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Chiral fermions and their phenomenological applications II

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Outline

- Introduction
- Domain-wall-overlap fermions
- Exact chiral symmetry and Ward identities
- Non-perturbative renormalization
- Meson spectrum

- The chiral condensate
- Topological susceptibility and WV relation
- Kaon matrix elements
- Conclusions and outlook

Exact chiral symmetry: some advantages

- The Dirac operator has an index at finite cut-off:
  - ▶ A natural definition for $Q(x)$
  - ▶ Identification of the topological charge

- Very light quark masses can be reached:
  - ▶ No exceptional configurations

- No mixing among operators of different chirality:
  - ▶ No additive quark renormalization
  - ▶ Simplified mixing for composite operators
  - ▶ $O(a)$ improvement straightforward

The chiral condensate from GMOR

- The chiral symmetry guarantees that

$$-\frac{1}{N_f}\langle\bar{\psi}\tilde{\psi}\rangle = \chi(a,m) + \alpha\frac{m(a)}{a^2}$$

renormalizes only multiplicatively in the chiral limit

- The chiral condensate is defined as

$$\Sigma(\mu) = \lim_{a\to 0} Z_S(a\mu)\Sigma(a) = \lim_{a,m\to 0} Z_S(a\mu)\chi(a,m)$$

- Large divergent subtractions for $m \neq 0$ can be avoided:

$$\frac{1}{N_f}\langle\bar{\psi}\tilde{\psi}\rangle = m\sum_x \langle P(x)P(0)\rangle$$

leads to the GMOR relation

$$\boxed{\frac{\Sigma(a)}{F^2} = \lim_{m\to 0}\frac{m}{F_P^2 M_P^2}\Big|\langle 0|P|P\rangle\Big|^2 = \lim_{m\to 0}\frac{M_P^2}{2m}}$$

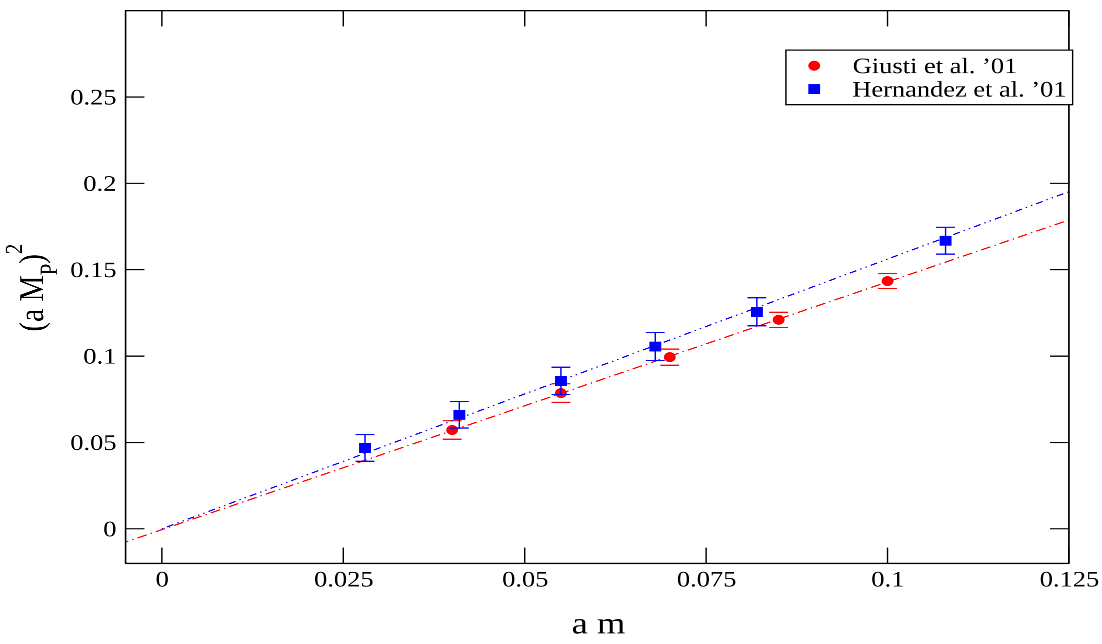
- We can extract $\Sigma(a)$ by studying M_P^2 as a function of m

- Quenched χPT at $O(p^4)$ $[M^2 = 2\Sigma m/F^2$, $\alpha_i = 8(4\pi)^2 L_i]$

$$\frac{M_P^2}{2m} = \frac{\Sigma}{F^2}\Big[1 - \frac{m_0^2}{3(4\pi F)^2}\Big(1 + \log\Big(\frac{M^2}{\mu_\chi^2}\Big)\Big)$$

$$+\frac{\alpha M^2}{3(4\pi F)^2}\Big(\log\Big(\frac{M^2}{\mu_\chi^2}\Big) + 1\Big) + \big(2\alpha_8 - \alpha_5\big)\frac{M^2}{(4\pi F)^2}\Big]$$

(G. Colangelo, E. Pallante '98)



- Linear behaviour in the range $m_s/2 < m < m_s$

Ref.	a (fm)	V	$\Sigma^{\overline{\rm MS}}(2\,\rm GeV)$ $(\rm GeV^3)$
L. G. et al. '01	$\simeq 0.093$	$16^3 \times 32$	$0.0190(11)(33)$
P. Hernández et al. '01	$\simeq 0.093$	$14^3 \times 24$	$0.0199(8)(15)$
P. Hernández et al. '01	$\simeq 0.12$	$10^3 \times 24$	$0.0195(8)(15)$

Light quarks in a box: ϵ expansion

(J. Gasser, H. Leutwyler '87)

- When $F^2 M^2 L^4 \simeq 1,\ L \gg 1/(4\pi F)$ and for $p^2 \simeq 1/L^2$

$$\frac{M}{\Lambda_\chi} \sim \frac{p^2}{\Lambda_\chi^2} \sim \frac{1}{(4\pi L F)^2} = \epsilon^2$$

and QCD Green's functions can be expanded in powers of ϵ

- The chiral expansion is reordered $\mathcal{S} = \mathcal{S}^{(0)} + \mathcal{S}^{(2)} + \dots$

$$\mathcal{S}^{(0)} = \int d^4x \frac{1}{2} \text{Tr}\,[\partial_\mu \xi(x) \partial_\mu \xi(x)] - \frac{m\Sigma V}{2} \text{Tr}\,[e^{i\theta/N_f} U_0 + \text{h.c.}]$$

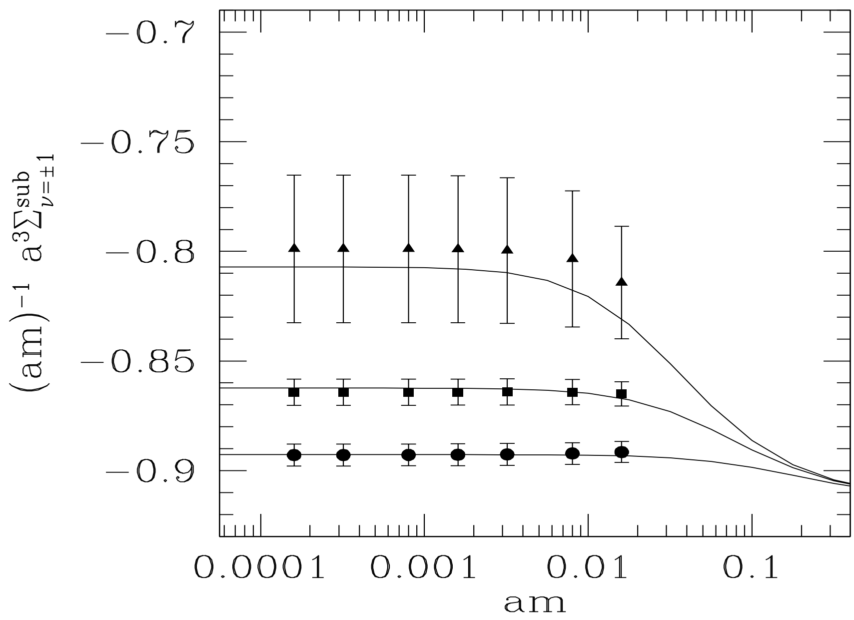
$$U = U_0 \exp\left(i\sqrt{2}\xi(x)/F\right) \qquad \int \xi(x) = 0$$

- The $O(1)$ zero-mode fluctuations have to be treated exactly

$$\int_{SU(N_f)} dU_0 \exp\Big(\frac{m\Sigma V}{2} \text{Tr}\,[e^{-i\theta/N_f} U_0^\dagger + U_0 e^{i\theta/N_f}]\Big)$$

- The expansion can also be performed in fixed-topology sectors
- As a consequence of reordering, $L_4 \to L_8$ do not enter $\mathcal{S}^{(2)}$
- Finite-volume effects can be predicted by χPT!

Quenched chiral condensate in the ϵ regime



- At leading order in ϵ ($z = m\Sigma V$)
  (J.C. Osborn et al. '99, P. H. Damgaard et al. '99)

$$\frac{\Sigma_\nu}{\Sigma} = z\Big(I_\nu(z)K_\nu(z) + I_{\nu+1}(z)K_{\nu-1}(z)\Big) + \frac{\nu}{z}$$

- One-loop corrections are obtained by replacing
  (P. H. Damgaard '01, P. H. Damgaard et al. '02)

$$\Sigma \to \Sigma_{eff}(V) = \Sigma\left[1 - \frac{m_0^2}{3(4\pi F)^2}\left(\tilde{\beta}_2 + \log\left(\frac{L_0^2}{L^2}\right)\right) - \frac{\alpha}{3(4\pi FL)^2}\tilde{\beta}_1\right]$$

- Numerical computations performed with GW fermions
  (P. H. Damgaard et al. '99, P. Hernández et al. '99)

LO Analyses	a (fm)	L	$\Sigma^{\overline{\rm MS}}(2\,{\rm GeV})$ (GeV^3)
P. Hernández et al. '01	$\simeq$ 0.12	8,10,12	0.0192(22)(2)(14)
T. DeGrand '01	$\simeq$ 0.11	8,10,12	0.0224(14)
P. Hasenfratz et al. '01	$\simeq$ 0.13	8,10	0.0180(15)(8)(17)

- The method is feasible, more studies are needed to reduce and properly assess the systematic errors

Comparing results for p and ϵ expansion

- For $M_P L \geq 1,\ L \gg 1/(4\pi F)$ [p expansion]

$$\frac{M_P^2}{2m} = \frac{\Sigma}{F^2}\Big[1 - \frac{m_0^2}{3(4\pi F)^2}\left(1 + \log\Big(\frac{M^2}{\mu_\chi^2}\Big)\right)$$

$$+\frac{\alpha M^2}{3(4\pi F)^2}\left(2\log\Big(\frac{M^2}{\mu_\chi^2}\Big) + 1\right) + \Big(2\alpha_8(\mu_\chi) - \alpha_5(\mu_\chi)\Big)\frac{M^2}{(4\pi F)^2}\Big]$$

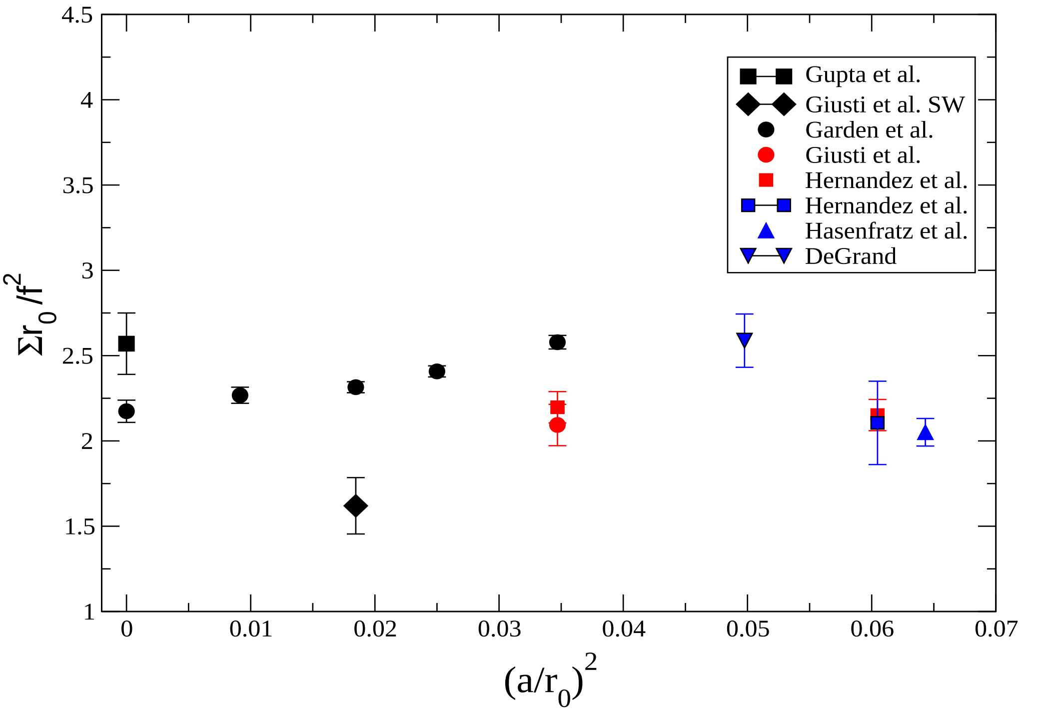
- For $F^2 M_P^2 L^4 \simeq 1,\ L \gg 1/(4\pi F)$ [ϵ expansion]

$$-\tfrac{1}{N_f}\langle\bar\psi\tilde\psi\rangle_\nu^{sub} = \Sigma_\nu(z)\Big[1 - \frac{m_0^2}{3(4\pi F)^2}\left(\tilde\beta_2 + \log\Big(\frac{L_0^2}{L^2}\Big)\right)$$

$$- \frac{\alpha}{3(4\pi F L)^2}\tilde\beta_1\Big]$$

- Higher-order corrections are expected to be different!

- Complementary ways to extract Σ, F, α_i, provided higher-order corrections are under control (Hernández et al. '01 and in preparation)



Topological susceptibility in full QCD ($m = 0$)

- For a given string of renormalized fundamental fields $\hat{O}$

$$\langle \partial_\mu^* A_\mu^0(x) \hat{O} \rangle = 2N_f \langle Q(x) \hat{O} \rangle + \langle \delta_A^x \hat{O} \rangle$$

$$2N_f \int d^4x \, \langle Q(x) \hat{O} \rangle + \langle \delta_A \hat{O} \rangle = 0$$

- As a consequence, properly renormalized operators are

$$\begin{aligned} \hat{Q}(x) &= \tfrac{1}{2}\mathrm{Tr}[\gamma_5 D(x,x)] - \tfrac{Z}{2N_f} \partial_\mu^* A_\mu^0(x) \\ \hat{A}_\mu^0(x) &= (1 - Z) A_\mu^0(x) \end{aligned}$$

- The renormalized AWIs read

$$\langle \partial_\mu^* \hat{A}_\mu^0(x) \hat{O} \rangle = 2N_f \langle \hat{Q}(x) \hat{O} \rangle + \langle \delta_A^x \hat{O} \rangle$$

- Taking $\hat{O} = \hat{Q}$ and defining

$$\begin{aligned} \chi_t(p) &= \tfrac{1}{2N_f} \int d^4x \, \mathrm{e}^{-ipx} \langle \partial_\mu^* \hat{A}_\mu^0(x) \hat{Q}(0) \rangle + \mathrm{CT(p)} \\ &= \int d^4x \, \mathrm{e}^{-ipx} \langle \hat{Q}(x) \hat{Q}(0) \rangle + \mathrm{CT(p)} \end{aligned}$$

in the full theory ($\mathrm{CT}(0) = 0$)

$$\chi_t(0) = \int d^4x \langle \tfrac{1}{2}\mathrm{Tr}[\gamma_5 D(x,x)] \tfrac{1}{2}\mathrm{Tr}[\gamma_5 D(0,0)] \rangle = 0$$

(L. G., G.C. Rossi, M. Testa, G. Veneziano '01)

The Witten-Veneziano formula

(E. Witten '79, G. Veneziano '79)

- A properly renormalized $\chi_t(p)$ satisfies
  (E. Seiler and I.O. Stamatescu '87, E. Seiler '02)

$$\chi_t(p) = b_1 + b_2 p^2 + b_3 (p^2)^2 - \frac{R^2_{\eta'}}{p^2 + m^2_{\eta'}} + (p^2)^3 \int_{\mathcal{M}^2}^{\infty} \frac{\rho(t)}{(t+p^2)t^3} dt$$

- For $p^2 \longrightarrow 0$, the "sum rule" $\chi_t(0) = 0 \left(R^2_{\eta'} = F^2_{\eta'} m^4_{\eta'} / 2N_f \right)$

$$b_1 = \frac{R^2_{\eta'}}{m^2_{\eta'}}$$

- Under the "smooth-quenching hypothesis"

$$\frac{F^2_\pi m^2_{\eta'}}{2N_f}\Big|_{\frac{N_f}{N_c}=0} = \int d^4x \left\langle \frac{1}{2}\mathrm{Tr}[\gamma_5 D(x,x)] \frac{1}{2}\mathrm{Tr}[\gamma_5 D(0,0)] \right\rangle \Big|_{\mathrm{YM}}$$

$$= \lim_{V\to\infty} \frac{\langle (n_{\mathrm{L}} - n_{\mathrm{R}})^2 \rangle}{V}$$

- With Wilson and staggered fermions it was argued
  (M. Bochicchio et al. '84, J. Smith and J.C. Vink '87)

$$\frac{F^2_\pi m^2_{\eta'}}{2N_f}\Big|_{\frac{N_f}{N_c}=0} = \lim_{m\to 0} \left(\frac{2m}{2N_f}\right)^2 \int d^4x \langle P^0(x) P^0(0) \rangle \Big|^{\mathrm{ZV}}_{\mathrm{quenched}}$$

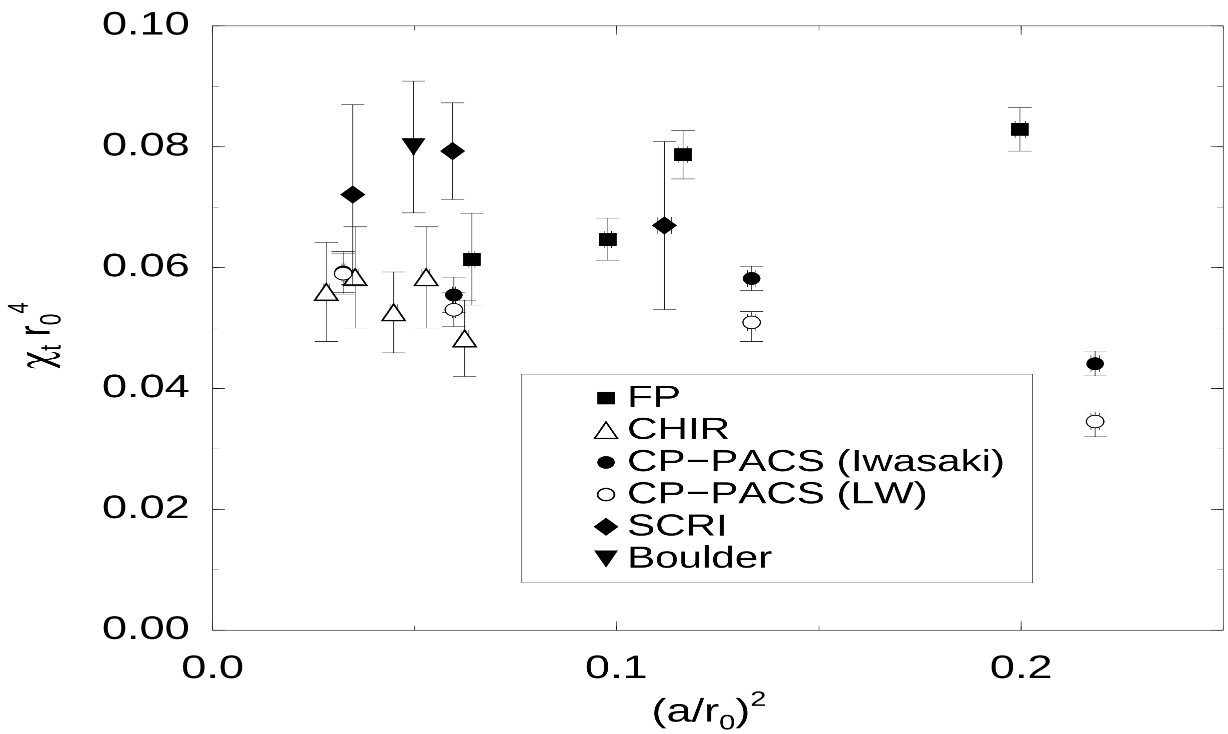
- With GW fermions obtained from the chirality of zero modes
  (S. Chandrasekharan'99, R.G. Edwards et al.'99, T. DeGrand et al. '02)

Numerical results for the topological susceptibility

- From the chirality of the zero modes

$$\chi_t = \lim_{V \to \infty} \frac{\langle (n_{\rm L} - n_{\rm R})^2 \rangle}{V}$$

(Summary taken from P. Hasenfratz et al. '02)



- The WA from other determinations (M. Teper '99)

$$\chi_t \; r_0^4 = 0.067 \pm 0.009$$

gives $\chi_t^{1/4} \simeq 200 \pm 18$ MeV, to be compared with $\chi_t^{1/4} \sim 180$ MeV

- Computations for $N_c > 3$ have been performed
  (B. Lucini M. Teper '01, L. Del Debbio et al. '02, N. Cundy et al. '02)

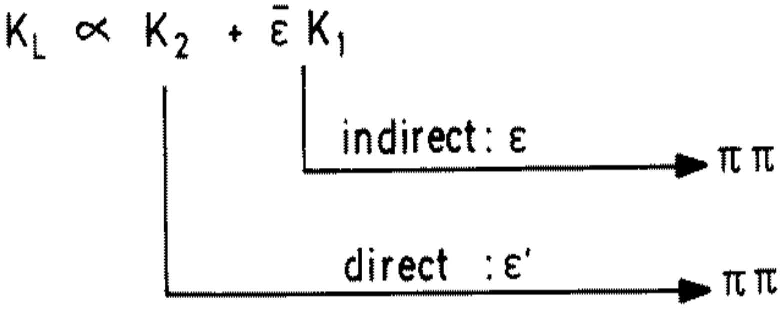
- Results compatible with smooth large-N_c limit and non-zero χ_t^∞

$K \to \pi\pi$ decays

- $K \to \pi\pi$ ampl. can be parametrized $\left({}_I\langle\pi\pi|S|\pi\pi\rangle_I = e^{2i\delta_I}\right)$

$$
\begin{aligned}
A[K^+ \to \pi^+\pi^0] &= \sqrt{\tfrac{3}{2}}A_2 e^{i\delta_2} \\
A[K^0 \to \pi^+\pi^-] &= \sqrt{\tfrac{2}{3}}A_0 e^{i\delta_0} + \sqrt{\tfrac{1}{3}}A_2 e^{i\delta_2} \\
A[K^0 \to \pi^0\pi^0] &= \sqrt{\tfrac{2}{3}}A_0 e^{i\delta_0} - 2\sqrt{\tfrac{1}{3}}A_2 e^{i\delta_2}
\end{aligned}
$$

- CP violation implies $A_I \neq A_I^*$



- Indirect CP violation can be parametrized as

$$\varepsilon \equiv \frac{A[K_L \to (\pi\pi)_{I=0}]}{A[K_S \to (\pi\pi)_{I=0}]}$$

- Direct CP violation can be parametrized as

$$\varepsilon' \simeq \frac{1}{\sqrt{2}} e^{i\frac{\pi}{4}} \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0}\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$

Experimental results

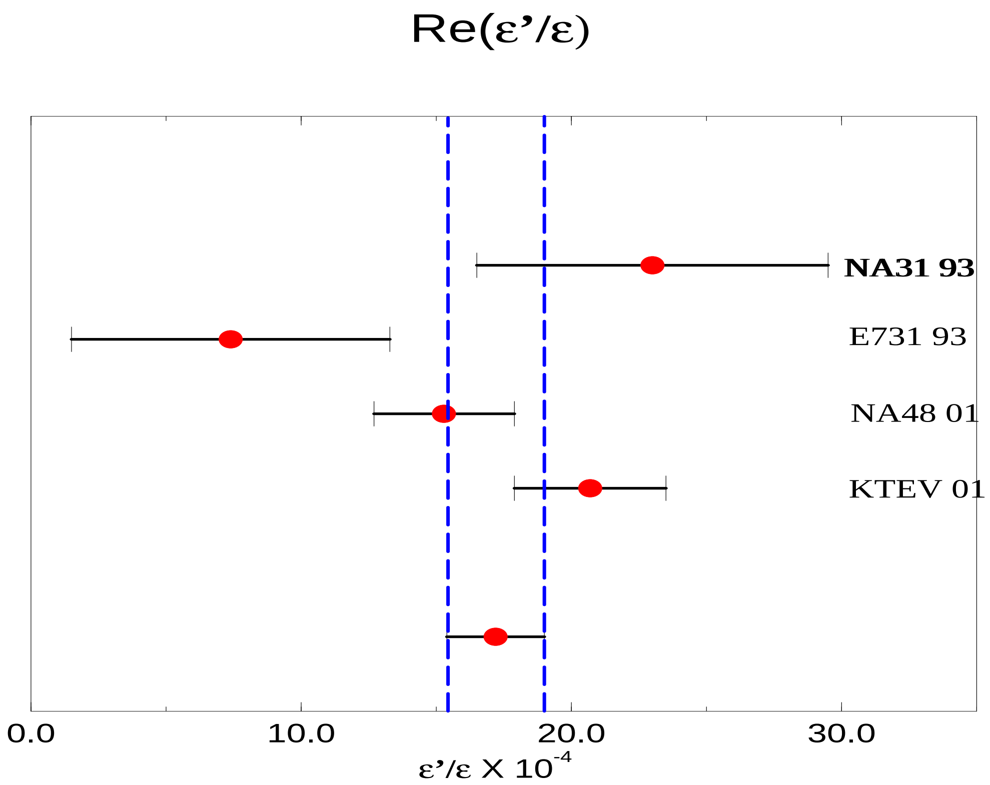
- $\Delta I = 1/2$ rule

$$\left|\frac{A_0}{A_2}\right| \simeq 22.2$$

- Indirect CP violation

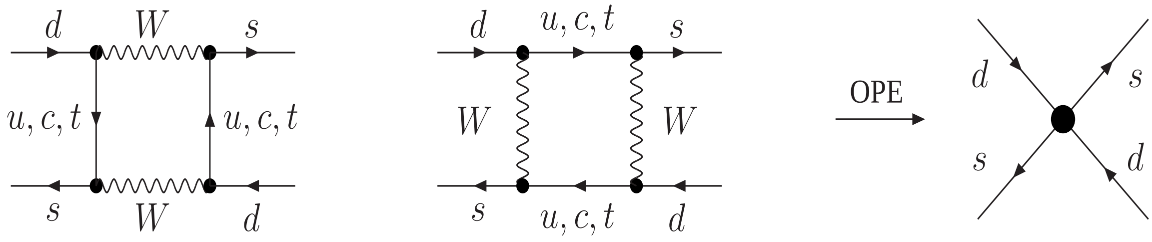
$$|\varepsilon| = (2.282 \pm 0.017) \times 10^{-3}$$

- Direct CP violation



$$\mathrm{Re}(\epsilon'/\epsilon) = (17.3 \pm 1.7) \times 10^{-4}$$

K^0–$\bar{K}^0$ mixing in the SM



- The particle–antiparticle mixing is given by

$$\varepsilon \simeq \frac{1}{\sqrt{2}} e^{i\frac{\pi}{4}} \frac{\mathrm{Im M_{12}}}{\Delta \mathrm{M_K}}$$

where

$$2 M_K M_{12}^* = \langle \bar{K}^0 | H_{\text{eff}}^{\Delta S=2} | K^0 \rangle$$

- In the Standard Model ($\lambda_i = V_{is}^* V_{id}$)

$$H_{\text{eff}}^{\Delta S=2} = \frac{G_F^2 M_W^2}{16\pi^2} \left[\lambda_c^2 \eta_1 S_0^c + \lambda_t^2 \eta_2 S_0^t + 2\lambda_c \lambda_t \eta_3 S_0^{ct} \right] \widehat{\mathcal{O}}^{\Delta S=2} + \text{h.c.}$$

- The QCD corrections η_1, η_2, η_3 are known at NLO (S. Herrlich and U. Nierste '94 '96)

- The long-distance QCD effects are parametrized as

$$\langle \bar{K}^0 | \hat{\mathcal{O}}^{\Delta S=2} | K^0 \rangle = \frac{16}{3} F_K^2 m_K^2 \hat{B}_K$$

Mixing with exact chiral symmetry

$$\hat{O}^{\Delta S=2}(a\mu) = Z_{11}(a\mu) O_1(a)$$

$$O_1(a) = [\bar{s}^\alpha \gamma_\mu (1-\gamma_5) \tilde{d}^\alpha][\bar{s}^\beta \gamma_\mu (1-\gamma_5) \tilde{d}^\beta]$$

- No mixing with other operators
- Proper chiral behaviour at finite lattice spacing!

Mixing with broken chiral symmetry

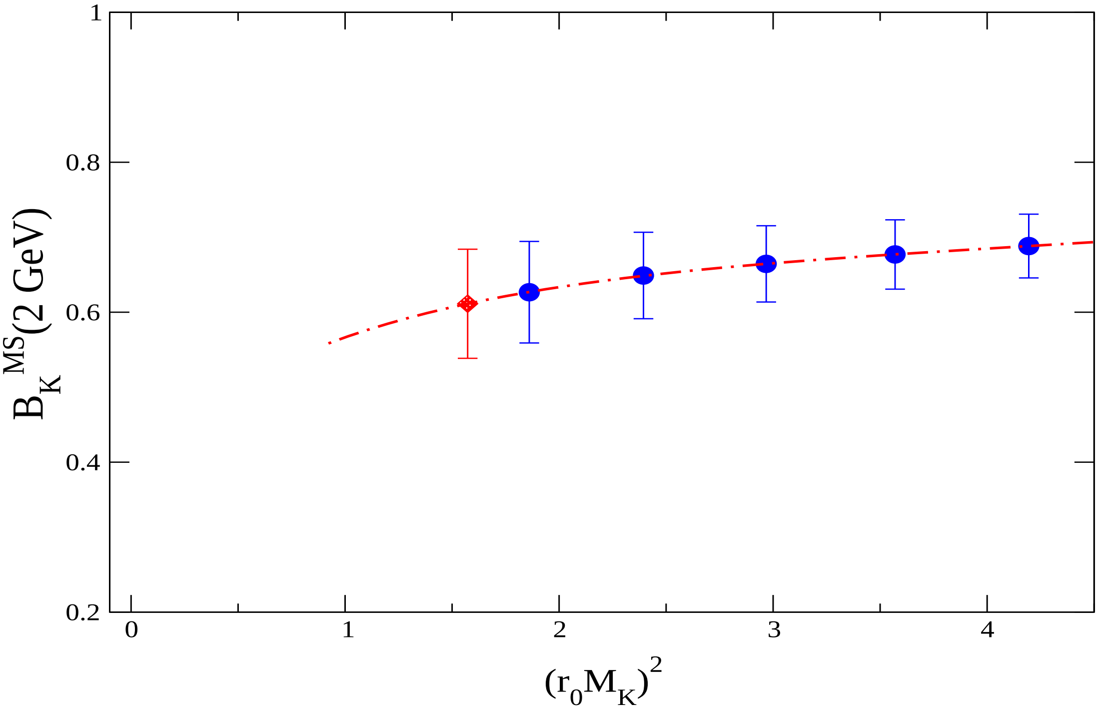
$$\hat{O}^{\Delta S=2}(a\mu)\Big|_{PC} = Z_{11}(a\mu) \Big[O_1(a) + \sum_{i=2}^{5} \Delta_{1i}(a) O_i(a) \Big]_{PC}$$

$$\begin{aligned}
O_1(a) &= [\bar{s}^\alpha \gamma_\mu (1-\gamma_5) d^\alpha][\bar{s}^\beta \gamma_\mu (1-\gamma_5) d^\beta] \\
O_2(a) &= [\bar{s}^\alpha (1-\gamma_5) d^\alpha][\bar{s}^\beta (1-\gamma_5) d^\beta] \\
O_3(a) &= [\bar{s}^\alpha (1-\gamma_5) d^\beta][\bar{s}^\beta (1-\gamma_5) d^\alpha] \\
O_4(a) &= [\bar{s}^\alpha (1-\gamma_5) d^\alpha][\bar{s}^\beta (1+\gamma_5) d^\beta] \\
O_5(a) &= [\bar{s}^\alpha (1-\gamma_5) d^\beta][\bar{s}^\beta (1+\gamma_5) d^\alpha]
\end{aligned}$$

- Subtractions most general $\Delta S = 2$ operators needed!
- Proper chiral behaviour recovered in the continuum limit
- Procedure feasible, reliable results in quenched approximation

B_K from overlap fermions

(N. Garron, L.G., C. Hoelbling, L. Lellouch, C. Rebbi)



- Lattice: $V = 16^3 \times 32$, $\beta = 6.0$, $m_s/2 \lesssim m \lesssim m_s$
- Renormalization: Non-perturbative RI/MOM
- Functional form to reach the physical point

$$B_K^{\overline{\text{MS}}}(2\,\text{GeV}) = \text{B}_0\left(1 - 3\frac{\text{M}_\text{K}^2}{(4\pi\text{F})^2}\log\left(\frac{\text{M}_\text{K}^2}{\mu_\chi^2}\right) + \text{C}\frac{\text{M}_\text{K}^4}{(4\pi\text{F})^4}\right)$$

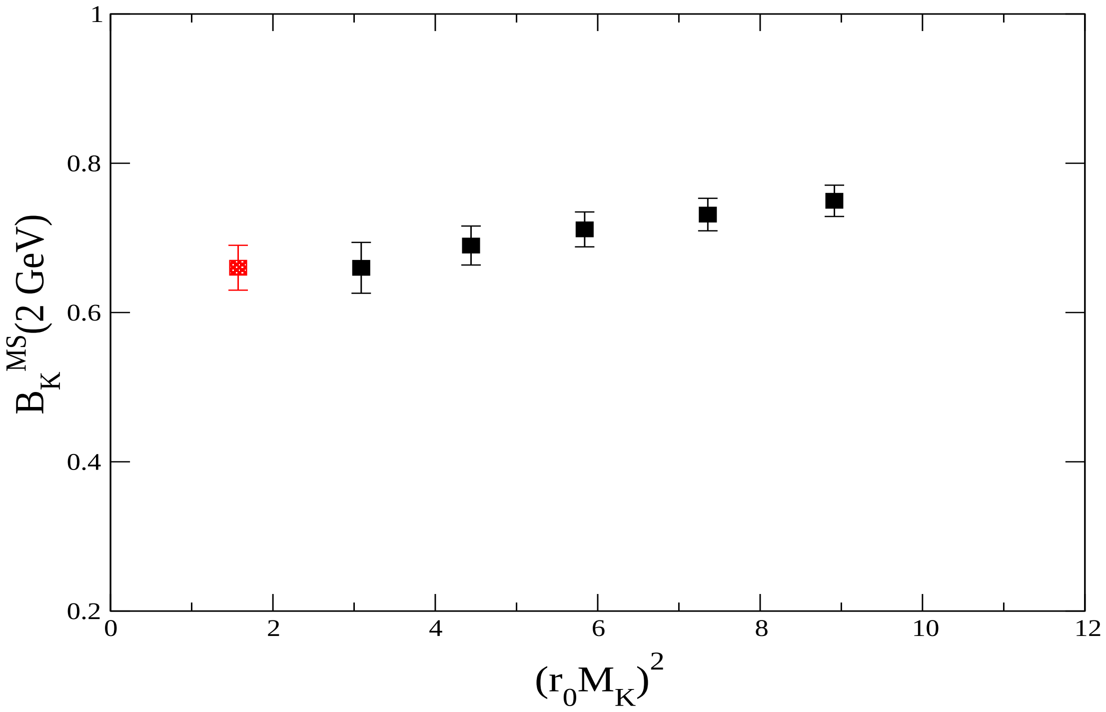
- Including the statistical error only

$$B_K^{\overline{\text{MS}}}(2\,\text{GeV}) = 0.61 \pm 0.07 \qquad \text{Preliminary !}$$

- More work needed to properly assess the systematic error

B_K from NNC-HYP overlap fermions

(T. DeGrand)



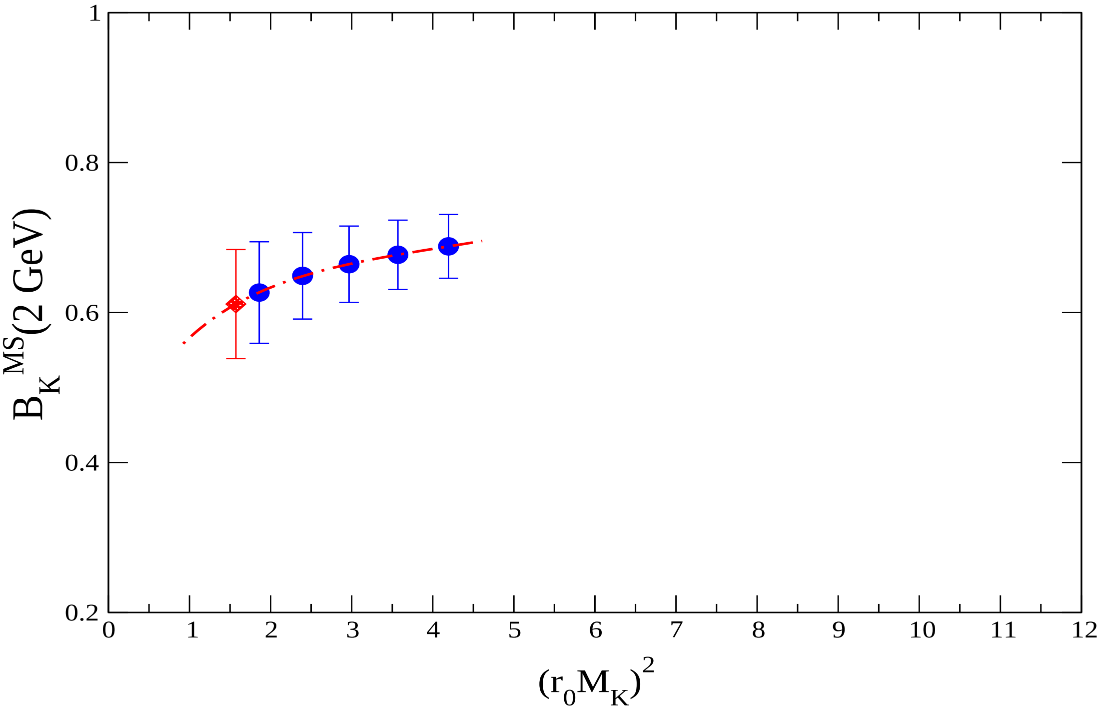
- Lattice: $V = 12^3 \times 36$, $\beta = 5.9$, $m_s \lesssim m \lesssim 2.5\, m_s$
- Renormalization: Perturbative
- Functional form to reach the physical point

$$B_K^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = \mathrm{B}_0 + \mathrm{D}\,\mathrm{M}_\mathrm{K}^2$$

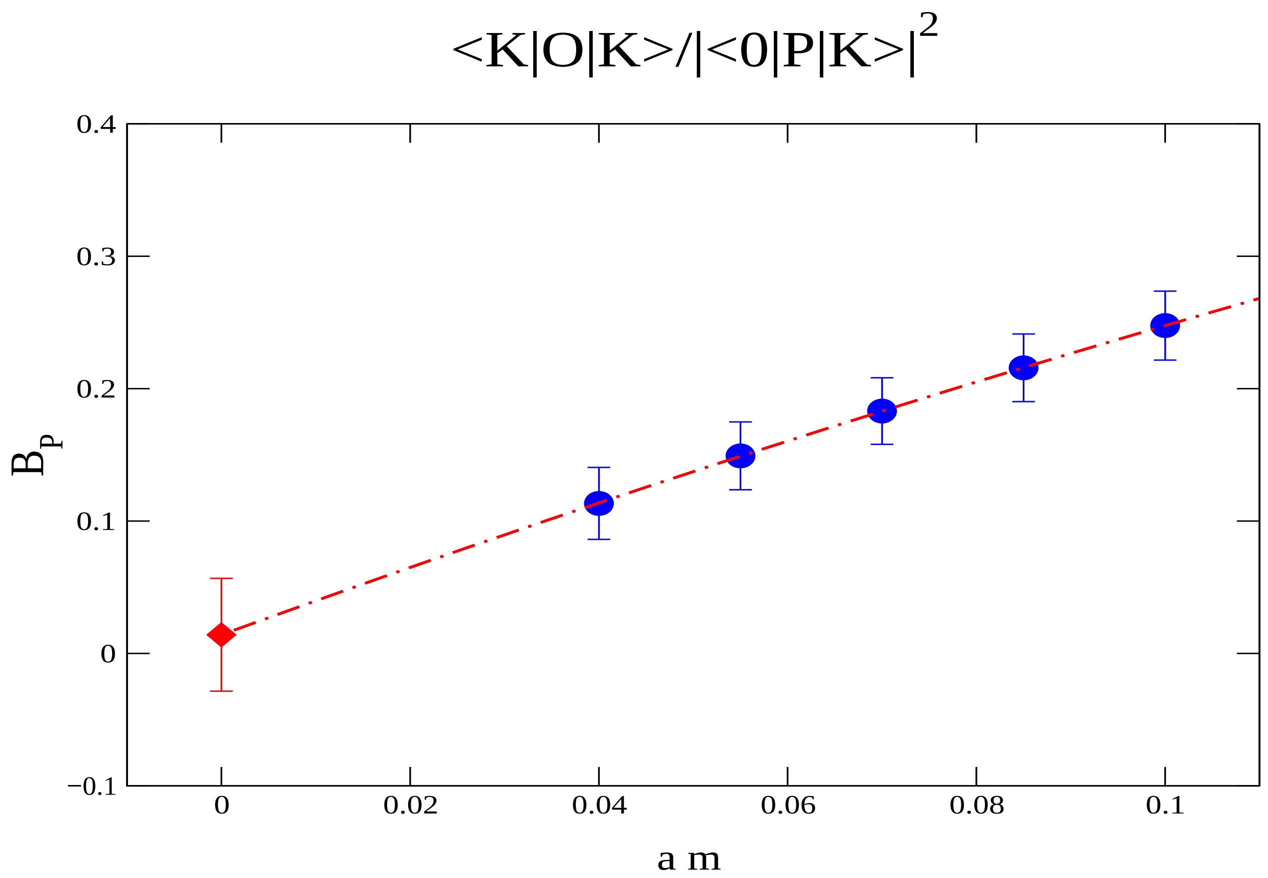
- Including the statistical error only

$$B_K^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 0.66 \pm 0.04 \qquad \text{Preliminary!}$$

- More to fix the systematic error

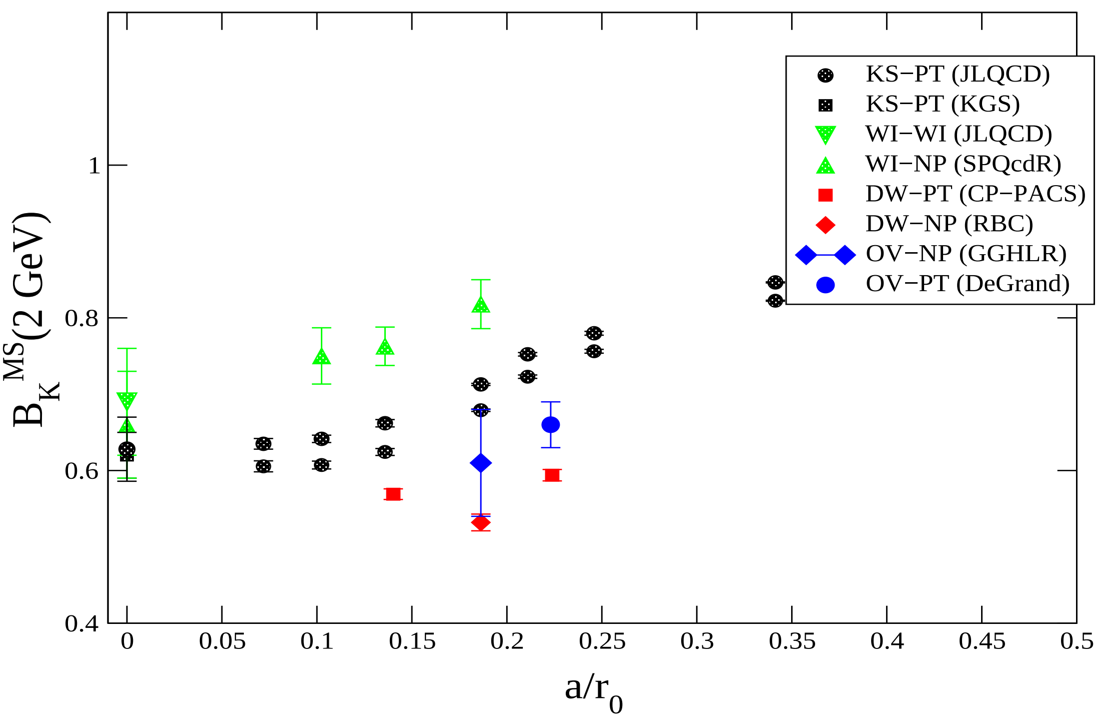


1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
7
8
9
10
11
12
B_K^MS(2 GeV)
(r_0M_K)^2



<K|O|K>/|<0|P|K>|2
0.4
0.3
0.2
0.1
0
−0.1
B_P
0
0.02
0.04
0.06
0.08
0.1
a m

B_K summary

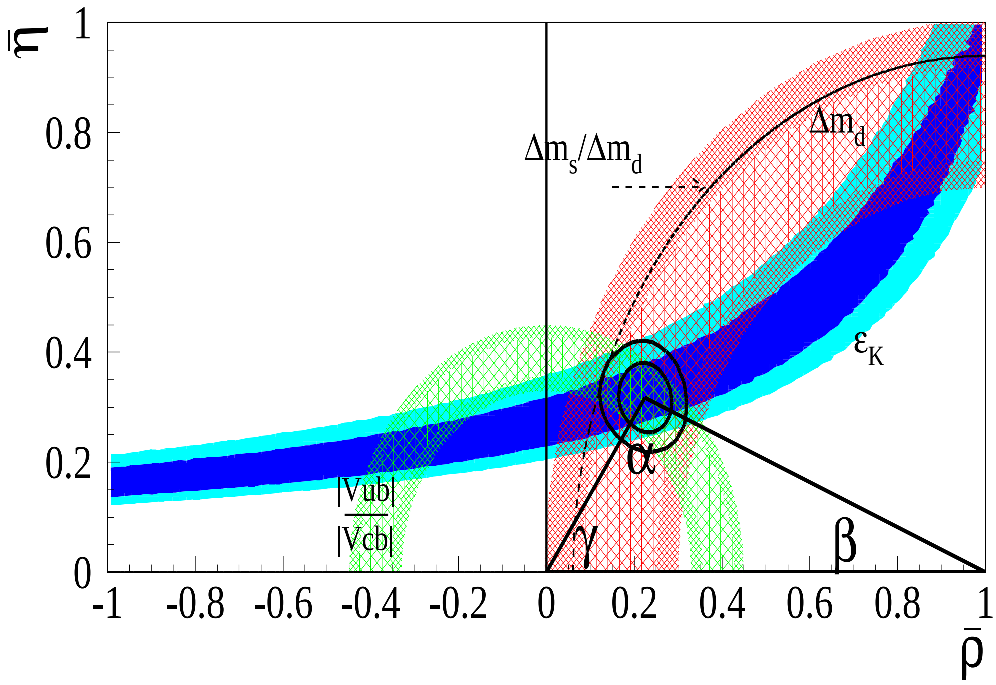


KS−PT (JLQCD)
KS−PT (KGS)
WI−WI (JLQCD)
WI−NP (SPQcdR)
DW−PT (CP−PACS)
DW−NP (RBC)
OV−NP (GGHLR)
OV−PT (DeGrand)
B_K^{MS}(2 GeV)
a/r_0
0.4
0.6
0.8
1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5

Unitarity triangle analysis

- Experimental input:

$$
\begin{aligned}
|\varepsilon_K^{\rm exp}| &= (2.271 \pm 0.017) \times 10^{-3} \\
\Delta M_d^{\rm exp} &= (0.489 \pm 0.008)\,{\rm ps}^{-1} \\
\Delta M_s^{\rm exp} &> 14.6\,{\rm ps}^{-1} \quad 95\%\ {\rm C.L.}
\end{aligned}
$$



(Most recent analyses: M. Ciuchini et al.'01, H. Höcker et al.'01)

- Fit results:

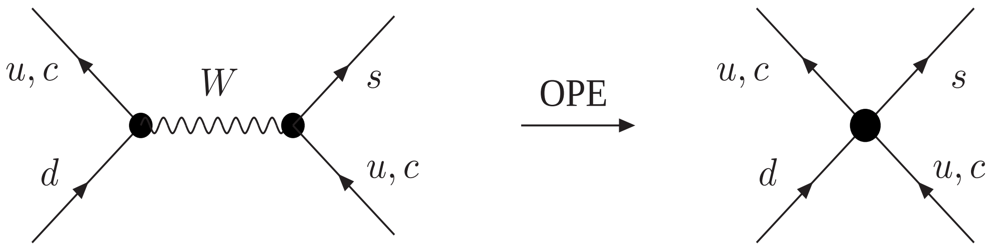
$$
\begin{aligned}
\sin(2\,\beta) &= 0.696 \pm 0.068 \\
\sin(2\,\alpha) &= -0.42 \pm 0.24 \\
\gamma &= (55.5 \pm 6.2)^\circ
\end{aligned}
$$

- Latest experimental average:

$$(\sin 2\beta)^{\rm exp} = 0.78 \pm 0.08$$

The $\Delta I = 1/2$ rule with an active charm

- By using the Operator Product Expansion



$$iA_I e^{i\delta_I} = {}_I\langle \pi\pi \mid H_{\text{eff}}^{\Delta S=1} \mid K^0 \rangle$$

- The CP-conserving $\Delta S = 1$ effective Hamiltonian is

$$H_{\text{eff}}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \Big[C_+(\mu) \widehat{\mathcal{O}}_+(\mu) + C_-(\mu) \widehat{\mathcal{O}}_-(\mu) \Big]$$

- The Wilson coefficients for the full $H_{\text{eff}}^{\Delta S=1}$ are known at NLO (see A.J. Buras et al. '92, M. Ciuchini et al. '94)

- A non-perturbative determination of ${}_I\langle \pi\pi | \widehat{\mathcal{O}}_\pm(\mu) | K^0 \rangle$ for the properly renormalized operators is needed

$$\mathcal{O}_\pm = \Big[(\bar{s}^\alpha \gamma_\mu^L u^\beta)(\bar{u}^\beta \gamma_\mu^L d^\alpha) \pm (\bar{s}^\alpha \gamma_\mu^L u^\alpha)(\bar{u}^\beta \gamma_\mu^L d^\beta) \Big] - (u \to c)$$

Renormalization pattern for $\mathcal{O}_{\pm}$

- To select operators with $d \leq 6$:

  - Flavour symmetry
  - CPS (S:$d \leftrightarrow s$)
  - Chiral symmetry

- At a non-zero physical distance (on-shell) one operator is left (S. Capitani, L. G. '00; L. G. et al. in preparation)

$$\mathcal{Q}_m = (m_c^2 - m_u^2)\Big(m_d(\bar{s}P_R d) + m_s(\bar{s}P_L d)\Big)$$

- Taking into account the quadratic GIM mechanism, no power divergent subtractions are needed for GW

$$\boxed{\widehat{\mathcal{O}}_{\pm} = Z_{\pm}\Big[\mathcal{O}_{\pm} + C_{\pm}^{m}\mathcal{Q}_m\Big]}$$

- For $m_s \neq m_d$

$$\mathcal{Q}_m = \frac{1}{2}\partial_\mu\left[\frac{m_d + m_s}{m_s - m_d}\bar{s}\gamma_\mu d - \frac{m_s - m_d}{m_s + m_d}\bar{s}\gamma_\mu\gamma_5 d\right]$$

No contributions in MEs which preserve four-momentum

The $H_{\text{eff}}^{\Delta S=1}$ with an integrated charm

- In either the CP-conserving or the CP-violating case

$$H_{\text{eff}}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} \sum_{i=1}^{10} C_i(\mu) \widehat{\mathcal{Q}}_i(\mu)$$

where a basis for QCD-penguin operators is

$$\begin{aligned} \mathcal{Q}_{3,5} &= (\bar{s}\gamma_\mu^L d) \sum_q (\bar{q}\gamma_\mu^{L,R} q) \\ \mathcal{Q}_{4,6} &= (\bar{s}^\alpha \gamma_\mu^L d^\beta) \sum_q (\bar{q}^\beta \gamma_\mu^{L,R} q^\alpha) \end{aligned}$$

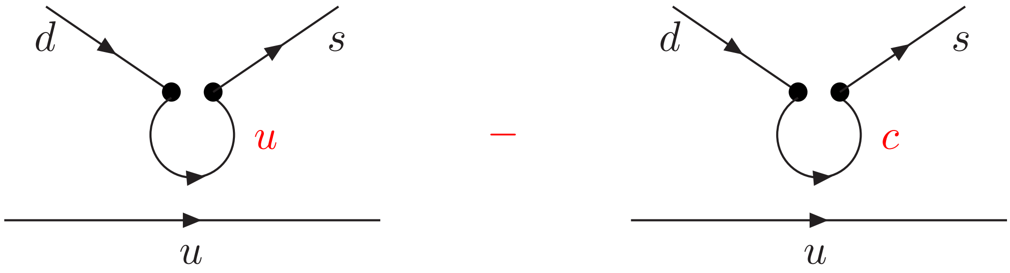
- At a non-zero physical distance two more operators can mix

$$\begin{aligned} \mathcal{Q}_m &= m_d(\bar{s}P_R d) + m_s(\bar{s}P_L d) \\ \mathcal{Q}_\sigma &= m_d(\bar{s}F_{\mu\nu}\sigma_{\mu\nu}P_R d) + m_s(\bar{s}F_{\mu\nu}\sigma_{\mu\nu}P_L d) \end{aligned}$$

- Without the GIM mechanism the mixing is power-divergent

$$\boxed{\widehat{\mathcal{Q}}_i = \widehat{Z}_{ij}\left[\mathcal{Q}_j + C_j^\sigma \mathcal{Q}_\sigma + \frac{C_j^m}{a^2}\mathcal{Q}_m\right]}$$

Active charm with Wilson fermions



- Parity-odd and parity-even components renormalize differently

- For parity conserving sector, using flavour and CPS symmetries (C. Bernard et al.'85, L. Maiani et al.'87, C. Dawson et al.'97)

$$
\begin{aligned}
\widehat{\mathcal{O}}_{\pm} &= Z_{\pm}\widetilde{\mathcal{O}}_{\pm} \\
\widetilde{\mathcal{O}}_{\pm} &= \mathcal{O}_{\pm} + (m_c - m_u) C^{\tau}_{\pm} \mathcal{Q}_{\tau} \\
&+ (m_c - m_u) \frac{C^{s}_{\pm}}{a^2} \mathcal{Q}_{s}
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{Q}_{\tau} &= \bar{s}\sigma_{\mu\nu}F_{\mu\nu}d \\
\mathcal{Q}_{s} &= \bar{s}d
\end{aligned}
$$

- With a broken chirality the GIM mechanism is only linear

K$\to \pi\pi$ from K$\to \pi$

(C. Bernard et al. '85)

- At leading order

$$f_\pi^3 \langle \pi^+\pi^- | \mathcal{O}^{(8,1)} | K^0 \rangle = i\alpha_1^{(8,1)} (M_K^2 - M_\pi^2)$$

$$f_\pi^2 \langle \pi^+(q) | \mathcal{O}^{(8,1)} | K^+(k) \rangle = \alpha_1^{(8,1)} (q \cdot k) - \alpha_2^{(8,1)} M_K^2$$

$$f_\pi \langle 0 | \mathcal{O}^{(8,1)} | K^0 \rangle = i\alpha_2^{(8,1)} (M_K^2 - M_\pi^2)$$

- First numerical results with DW fermions and the charm integrated out (RBC Coll. '01, CP-PACS Coll. '01)

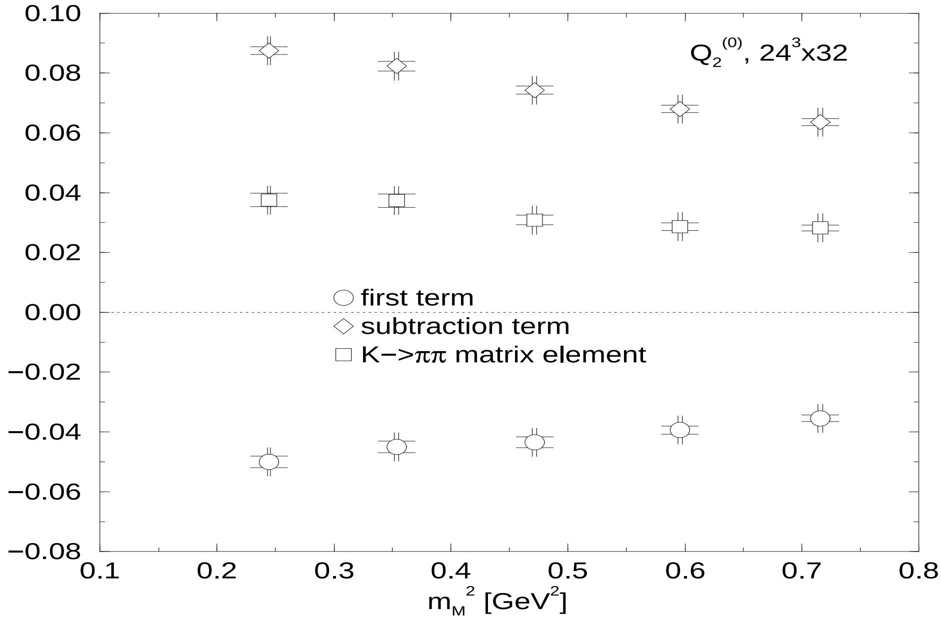
- Both groups: a signal after the power-divergent subtractions

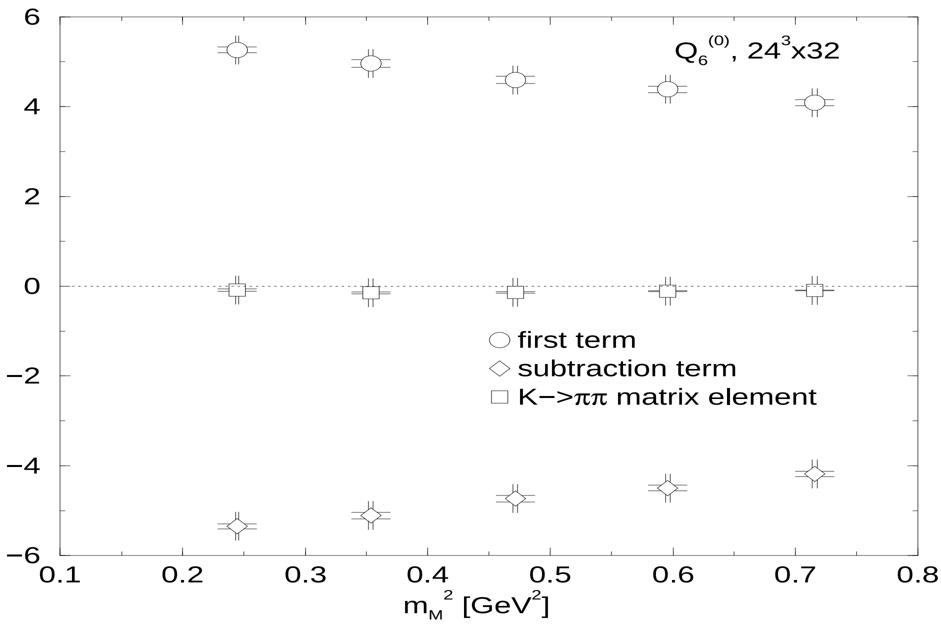
- Possible problems:

    - Approximate chiral symmetry only

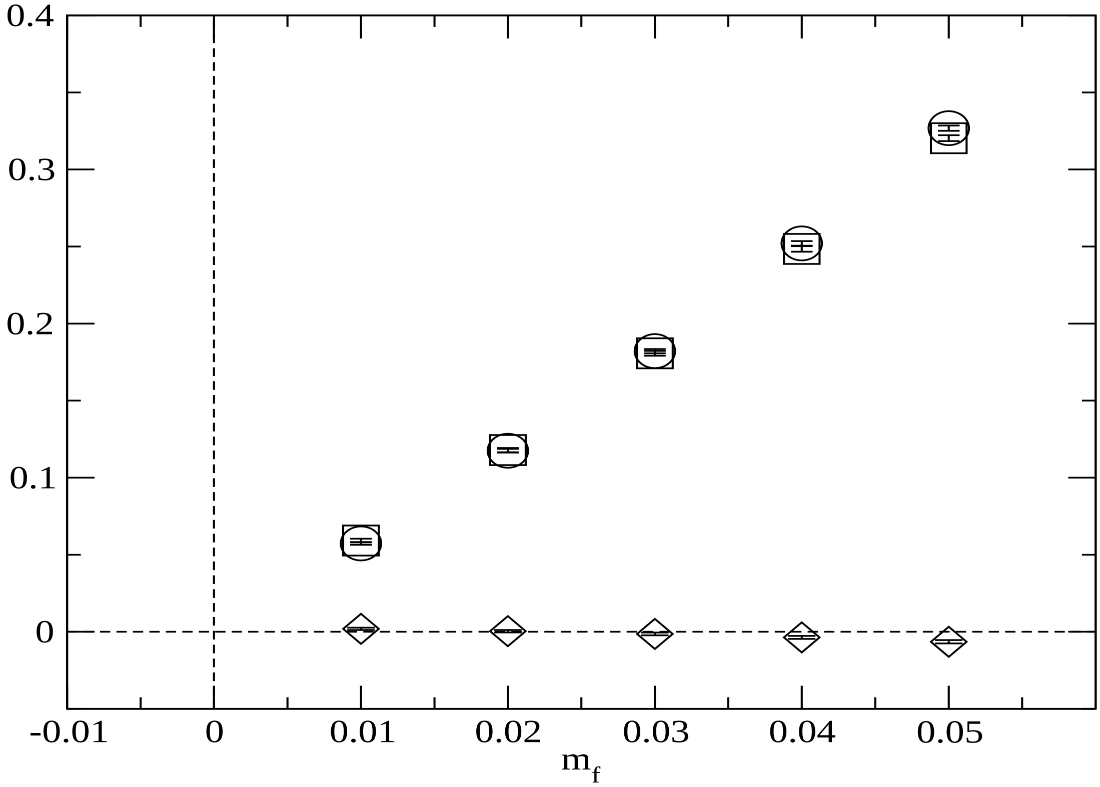
    - Quark masses quite heavy

CP-PACS: subtractions for $\mathcal{Q}_2$ and $\mathcal{Q}_6$

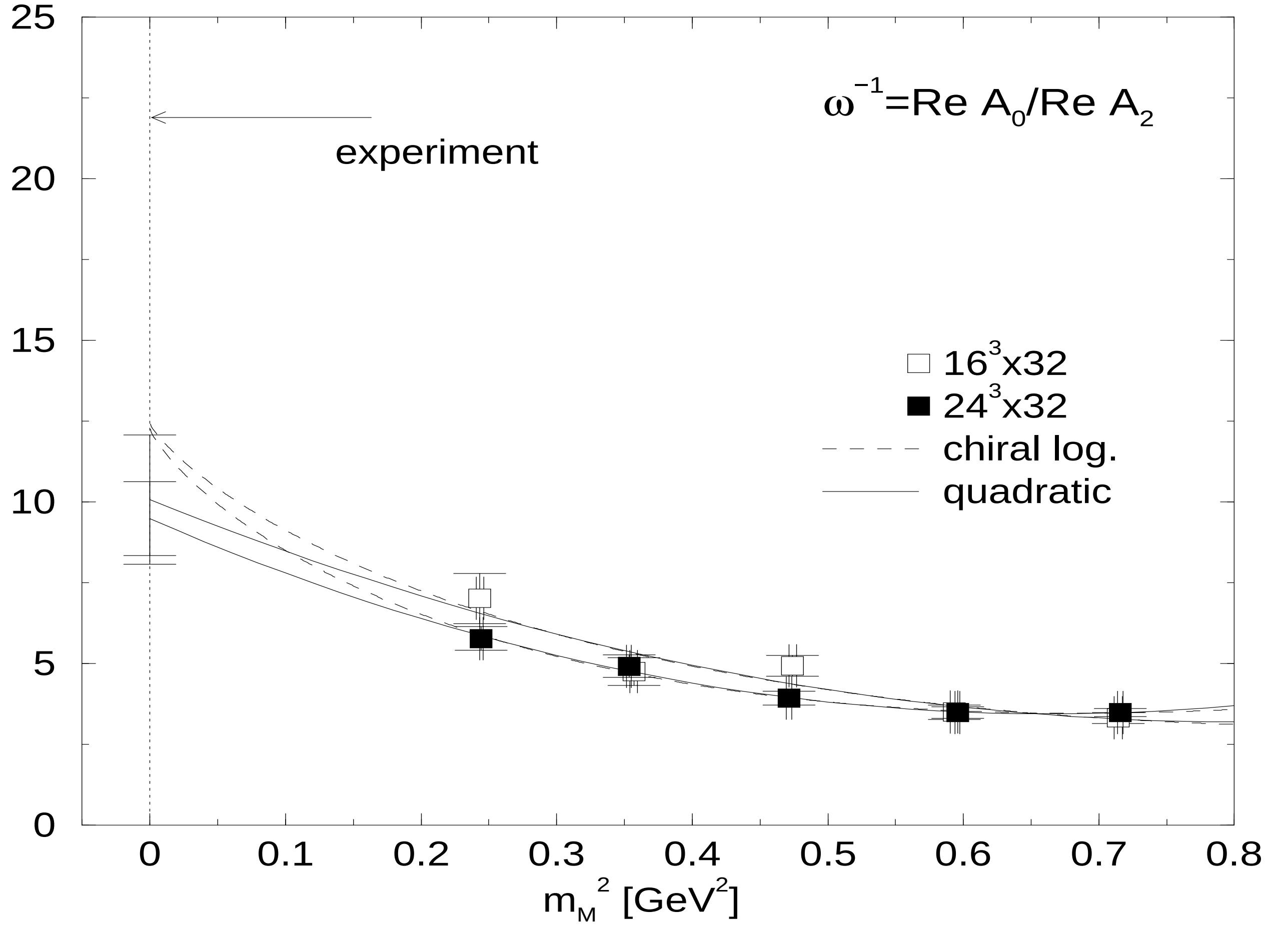




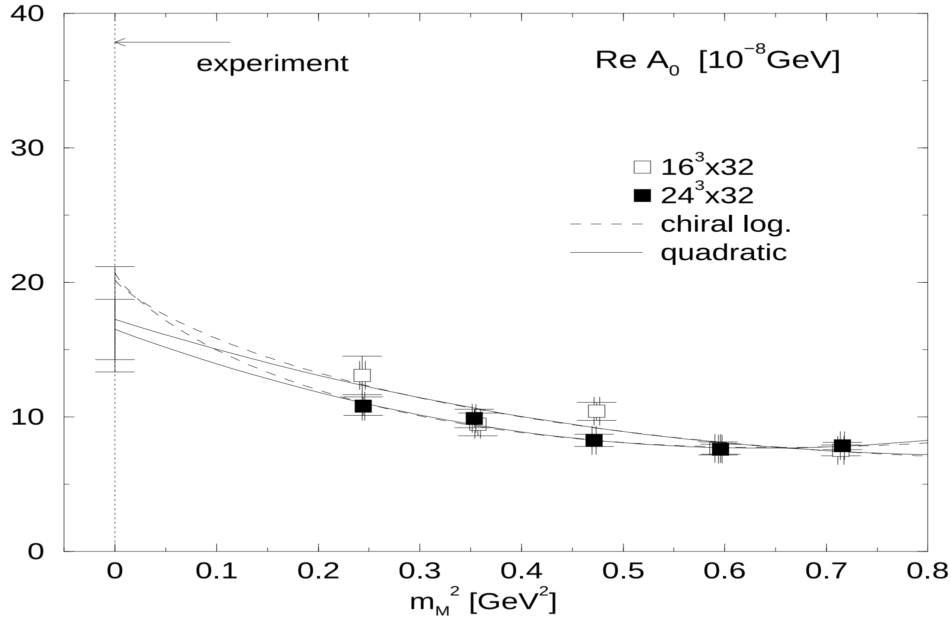
RBC: subtractions for $\mathcal{Q}_6$

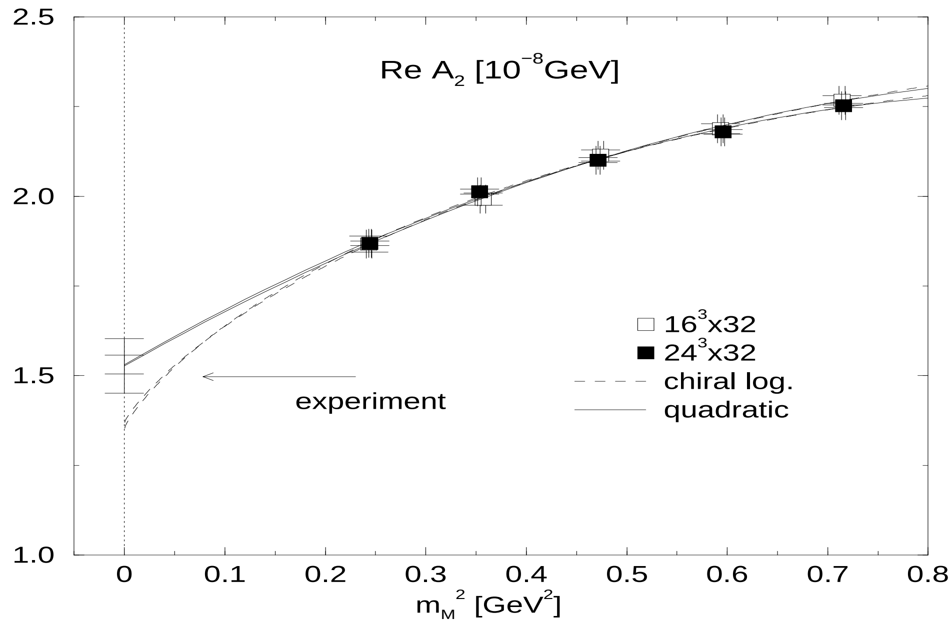


CP-PACS: $\mathrm{Re}\,A_0/\mathrm{Re}\,A_2$



CP-PACS: $\mathrm{Re}\,A_0$ and $\mathrm{Re}\,A_2$





Weak interactions in the ϵ regime

(L. G., P. Hernández, C. Hoelbling, K. Jansen, M. Laine, L. Lellouch, M. Lüscher, P. Weisz, H. Wittig in preparation)

- It is conceivable to extract the LECs of the weak chiral Lagrangian by studying the weak interactions in the ϵ regime

- A numerical study on the lattice is under way

- It can be interesting to extract LECs in standard regime with

  - Fermions with exact chiral symmetry
  - Active charm
  - Lighter quark masses

and to compare the results in the two regimes

Conclusions

- Exact chiral symmetry on the lattice at finite cut-off

- Domain-wall-overlap: explicit chirally symmetric regularization

- Quenched large-scale numerical simulations are feasible
  Regime of quark masses not reachable with Wilson fermions

- First phenomenological computations performed
  Results indicate small discretization errors

- QCD results in the ϵ regime are very encouraging

- Weak matrix elements:
  - No power-divergent subtractions for the $\Delta I = 1/2$ rule
  - First results obtained (B_K, $K \to \pi$)
  - A study in the ϵ regime started