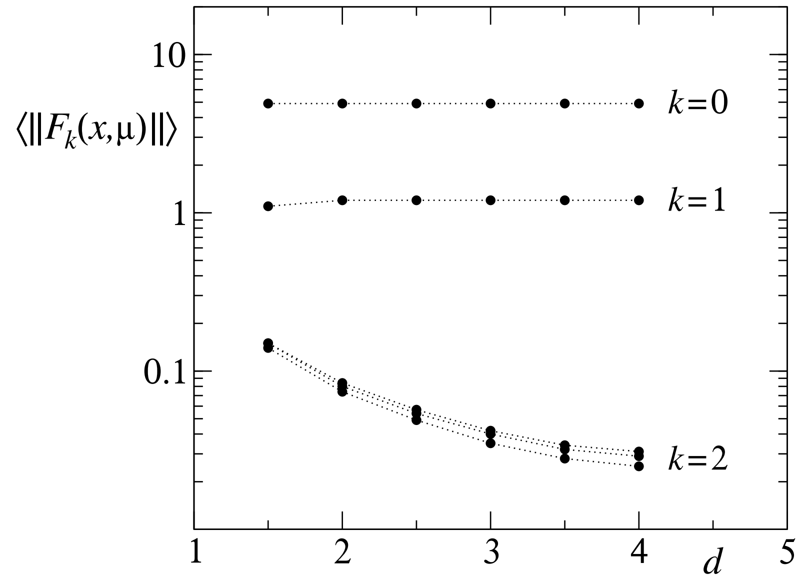
$$S_{\phi\chi} = \sum_{\text{all }\Lambda} ||\hat{D}_\Lambda^{-1}\phi_\Lambda||^2 + ||R^{-1}\chi||^2$$

where ϕ_Λ defined on Λ and χ on $\partial\Omega^*$

- In molecular dynamics force naturally split

$$\frac{d}{dt}\Pi(x,\mu) = -F_{\rm G}(x,\mu) - F_{\Lambda}(x,\mu) - F_{\rm R}(x,\mu)$$

$$\frac{d}{dt}U(x,\mu) = \Pi(x,\mu)U(x,\mu)$$



- Integration step-sizes chosen such that

$$\epsilon_{\rm G}||F_{\rm G}|| \sim \epsilon_{\Lambda}||F_{\Lambda}|| \sim \epsilon_{\rm R}||F_{\rm R}||$$

i.e. the most expensive force computed less often!

- Do not give up first-principles: teach Physics to exact algorithms for being smarter (*faster*)!

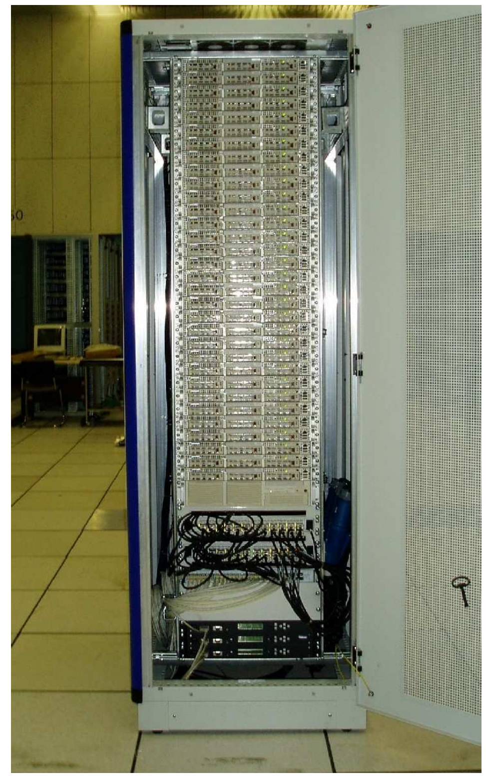
$$\text{Cost} \propto m_q^{-1}$$

Collaboration and PC clusters

Collaboration: L. Del Debbio (Edinburgh), L. G. and M. Lüscher (CERN), R. Petronzio and N. Tantalo (Tor Vergata)

Fermi Institute PC cluster with 80 nodes (160 Xeon procs)

64 nodes used for this project (∼200 Gflops sustained)



CERN PC cluster with 32 nodes (64 Xeon procs)

All nodes used for this project (∼160 Gflops sustained)

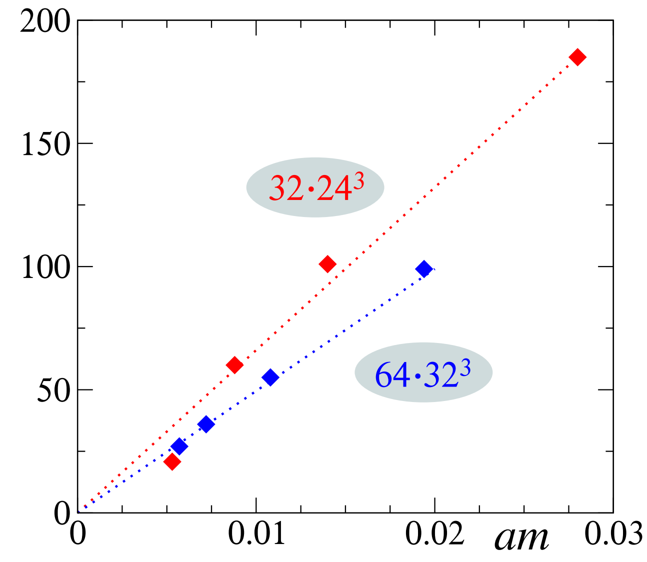
Bern Physics Institute PC cluster with 32 nodes (64 Xeon procs)

8 nodes used for this project (∼25 Gflops sustained)

Parameters of the runs with the Wilson action

	k	N_{trj}	N_{sep}	N_{conf}
	0.15750	6400	100	64
$V = 24^3 \times 32$	0.15800	10900	100	109
$\beta = 5.6$	0.15825	10000	100	100
	0.15835	5000	50	100
	0.15410	5000	50	100
$V = 32^3 \times 64$	0.15440	5050	50	101
$\beta = 5.8$	0.15455	5200	50	104
	0.15462	5100	50	102

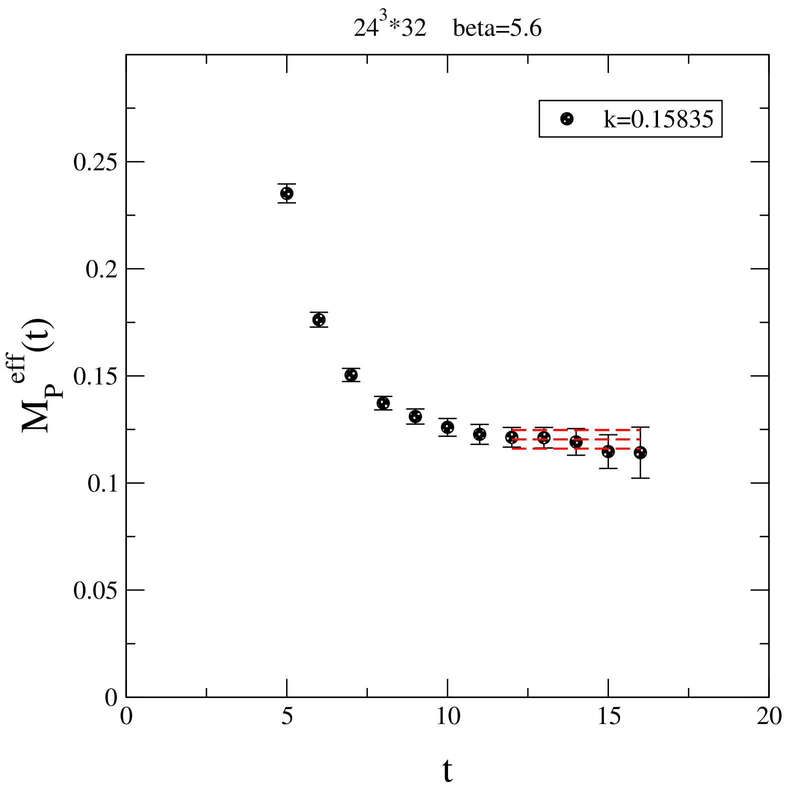
- Parameter ranges:
  1. $m \sim \frac{1}{4} m_s - m_s$
  2. $a \sim 0.050 - 0.075$ fm
  3. $L \sim 1.75$ fm
- All confs archived @ CERN
- All following results preliminary!



Accepted gauge field configurations generated per day

Pseudoscalar meson mass

	k	aM_P
	0.15750	0.2744(21)
$V = 24^3 \times 32$	0.15800	0.1969(16)
$\beta = 5.6$	0.15825	0.1554(31)
$t_1 - t_2 = 12 - 16$	0.15835	0.1204(44)
	0.15410	0.1965(8)
$V = 32^3 \times 64$	0.15440	0.1481(11)
$\beta = 5.8$	0.15455	0.1151(12)
$t_1 - t_2 = 18 - 32$	0.15462	0.1040(12)



- Pseudoscalar meson mass extracted from

$$C_{PP}(t) = \sum_{\vec{x}} \langle P^a(x) P^a(0) \rangle$$

by fitting the effective mass to a plateaux

Pseudoscalar meson mass

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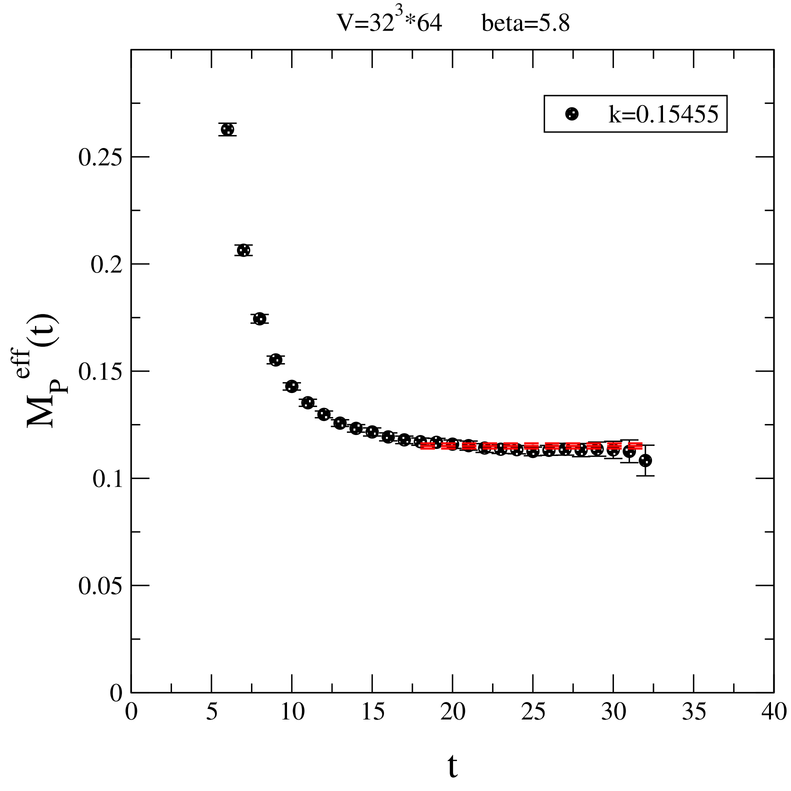

- Pseudoscalar meson mass extracted from

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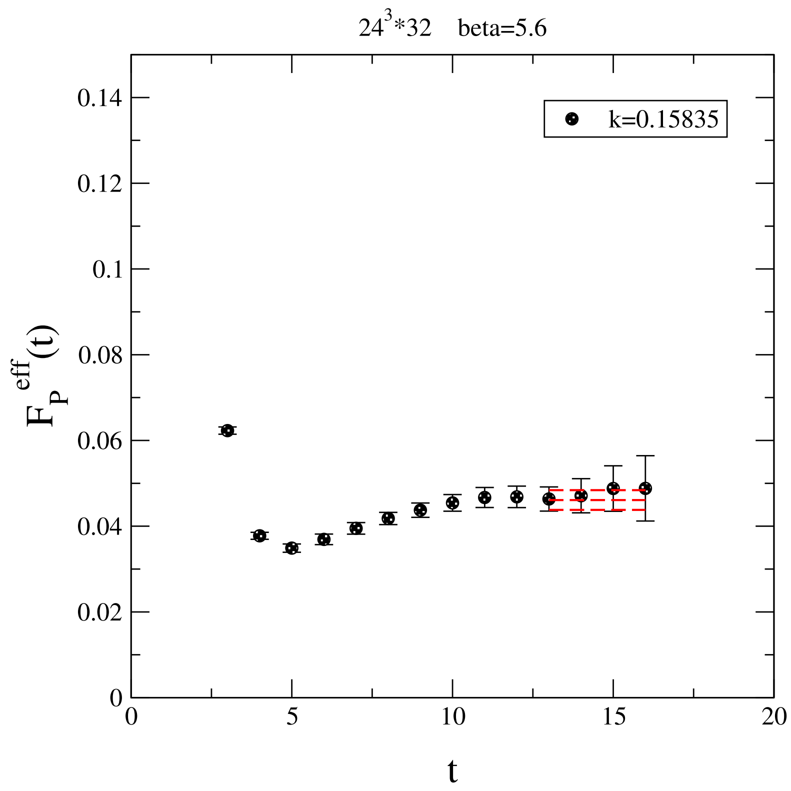
by fitting the effective mass to a plateaux

Pseudoscalar decay constant

	k	aF_P
	0.15750	0.0648(8)
$V = 24^3 \times 32$	0.15800	0.0544(9)
$\beta = 5.6$	0.15825	0.0500(17)
$t_1 - t_2 = 13 - 16$	0.15835	0.0461(23)
	0.15410	0.0457(4)
$V = 32^3 \times 64$	0.15440	0.0379(4)
$\beta = 5.8$	0.15455	0.0347(4)
$t_1 - t_2 = 18 - 32$	0.15462	0.0339(6)



- Pseudoscalar decay constant extracted by combining $C_{PP}(t)$ with

$$C_{AP}(t) = \sum_{\vec{x}} \langle A_0^a(x) P^a(0) \rangle$$

and by fitting the effective decay constant to a plateaux

Pseudoscalar decay constant

	k	aF_P
	0.15750	0.0648(8)
$V = 24^3 \times 32$	0.15800	0.0544(9)
$\beta = 5.6$	0.15825	0.0500(17)
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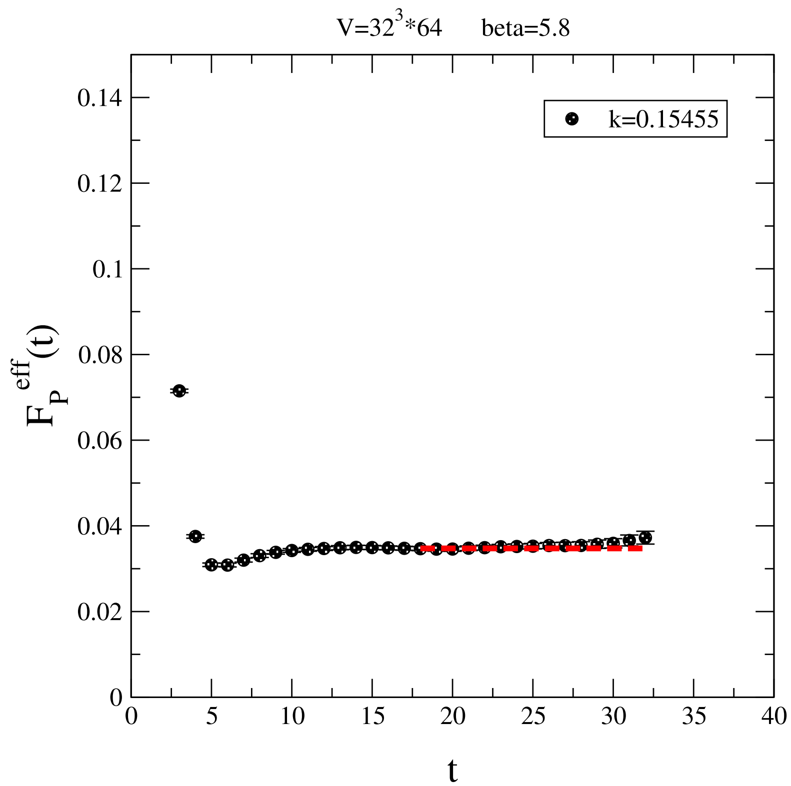


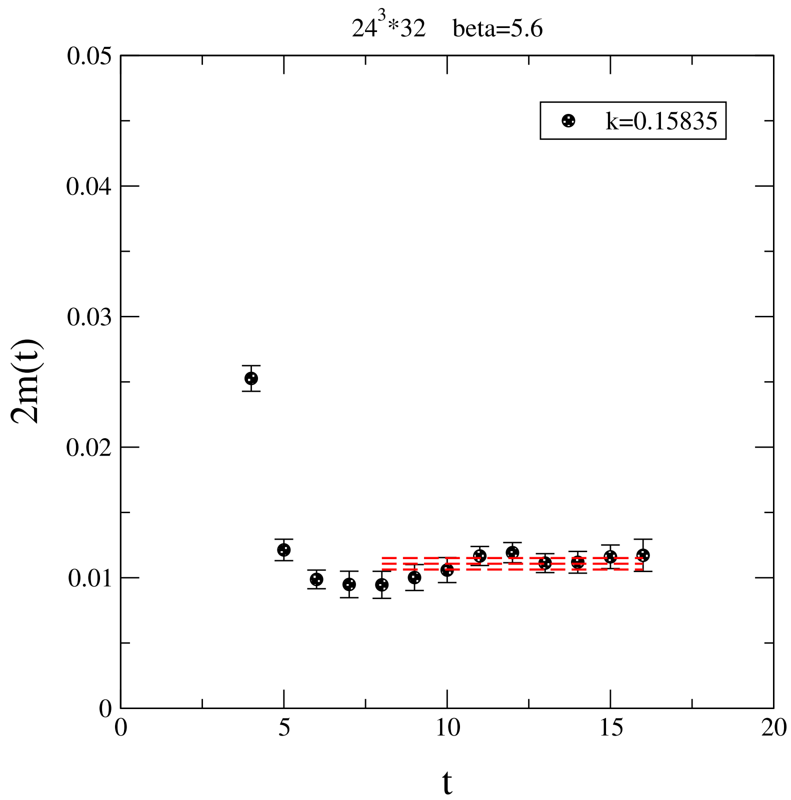
- Pseudoscalar decay constant extracted by combining $C_{PP}(t)$ with

$$C_{AP}(t) = \sum_{\vec{x}} \langle A_0^a(x) P^a(0) \rangle$$

and by fitting the effective decay constant to a plateaux

Quark mass

	k	$2\,am$
	0.15750	0.05477(53)
$V = 24^3 \times 32$	0.15800	0.02853(31)
$\beta = 5.6$	0.15825	0.01724(42)
$t_1 - t_2 = 8 - 16$	0.15835	0.01107(44)
	0.15410	0.03898(16)
$V = 32^3 \times 64$	0.15440	0.02170(11)
$\beta = 5.8$	0.15455	0.01417(12)
$t_1 - t_2 = 7 - 32$	0.15462	0.01139(16)



- Quark mass extracted from

$$2m(t) = \frac{\partial_t^* C_{AP}(t)}{C_{PP}(t)}$$

by fitting to a plateaux

Quark mass

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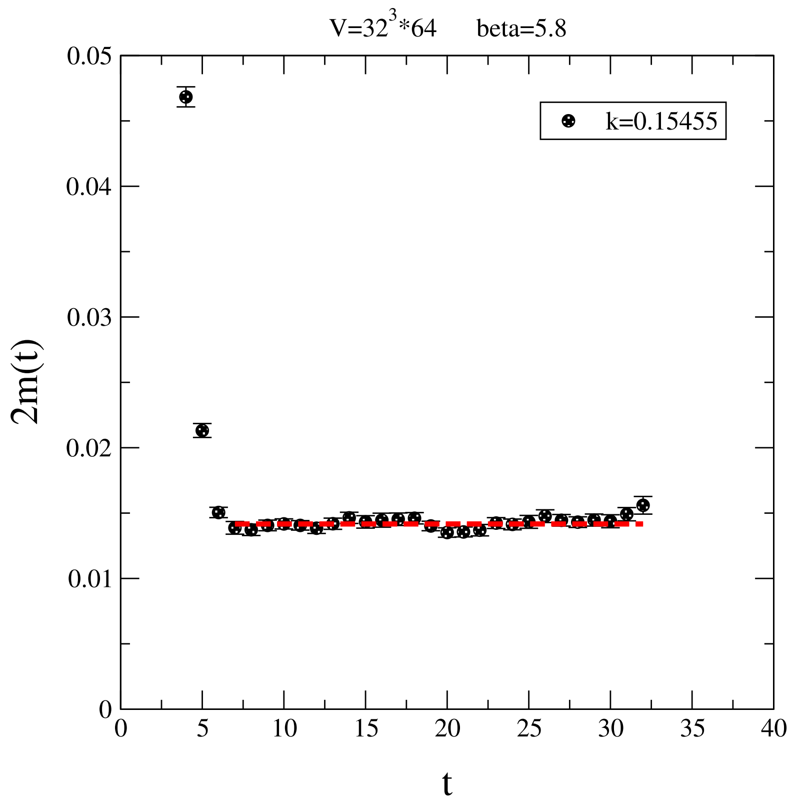


- Quark mass extracted from

$$2m(t) = \frac{\partial_t^* C_{AP}(t)}{C_{PP}(t)}$$

by fitting to a plateaux

Statistical gain with five sources

- Two-point pseudoscalar correlation functions computed for 5 sources

	k	$2\,am$	aM_P	aF_P
	0.15750	0.05477(53)[58][71]	0.2744(21)[27][31]	0.0648(8)[11][14]
$V = 24^3 \times 32$	0.15800	0.02853(31)[41][47]	0.1969(16)[19][29]	0.0544(9)[12][18]
$\beta = 5.6$	0.15825	0.01724(42)[49][55]	0.1554(31)[38][33]	0.0500(17)[23][30]
	0.15835	0.01107(44)[53][52]	0.1204(44)[49][66]	0.0461(23)[28][31]
	0.15410	0.03898(16)[18][19]	0.1965(8) [9][13]	0.0457(4)[6][8]
$V = 32^3 \times 64$	0.15440	0.02170(11)[13][15]	0.1481(11)[12][14]	0.0379(4)[5][8]
$\beta = 5.8$	0.15455	0.01417(12)[13][14]	0.1151(12)[14][15]	0.0347(4)[6][8]
	0.15462	0.01139(16)[16][19]	0.1040(12)[13][16]	0.0339(6)[8][10]

- A general error reduction observed

- A clear pattern of error reduction in F_P

Finite volume corrections

	k	$2\,am$	$a^2M_P^2$	aF_P
	0.15750	0.05477(53)	0.0753(11)	0.0648(8)
$V = 24^3 \times 32$	0.15800	0.02853(31)	0.0388(6)	0.0544(9)
$\beta = 5.6$	0.15825	0.01724(42)	0.0241(10)	0.0500(17)
	0.15835	0.01107(44)	0.0145(11)	0.0461(23)
	0.15410	0.03898(16)	0.0386(3)	0.0457(4)
$V = 32^3 \times 64$	0.15440	0.02170(11)	0.0219(3)	0.0379(4)
$\beta = 5.8$	0.15455	0.01417(12)	0.0132(3)	0.0347(4)
	0.15462	0.01139(16)	0.0108(2)	0.0339(6)

- Meson masses and decay constants at $\mathcal{O}(p^4)$ in finite volume

$$M_P^2 = M^2 \left\{1 + \frac{M^2}{32\pi^2 F^2}\log\left(\frac{M^2}{\Lambda_\pi^2}\right) + \frac{1}{2F^2}\, g_1^4(M^2)\right\}$$

$$F_P = F\left\{1 - \frac{M^2}{16\pi^2 F^2}\log\left(\frac{M^2}{\Lambda_F^2}\right) - \frac{1}{F^2}\, g_1^4(M^2)\right\}$$

- The finite volume corrections in M_P^2 for the various masses are

$\beta = 5.6$	$\beta = 5.8$
$\{0\%, 0.2\%, 0.7\%, 2.1\%\}$	$\{0\%, 0.6\%, 0.9\%, 1.3\%\}$

Reference point

- Reference point defined to be

$$\left(\frac{M_P}{M_V}\right)^2\Big|_{m=m_{\rm ref}} = \left(\frac{M_K^{\rm exp}}{M_{K^*}^{\rm exp}}\right)^2 = 0.30657$$

- If we fix $M_{\rm ref} = M_K^{\rm exp}$ to fix the lattice spacing

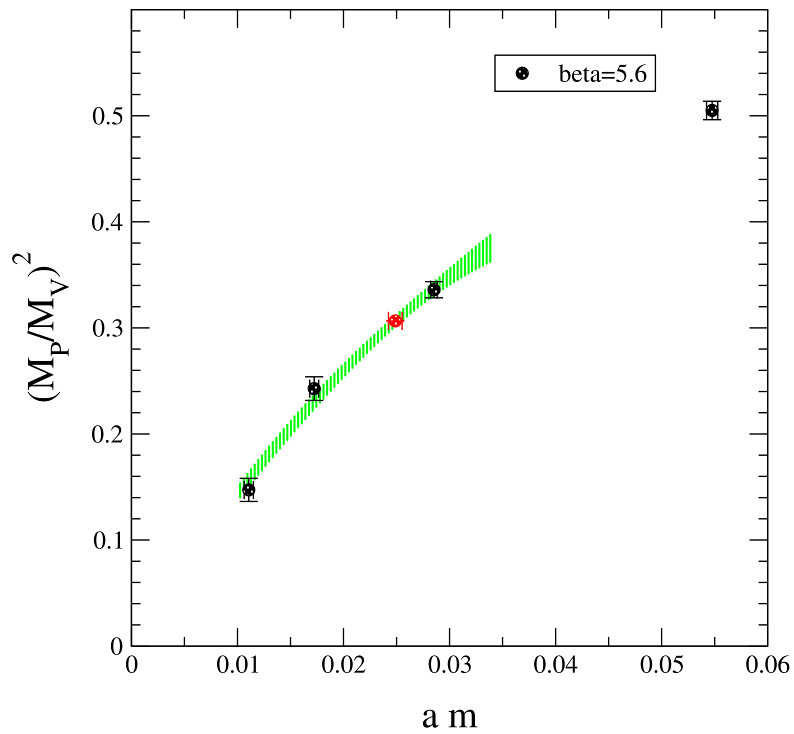
$$a^{-1} = 2.70(3)\ \text{GeV} \qquad \beta = 5.6$$

$$a^{-1} = 3.77(4)\ \text{GeV} \qquad \beta = 5.8$$

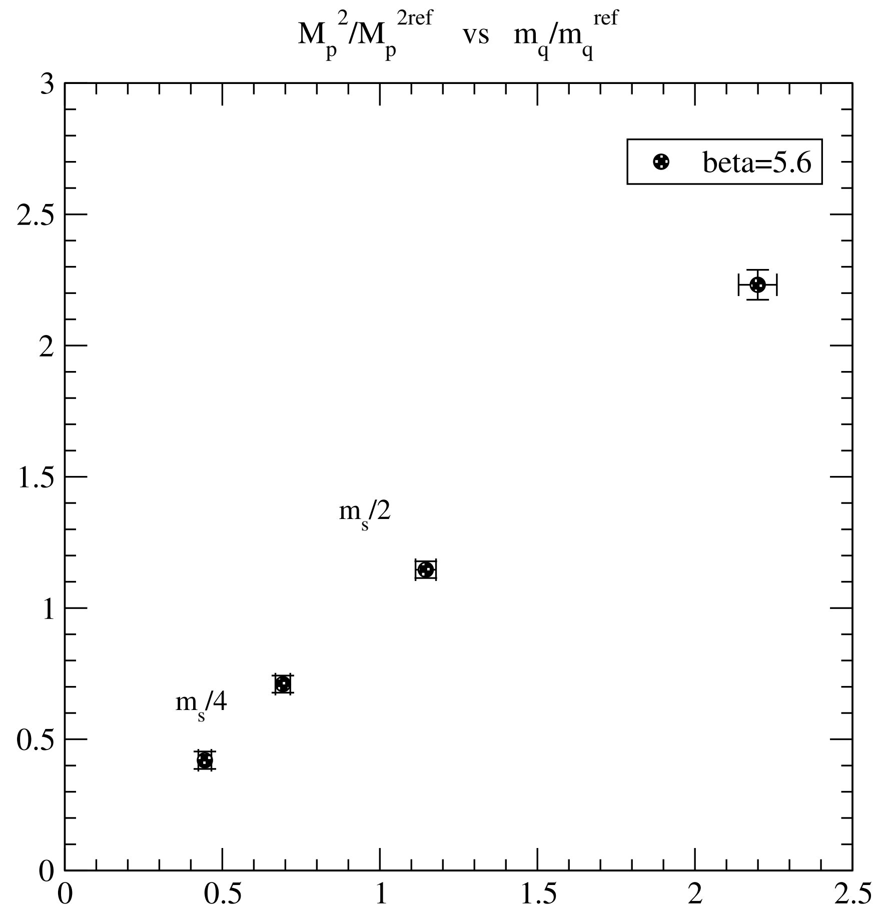
- If we use Z_A from RI-MOM D. Bećirević et al 05

$$F_{\rm ref} = 111(2) \qquad \beta = 5.6$$

$$F_{\rm ref} = 108(2) \qquad \beta = 5.8$$

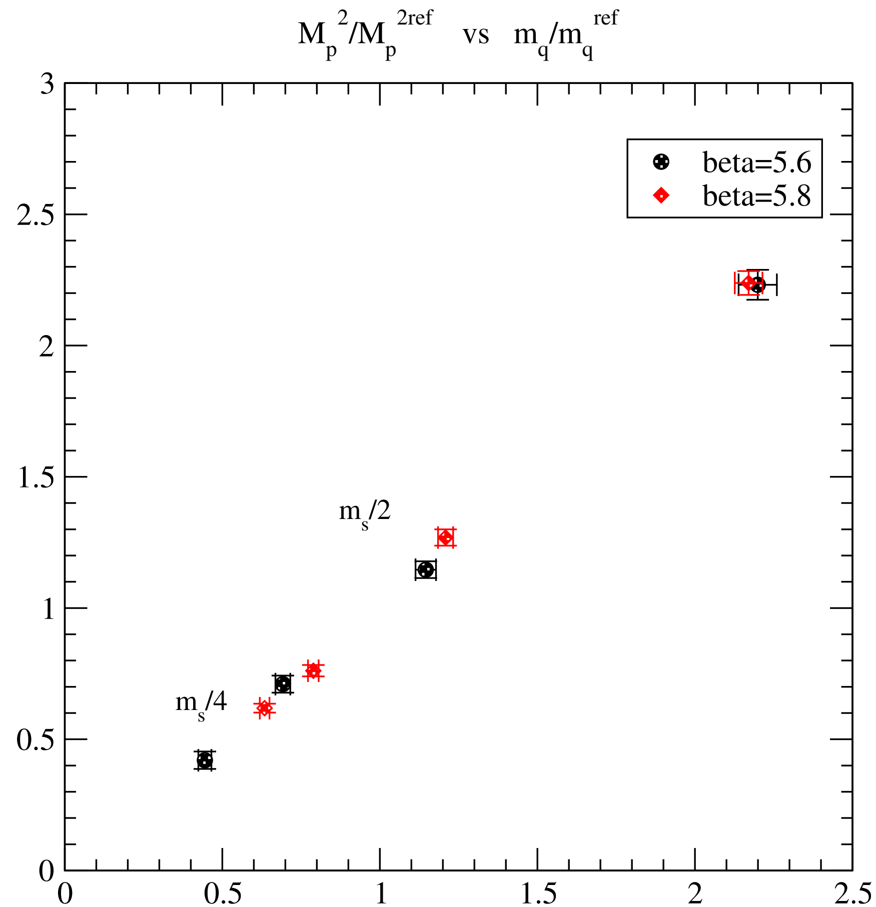


Pseudoscalar meson mass versus the quark mass



- A remarkable linear behavior is observed

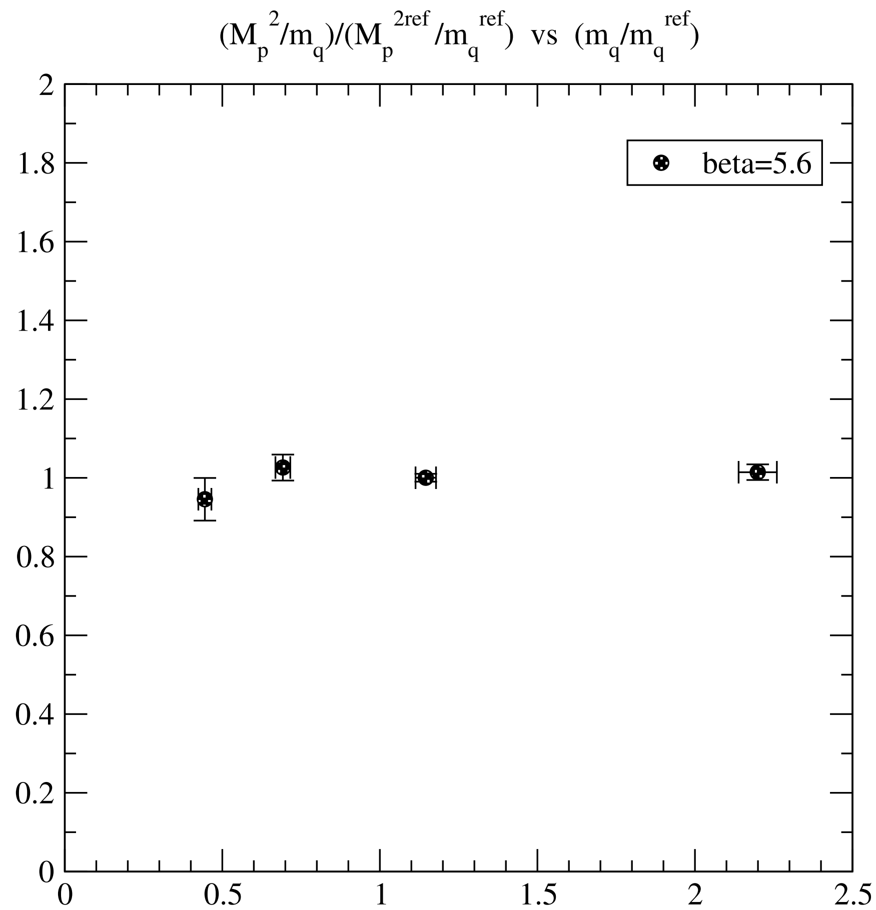
Pseudoscalar meson mass versus the quark mass



- A remarkable linear behavior is observed

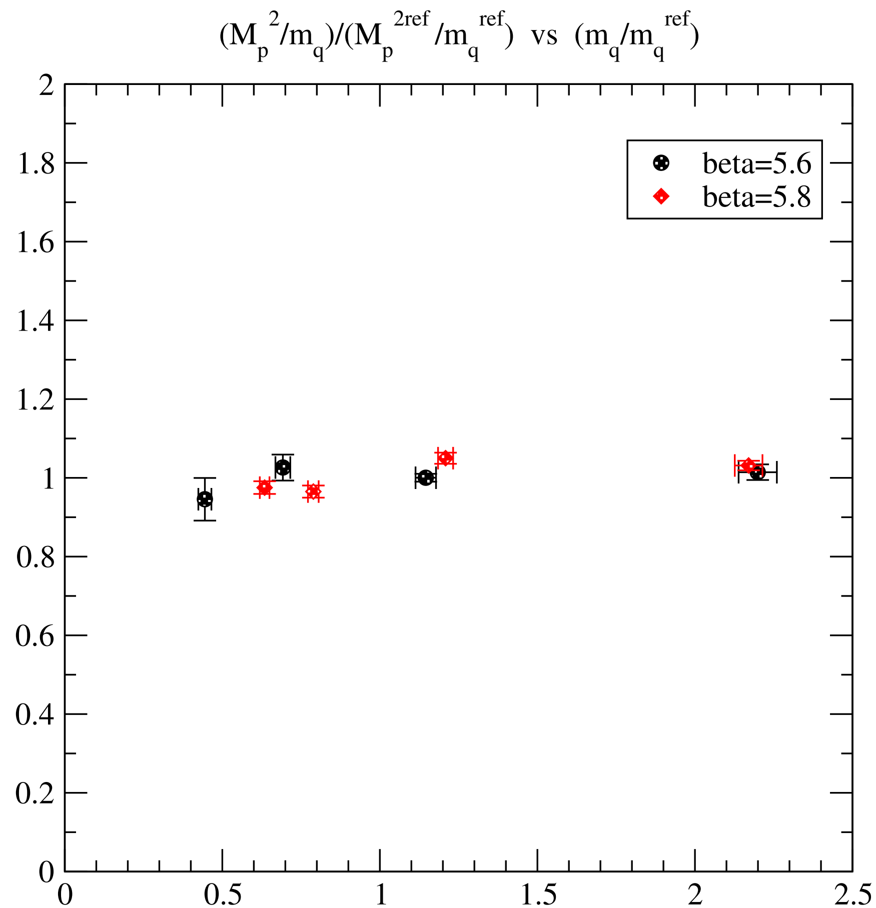
  . . . . . . and results from the two lattices are consistent

Pseudoscalar meson mass versus the quark mass



- In QCD with two light flavors the mass of the light pseudoscalar meson shows a remarkable linearity in the quark mass

Pseudoscalar meson mass versus the quark mass



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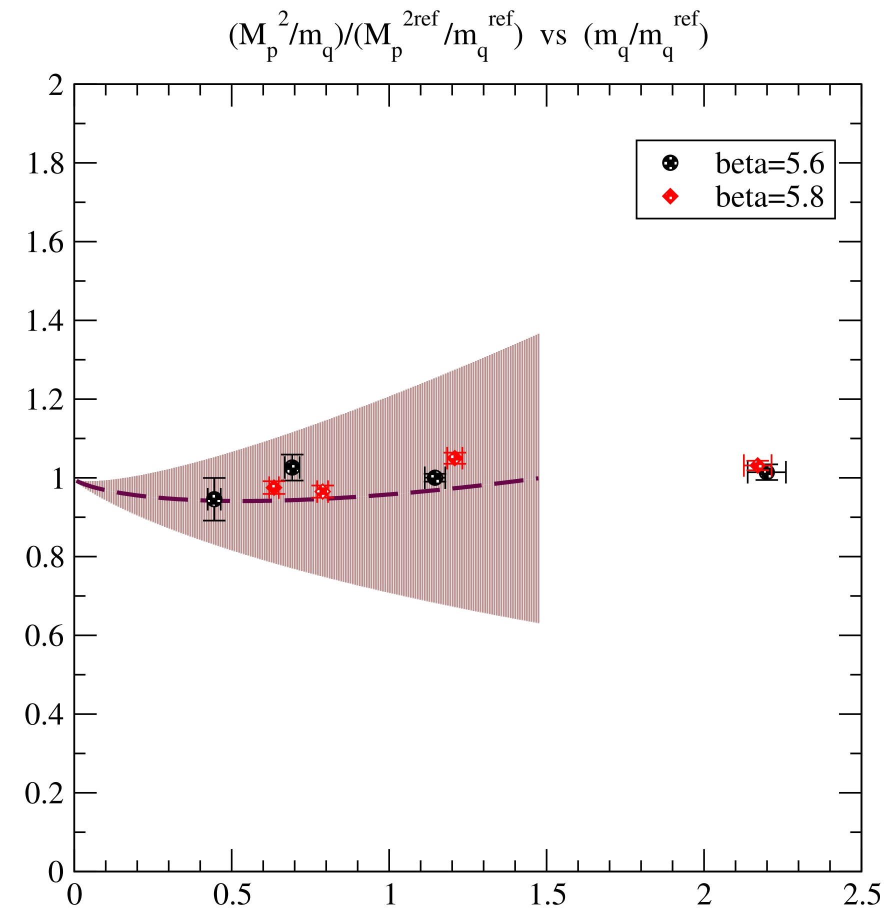
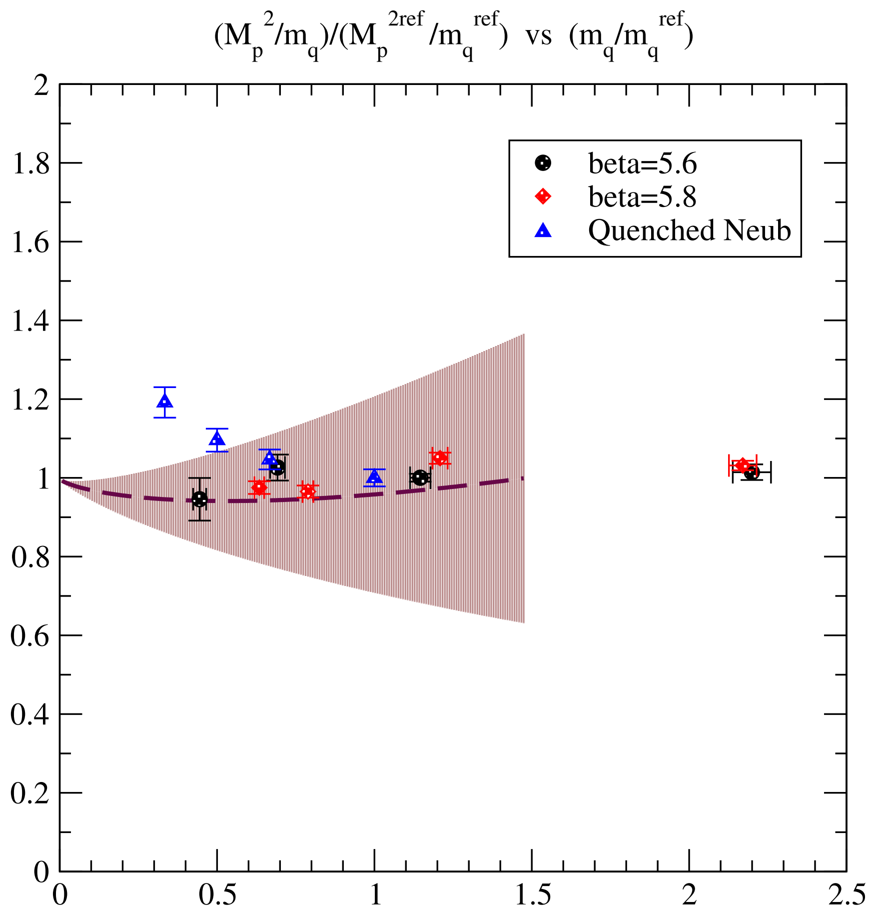
Pseudoscalar meson mass versus the quark mass



- In QCD with two light flavors the mass of the light pseudoscalar meson shows a remarkable linearity in the quark mass

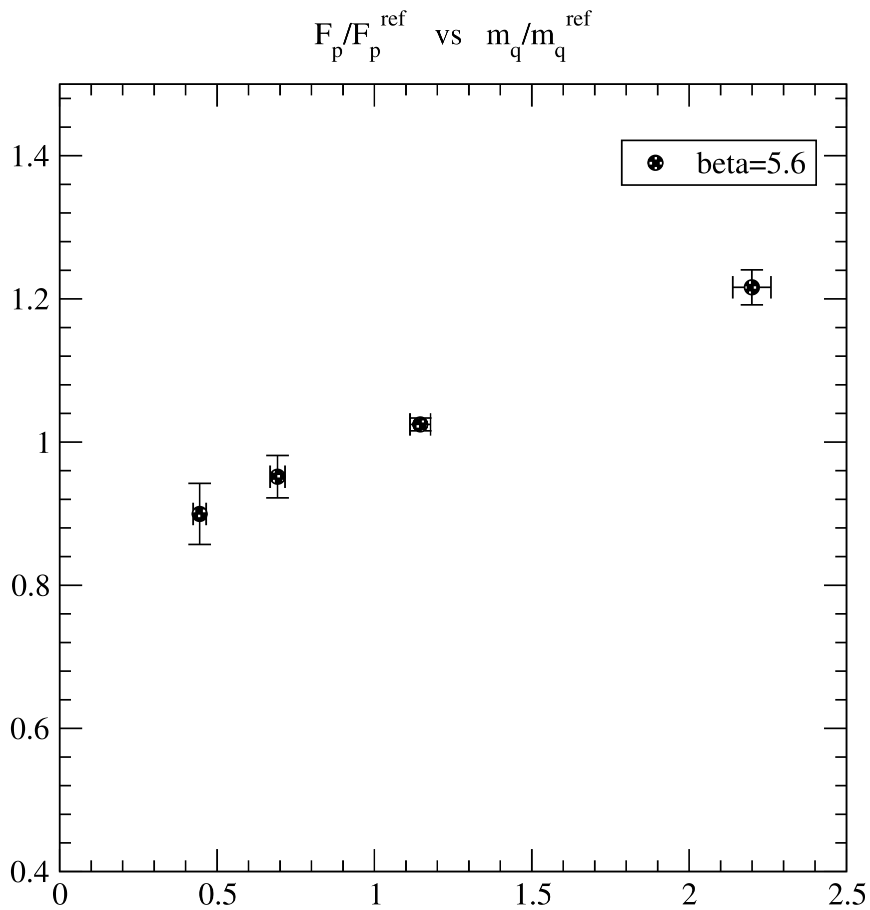
- The mass dependence is also compatible with the “experimental” curve

Comparison with quenched data



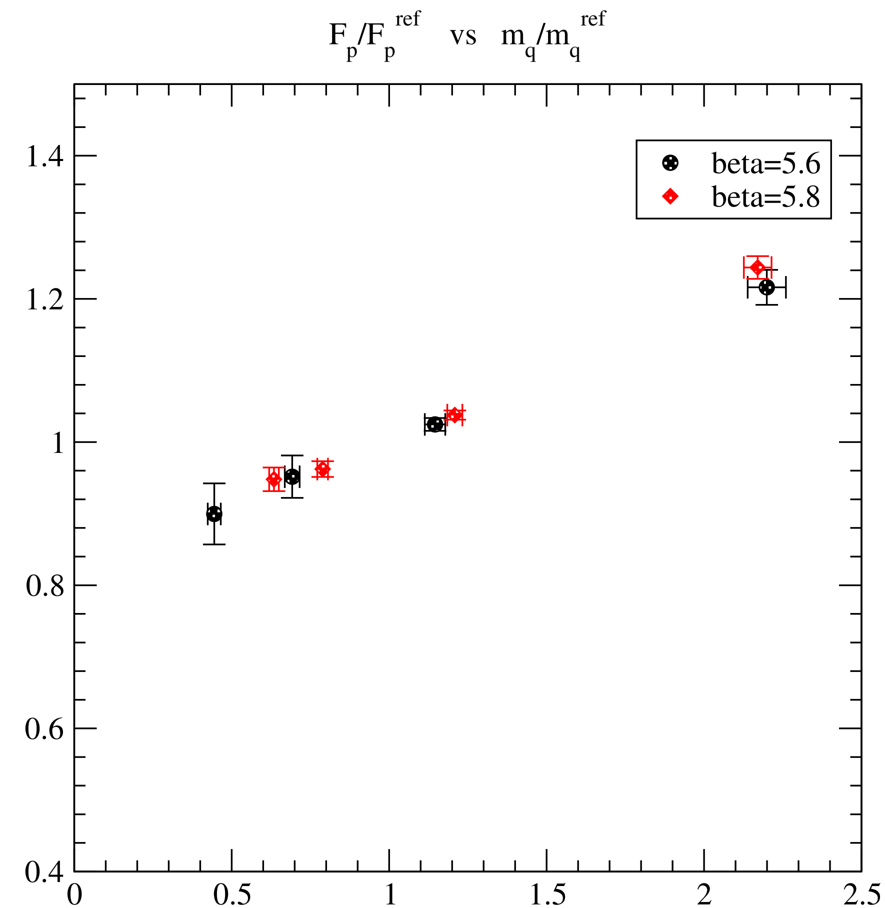
- For quenched data thanks to: P. Hernández, C. Pena, J. Wennekers and H. Wittig

Pseudoscalar decay constant versus the quark mass



- In QCD with two light flavors the decay constant of the light pseudoscalar meson shows a clear dependence on the quark mass

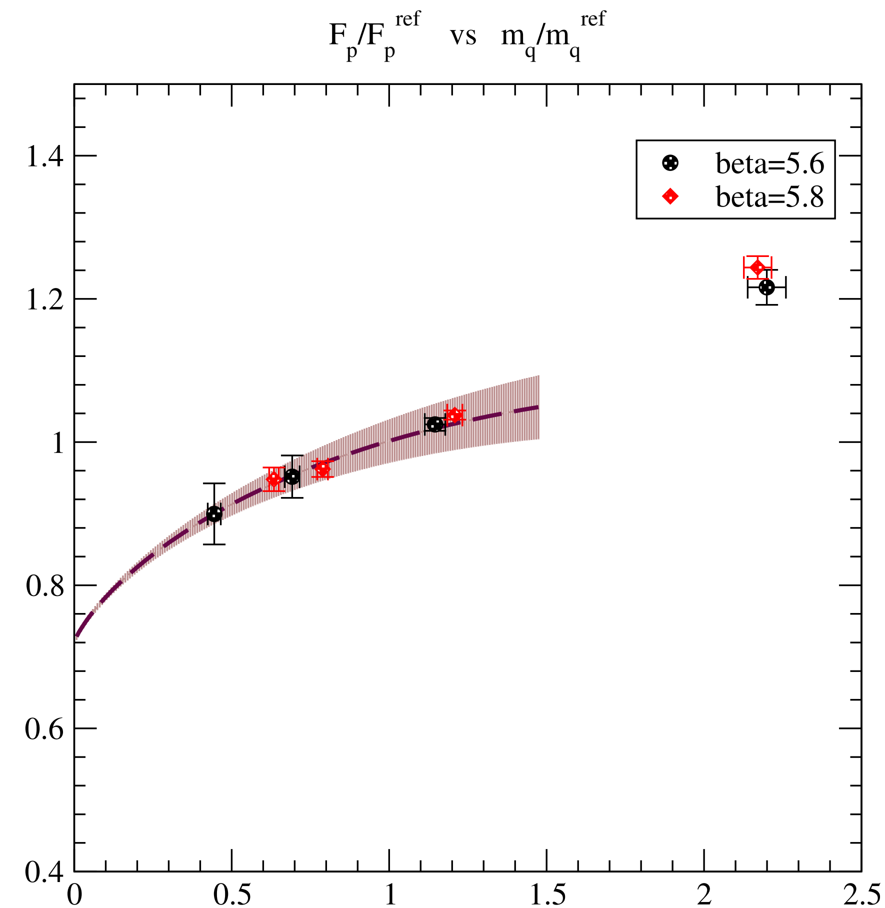
Pseudoscalar decay constant versus the quark mass



- In QCD with two light flavors the decay constant of the light pseudoscalar meson shows a clear dependence on the quark mass

. . . . . . and results from the two lattices are consistent

Pseudoscalar decay constant versus the quark mass



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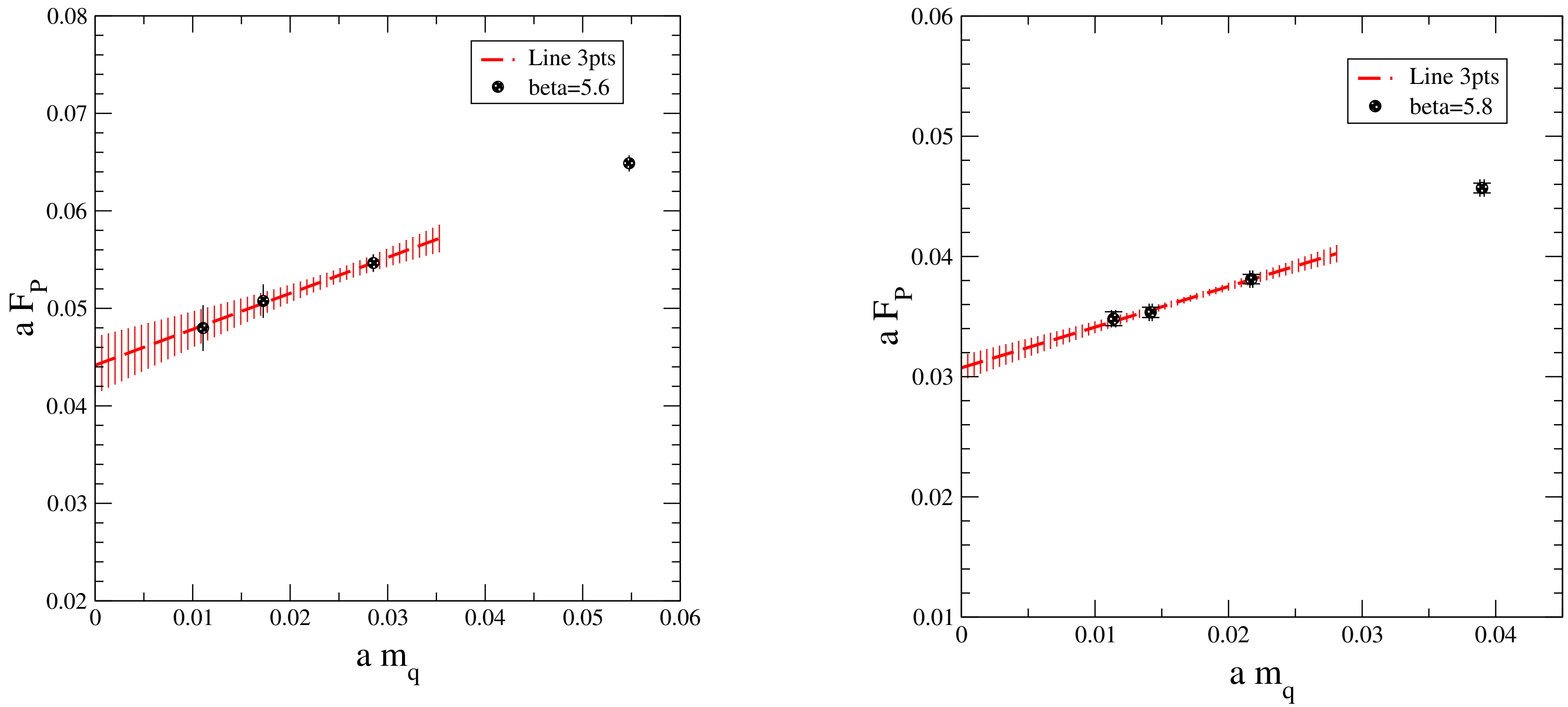
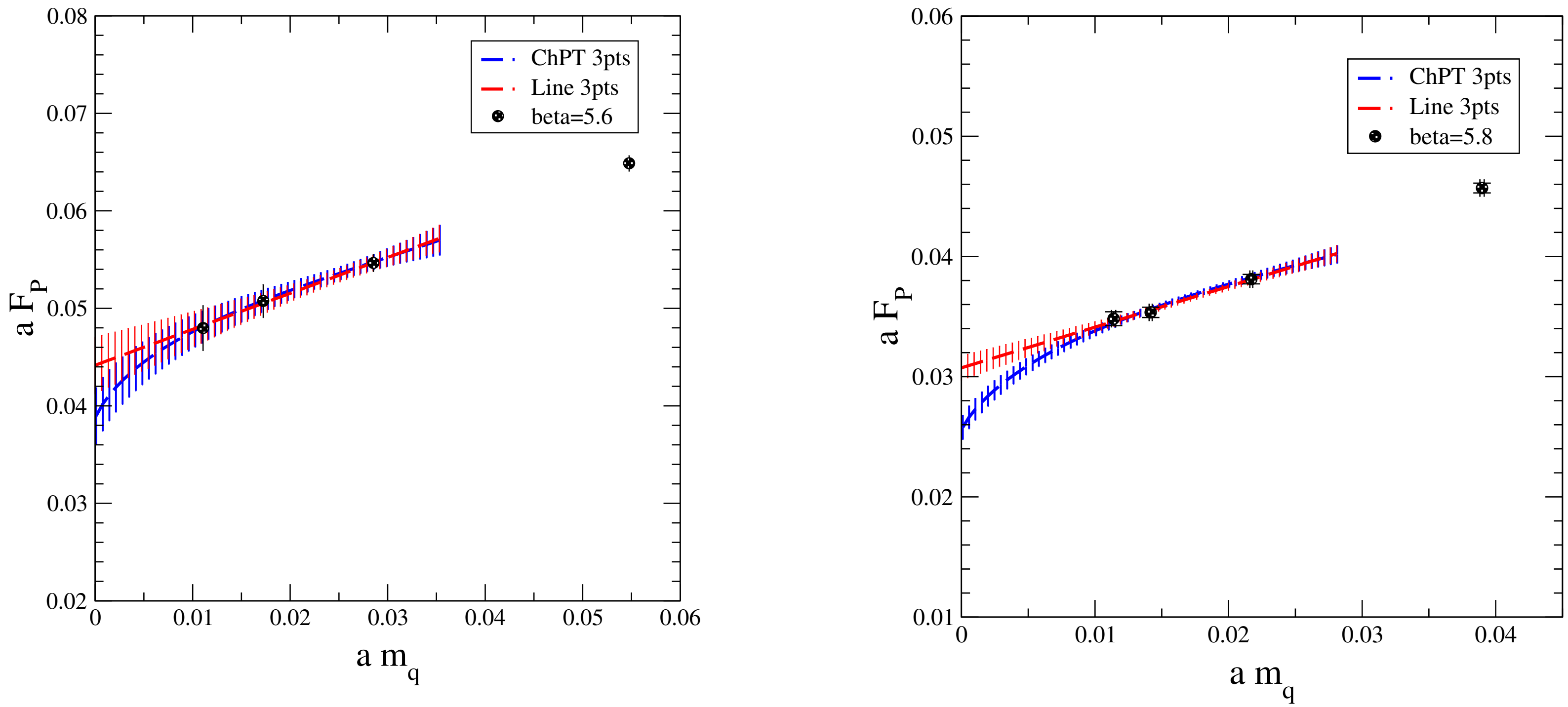
ChPT fits for the pseudoscalar decay constant



- The lightest three points are compatible with a linear behavior

ChPT fits for the pseudoscalar decay constant



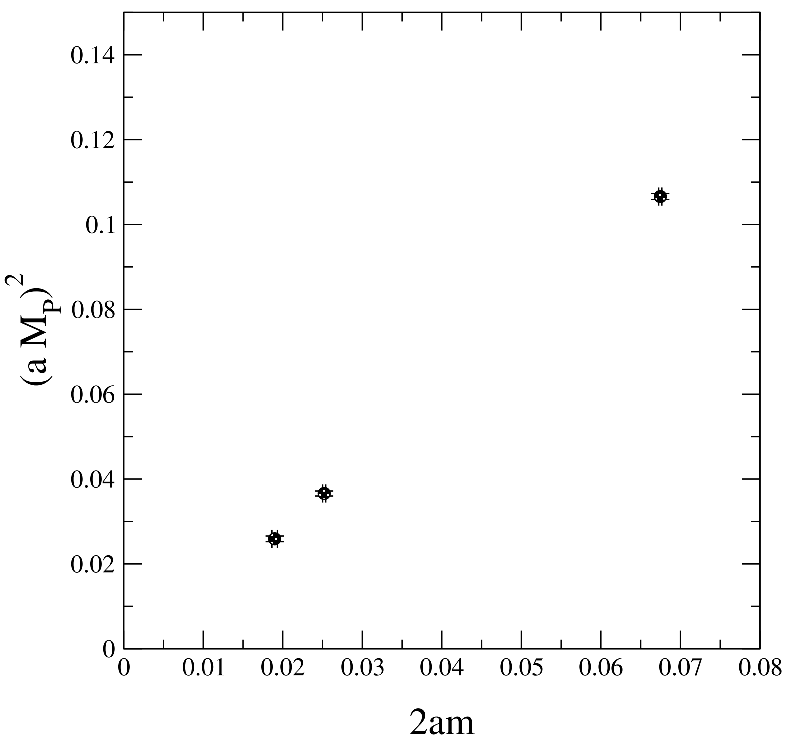
- The lightest three points are compatible with a linear behavior

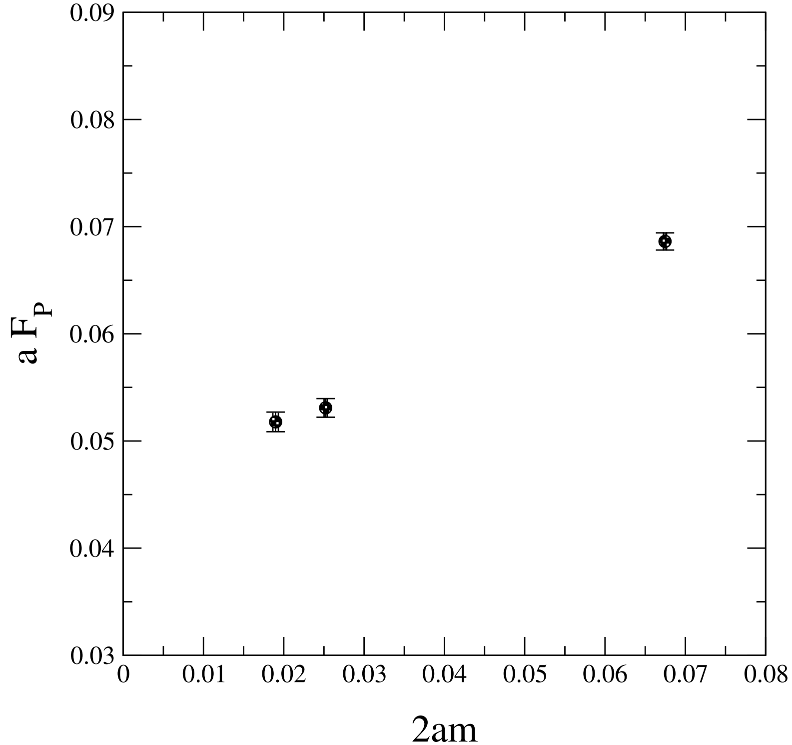
  . . . . . . and also with the NLO ChPT fit function

- Light and precise points are needed for an accurate determination of F

First clover run

	k	$\mathrm{N_{trj}}$	$\mathrm{N_{sep}}$	$\mathrm{N_{conf}}$
	0.13550	5200	50	104
$V = 24^3 \times 48$	0.13590	4620	30	154
$\beta = 5.3$	0.13610	5070	30	169
$c_{sw} = 1.90952$	0.13620	1770	30	59
	TBD			





Conclusions

- Our experience for two flavor QCD shows that SAP is very stable in the ranges
  1. $m \sim \frac{1}{4}m_s - m_s$
  2. $a \sim 0.050 - 0.075$ fm
  3. $L \sim 1.75$ fm
- The production for two Wilson lattices completed. The first clover run is finishing
- Discretization effects in the quark mass dependence of M_P^2 and F_P are small
- The mass dependence of M_P^2 turns out to be very linear for $M_P = 300 - 600$ MeV
  Data compatible with NLO ChPT + exp.
- F_P shows a clear quark mass dependence. Data compatible with NLO ChPT + exp.
- Precise points at light quark masses are necessary to extract the LECs reliably