Broken symmetries and lattice gauge theory (II and III): chiral anomaly and the Witten–Veneziano mechanism

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- Octet compatible with SSB pattern

$$\mathrm{SU}(3)_{\mathrm{L}} \times \mathrm{SU}(3)_{\mathrm{R}} \to \mathrm{SU}(3)_{\mathrm{L+R}}$$

and soft explicit symmetry breaking

$$m_u, m_d \ll m_s < \Lambda$$

- $m_u, m_d \ll m_s \Longrightarrow m_\pi \ll m_{\mathrm{K}}$

- A 9^{th} pseudoscalar with $m_{\eta'} \sim \mathcal{O}(\Lambda)$

I	I_3	S	Meson	Quark Content	Mass (GeV)
1	1	0	π^+	$u\bar{d}$	0.140
1	-1	0	π^-	$d\bar{u}$	0.140
1	0	0	π^0	$(d\bar{d} - u\bar{u})/\sqrt{2}$	0.135
$\frac{1}{2}$	$\frac{1}{2}$	+1	K^+	$u\bar{s}$	0.494
$\frac{1}{2}$	-$\frac{1}{2}$	+1	K^0	$d\bar{s}$	0.498
$\frac{1}{2}$	-$\frac{1}{2}$	-1	K^-	$s\bar{u}$	0.494
$\frac{1}{2}$	$\frac{1}{2}$	-1	$\overline{\mathrm{K}}^0$	$s\bar{d}$	0.498
0	0	0	η	$\cos\vartheta\eta_8 - \sin\vartheta\eta_0$	0.548
0	0	0	η'	$\sin\vartheta\eta_8 + \cos\vartheta\eta_0$	0.958

$$\eta_8 = (d\bar{d} + u\bar{u} - 2s\bar{s})/\sqrt{6}$$
$$\eta_0 = (d\bar{d} + u\bar{u} + s\bar{s})/\sqrt{3}$$
$$\vartheta \sim -10^\circ$$

QCD action and its (broken) symmetries

- QCD action for $N_F = 2$, $M^\dagger = M = \text{diag}(m, m)$

$$S = S_G + \int d^4x \left\{ \bar\psi D \psi + \bar\psi_{\rm R} M^\dagger \psi_{\rm L} + \bar\psi_{\rm L} M \psi_{\rm R} \right\}\, , \qquad D = \gamma_\mu(\partial_\mu - iA_\mu)$$

- For $M = 0$ chiral symmetry

$$\psi_{\rm R,L} \to V_{\rm R,L}\psi_{\rm R,L} \quad \psi_{\rm R,L} = \left(\frac{1 \pm \gamma_5}{2}\right)\psi$$

Chiral anomaly: measure not invariant

SSB: vacuum not symmetric

$$\text{SU}(3)_c \times \text{SU}(2)_{\rm L} \times \text{SU}(2)_{\rm R} \times \text{U}(1)_{\rm L} \times \text{U}(1)_{\rm R} \times \mathcal{R}_{\rm scale}$$

↓ (dim. transm., chiral anomaly)

$$\text{SU}(3)_c \times \text{SU}(2)_{\rm L} \times \text{SU}(2)_{\rm R} \times \text{U}(1)_{\rm B=L+R}$$

↓ (Spont. Sym. Break.)

$$\text{SU}(3)_c \times \text{SU}(2)_{\rm L+R} \times \text{U}(1)_{\rm B}$$

↓ (Confinement)

$$\text{SU}(2)_{\rm L+R} \times \text{U}(1)_{\rm B}$$

- Breaking due to non-perturbative dynamics. Precise quantitative tests are being made on the lattice

Numerical challenge

- A Monte Carlo computation of

$$\chi_{\rm L}^{\rm YM} = \frac{1}{V}\Big\langle (n_+ - n_-)^2 \Big\rangle^{\rm YM}$$

is challenging for several reasons

- $L \sim 1$ fm and $a \sim 0.08$ fm $\Longrightarrow$ $\boxed{\dim[D] \sim 2.5\ 10^5}$: computing and diagonalizing the full matrix not feasible

- A standard minimization would require high precision to beat contamination from quasi-zero modes

- At large V the probability distribution has a width which increases linearly with V

$$P_Q = \frac{1}{\sqrt{2\pi V \chi_{\rm L}^{\rm YM}}} e^{-\frac{Q^2}{2V\chi_{\rm L}^{\rm YM}}} \{1 + O(V^{-1})\}$$

$\Longrightarrow$ computing $\chi_{\rm L}^{\rm YM}$ requires very high statistics

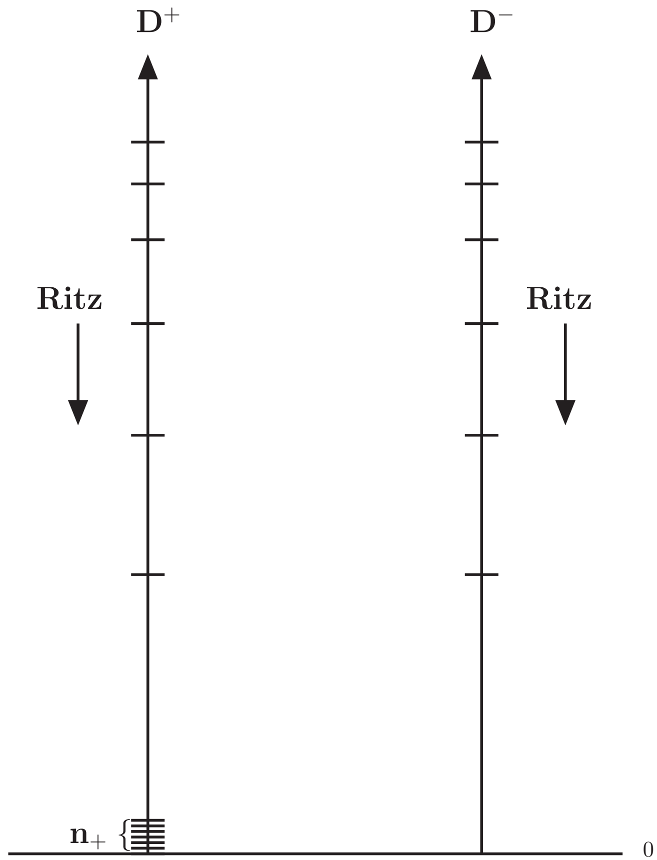
Algorithm for zero-mode counting

- In finite V null prob. for $n_+ \neq 0$ and $n_- \neq 0$
- Simultaneous minimization of Ritz functionals associated to

$$D^{\pm} = P_{\pm} D P_{\pm} \qquad P_{\pm} = \frac{1 \pm \gamma_5}{2}$$

to find the gap in one of the sectors

- Run again the minimization in the sector without gap and count zero modes
- No contamination from quasi-zero modes



Non-perturbative computation for $N_c = 3$ [Del Debbio et al. 04; Cè et al. 14]

- With the GW definition a fit of the form

$$r_0^4 \chi^{\rm YM}(a,s) = r_0^4 \chi^{\rm YM} + c_1(s)\,\frac{a^2}{r_0^2}$$

gives

$$r_0^4 \chi^{\rm YM} = 0.059 \pm 0.003$$

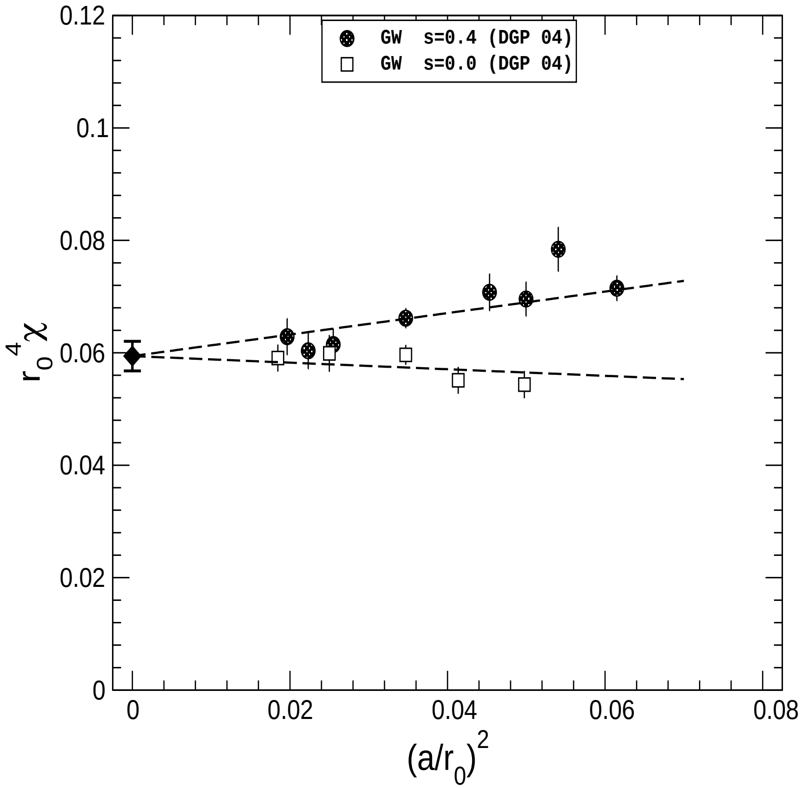
- By setting the scale $F_{\rm K} = 109.6$ MeV

$$\chi^{\rm YM} = (0.185 \pm 0.005 \ {\rm GeV})^4$$

to be compared with

$$\frac{F^2}{2N_F}(M_\eta^2 + M_{\eta'}^2 - 2M_K^2) \underset{\rm exp}{\approx} (0.180 \ {\rm GeV})^4$$

- The (leading) QCD anomalous contribution to $M_{\eta'}^2$ supports the Witten–Veneziano explanation for its large experimental value



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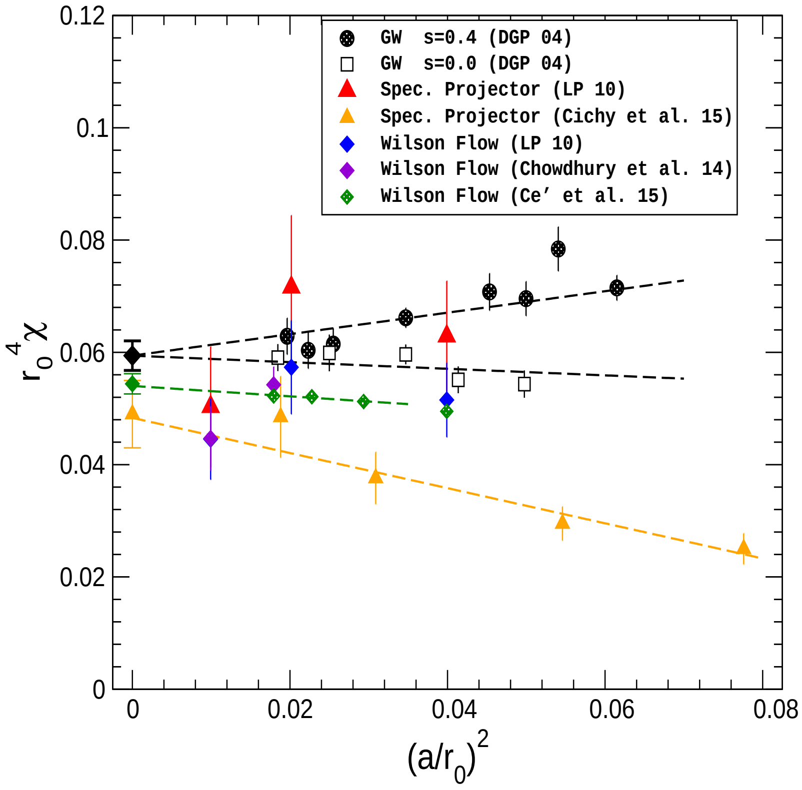
- With the Wilson flow definition

$$r_0^4 \chi^{\rm YM} = 0.054 \pm 0.002$$

which corresponds to

$$\chi^{\rm YM} = (0.181 \pm 0.004\ {\rm GeV})^4$$



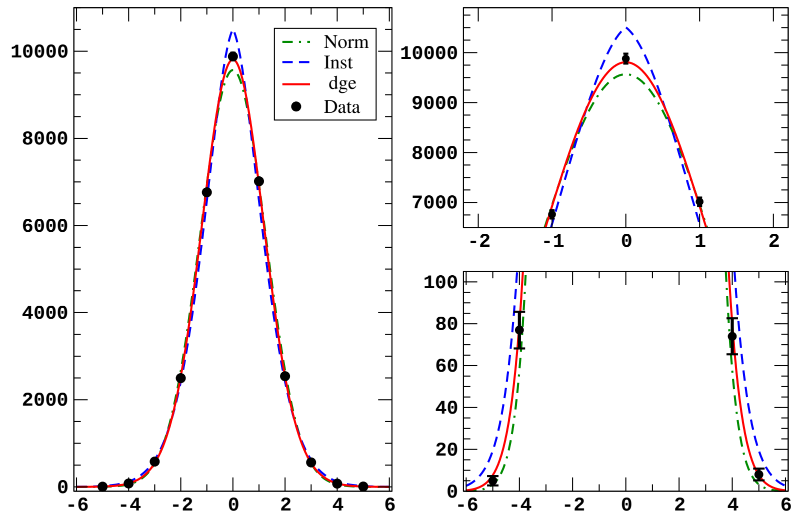
- From an unsolved problem to a universality test of lattice gauge theory!

How the WV mechanism works ? [LG, Petrarca, Taglienti 07; Cè et al. 14]

- Vacuum energy and charge distribution are

$$e^{-F(\theta)} = \langle e^{i\theta Q}\rangle, \; P_Q = \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} e^{-i\theta Q} e^{-F(\theta)}$$

Their behaviour is a distinctive feature of the configurations that dominate the path integr.



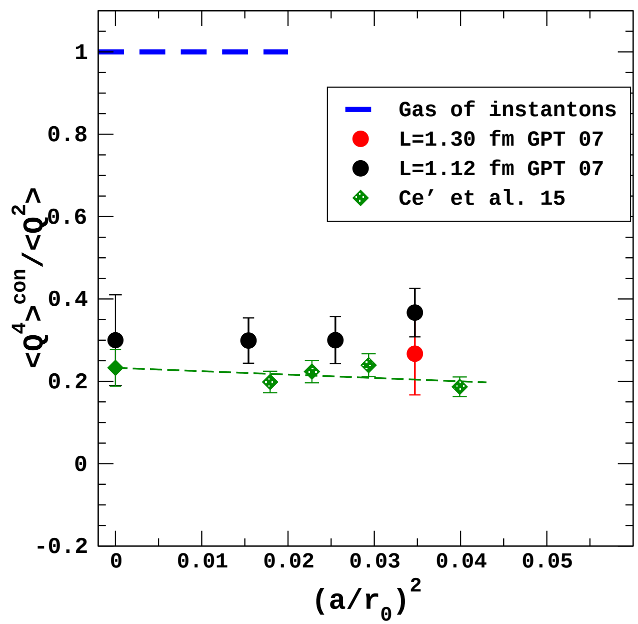
- Large N_c predicts ['t Hooft 74; Witten 79; Veneziano 79]

$$\frac{\langle Q^{2n}\rangle^{\rm con}}{\langle Q^2\rangle} \propto \frac{1}{N_c^{2n-2}}$$

- Various conjectures. For example, dilute-gas instanton model gives ['t Hooft 76; Callan et al. 76; . . .]

$$F^{\rm Inst}(\theta) = -VA\{\cos(\theta) - 1\}$$

$$\frac{\langle Q^{2n}\rangle^{\rm con}}{\langle Q^2\rangle} = 1$$



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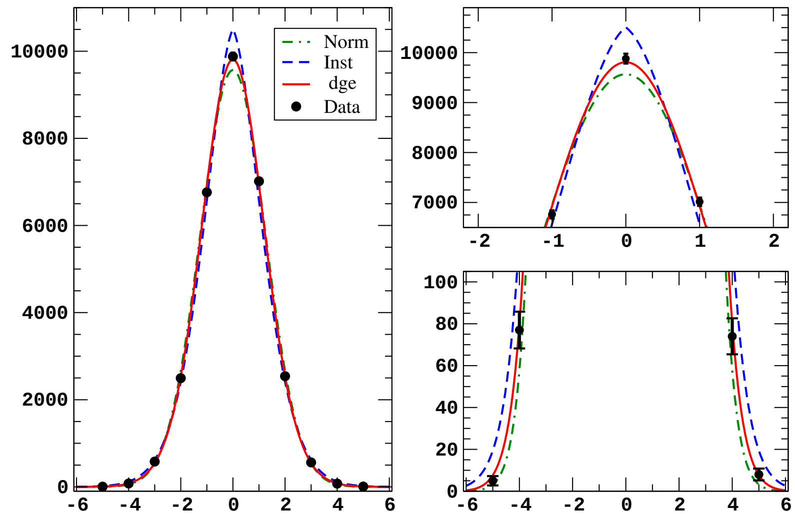
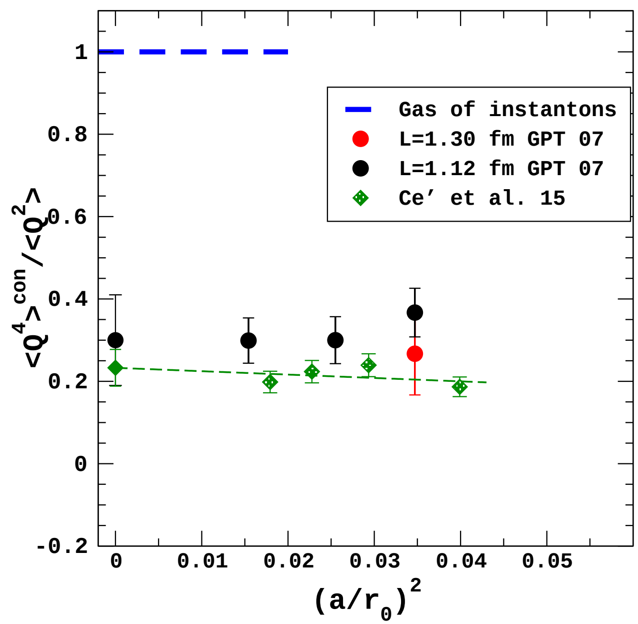


- Lattice computations give

$$\frac{\langle Q^4\rangle^{\rm con}}{\langle Q^2\rangle} = 0.30 \pm 0.11 \quad \text{Ginsparg–Wilson}$$
$$= 0.23 \pm 0.05 \quad \text{Wilson-Flow}$$



i.e. supports large N_c and disfavours a dilute gas of instantons

- The anomaly gives a mass to the η' thanks to the NP quantum fluctuations of Q