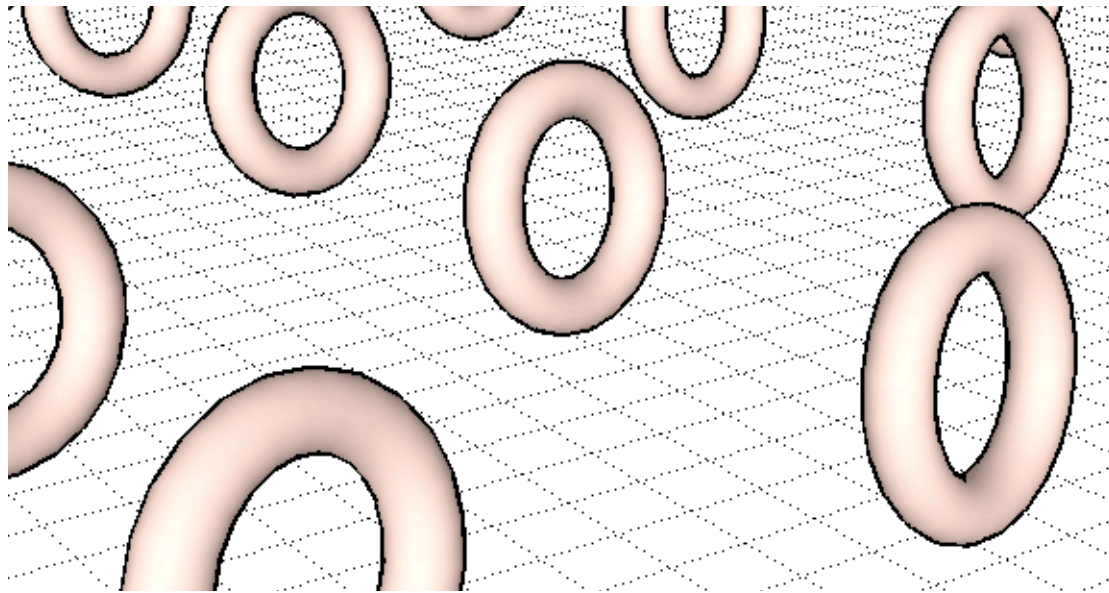
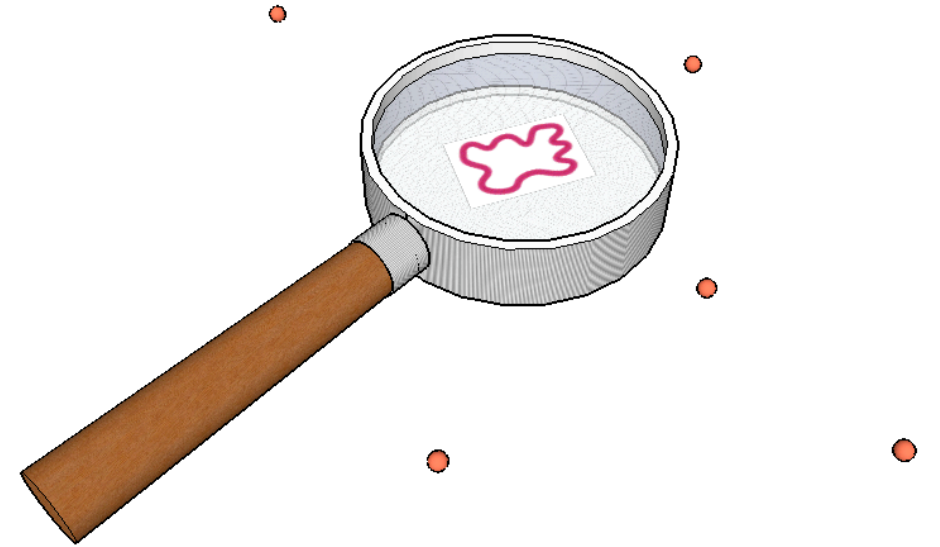


# The Extra Dimensions of String Theory

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# String theory

is a proposal for the unification of forces



Essential ingredient:  
extra dimensions

My work tries to characterize  
the **shape** of the extra dimensions

- I have obtained  
general classification results; [Graña, Minasian, Petrini, AT'05]
- I have found many examples  
of allowed shapes. [AT'07]

The internal shape is constrained  
by the laws of gravity that we are familiar with.

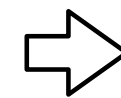
[Einstein's equations, string-corrected]

Only.. in ten dimensions.

Before my work, two stages:

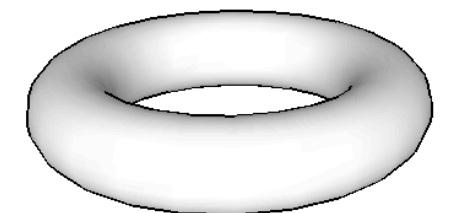
- 1. take the internal space to be **empty**

[even that can be challenging!]



internal space is  
'Ricci-flat'

Einstein's  
equations



“Calabi-Yau”

[Calabi'57; Yau'77;  
Candelas, Horowitz, Strominger, Witten '85]

Such spaces can rescale at no cost in energy



Many massless scalar particles!

- 2. Fix the problem by making the rescaling cost energy:

EM field strength

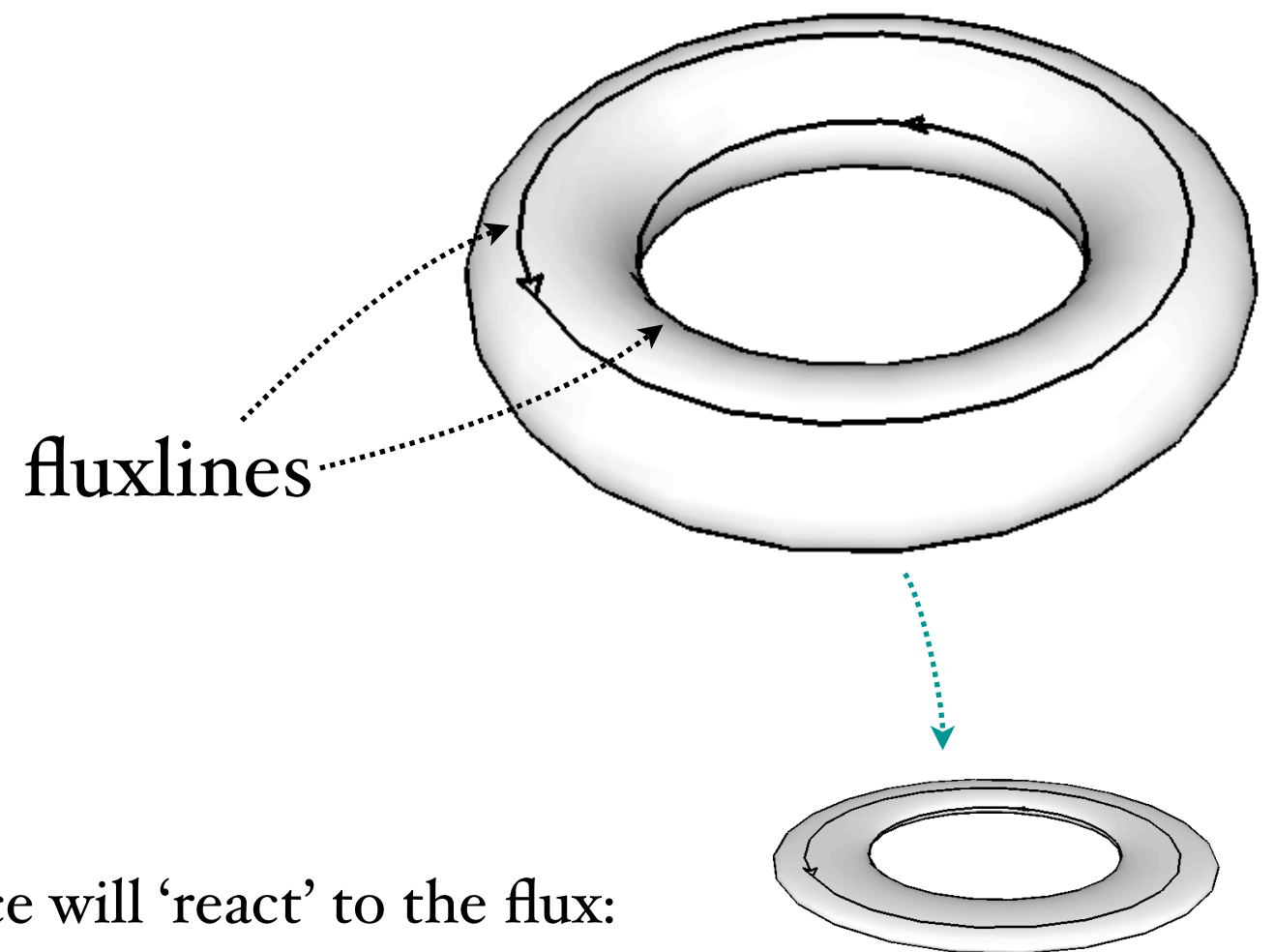
$$F_{mn} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_y & -B_z \\ E_y & -B_y & 0 & B_x \\ E_z & B_z & -B_x & 0 \end{pmatrix}$$

string theory has generalizations  $F_{m_1 \dots m_k}$   
antisymmetric

fill the space with EM-like flux

$$E = - \int \cancel{R} + F^2$$

no curvature  
in this case

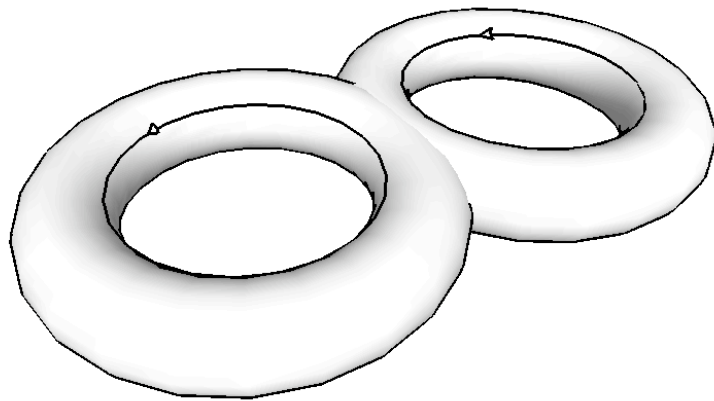


the space will 'react' to the flux:

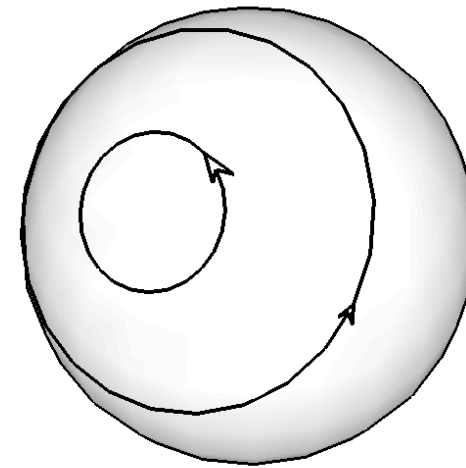
no longer 'Ricci flat'

My idea:

why not look among all spaces from  
the start?



?



A general classification looked too difficult

until

we assumed **supersymmetry** in the vacuum

We obtained:

[Graña, Minasian, Petrini, AT '05, '06]

- 1. Elegant 1st order equations in which gravity appears EM-like

$$\begin{aligned} d\Phi_- &= F \\ d\Phi_+ &= 0 \end{aligned}$$

'gravity fluxes!' (pointing to  $d\Phi_-$ )  
string theory 'EM' (pointing to  $F$ )

- 2. A general mathematical statement:  
internal space is 'generalized complex'

name given in [Hitchin '02]

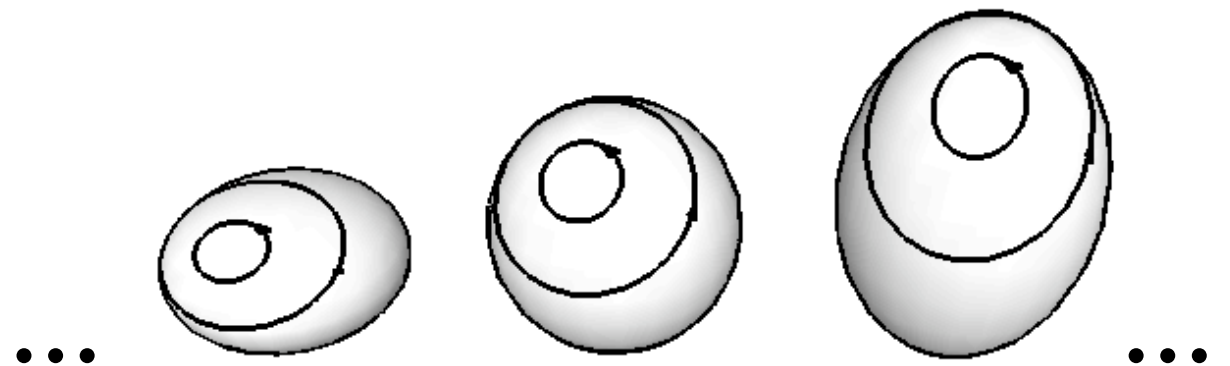
Recently, I found a new class of examples:

[AT'07]

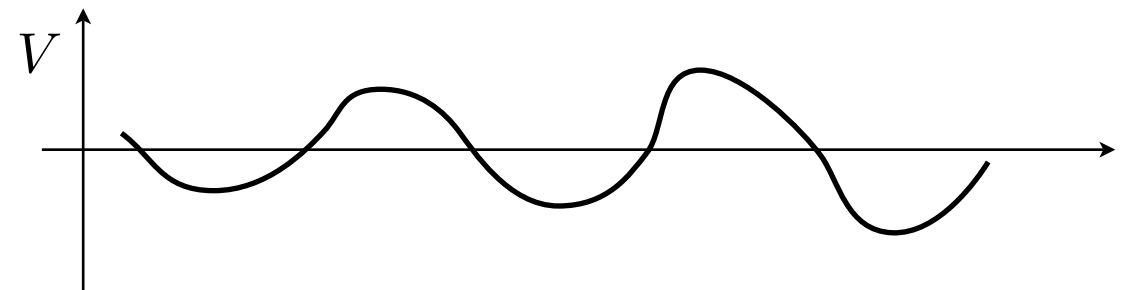
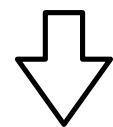
two families :      internal space is  $\mathbb{C}\mathbb{P}^3$       or       $\frac{SU(3)}{U(1) \times U(1)}$

topologies are almost  
as simple as they can get!

with many different metrics



few parameters;  
**discretized** by the fluxes



there are indeed no massless scalars!



# Conclusions

- General characterization of the internal space in string theory.
- Many new examples, that systematically avoid old problems.