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Chiral fermions and their phenomenological applications I

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Outline

- Introduction
- Domain-wall-overlap fermions
- Exact chiral symmetry and Ward identities
- Non-perturbative renormalization
- Meson spectrum

- The chiral condensate
- Topological susceptibility and WV relation
- Kaon matrix elements
- Conclusions and outlook

Plenary talks at lattice conferences

- T. Blum '98
  "Domain wall fermions in vector gauge theories"
- F. Niedermayer '98
  "Exact chiral symmetry, topological charge and related topics"
- M. Lüscher '99
  "Chiral gauge theories on the lattice with exact gauge invariance"
- H. Neuberger '99
  "Chiral fermions on the lattice"
- P.M. Vranas '00
  "Domain wall fermions and applications"
- P. Hernández '01
  "Ginsparg-Wilson fermions: practical aspects and applications"
- Y. Kikukawa '01
  "Analytic progress on exact lattice chiral symmetry"
- C. Gattringer '02
  "Recent results using systematic parameterizations of Ginsparg-Wilson fermions"
- L. G. '02
  "Exact chiral symmetry on the lattice: QCD applications"

Selected reviews

- M. Lüscher '00
  "Chiral gauge theories revisited"
- H. Neuberger '01
  "Exact chiral symmetry on the lattice"

The Ginsparg-Wilson relation

• In '82 Ginsparg and Wilson proposed the "mildest way" of breaking standard chiral symmetry on the lattice

$$\{\gamma_5, D\} = \bar{a} D \gamma_5 D \quad \Longleftrightarrow \quad \{\gamma_5, D^{-1}\} = \bar{a}\gamma_5$$

• An exact symmetry at finite cut-off is implied (M. Lüscher '98)

$$\delta q = \hat{\gamma}_5 q \quad \delta \bar{q} = \bar{q}\gamma_5$$

$$\hat{\gamma}_5 = \gamma_5(1 - \bar{a}D)$$

where $\hat{\gamma}_5^\dagger = \hat{\gamma}_5$, $\hat{\gamma}_5^2 = 1$.

• The anomaly is recovered à la Fujikawa (M. Lüscher '98)

$$a^4 Q(x) = \frac{\bar{a}}{2a} \mathrm{Tr}[\gamma_5 \mathrm{D}(\mathrm{x}, \mathrm{x})]$$

$$n_{\mathrm{L}} - n_{\mathrm{R}} = \mathrm{index(D)} = \mathrm{a}^4 \sum_{\mathrm{x}} \mathrm{Q(x)}$$

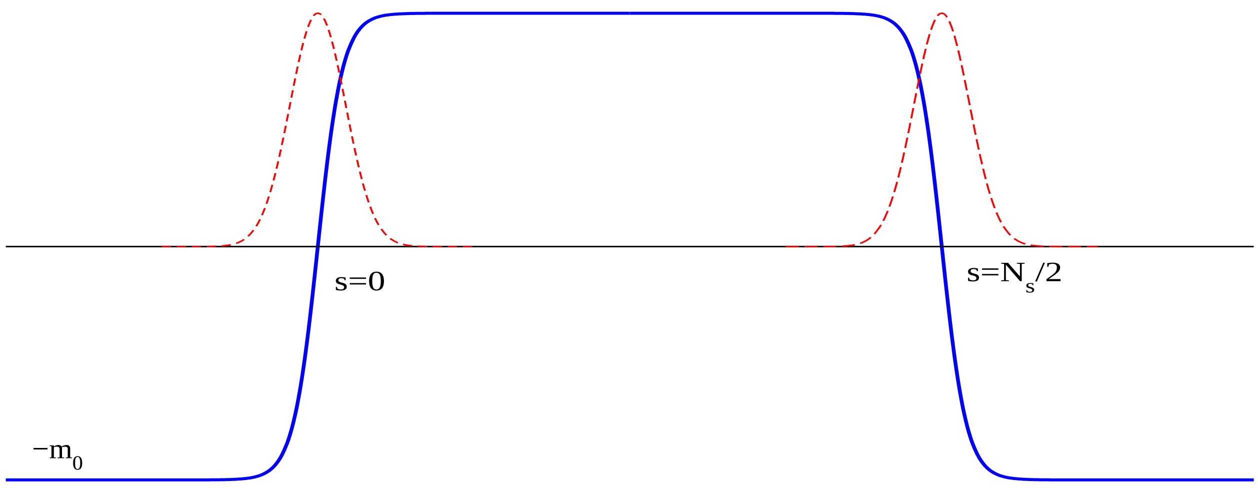
(see also H. Neuberger '97, P. Hasenfratz et al. '98)

• After more than a decade, a Dirac operator that satisfies the GW relation, is local and leads to the correct continuum limit was found

Domain-wall-overlap fermions

(D. B. Kaplan '92, H. Neuberger '97)

• A five-dimensional fermion with a domain-wall mass term generates a 4D fermion with the two chiral components separated by a distance $N_s/2$ and with exponentially small overlap
(Rubakov-Shaposhnikov '83, Callan-Harvey '85, Kaplan '92)



• Light 4D states interpolated by fermion fields on the walls

• In the $N_s \to \infty$ limit we expect a massless 4D fermion!

- The 5D domain-wall Dirac operator (Y. Shamir '93)

$$\mathcal{D} = \frac{1}{2}\Big[\gamma_5(\partial_s^* + \partial_s) - a_s \partial_s^* \partial_s\Big] + X$$

where ∂_s^* and ∂_s are forward and backward derivatives

$$X = D_W - \frac{\rho}{a} \;, \qquad 0 < \rho < 2$$

- The 4D Wilson-Dirac operator is

$$D_W = \frac{1}{2}\Big[\gamma_\mu(\nabla_\mu + \nabla_\mu^*) - a\nabla_\mu^* \nabla_\mu\Big]$$

$$\begin{aligned} \nabla_\mu q(x) &= \tfrac{1}{a}\Big[U_\mu(x) q(x + a\hat{\mu}) - q(x)\Big] \\ \nabla_\mu^* q(x) &= \tfrac{1}{a}\Big[q(x) - U_\mu^\dagger(x - a\hat{\mu}) q(x - a\hat{\mu})\Big] \end{aligned}$$

- The system is supplemented with open boundary conditions

$$P_+ q(0,x) = P_- q(a_s N_s + a_s, x) = 0 \;, \quad P_\pm = \frac{1}{2}(1 \pm \gamma_5)$$

• We are studying QCD with many flavors mixed in a given way

• We can integrate out the heavy flavors and remain with a four-dimensional effective action of the light boundary fields
(R. Narayanan, H. Neuberger '92 '94; H. Neuberger '97)

$$\bar{a}D_{\mathrm{N_s}} = 1 + \gamma_5 \frac{(1+\tilde{H})^{\mathrm{N_s}} - (1-\tilde{H})^{\mathrm{N_s}}}{(1+\tilde{H})^{\mathrm{N_s}} + (1-\tilde{H})^{\mathrm{N_s}}}$$

$$\tilde{H} \equiv \gamma_5 \tilde{X} \quad , \qquad \tilde{X} = \frac{a_s X}{2 + a_s X}$$

• For $N_s \to \infty$

$$\bar{a}D_{\mathrm{DW}} = 1 + \frac{\tilde{X}}{\sqrt{\tilde{X}^\dagger \tilde{X}}}$$

• Neuberger's operator is obtained in the limit

$$\bar{a}D_{\mathrm{N}} \equiv \lim_{a_s \to 0} \lim_{N_s \to \infty} \bar{a}D_{N_s} = 1 + \frac{X}{\sqrt{X^\dagger X}}$$

• Most remarkably both operators satisfy

$$\{\gamma_5, D\} = \bar{a}D\gamma_5 D$$

• If the gauge field is sufficiently smooth, the operators are local
(P. Hernández, K. Jansen and M. Lüscher '98)

Fixed-point Dirac operator

• An operator that satisfies the GW relation can be constructed iteratively with a RG blocking procedure from the continuum
(P. Hasenfratz, F. Niedermayer '94 '98)

• In the simulations an approximate explicit solution $D_{\rm FP}$ of the fixed-point equations can be used

• For measurements where an exact chiral symmetry is crucial, the residual breaking can be removed by defining

$$D^{\rm ov}_{\rm FP} = 1 + \frac{X_{\rm FP}}{\sqrt{X^{\dagger}_{\rm FP} X_{\rm FP}}}$$

(W. Bietenholz '99, P. Hasenfratz et al. '01)

Massive action

- The massive action is defined as

$$S_f = a^4 \sum_x \bar{\psi}(x) \left[(D + P_- M^\dagger \hat{P}_- + P_+ M \hat{P}_+) \psi \right](x)$$

- If we define

$$\hat{P}_\pm = \frac{1}{2}(1 \pm \hat{\gamma}_5) \qquad P_\pm = \frac{1}{2}(1 \pm \gamma_5)$$

and

$$\psi_{R,L} = \hat{P}_\pm \psi \qquad \bar{\psi}_{R,L} = \bar{\psi} P_\mp$$

the $U(N_f)_\mathrm{L} \times U(N_f)_\mathrm{R}$ transformations are defined as

$$\boxed{\begin{array}{ll} \psi_L \to V_L \psi_L & \bar{\psi}_L \to \bar{\psi}_L V_L^\dagger \\ \psi_R \to V_R \psi_R & \bar{\psi}_R \to \bar{\psi}_R V_R^\dagger \end{array}}$$

- The action is invariant if also

$$M \to V_L M V_R^\dagger$$

- The anomaly is recovered à la Fujikawa

• The exact chiral symmetry forbids operators of $d=5$ in the action which is $O(a)$-improved

• Chiral symmetry forbids additive quark renormalization

• Bilinears with correct chiral properties are $O(a)$-improved

$$\mathcal{O}^{\Gamma}_{\alpha\beta}(x) = \bar{\psi}_\alpha(x)\Gamma\tilde{\psi}_\beta(x) \qquad \tilde{\psi}_\beta(x) = \Big[(1-\frac{\bar{a}}{2}D)\psi_\beta\Big](x)$$

• Apparently no simple transformation of $\mathcal{O}^{\Gamma}$ under CP
(M. Lüscher '98, P. Hasenfratz '01, K. Fujikawa et al. '02)

• In correlations of operators at non-zero physical distance

$$\mathcal{O}^{\Gamma}_{\alpha\beta}(x) = \frac{1}{(1-\frac{\bar{a}}{2}m_\beta)}\bar{\psi}_\alpha(x)\Gamma\psi_\beta(x)$$

and therefore under CP (L.G. et al. in preparation)

$$\mathcal{O}^{\Gamma}_{\alpha\beta}(x) \xrightarrow{\text{CP}} \frac{1-\frac{\bar{a}}{2}m_\alpha}{1-\frac{\bar{a}}{2}m_\beta}\mathcal{O}^{\Gamma}_{\beta\alpha}(\tilde{x})$$

• The generalization to four-fermion operators is straightforward

Ward identities and quark masses

- By performing a non-singlet local rotation $[\epsilon_{V,A} = \epsilon_{V,A}(x)\delta_{xy}]$

$$-i\delta_{V,A}\psi = \left[\hat{P}_R \epsilon_{V,A} \hat{P}_R \pm \hat{P}_L \epsilon_{V,A} \hat{P}_L\right] \psi$$

$$i\delta_{V,A}\bar{\psi} = \bar{\psi}\left[P_L \epsilon_{V,A} \pm P_R \epsilon_{V,A}\right]$$

exact vector and axial WIs are obtained

$$\langle \partial^*_\mu V_\mu(x) \mathcal{O} \rangle = (m_1 - m_2) \langle S(x) \mathcal{O} \rangle + \mathrm{CT}$$

$$\langle \partial^*_\mu \mathcal{A}_\mu(x) \mathcal{O} \rangle = (m_1 + m_2) \langle P(x) \mathcal{O} \rangle + \mathrm{CT}$$

• By Fourier transforming at zero momentum transfer $q = 0$, choosing $\mathcal{O}(x_\alpha, x_\beta) = \tilde{\psi}_\alpha(x_\alpha)\bar{\psi}_\beta(x_\beta)$, defining

$$\mathcal{S}(p) = \sum_x e^{-ipx} \langle \tilde{\psi}(x)\bar{\psi}(0)\rangle$$

$$G_O(p) = \sum_{x_1,x_2} e^{-ip(x_1-x_2)} \langle \tilde{\psi}(x_1)O(0)\bar{\psi}(x_2)\rangle$$

and

$$\Lambda_O(p) = \mathcal{S}^{-1}(p)G_O(p)\mathcal{S}^{-1}(p)$$

the following identity can be obtained

$$(m_1 + m_2)\mathrm{Tr}\Big[\gamma_5 \Lambda_{\mathrm{P}}\left(\mathrm{p}, \mathrm{m}_1, \mathrm{m}_2\right)\Big] = \mathrm{Tr}\Big[\mathcal{S}^{-1}\left(\mathrm{p}, \mathrm{m}_1\right) + \mathcal{S}^{-1}\left(\mathrm{p}, \mathrm{m}_2\right)\Big]$$

• At variance with the Wilson case, the very same definition of the quark mass appears in the vector and axial WIs and in the quark propagator

Conserved axial current

(P.H. Ginsparg, K. G. Wilson '82; Y. Kikukawa, A. Yamada '99)

- Under a non-singlet local chiral rotation ($\epsilon = \epsilon^a(x)T^a\delta_{xy}$)

$$-i\delta_A\psi = (\hat{P}_R\epsilon\hat{P}_R - \hat{P}_L\epsilon\hat{P}_L)\psi \qquad -i\delta_A\bar{\psi} = \bar{\psi}\gamma_5\epsilon$$

$$-i\delta_A S = -\sum_x \epsilon(x)\partial_\mu^* A_\mu(x) = \sum_x \partial_\mu\epsilon(x)A_\mu(x)$$

- By extending the gauge group $SU(N_c) \to SU(N_c) \times U(1)$ and performing a $U(1)$ gauge rotation

$$U_\mu(x) \to U_\mu^{(\alpha)} = e^{i\alpha_\mu(x)}U_\mu(x) = e^{i\epsilon(x)}U_\mu(x)e^{-i\epsilon(x+\hat{\mu})}$$

we can define the kernel

$$K_\mu = -i\frac{\delta D(U_\mu^{(\alpha)})}{\delta\alpha_\mu(x)}\Big|_{\alpha=0}$$

and the corresponding conserved axial current

$$\mathcal{A}_\mu^a(x) = \bar{\psi}\Big(P_L K_\mu(x)\hat{P}_R - P_R K_\mu(x)\hat{P}_L\Big)T^a\psi$$

- In this form it can be implemented numerically (P. Hasenfratz et al. '02)

- For CC correlations (and generalizations) the propagator from any point to any point is required (L.G. et al. in prep.)

Singlet axial Ward identities

- For a given string of renormalized fundamental fields $\hat{O}$

$$\langle \partial_\mu^* \mathcal{A}_\mu^0(x) \hat{O} \rangle = 2N_f \langle Q(x) \hat{O} \rangle + \langle \delta_A^x \hat{O} \rangle$$

$$2N_f \int d^4x \, \langle Q(x) \hat{O} \rangle + \langle \delta_A \hat{O} \rangle = 0$$

- As a consequence, properly renormalized operators are

$$\begin{aligned} \hat{Q}(x) &= \tfrac{1}{2} \mathrm{Tr}[\gamma_5 D(x,x)] - \tfrac{Z}{2N_f} \partial_\mu^* \mathcal{A}_\mu^0(x) \\ \hat{\mathcal{A}}_\mu^0(x) &= (1-Z) \mathcal{A}_\mu^0(x) \end{aligned}$$

- The renormalized AWIs read

$$\langle \partial_\mu^* \hat{\mathcal{A}}_\mu^0(x) \hat{O} \rangle = 2N_f \langle \hat{Q}(x) \hat{O} \rangle + \langle \delta_A^x \hat{O} \rangle$$

Exact chiral symmetry: some advantages

- The Dirac operator has an index at finite cut-off:
  - A natural definition for $Q(x)$
  - Identification of the topological charge
- Very light quark masses can be reached:
  - No exceptional configurations
- No mixing among operators of different chirality:
  - No additive quark renormalization
  - Simplified mixing for composite operators
  - $O(a)$ improvement straightforward

“Local” axial current

- The “local” axial current

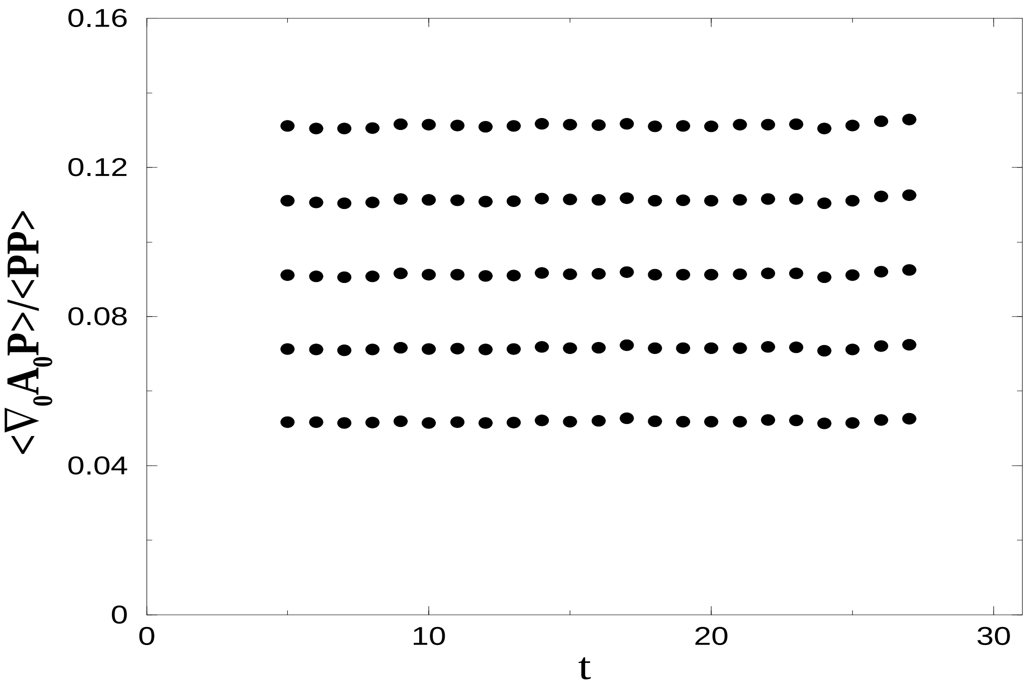
$$A_\mu(x) = \bar{\psi}_\alpha(x)\gamma_\mu\gamma_5\tilde{\psi}_\beta(x)$$

is not conserved but has the correct transformation properties

- On shell

$$Z_A\langle\nabla_\mu A_\mu(x)P(0)\rangle = (m_1 + m_2)\langle P(x)P(0)\rangle + O(a^2)$$

and Z_A can be extracted from the ratio of correlation functions (L.G., C. Hoelbling, C. Rebbi '01; S.J. Dong et al. '01, BGR Coll. '02)

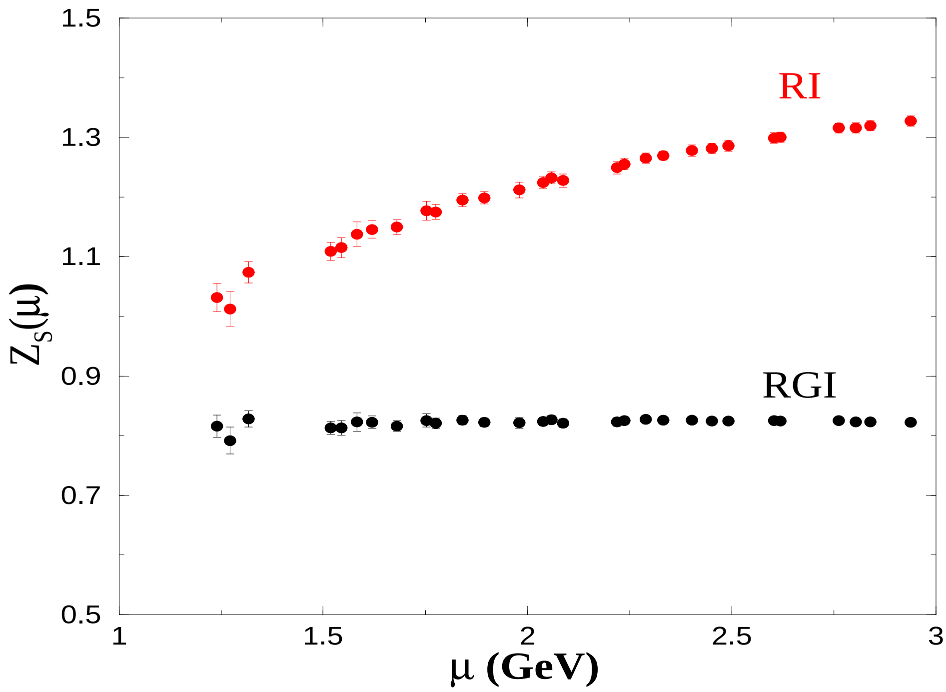


- For standard overlap ($\beta = 6.0$, $\rho = 1.4$), $Z_A = 1.55(4)$ to be compared with PT $Z_A^{\rm PT} = 1.15 - 1.35$

RI/MOM non-perturbative renormalization

(G. Martinelli et al. '95)

- Extensive applications for DW (T. Blum et al. '99-'01)
- First applications for overlap (L.G., C. Hoelbling, C. Rebbi '01)



- A RI/MOM renormalization condition is fixed on amputated off-shell Green's functions computed in the Landau gauge

$$Z_{\mathcal{O}}^{-1}(\mu a) Z_q(\mu a) = \lim_{m \to 0} \mathrm{Tr}\left[\mathrm{P}_{\mathcal{O}} \Lambda_{\mathcal{O}}(\mathrm{p}, \mathrm{m})\right]_{\mathrm{p}^2 = \mu^2}$$

- For the scalar density

$$Z_S(a\mu) = Z_A \lim_{m=0} \frac{\mathrm{Tr}[\mathrm{P_A} \Lambda_\mathrm{A}(\mathrm{p}, \mathrm{m})]}{\mathrm{Tr}[\mathrm{P_S} \Lambda_\mathrm{S}(\mathrm{p}, \mathrm{m})]}\bigg|_{p^2 = \mu^2}$$

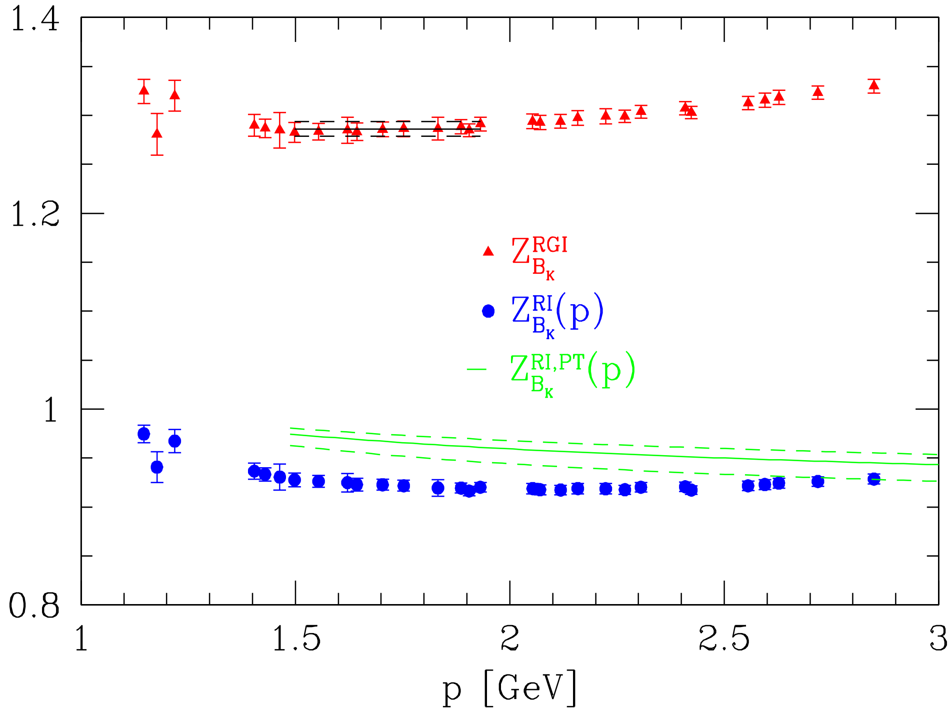
- For bilinears there is a "renormalization window"
- Matching with $\overline{\mathrm{MS}}$ needs continuum PT only

RI/MOM: four-quark operators

(A. Donini at al. '99)

- First application for overlap (see N. Garron, C. Hoelbling talks)

Preliminary !



- For the $\Delta S = 2$ Standard Model Operator

$$O_1 = [\bar{s}\gamma_\mu(1-\gamma_5)\tilde{d}][\bar{s}\gamma_\mu(1-\gamma_5)\tilde{d}]$$

the renormalization constant is given by

$$Z_{B_K}(a\mu) = \frac{Z_{11}(a\mu)}{Z_A^2} = \lim_{m=0} \frac{\mathrm{Tr}^2[\mathrm{P_A}\Lambda_\mathrm{A}(\mathrm{p},\mathrm{m})]}{\mathrm{Tr}[\mathrm{P_1}\Lambda_1(\mathrm{p},\mathrm{m})]}\bigg|_{p^2=\mu^2}$$

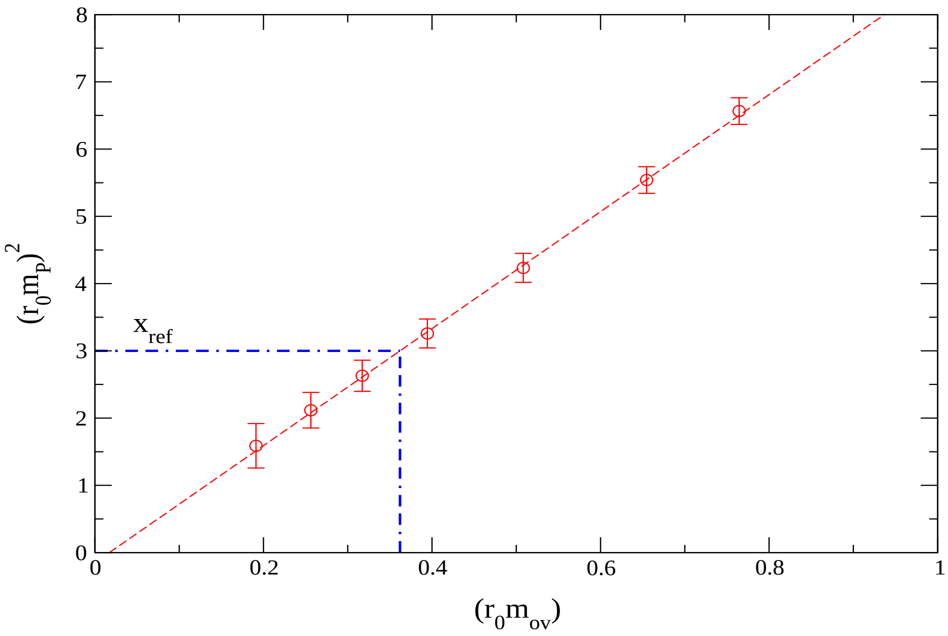
- Also for $Z_{B_K}^{\mathrm{RI}}$ there is a "renormalization window"

Matching to continuum Wilson data

(P. Hernández et al. '01)

- Fixing the RGI quark mass to the continuum extrapolated Wilson value (S. Capitani et al. '98, J. Garden et al. '99)

$$M_W^{\rm RGI}\Big|_{(r_0 M_P)^2 = x_{\rm ref}} = \frac{1}{Z_S^{\rm RGI}(a)} m_{\rm ov}(a)\Big|_{(r_0 M_P)^2 = x_{\rm ref}}$$



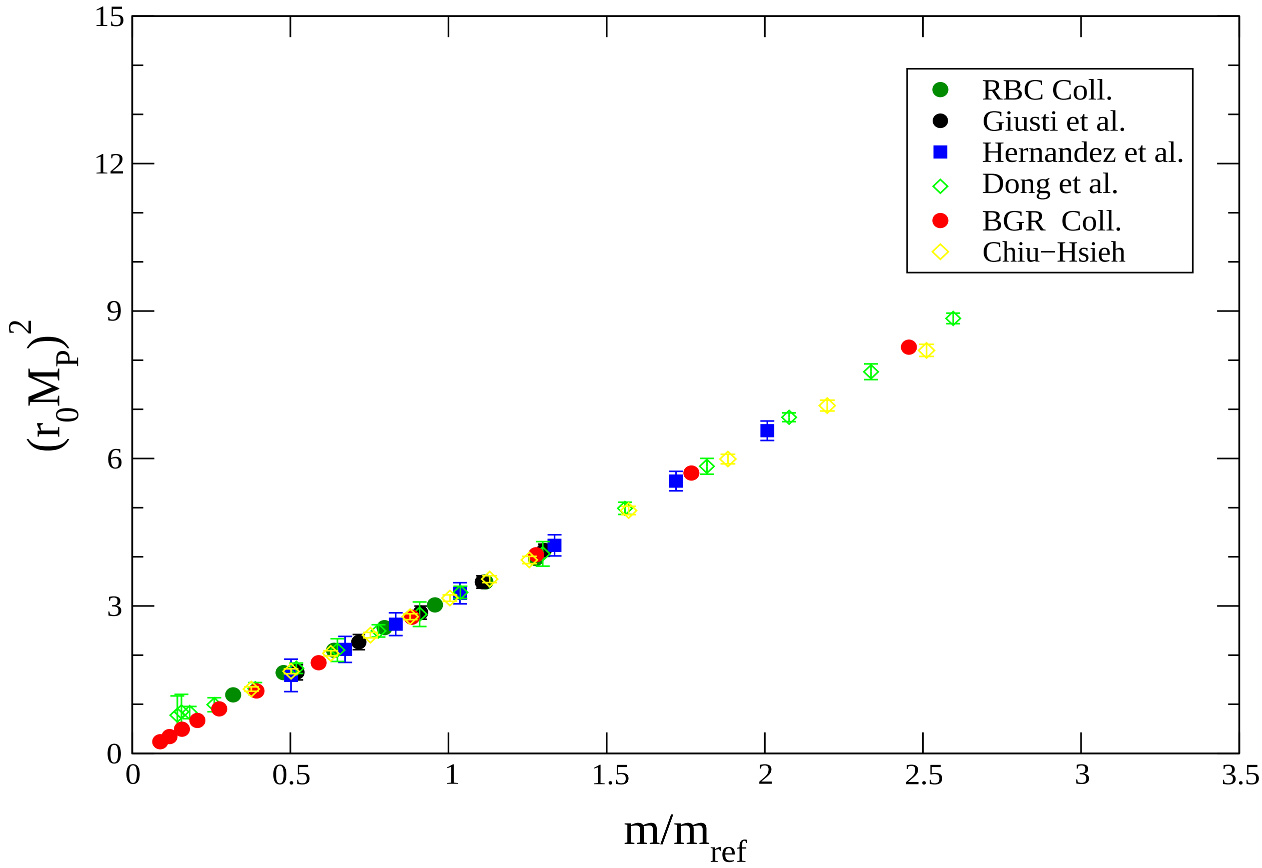
. . . or fixing

$$\langle 0|P^{\rm RGI}|\pi\rangle_W\Big|_{(r_0 M_P)^2 = x_{\rm ref}} = Z_S^{\rm RGI}(a)\langle 0|P_{\rm ov}(a)|\pi\rangle\Big|_{(r_0 M_P)^2 = x_{\rm ref}}$$

- Warning: the prediction for a low energy hadronic quantity is lost in the renormalization procedure

$Z_S^{\overline{\rm MS}}(2{\rm GeV})$ Overlap, $\beta = 6.0$, $\rho = 1.4$		
PT	RI/MOM	Wilson
1.1-1.3	1.41(6)	1.43(11)

Overlap operator: M_π^2 vs. m

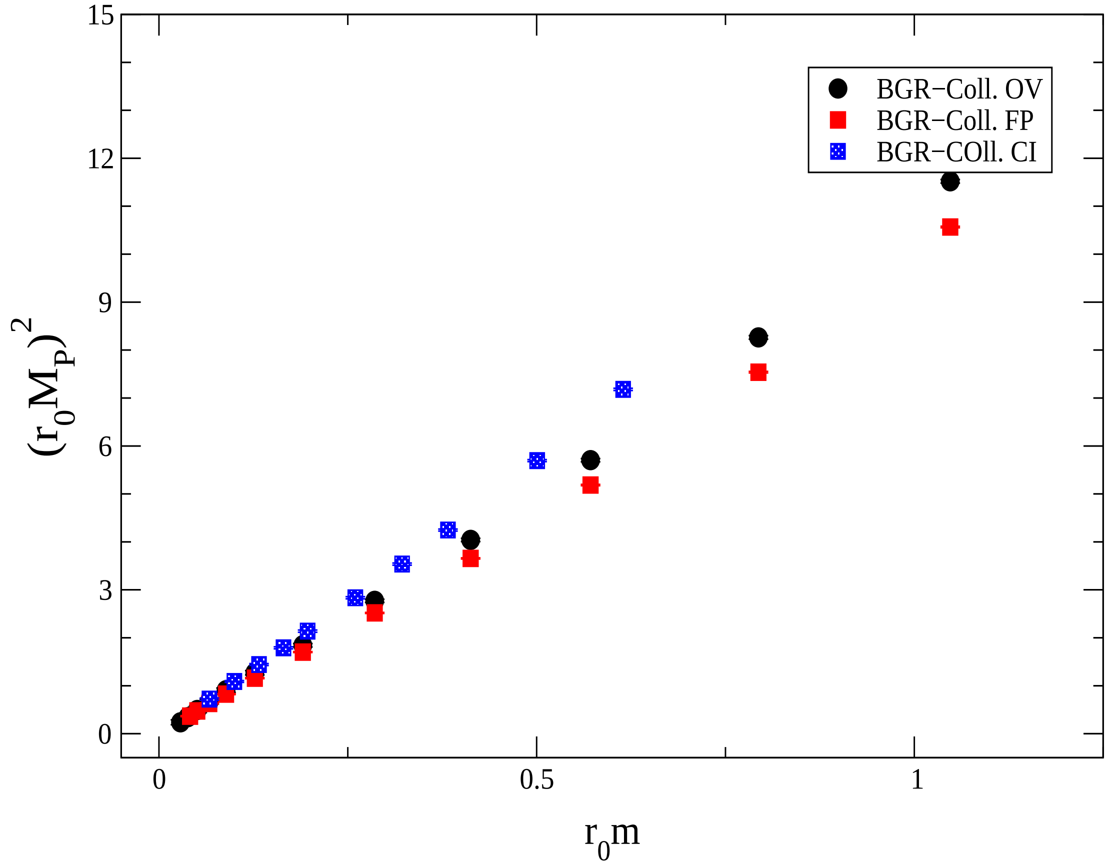


$$(r_0 M_P)^2\Big|_{m_{\rm ref}} = 2(r_0 M_K)^2 \quad , \qquad \begin{array}{rcl} M_K & = & 495\,\text{MeV} \\ r_0 & = & 0.5\,\text{fm} \end{array}$$

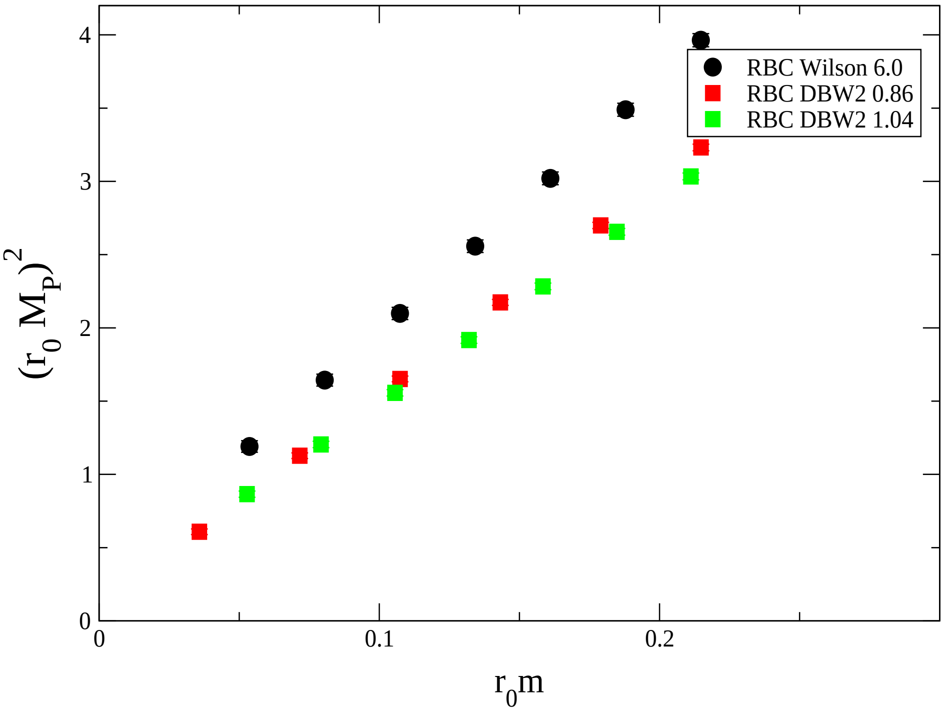
Collaboration	a (fm)	V
L. G. et al. '01	~ 0.093	$16^3 \times 32$
P. Hernández et al. '01	~ 0.093	$14^3 \times 24$
P. Hernández et al. '01	~ 0.12	$10^3 \times 24$
S. J. Dong et al. '01	~ 0.14	$20^3 \times 20$
BGR-Coll. '02	~ 0.16	$12^3 \times 24$
Chiu-Hsieh '02	~ 0.14	$8^3 \times 24$

- Good control over chiral symmetry!

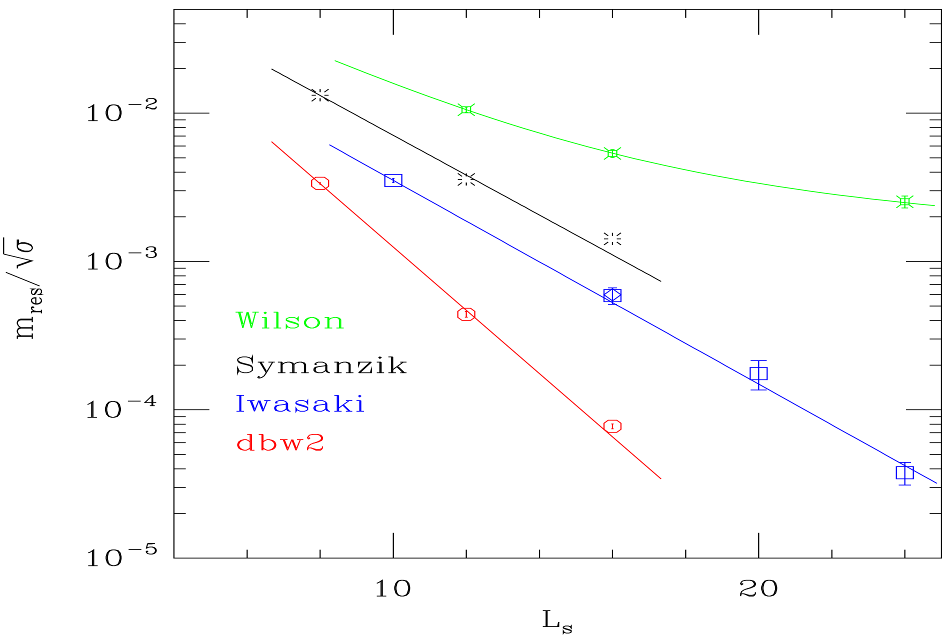
Fixed-point and chirally-improved operators: M_π^2 vss m



Domain-wall fermions with DBW2 action: M_π^2 vs. m



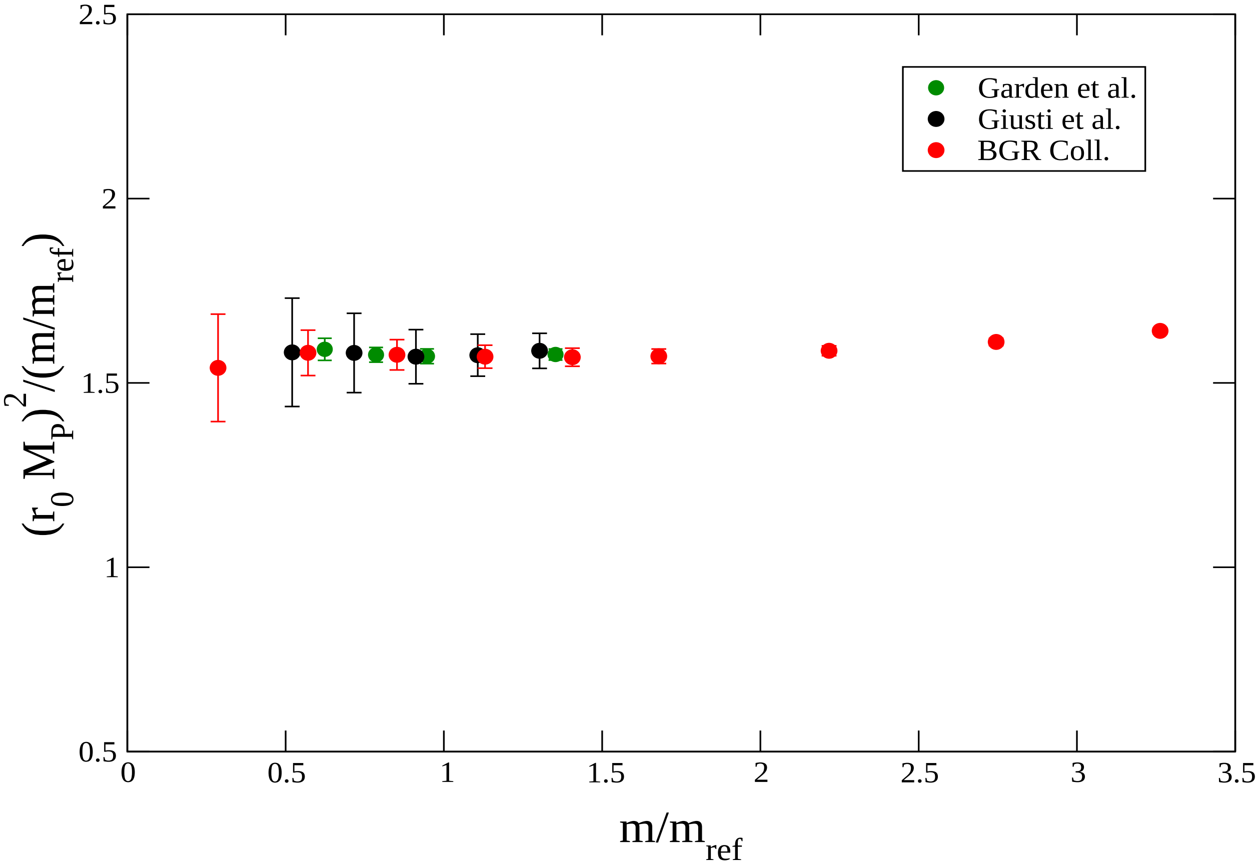
- With the DBW2 action improved m_{res}



- A potential difficulty: samples with proper distribution for Q
- Other properties need to be investigated (scaling, etc.)

Comparison with NP-improved Wilson data

$$(r_0 M_P)^2\Big|_{m_{\rm ref}} = 2(r_0 M_K)^2 \ , \qquad \begin{array}{rcl} M_K &=& 495\,\mathrm{MeV} \\ r_0 &=& 0.5\,\mathrm{fm} \end{array}$$



$$\frac{M_P^2}{2m} = \frac{\Sigma}{F^2}\Big[1 - \frac{m_0^2}{3(4\pi F)^2}\Big(1+\log\Big(\frac{M^2}{\mu_\chi^2}\Big)\Big)$$

$$+\frac{\alpha M^2}{3(4\pi F)^2}\Big(2\log\Big(\frac{M^2}{\mu_\chi^2}\Big)+1\Big) + \Big(2\,\alpha_8 - \alpha_5\Big)\frac{M^2}{(4\pi F)^2}\Big]$$

- Linear behaviour for M_P^2 in the range $500 \lesssim M_P \lesssim 800$ MeV

Mesons with non-degenerate quarks

- For the ratio (C. W. Bernard et al. '92, S. Aoki at al. '02)

$$y = \frac{4m_1 m_2}{(m_1+m_2)^2} \frac{M_P^4(m_1,m_2)}{M_P^2(m_1,m_1)M_P^2(m_2,m_2)}$$

χPT expectations are ($\delta = m_0^2/3(4\pi F)^2$)

$$y = 1 + \delta x + \frac{\alpha}{3(4\pi F)^2} \frac{2\Sigma}{F^2} z + O(m_1^2, m_2^2)$$

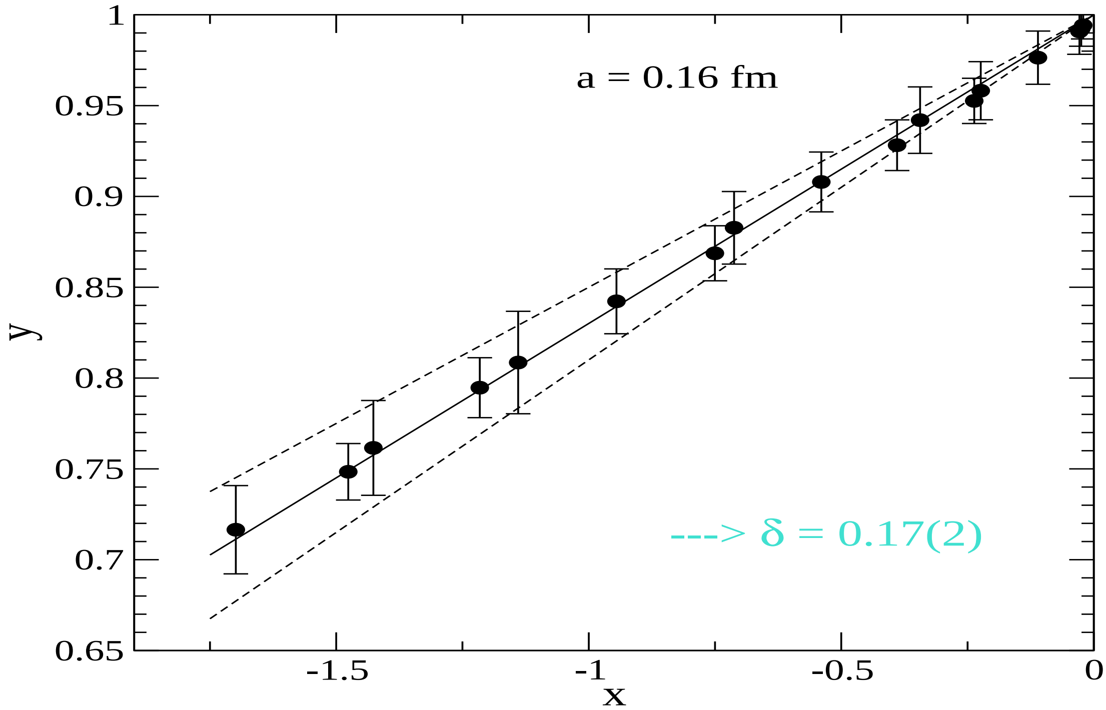
where

$$x = 2 + \tfrac{m_1+m_2}{m_1-m_2} \log\left(\tfrac{m_2}{m_1}\right)$$

$$z = \left(\tfrac{2m_1 m_2}{m_2-m_1} \log\left(\tfrac{m_2}{m_1}\right) - m_1 - m_2 \right)$$

- The explicit dependence on μ_χ is removed
- This ratio has been re-analyzed from the BGR coll. to extract δ

BGR collaboration: quenched chiral logs

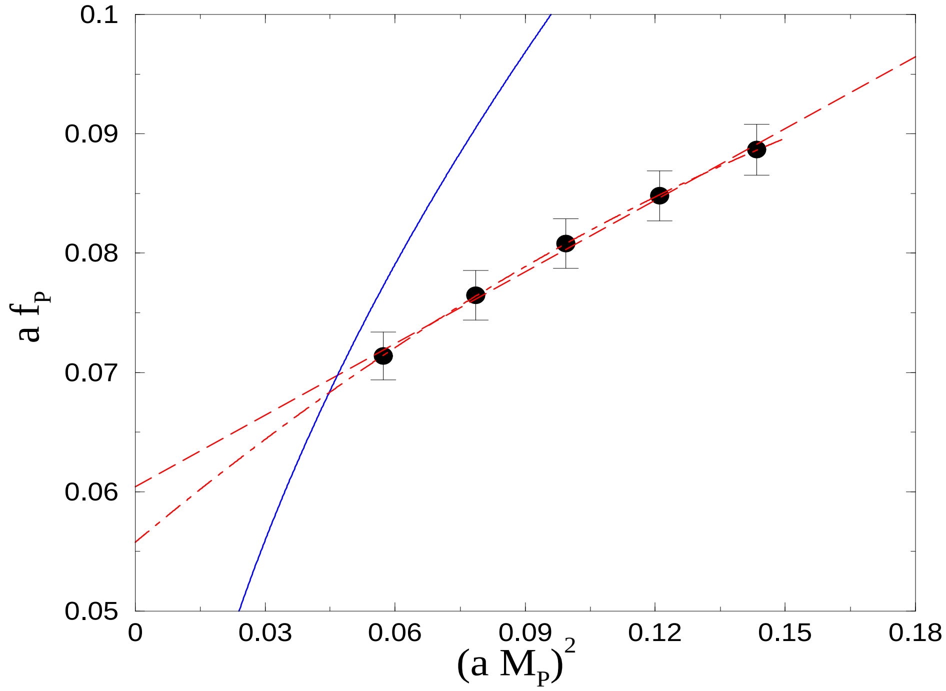


Collaboration	Action	δ
CP-PACS '98	Wilson	$0.10(2)$
Bardeen et al. '00	modified Wilson	$0.065(13)$
Kentucky '01	overlap	$0.23 - 0.48$
RBC '01	DW	$0.05(2)$
BGR-Coll. '02	Fixed-Point	$0.17(2)$
BGR-Coll. '02	Chirally-Improved	$0.18(4)$
Kentucky '02	overlap	$0.248(12)$
Chiu-Hsieh	overlap	0.203(14)

- More studies to properly assess the systematics

Light quark masses

(L. G., C. Hoelbling, C. Rebbi)



- On the plane $[(af_P), (aM_P)^2]$, $f_P/M_P = f_K^{\rm exp}/M_K^{\rm exp}$

$$aM_K = 0.216(8) \qquad af_K = 0.0698(26)$$

- From $[(aM_P)^2, (am)]$ with $a_{f_K}^{-1} = 2.29(9)$

$$(m_s + \hat{m})^{\rm RI}(2\,{\rm GeV}) = 120 \pm 7 \pm 21\,{\rm MeV}$$

- By using χPT and N^{2}LO PT

$$m_s^{\overline{\rm MS}}(2{\rm GeV}) = 102 \pm 6 \pm 18 \;\;{\rm MeV}$$

Conclusions

- Exact chiral symmetry on the lattice at finite cut-off
- Domain-wall-overlap: explicit chirally symmetric regularization
- Quenched large scale numerical simulations are feasible
  Regime of quark masses not reachable with Wilson fermions
- First phenomenological computations performed
  Results indicate small discretization errors

$\cdots\cdots$ More on Friday !!