Broken symmetries and lattice gauge theory (I): LGT, a theoretical femtoscope for non-perturbative strong dynamics

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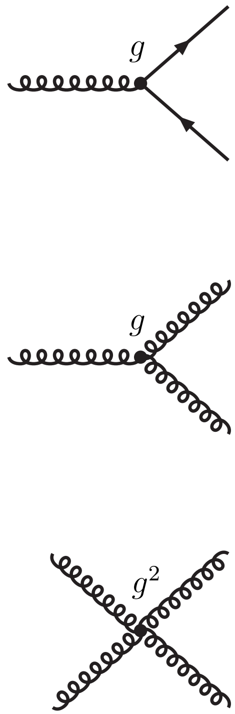
Quantum Chromodynamics (QCD)

- QCD is the quantum field theory of strong interactions in Nature. Its action

[Fritzsch, Gell-Mann, Leutwyler 73; Gross, Wilczek 73; Weinberg 73]

$$S[A, \bar{\psi}_i, \psi_i; g, m_i, \theta]$$

is fixed by few simple principles:



  - ∗ SU(3)$_c$ gauge (local) invariance
  - ∗ Quarks in fundamental representation $\psi_i = u, d, s, c, b, t$
  - ∗ Renormalizability

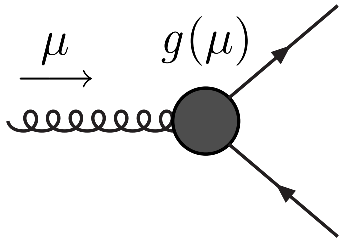
- Present experimental results compatible with $\theta = 0$

- It is fascinating that such a simple action and few parameters $[g, m_i]$ can account for the variety and richness of strong-interaction physics phenomena

Asymptotic freedom

- The renormalized coupling constant is scale dependent

$$\mu \frac{d}{d\mu} g = \beta(g)$$



and QCD is asymptotically free [$b_0 > 0$]
[Gross, Wilczek 73; Politzer 73]

$$\beta(g) = -b_0 g^3 - b_1 g^5 + \dots$$

- The theory develops a fundamental scale

$$\Lambda = \mu \left[b_0 g^2(\mu)\right]^{-b_1/2b_0^2} e^{-\frac{1}{2b_0 g^2(\mu)}} e^{-\int_0^{g(\mu)} dg \left[\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g}\right]}$$

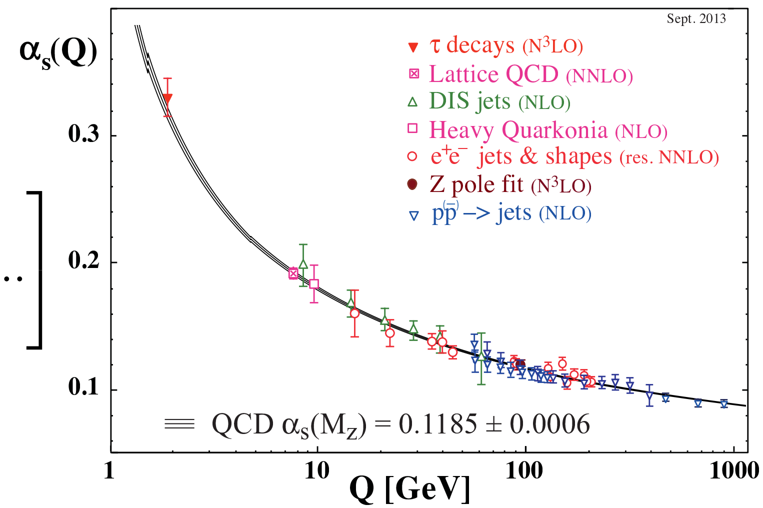
which is a non-analytic function of the coupling constant at $g^2 = 0$. Quantization breaks scale invariance at $m_i = 0$

Perturbative corner: hard processes

- Processes where the relevant energy scale is $\mu \gg \Lambda$ can be studied by pert. expansion

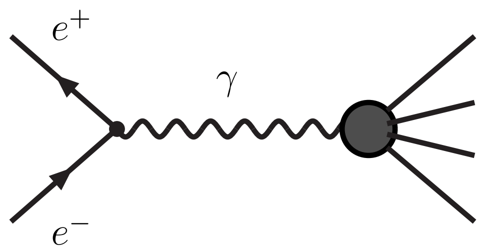
$$\alpha_s(\mu) = \frac{g^2(\mu)}{4\pi} = \frac{1}{4\pi b_0 \ln\left(\frac{\mu^2}{\Lambda^2}\right)}\left[1 - \frac{b_1}{b_0^2}\frac{\ln(\ln(\frac{\mu^2}{\Lambda^2}))}{\ln(\frac{\mu^2}{\Lambda^2})} + \ldots\right]$$

[PDG 2014]



- An example is given by

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_i Q_i^2 \cdot \left[1 + \frac{\alpha_s(\mu)}{\pi} + C_2\left(\frac{\alpha_s(\mu)}{\pi}\right)^2 + \cdots\right]$$



- Experimental results significantly prove the logarithmic dependence in μ/Λ predicted by perturbative QCD

Scale of the strong interactions

- By comparing these measurements to theory

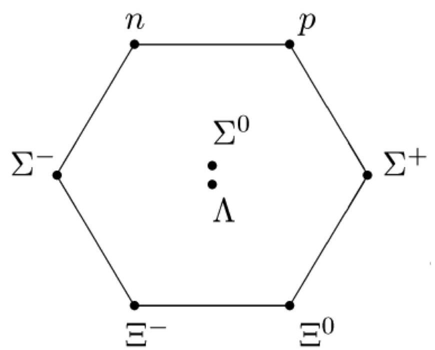
$$\Lambda \sim 0.2 \text{ GeV} \qquad 1/\Lambda \sim 1 \text{ fm} = 10^{-15} \text{ m}$$

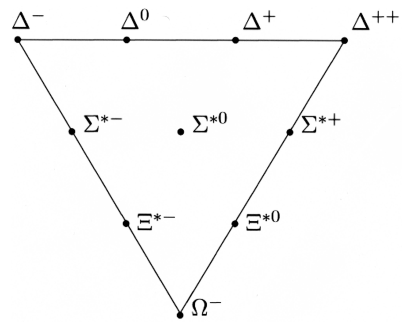
- At these distances the dynamics of QCD is non-perturbative

- A rich spectrum of hadrons is observed at these energies. Their properties such as

$$M_n = b_n \, \Lambda$$

need to be computed non-perturbatively





- The theory is highly predictive: in the (interesting) limit $m_{u,d,s} = 0$ and $m_{c,b,t} \to \infty$, for instance, dimensionless quantities are parameter-free numbers

Light pseudoscalar meson spectrum

- Octet compatible with SSB pattern

$$\mathrm{SU}(3)_{\mathrm{L}} \times \mathrm{SU}(3)_{\mathrm{R}} \to \mathrm{SU}(3)_{\mathrm{L+R}}$$

and soft explicit symmetry breaking

$$m_u, m_d \ll m_s < \Lambda$$

- $m_u, m_d \ll m_s \Longrightarrow m_\pi \ll m_{\mathrm{K}}$

- A 9^{th} pseudoscalar with $m_{\eta'} \sim \mathcal{O}(\Lambda)$

I	I_3	S	Meson	Quark Content	Mass (GeV)
1	1	0	π^+	$u\bar{d}$	0.140
1	-1	0	π^-	$d\bar{u}$	0.140
1	0	0	π^0	$(d\bar{d} - u\bar{u})/\sqrt{2}$	0.135
$\frac{1}{2}$	$\frac{1}{2}$	+1	K^+	$u\bar{s}$	0.494
$\frac{1}{2}$	-$\frac{1}{2}$	+1	K^0	$d\bar{s}$	0.498
$\frac{1}{2}$	-$\frac{1}{2}$	-1	K^-	$s\bar{u}$	0.494
$\frac{1}{2}$	$\frac{1}{2}$	-1	$\overline{\mathrm{K}}^0$	$s\bar{d}$	0.498
0	0	0	η	$\cos\vartheta\eta_8 - \sin\vartheta\eta_0$	0.548
0	0	0	η'	$\sin\vartheta\eta_8 + \cos\vartheta\eta_0$	0.958

$$\eta_8 = (d\bar{d} + u\bar{u} - 2s\bar{s})/\sqrt{6}$$
$$\eta_0 = (d\bar{d} + u\bar{u} + s\bar{s})/\sqrt{3}$$
$$\vartheta \sim -10^\circ$$

QCD action and its (broken) symmetries

- QCD action for $N_F = 2$, $M^\dagger = M = \mathrm{diag}(m, m)$

$$S = S_G + \int d^4x \left\{ \bar\psi D\psi + \bar\psi_{\rm R} M^\dagger \psi_{\rm L} + \bar\psi_{\rm L} M \psi_{\rm R} \right\}, \qquad D = \gamma_\mu(\partial_\mu - iA_\mu)$$

- For $M = 0$ chiral symmetry

$$\psi_{\rm R,L} \to V_{\rm R,L}\psi_{\rm R,L} \quad \psi_{\rm R,L} = \left(\frac{1 \pm \gamma_5}{2}\right)\psi$$

Chiral anomaly: measure not invariant

SSB: vacuum not symmetric

$$\mathrm{SU(3)}_c\times \mathrm{SU(2)_L} \times \mathrm{SU(2)_R}\times \mathrm{U(1)_L} \times \mathrm{U(1)_R} \times\mathcal{R}_{\rm scale}$$

$$\Big\downarrow \quad \textbf{(dim. transm., chiral anomaly)}$$

$$\mathrm{SU(3)}_c\times \mathrm{SU(2)_L} \times \mathrm{SU(2)_R} \times\mathrm{U(1)_{B=L+R}}$$

$$\Big\downarrow \quad \textbf{(Spont. Sym. Break.)}$$

$$\mathrm{SU(3)}_c\times \mathrm{SU(2)_{L+R}} \times\mathrm{U(1)_B}$$

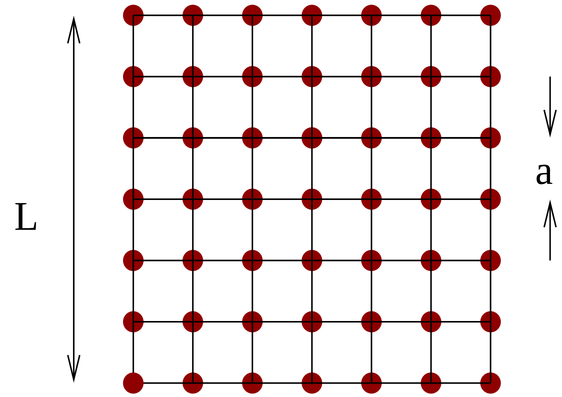
$$\Big\downarrow \quad \text{(Confinement)}$$

$$\mathrm{SU(2)_{L+R}} \times\mathrm{U(1)_B}$$

- Breaking due to non-perturbative dynamics. Precise quantitative tests are being made on the lattice

Lattice QCD: action [Wilson 74]

- QCD can be defined on a discretized space-time so that gauge invariance is preserved
- Quark fields reside on four-dimensional lattice, the gauge field $U_\mu \in \mathrm{SU}(3)$ resides on links

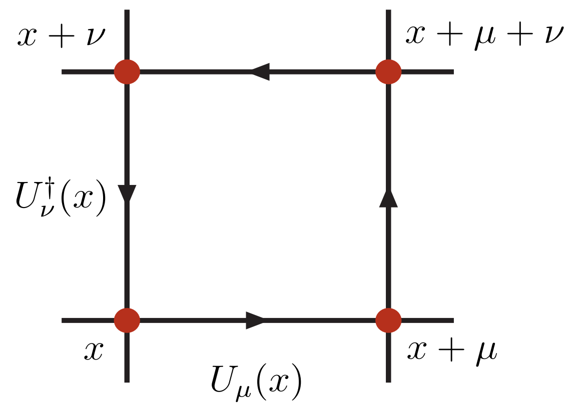


- The Wilson action for the gauge field is

$$S_G[U] = \frac{\beta}{2} \sum_x \sum_{\mu,\nu} \Big[1 - \frac{1}{3} \mathrm{ReTr} \Big\{ U_{\mu\nu}(x) \Big\} \Big]$$

where $\beta = 6/g^2$ and the plaquette is

$$U_{\mu\nu}(x) = U_\mu(x)\, U_\nu(x+\hat{\mu})\, U^\dagger_\mu(x+\hat{\nu})\, U^\dagger_\nu(x)$$



- Popular discretizations of fermion action: Wilson, Domain-Wall-Neuberger, tmQCD

Lattice QCD: path integral [Wilson 74]

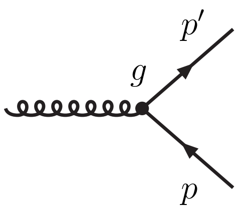
- The lattice provides a non-perturbative definition of QCD. The path integral at finite spacing and volume is mathematically well defined (Euclidean time)

$$Z = \int DU\, D\bar{\psi}_i D\psi_i \; e^{-S[U,\bar{\psi}_i,\psi_i;g,m_i]}$$

- Nucleon mass, for instance, can be extracted from the behaviour of a suitable two-point correlation function at large time-distance

$$\langle O_N(x)\bar{O}_N(y)\rangle = \frac{1}{Z}\int DU\, D\bar{\psi}_i D\psi_i \; e^{-S}\, O_N(x)\bar{O}_N(y) \longrightarrow R_N \; e^{-M_N\,|x_0-y_0|}$$

- For small gauge fields, the pert. expansion differs from usual one for terms of $O(a)$



$$= -igT^a \left\{\gamma_\mu - \tfrac{i}{2}(p_\mu + p'_\mu)a + O(a^2)\right\}$$

Consistency of lattice QCD with standard perturbative approach is thus guaranteed

Lattice QCD: universality

- Continuum and infinite-volume limit of Lattice QCD is the *non-perturbative definition* of QCD

- Details of the discretization become irrelevant in the continuum limit, and any reasonable lattice formulation tends to the same continuum theory

$$M_N(a) = M_N + c_N a + \ldots$$

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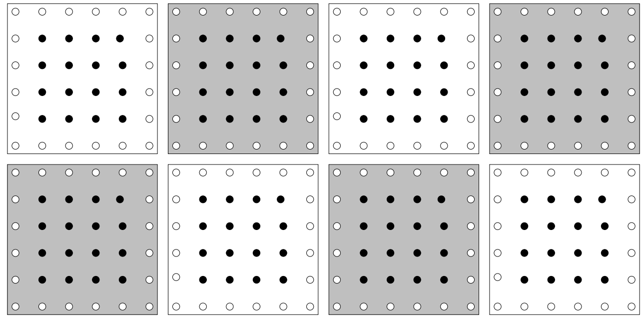
- By a proper tuning of the action and operators, convergence to continuum can be accelerated without introducing extra free-parameters
  [Symanzik 83; Sheikholeslami Wohlert 85; Lüscher et al. 96]

- Finite-volume effects are proportional to $\exp(-M_\pi L)$ at asymptotically large volumes

Numerical lattice QCD: machines

- Correlation functions at $finite\ volume$ and $finite\ lattice\ spacing$ can be computed by Monte Carlo techniques $exactly$ up to statistical errors



- Look at quantities not accessible to experiments:

  $*$ quark mass dependence

  $*$ volume dependence

  $*$ unphysical quantities $\Sigma, \chi, \ldots$

  for understanding...

[Galileo – CINECA]

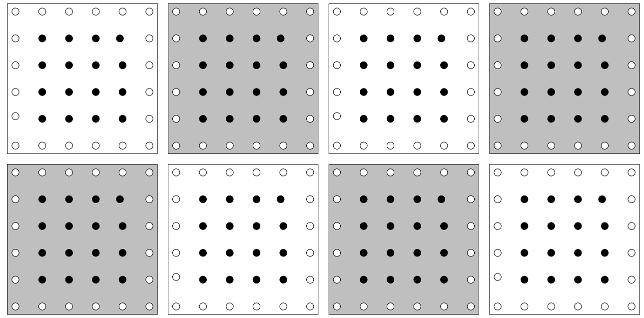
Numerical lattice QCD: machines

- Typical lattice parameters:

$a \;= 0.05$ fm $\qquad (a\Lambda)^2 \sim 0.25\%$

$L = 3.2$ fm $\qquad \Longrightarrow \quad M_\pi L \geq 4, \quad M_\pi \geq 0.25$ GeV

$V = 2L \times L^3 \qquad$ #points $= 2^{25} \sim 3.4 \cdot 10^7$



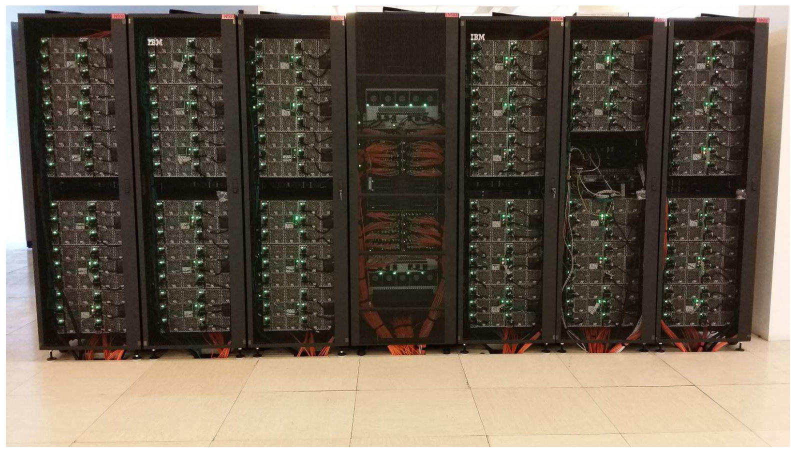
- Monte Carlo algorithms integrate over 10^7–10^9 SU(3) link variables

- A typical cluster of PCs:

  $*$ Standard CPUs [Intel, AMD]

  $*$ Fast connection [40Gbit/s]

[Galileo – CINECA]



- Lattice partitioned in blocks which are distributed over the nodes (256×16 a good example)

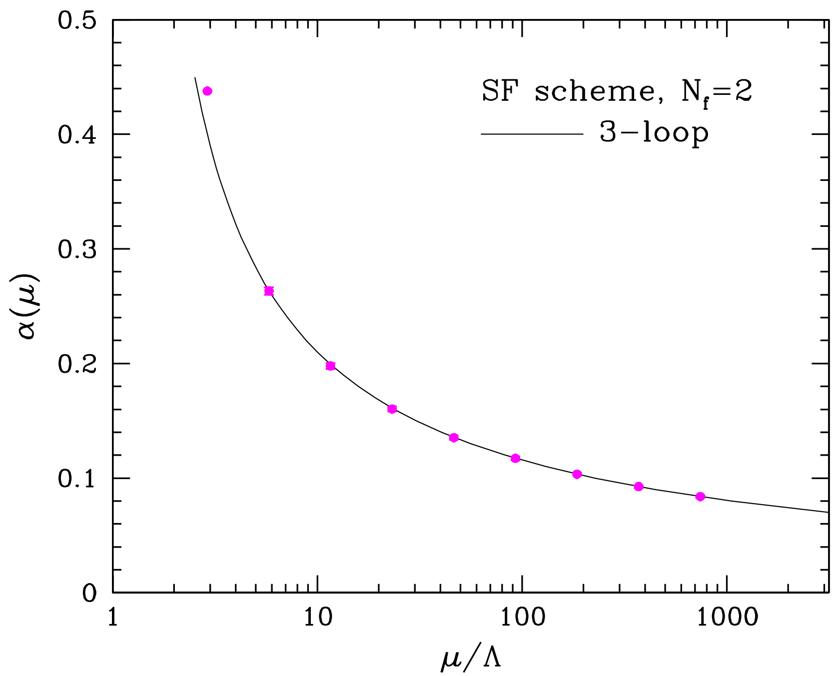
- Data exchange among nodes minimized thanks to the locality of the action

Numerical lattice QCD: algorithms

- Extraordinary algorithmic progress over the last 30 years, keywords:

  * Hybrid Monte Carlo (HMC)
    Duane et al. 87
  * Multiple time-step integration
    Sexton, Weingarten 92
  * Frequency splitting of determinant
    Hasenbusch 01
  * Domain Decomposition
    Lüscher 04
  * Mass preconditioning and rational HMC
    Urbach et al 05; Clark, Kennedy 06
  * Deflation of low quark modes
    Lüscher 07
  * Avoiding topology freezing
    Lüscher, Schaefer 12

[Della Morte et al. 05]



- Light dynamical quarks can be simulated. Chiral regime of QCD is accessible

- Algorithms are designed to produce exact results up to statistical errors

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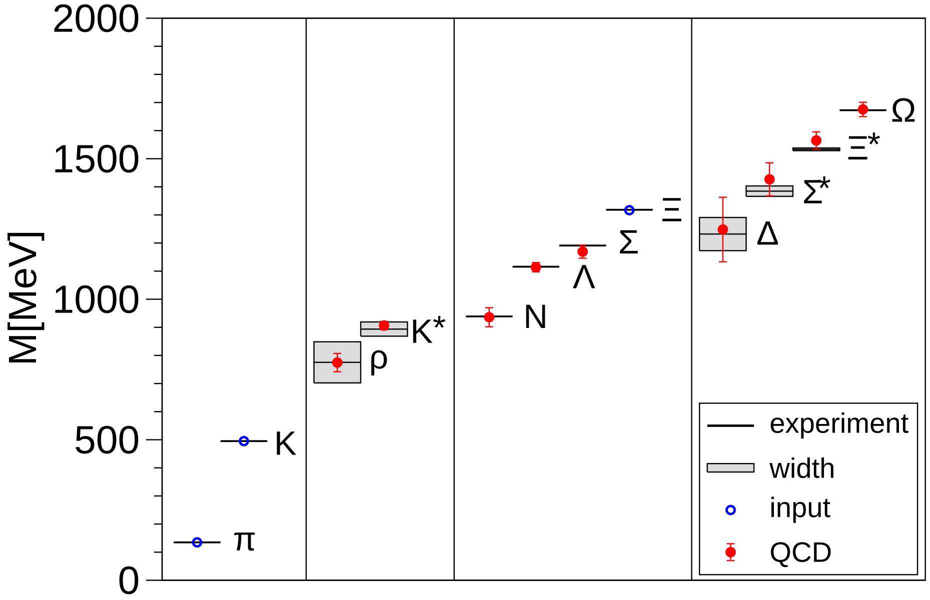
[BMW Collaboration 09]



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- Lattice QCD is the femtoscope for studying strong dynamics. Its lenses are made of quantum field theory, numerical techniques and computers

- It allows us to look also at quantities not accessible to experiments which may help understanding the underlying mechanisms

- Femtoscope still rather crude. Often we compute what we can and not what would like to

- An example: the signal-to-noise ratio of the nucleon two-point correlation function

$$\frac{\langle O_N \bar{O}_N \rangle^2}{\Delta^2} \propto n\, e^{-(2M_N - 3M_\pi)|x_0 - y_0|}$$

  decreases exp. with time-distance of sources. At physical point $2M_N – 3M_\pi \simeq$ 7 fm^{-1}

Lattice quantum field theory

$\downarrow\uparrow$

Observables (probes)

$\downarrow\uparrow$

Algorithms

$\downarrow\uparrow$

Computers

Lattice QCD: a theoretical femtoscope

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- Femtoscope still rather crude. Often we compute what we can and not what would like to
- A rather general strategy is emerging: design special purpose algorithms which exploit known math. and phys. properties of the theory to be faster
- Results from first-principles when all syst. uncertainties quantified. This achieved without introducing extra free parameters or dynamical assumptions but just by improving the femtoscope