Dalitz plot
an introduction

Particelle III 2016

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Dalitz plot

- A Dalitz plot is a visual representation of the phase space of a three body decay involving only spin-0 particles
- Named after its inventor Richard Dalitz (1925-2006) who first applied it to the "τ–θ puzzle" (strange particles that decay to 2 or 3 pions, now understood as different decay modes of kaons) R.H. Dalitz, Phil. Mag 44 (1953) 1068

- For a 3 body decay M→1+2+3 there are
  - three 4-vectors for momenta of final-state particles
  - 10 constraints
  - → only 2 independent quantities

- Can chose the two most convenient variables

Constraints	Degree of freedom
3 four-vectors	12
4-momentum conservation	-4
3 masses	-3
3 Euler angles	-3
TOT	2

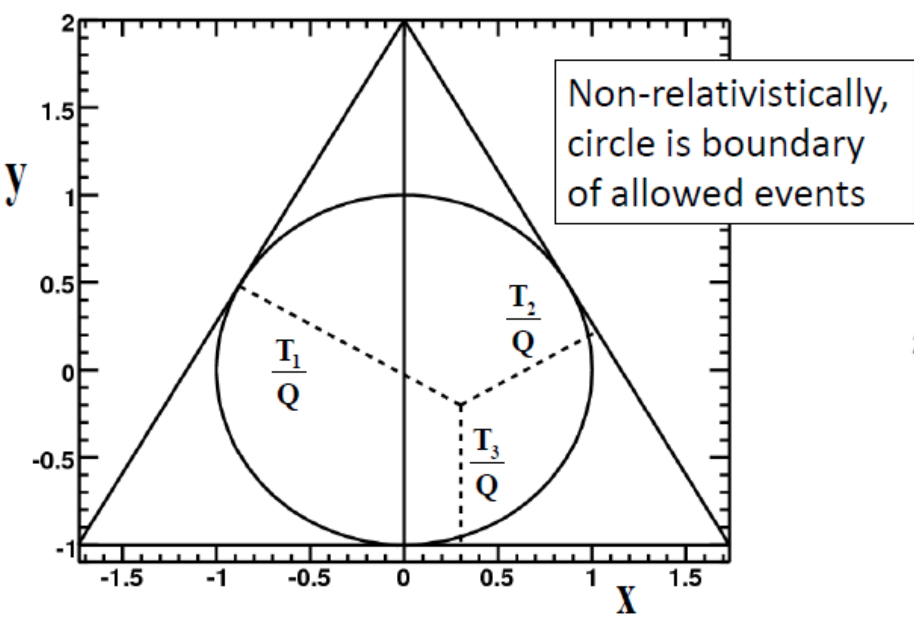
The original Dalitz plot

- Choose kinetic energies of 2 different final state particles (non relativistic regime)

  Constraint: $T_1+T_2+T_3 = Q$ energy release in the decay

  $Q = M-m_1-m_2-m_3$

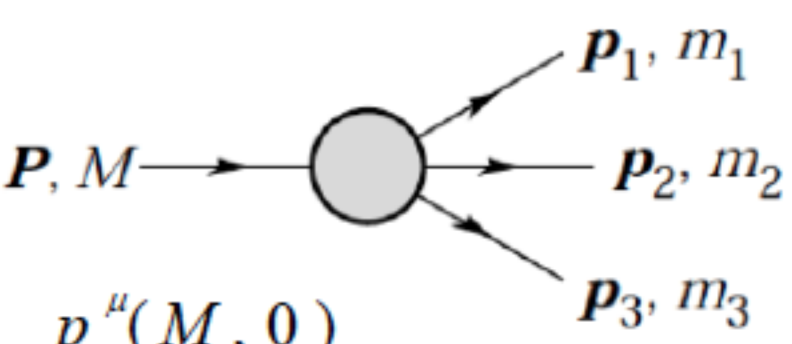
- Choose an independent set of T_i/Q
- Points lie within boundary of circle of unit radius $T_1/Q+T_2/Q+T_3/Q=1$

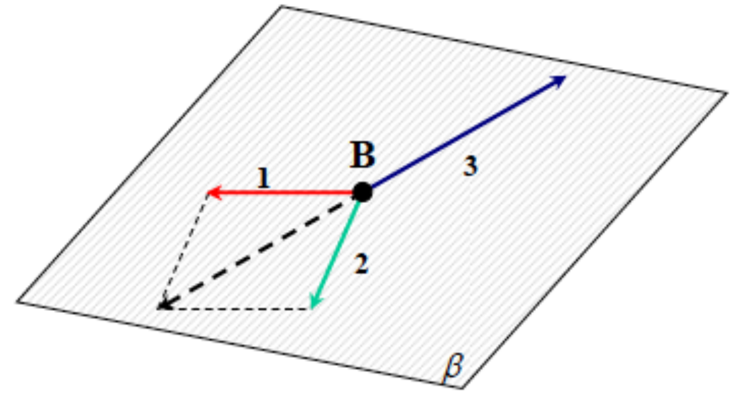


$$x = \frac{\sqrt{3}(T_1 - T_2)}{Q}; \qquad y = \frac{2T_3 - T_1 - T_2}{Q}$$

$$Q = T_1 + T_2 + T_3 = M - m_1 - m_2 - m_3$$

Three bodies decay $M \rightarrow 1+2+3$





$$p^{\mu}(M, 0)$$
$$p_1^{\mu}(E_1, \vec{p}_1)$$
$$p_2^{\mu}(E_2, \vec{p}_2)$$
$$p_3^{\mu}(E_3, \vec{p}_3)$$

$$m_{12}^2 = (p_1^{\mu} + p_2^{\mu})^2 = (p^{\mu} + p_3^{\mu})^2 = M^2 + m_3^2 - 2ME_3$$
$$m_{23}^2 = (p_2^{\mu} + p_3^{\mu})^2 = (p^{\mu} + p_1^{\mu})^2 = M^2 + m_1^2 - 2ME_1$$
$$m_{13}^2 = (p_1^{\mu} + p_3^{\mu})^2 = (p^{\mu} + p_2^{\mu})^2 = M^2 + m_2^2 - 2ME_2$$

$$m_{12}^2 + m_{23}^2 + m_{13}^2 = 3M^2 + m_1^2 + m_2^2 + m_3^2 - 2M(E_1 + E_2 + E_3)$$
$$= 3M^2 + m_1^2 + m_2^2 + m_3^2 - 2M^2 = M^2 + m_1^2 + m_2^2 + m_3^2 = \text{costant}$$

- Lorentz invariant phase space volume

$$dN \propto \int \frac{p_1 dp_1 p_2 dp_2 p_3 dp_3}{E_1 E_2 E_3} \delta(E_1 + E_2 + E_3 - M)$$

$$E_1^2 = p_1^2 + m_1^2 \rightarrow 2E_1 dE_1 = 2p_1 dp_1$$

$$dN \propto \int dE_1\, dE_2 dE_3 \delta(E_1 + E_2 + E_3 - M) = \int dE_1\, dE_2$$

or any other two variables

- Therefore $\boxed{dN \propto dm_{12}^2 dm_{23}^2}$

- Decay rate $$\boxed{d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{32M^3} \overline{|\mathcal{M}|^2} dm_{12}^2 dm_{23}^2}$$

Invariant amplitude

- The three body decay is described by the square of an invariant Amplitude
- If the invariant amplitude is constant the Dalitz plot will be uniformly populated

  Non-uniformity in population of Dalitz plot gives information on final-state interactions.

  – In particular 2-body resonances show up

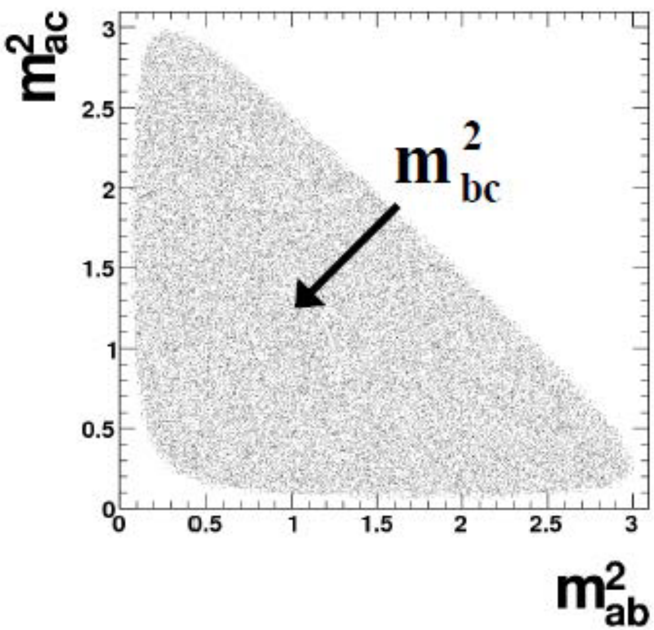
Dalitz plot Relativistic Kinematics

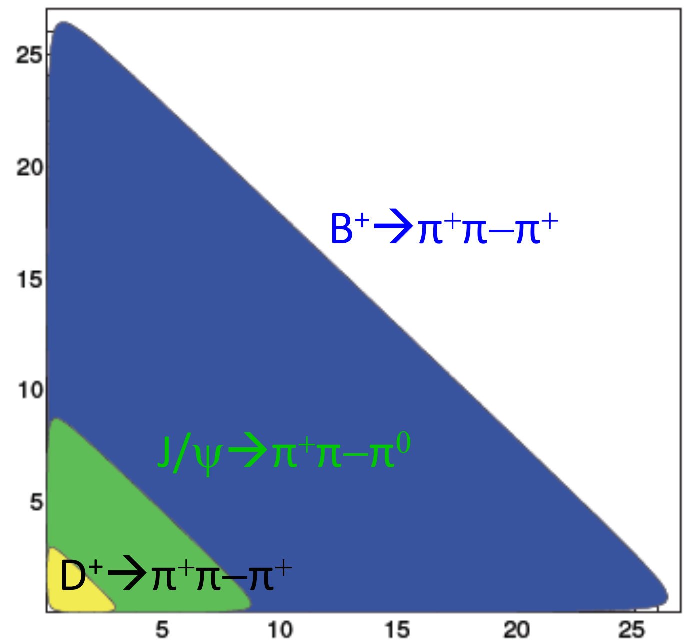
M→a+b+c

- Choose invariant mass squared of two different pairs
- In relativistic regime Q→∞ DP shape is more similar to triangle

$$x = m_{ab}^2 = s_{ab} = (p_a^\mu + p_b^\mu)^2$$

$$y = m_{ac}^2 = s_{ac} = (p_a^\mu + p_c^\mu)^2$$



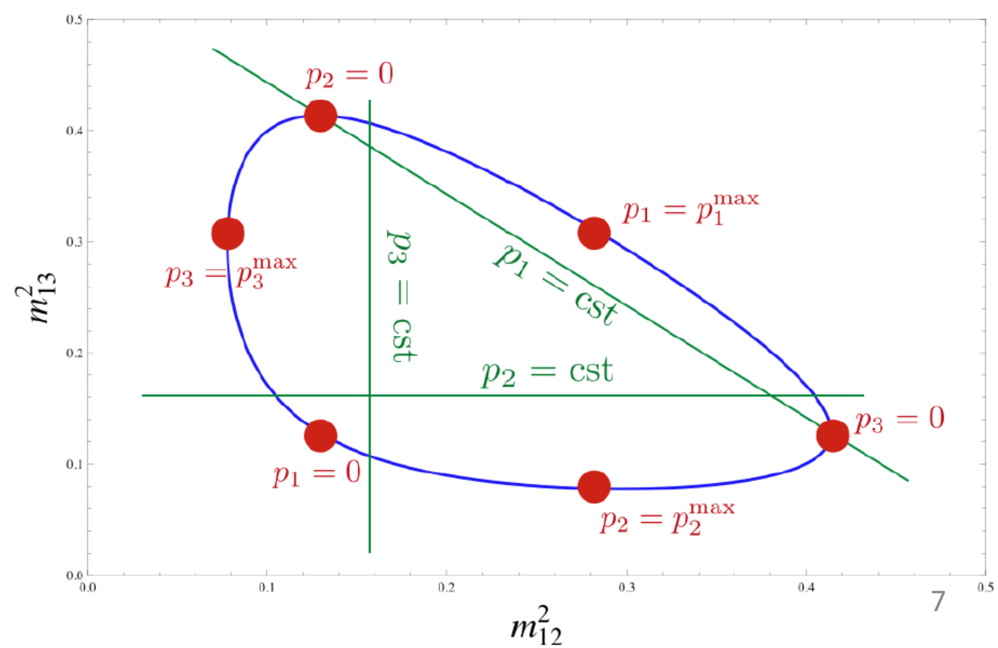


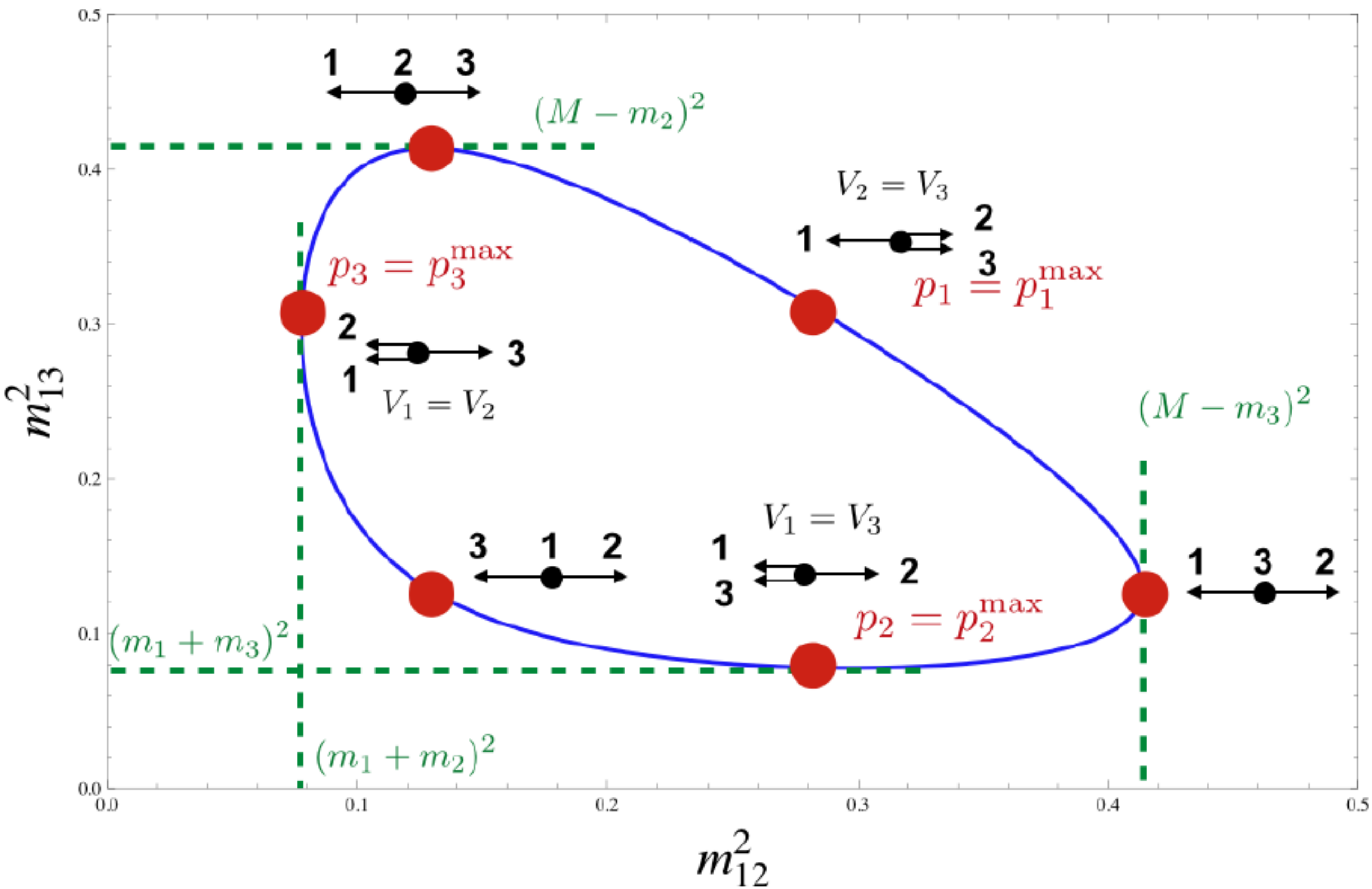
Dalitz plot Relativistic Kinematics

In CM frame of M→1+2+3

- Boundaries of Dalitz plot determined by kinematic conditions.
- Maximum m_{12} corresponds to $p_3 = 0$
  - Similarly max m_{13} (m_{23}) corresponds to $p_2=0$ ($p_1=0$)
  - For a given $\mathbf{p_1}$ and $\mathbf{p_2}$, the maximum m_{12} is when $\mathbf{p_1}$ and $\mathbf{p_2}$ are in opposite directions
- Constant m_{13} line correspond to constant p_2 and constant $m_{12}^2+m_{23}^2$ line
  - Similarly constant m_{12} (m_{23}) line correspond to constant p_3 (p_1)

$$s_{12} = (p_1^\mu + p_2^\mu)^2 = p_1^2 + p_2^2 + p_1 \cdot p_2$$
$$= m_1^2 + m_2^2 + E_1 E_2 - \vec{\mathbf{p}}_1 \cdot \vec{\mathbf{p}}_2$$

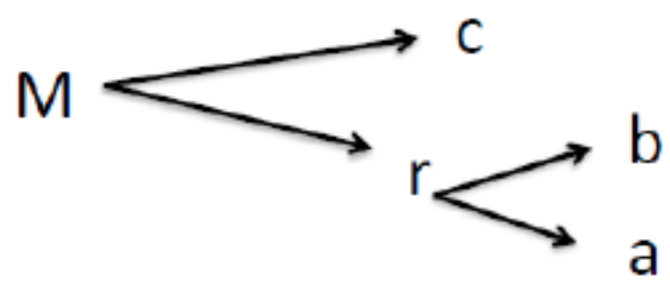




0.5
0.4
0.3
0.2
0.1
0.0
m_{13}^2
0.0
0.1
0.2
0.3
0.4
0.5
m_{12}^2
1 2 3
$(M-m_2)^2$
$V_2 = V_3$
1 2 3
$p_3 = p_3^{\max}$
$p_1 = p_1^{\max}$
2 3 1
$V_1 = V_2$
$(M-m_3)^2$
$V_1 = V_3$
3 1 2
1 2 3
1 3 2
$p_2 = p_2^{\max}$
$(m_1+m_3)^2$
$(m_1+m_2)^2$

Three-Body Decay Through Intermediate Resonance

Three-body decays will often take place through an intermediate resonant state "r" that subsequently undergoes two-body decay,



We can describe the behavior of the resonance by using a relativistic Breit-Wigner amplitude

$$\mathcal{A}_{BW} \sim \frac{1}{M_r^2 - s_{ab} - i\Gamma M_r} \; ; \quad \Gamma = \frac{\hbar}{\tau}$$

Γ is inverse of lifetime τ of resonant state

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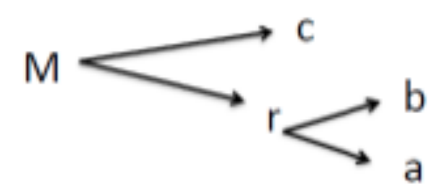
- When the decay has one intermediate resonant state [ab]

2-Body Resonances in Dalitz Plots



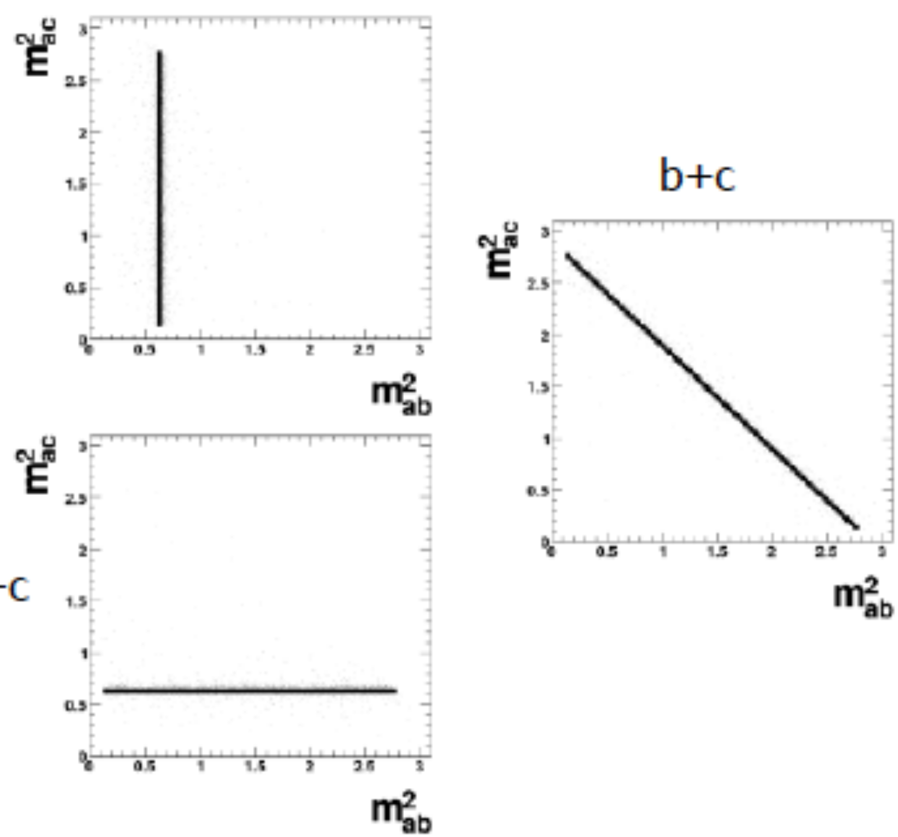
A 2-body resonance will appear on a Dalitz plot as a sharp enhancement, corresponding to the pair of particles that forms the resonance

$$\mathcal{A}_{BW}(ab) \sim \frac{1}{M_r^2 - s_{ab} - i\Gamma M_r}$$

a+b

b+c

a+c



If Γ=0 events from the resonance appear in the DP as a line at $m_{ab}=M_r$

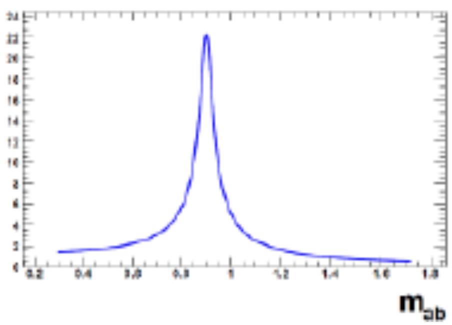
Identification of Resonances in Dalitz Plot

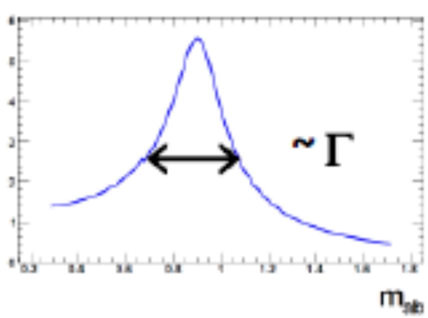
Use relativistic Breit-Wigner form for resonance,

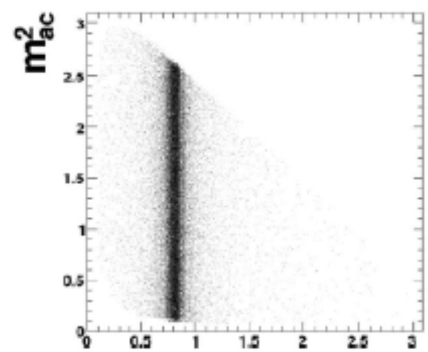
$$\mathcal{A}_{BW}(ab) \sim \frac{1}{M_r^2 - s_{ab} - i\Gamma M_r}$$

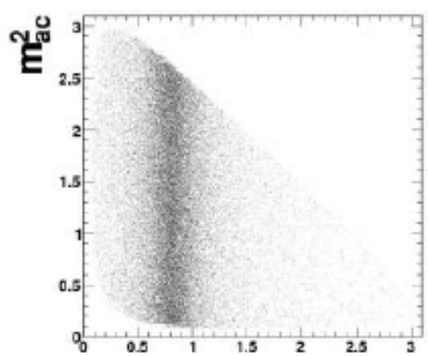
Γ is inverse of lifetime of resonant state

Magnitude









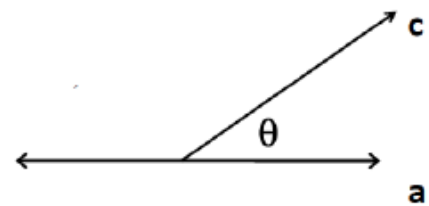
Width of resonance can be inferred from sharpness of resonant line; mass M_r from location on axis.

- If a,b,c are spinless particles and resonance [ab] has spin S, then the decay amplitude will have zeros corresponding to Legendre Polynomials
- Spin-S resonance will have S zeros in Dalitz plot.

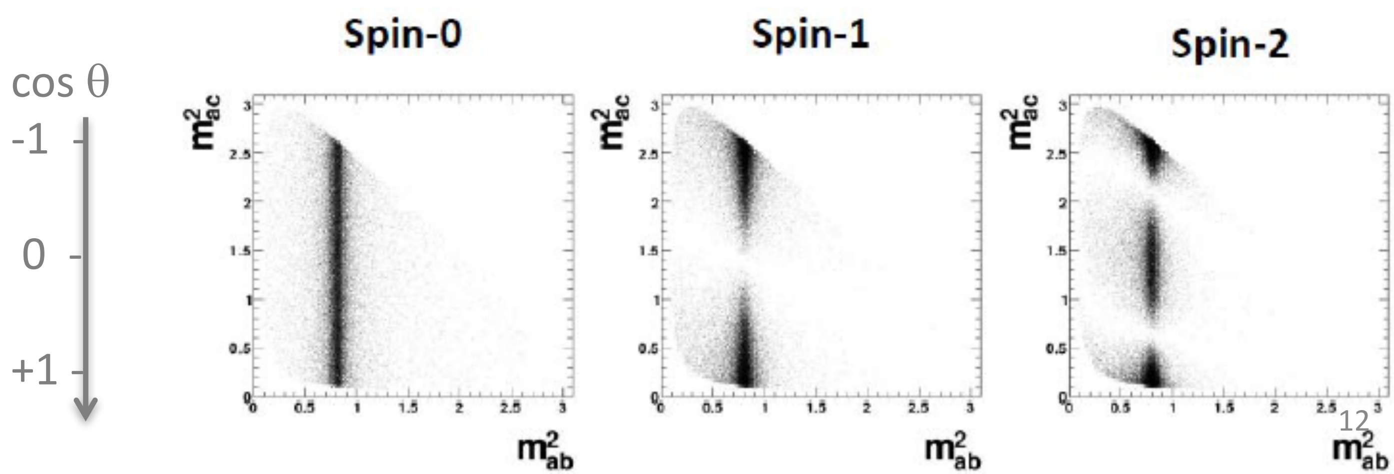
$$\mathcal{A} \sim \mathcal{A}_{BW}\, P_S(\cos\theta)\ ;$$
$$P_0(\cos\theta) = 1$$
$$P_1(\cos\theta) = \cos\theta$$
$$P_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1)$$

- Helicity angle (in the rest frame of [ab] pair)

$$\cos\theta = \frac{(m_{ac}^2)_{\max} + (m_{ac}^2)_{\min} - 2m_{ac}^2}{(m_{ac}^2)_{\max} - (m_{ac}^2)_{\min}}$$



$$m_{ac}^2 = (m_{ac}^2)_{\max} \rightarrow \cos\theta = -1$$
$$m_{ac}^2 = [(m_{ac}^2)_{\max} + (m_{ac}^2)_{\min}]/2 \rightarrow \cos\theta = 0$$
$$m_{ac}^2 = (m_{ac}^2)_{\min} \rightarrow \cos\theta = +1$$

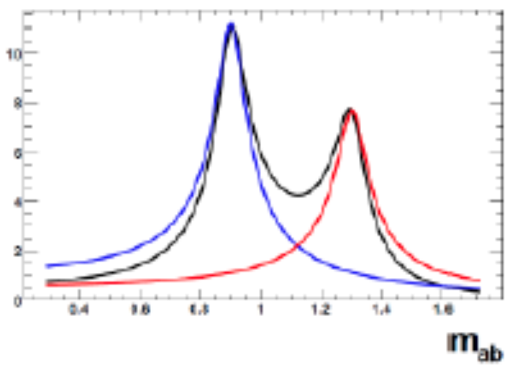


- When the decay can go through two [ab] resonant states, with different masses, width and phases, interference can be constructive or destructive.

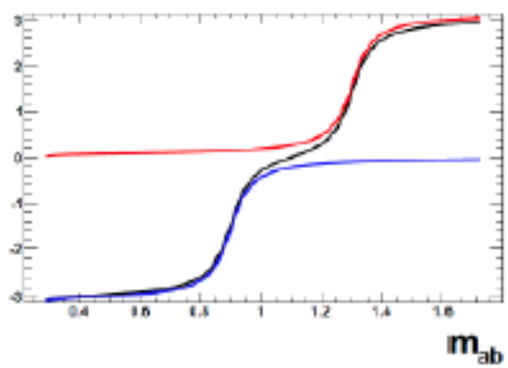
Interference of Overlapping Resonances in Dalitz Plot

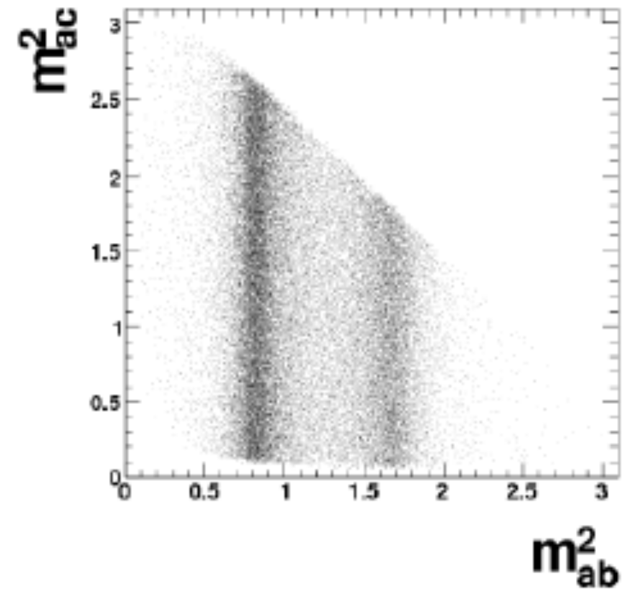
Constructive interference between a pair of resonances in [ab]

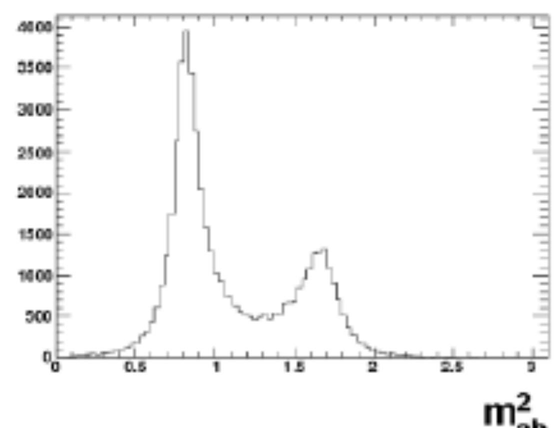
Magnitude



Phase



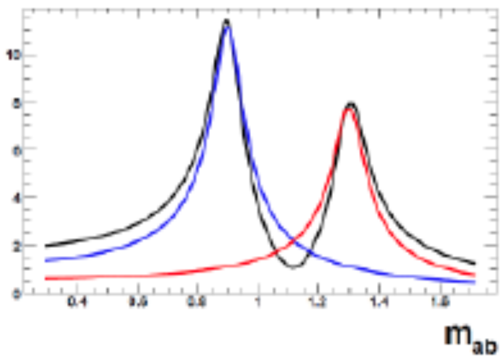




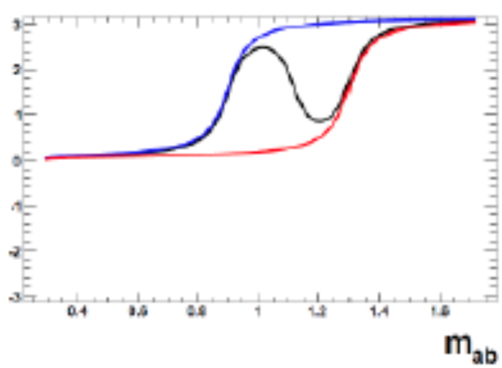
Interference of Resonances in Dalitz Plot (2)

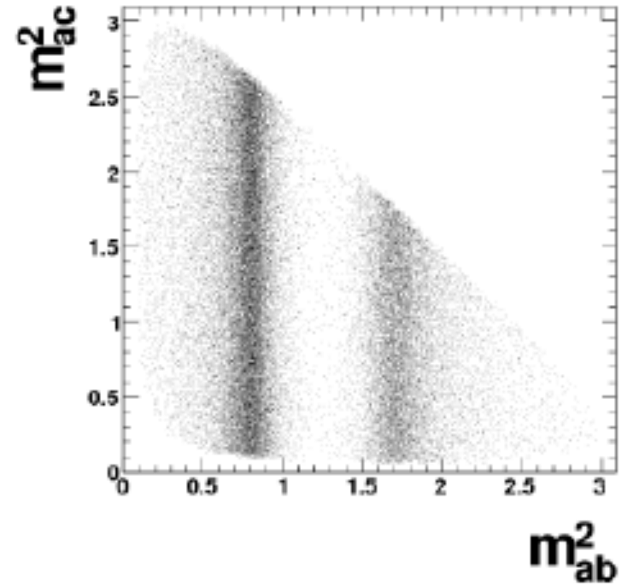
Destructive interference between a pair of resonances in [ab]

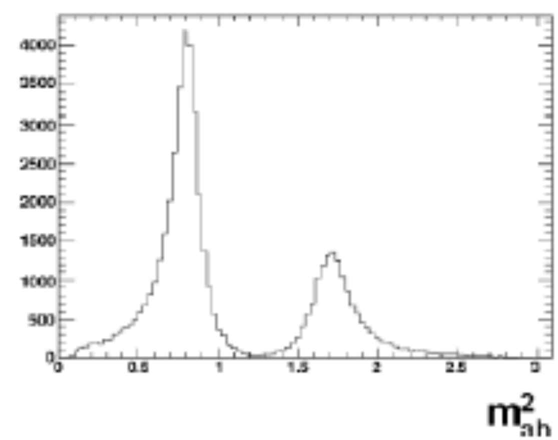
Magnitude



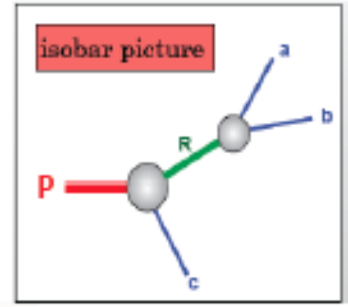
Phase



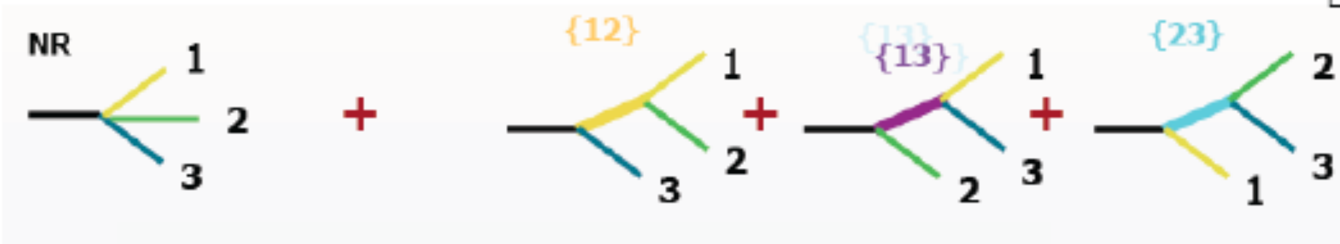




Applications of Isobar Model



Total 3-body decay amplitude = coherent sum of 2-body resonant processes plus background?



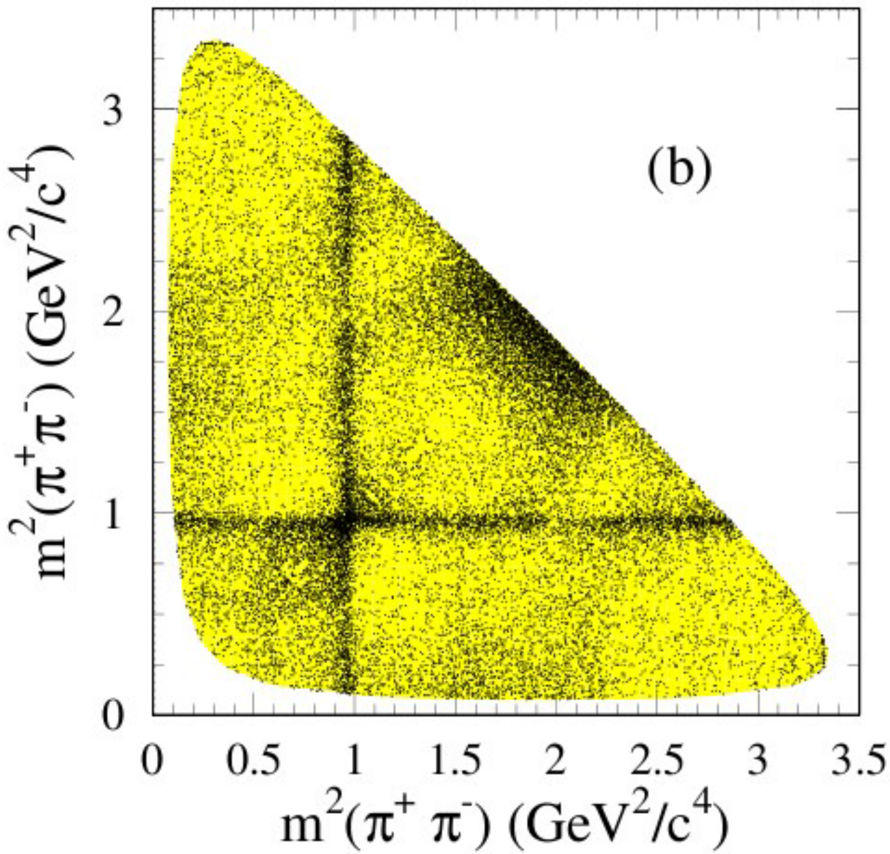
$$\mathcal{A}_{\mathcal{D}}(s_{12},s_{13}) = a_0 e^{i\delta_0} + \sum_r a_r e^{i\delta_r} \mathcal{A}_r(s_{12},s_{13})$$

NR term(direct 3 body decay)

Relativistic Breit-Wigner parameters extracted from resonance properties (or from particle data tables);
a_i, δ_i are constants that are determined through a maximum-likelihood fit
Can measure fractions and relative phases of different isobars

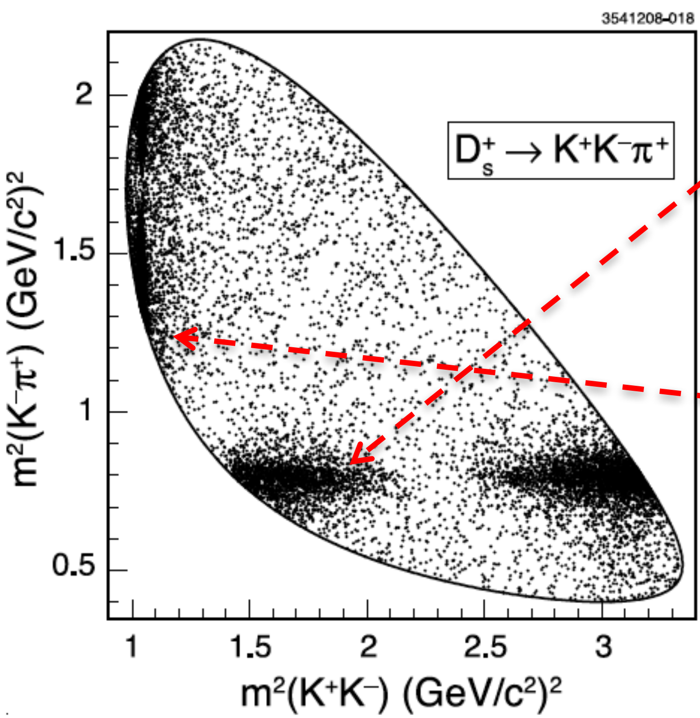
- Final state has 3 identical particles: the DP is symmetric..

$D_s^+ \to \pi^+\pi^-\pi^+$
BaBar PRD 79 (2009) 032003



Dalitz Plot Analysis of $D_s^+ \to K^+K^-\pi^+$

CLEO Phys.Rev.D79:072008,2009



Resonance	Parameter (MeV/c^2)	Central Fit	Floated	PDG [8]
$K^*(892)$	m	895.8±0.5	895.8±0.5	896.00±0.25
	Γ	44.2±1.0	44.2±1.0	50.3±0.6
$K_0^*(1430)$	m	1414	1422±23	1414±6
	Γ	290	239±48	290±21
$f_0(980)$	m	965	933±21	980±10
	$g_{\pi\pi}$	406	393±36	Γ=40 to 100
	g_{KK}	800	557±88	
$\phi(1020)$	m	1019.460	1019.64±0.05	1019.460±0.019
	Γ	4.26	4.780±0.14	4.26±0.05
$f_0(1370)$	m	1350	1315±34	1200 to 1500
	Γ	265	276±39	200 to 500
$f_0(1710)$	m	1718	1749±12	1718±6
	Γ	137	175±29	137±8