Dalitz Plot analysis of $D_s^+ \longrightarrow \pi^-\pi^+\pi^+$

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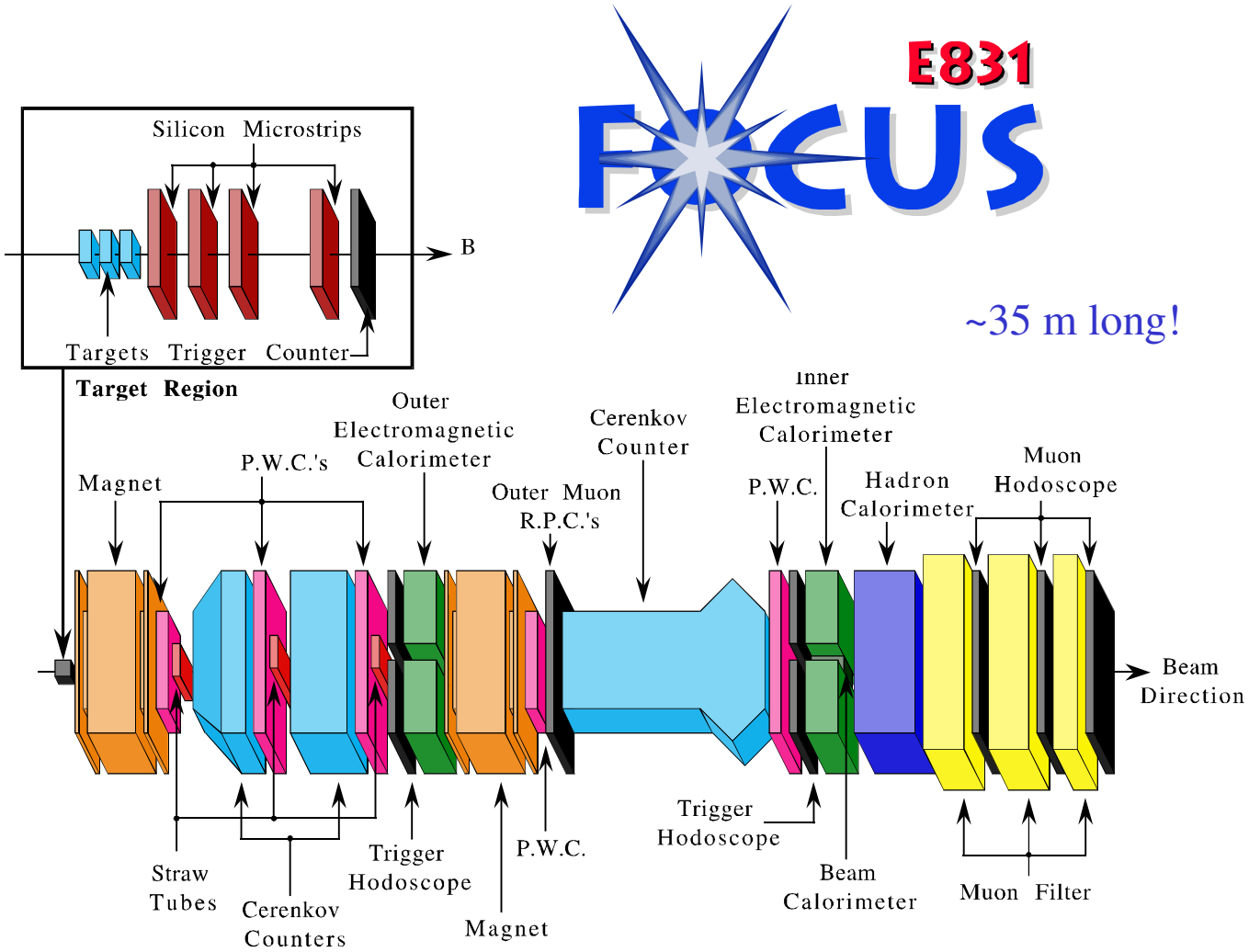
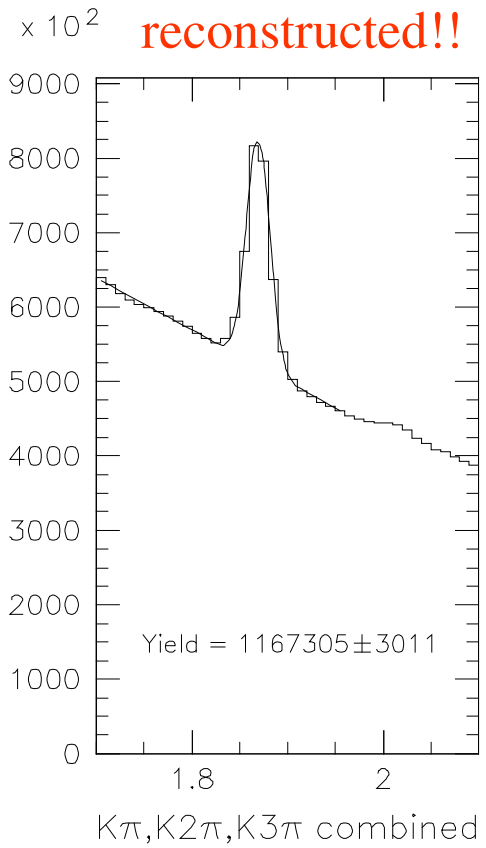
Talk outline

- The E831-FOCUS experiment.
- Why are we interested in $D_s^+ \longrightarrow \pi^-\pi^+\pi^+$?
- Dalitz Plot, coherent amplitude analysis and formalism used in this work.
- The difficulties of this analysis and the strategy that we have used to solve them.
- Discussion about choices made for the fit model.
- Final results of $D_s^+ \longrightarrow \pi^-\pi^+\pi^+$ Dalitz Plot analysis.
- Besides the three pions analysis:
  - Dalitz Plot analyses of the $D \longrightarrow KK\pi$ decays

    • **Discovery** of DCS decay $D^+ \rightarrow K^-K^+K^+$ and SCS decay $D_s^+ \rightarrow K^-K^+K^+$
- Conclusions





Successor to E687. Designed to study charm particles produced by ~200 GeV photons using a fixed target spectrometer with upgraded **Vertexing**, **Cerenkov**, **E+M Calorimetry**, and **Muon id** capabilities. Includes groups from USA, Italy, Brazil, Mexico, Korea

1 million charm particles reconstructed into D→Kπ , K2π , K3π

Why are we interested to investigate $D_s^+ \longrightarrow \pi^-\pi^+\pi^+$ dynamics?

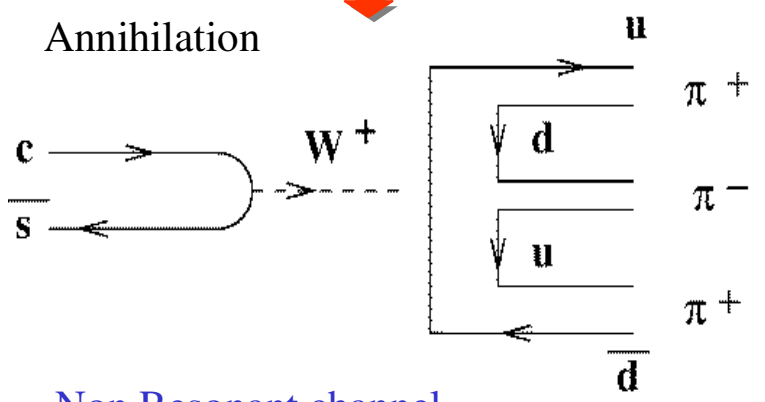
- $\Delta c = \Delta s \Rightarrow$ Cabibbo allowed at 2 vertices
- $Br(\pi^-\pi^+\pi^+) = (1.4 \pm 0.4)\%$ (High)
- In $D_s^+ \longrightarrow \pi^-\pi^+\pi^+$ we have

  initial state (c,s) $\Rightarrow$ (u,d) final state

Which mechanism could make the quarks (c,s) to "vanish" ?

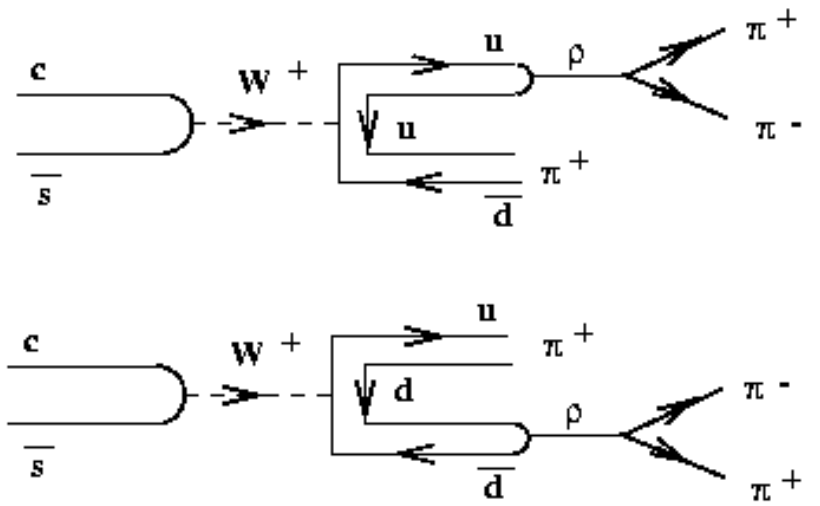
The decay $D_s^+ \longrightarrow \pi^-\pi^+\pi^+$ is the best candidate to proceed through annihilation

Annihilation



Non Resonant channel

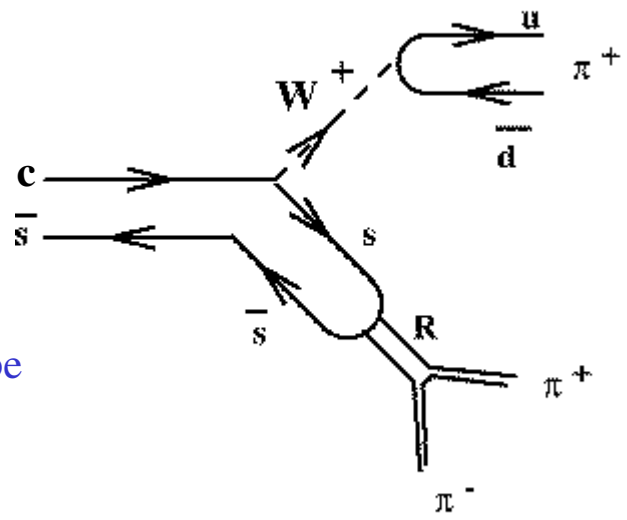
Annihilation through the resonance ρ



Why are we interested to investigate $D_s^+ \longrightarrow \pi^-\pi^+\pi^+$ dynamics?

However we know that the annihilation diagram is suppressed by helicity and $\Gamma \propto M^2_c$, while the Branching Ratio is quite high.

There is another mechanism
- not suppressed
- $\Gamma \propto M^5_c$



Spectator through resonance R

… and one can guess that it could be the dominant process in this decay

Resonance R must couple simultaneously to $s\,\bar{s}$ and $\pi\,\pi$

Look for G = +1 resonances with simultaneous KK & $\pi\,\pi$ decay modes

$f_0(980)$ → $\pi\,\pi$ 78%, K K 22%

$f_2(1270)$ → $\pi\,\pi$ 85%, K K 5%

The Dalitz Plot

For a decaying particle with spin = 0 (D mesons):

$$D \rightarrow 1 + 2 + 3$$

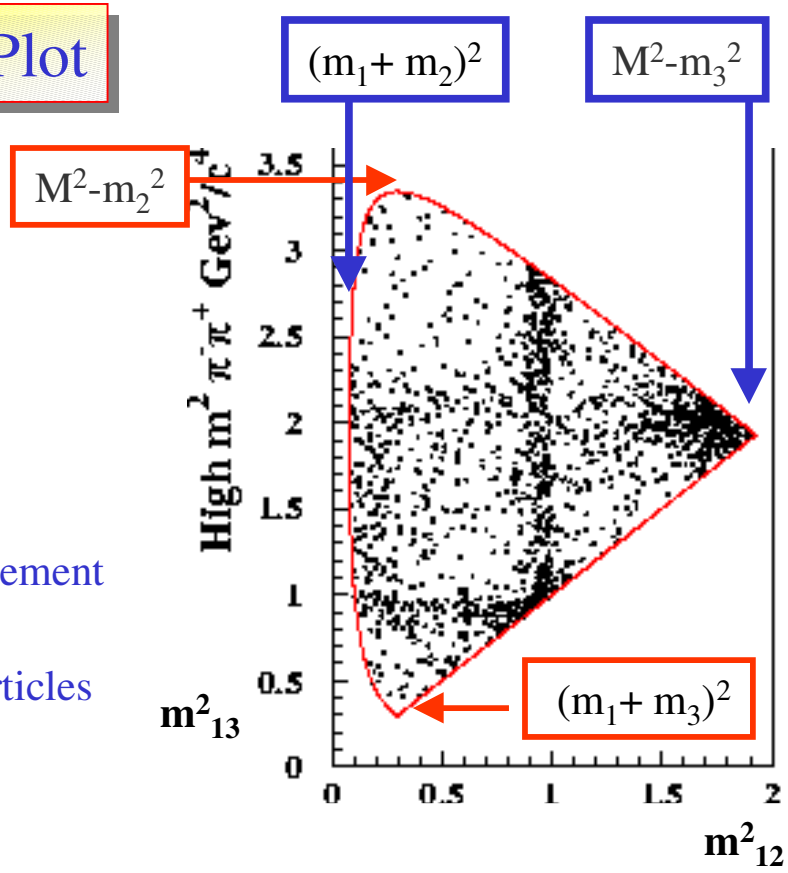
the decay rate can be written as :

$$d\Gamma = \frac{1}{(2\pi)^3}\frac{1}{32M^3}|\mathcal{M}|^2\, dm_{12}^2 dm_{13}^2$$

Where M is the decaying particle mass, ***M*** is the matrix element that keeps into account the dynamics of the process, $\mathbf{m^2_{12}}$ and $\mathbf{m^2_{13}}$ are the square masses of the pairs of the particles produced in the decay. One can expect that, without any resonant states like the following:

$$D \rightarrow \textit{resonance} + 3$$
$$\hookrightarrow 1 + 2$$

the yield on the Dalitz Plot surface should be uniform. Otherwise, the presence of any structure on the Dalitz Plot can be the signal of the contribution of resonant amplitude in the dynamics of the decay.



Dalitz Plot and its kinematic limits

The Dalitz Plot fit formalism.

We performed a continuous maximum Likelihood fit :

$$\mathcal{L} = \prod_{j=1}^{Nev} p_j \qquad (0.1)$$

For every event "j" we can define the probability P_j :

$$P_j = \left[\alpha \frac{|A_T(j)|^2}{\int |A_T|^2 dm_{12}^2 dm_{13}^2} + (1-\alpha)\frac{B}{\int B(j) dm_{12}^2 dm_{13}^2}\right] \qquad (0.2)$$

Identical particles: amplitudes must be symmetric for the exchange of $\mathbf{m^2_{12}}$,$\mathbf{m^2_{13}}$ (Bose - Einstein)

$D_s^+ \rightarrow \pi_1^- \pi_2^+ \pi_3^+$

Where:

- $A_T(j)$ is the theoretical amplitude evaluated for the event j =($\mathbf{m^2_{12}}$,$\mathbf{m^2_{13}}$). It is the sum of the amplitudes of all the modes (resonant and non resonant):

The theoretical model

$$A_T = \sum_{j=1}^{N_R} a_j A_j e^{i\phi_j} \qquad (0.3)$$

Normalized by f_0:
$\phi(f_0) = 0$ fixed
$a(f_0) = 1$ fixed

Where N_R is the number of the modes which the decay could proceed through, a_j are the coefficients of the normalized amplitudes A_j , and ϕ_j the associated phases. a_j and ϕ_j are free parameters of the fit.

Amplitude for Non Resonant channel is not depend on energy and it is a constant $A_{NR} = 1$

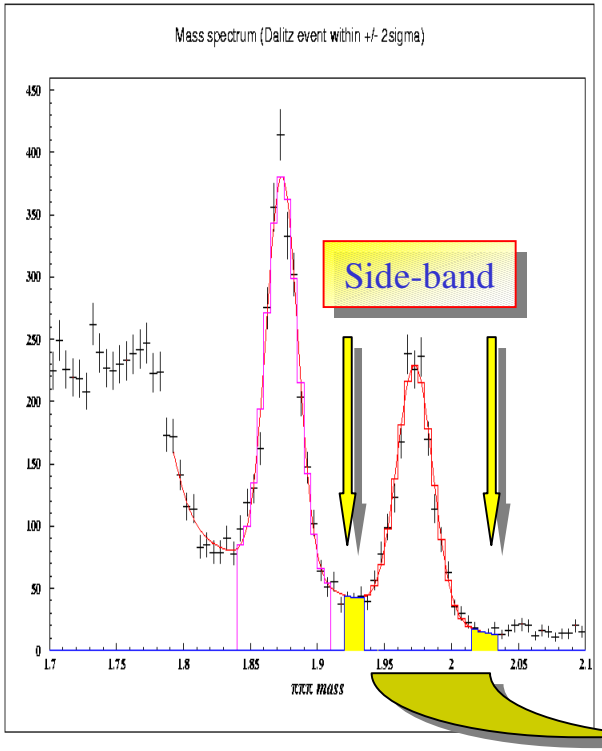
- α represents the signal fraction of the Dalitz Plot events and it is bounded to move inside a Gaussian centered on the measured value $\bar{\alpha}$ and with width $\sigma(\bar{\alpha})$.

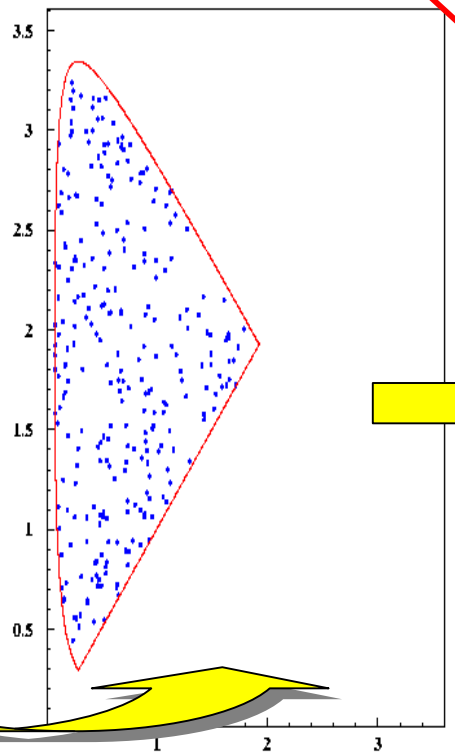
$\bar{\alpha}$ = **Signal / (Signal+Background)**

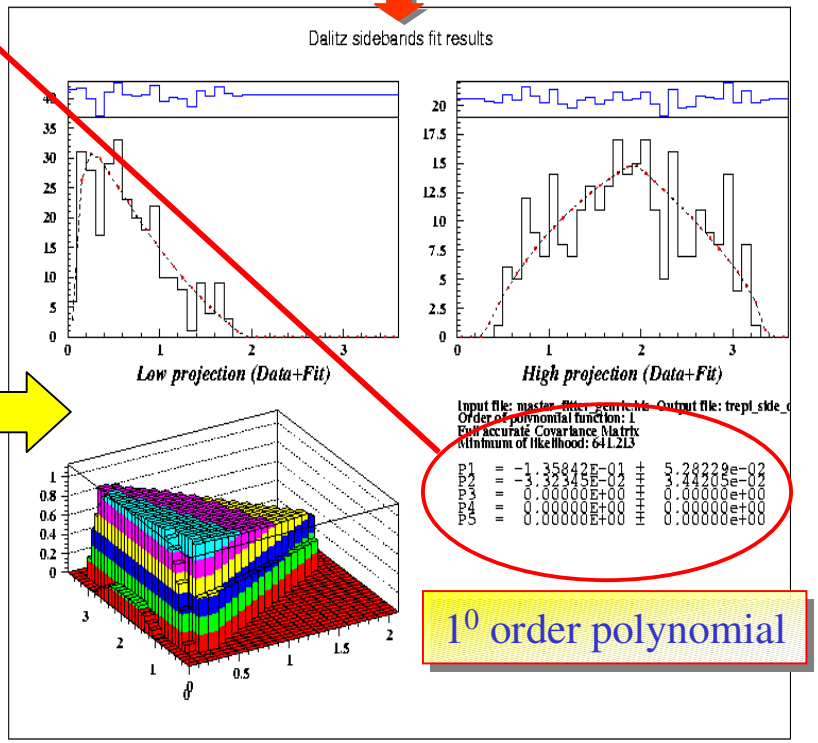
The Dalitz Plot fit formalism: the *side-band modeling*.

- ***B*** is the probability for the background, parameterized as a polynomial.
  The coefficients of the polynomial are gaussianly constrained to the values measured in the side-band fit.

The *side bands modeling* :
The distribution on the Dalitz Plot of background events pick up far enough from the signal can be considered similar to the distribution of bkg events under the signal (side band events).

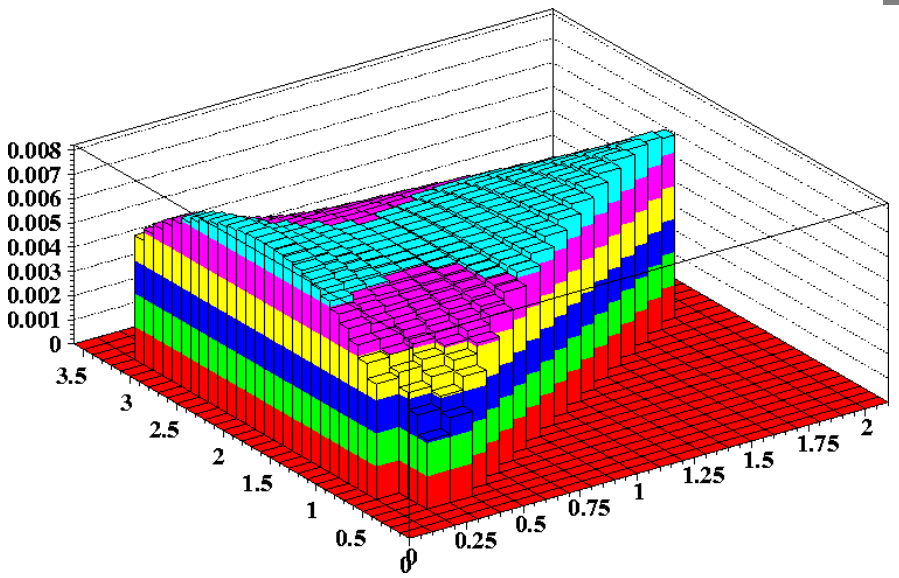






1^0 order polynomial

The Dalitz Plot fit formalism: efficiency correction to the model.



Efficiency function

We must take also reconstruction efficiency in account ! We need a :

Efficiency correction for the model

- Amplitudes are corrected with a function that parameterize the efficiency on the Dalitz Plot phase space.
- Efficiency is parameterized by a 4^{th} order polynomial.
- The coefficients of this polynomial have been defined by a binned maximum likelihood fit.
- For every bin of an appropriate built lattice dividing the phase space of the Dalitz Plot, has been calculated the ratio between reconstructed MC events and generated MC events that drop in the bin.
- **10^6** MC events of $D_s^+ \longrightarrow \pi^-\pi^+\pi^+$ have been generated

The Dalitz Plot fit formalism: amplitude parameterization

- $$A_j = BW(1\ 2\ 3|r) = BW(1\ 2|r)\,\mathcal{S}(1\ 3) \qquad (0.1)$$

where ***BW*** **(1 2 | *r***) is a Breit-Wigner function:

$$BW(1\ 2|r) = \frac{F_D F_r}{M_r^2 - m_{12}^2 - i\Gamma M_r} \qquad (0.2)$$

and :

$\mathcal{S}(1\ 3) = 1$ **resonance of spin = 0**

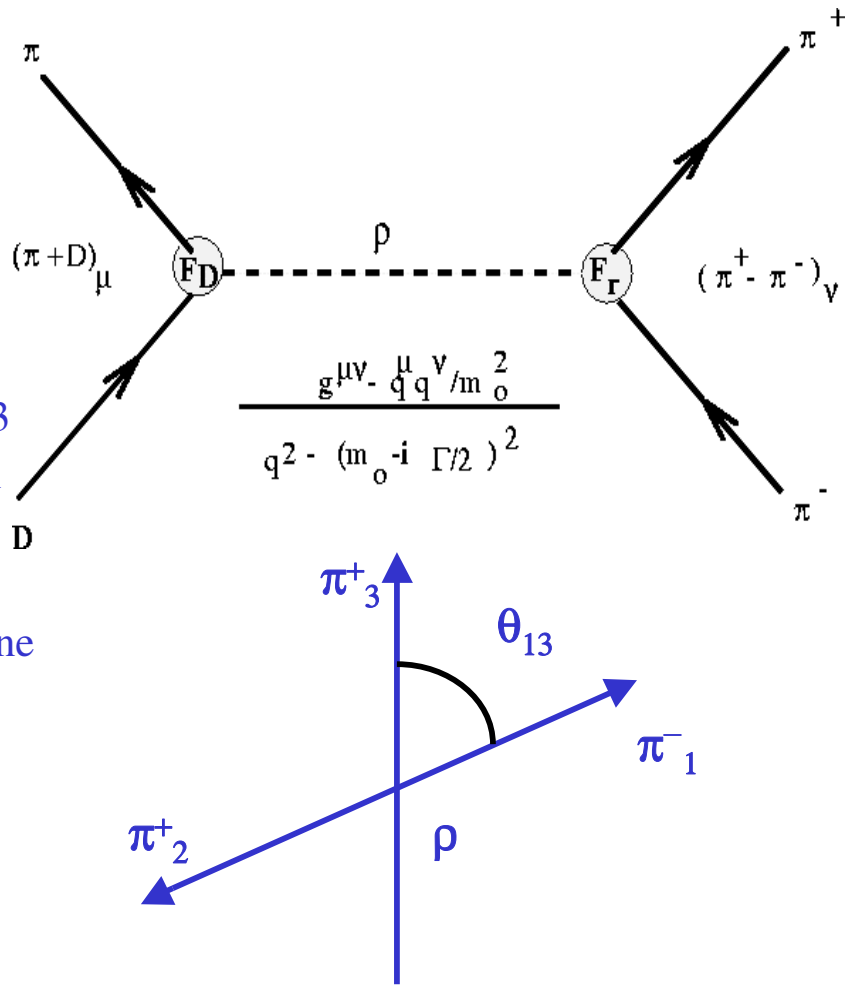
$\mathcal{S}(1\ 3) = (-2\vec{p}_3\vec{p}_1)$ **resonance of spin = 1**

$\mathcal{S}(1\ 3) = 2(p_3p_1)^2(3\cos^2\theta_{13} - 1)$ **resonance of spin = 2**

where p_1 and p_3 are the momenta of the particles 1 and 3 in the system (1, 2), θ_{13} is the angle between 1 and 3 in the system (1, 2) (helicity angle). F_D and F_r factorize the strong coupling at each vertex of the diagram. For every resonance with mass M_r and spin j we can define a width Γ :

$$\Gamma = \Gamma_r \left[\frac{p}{p_0}\right]^{2j+1} \frac{M_r}{m_{12}} \frac{F_r^2(p)}{F_r^2(p_0)} \qquad (0.4)$$

where Γ_o is the PDG width for the resonance, p is the momentum of the particle in the system (1, 2), p_0 is the momentum in the system where the resonance is at rest.



The Dalitz Plot fit formalism: coupled channel formalism.

- $$BW(1\ 2|f_0(980)) = \frac{F_D F_{f_0}}{M_{f_0}^2 - m_{12}^2 - i(\Gamma_\pi + \Gamma_K) M_{f_0}}$$

The partial widths Γ_π and Γ_K are defined as :

$$\Gamma_\pi = g_\pi \left[\frac{1}{4} m_{12}^2 - M_\pi^2\right]^{\frac{1}{2}}$$

f_0 coupled channel

$$\Gamma_K = \frac{1}{2} g_K \left[\left(\frac{1}{4} m_{12}^2 - M_{K^+}^2\right)^{\frac{1}{2}} + \left(\frac{1}{4} m_{12}^2 - M_{K^0}^2\right)^{\frac{1}{2}}\right]$$

I try to use for the coupling constants g_π and g_K the value measured by **WA76** (used in E687 Dalitz Plot analyses) ...

$$g_\pi = 0.28 \pm 0.04$$
$$g_K = 0.56 \pm 0.18$$

…and the most recently values measured by WA102...

$$g_\pi = 0.19 \pm 0.03 \pm 0.04$$
$$g_K = 0.40 \pm 0.04 \pm 0.04$$

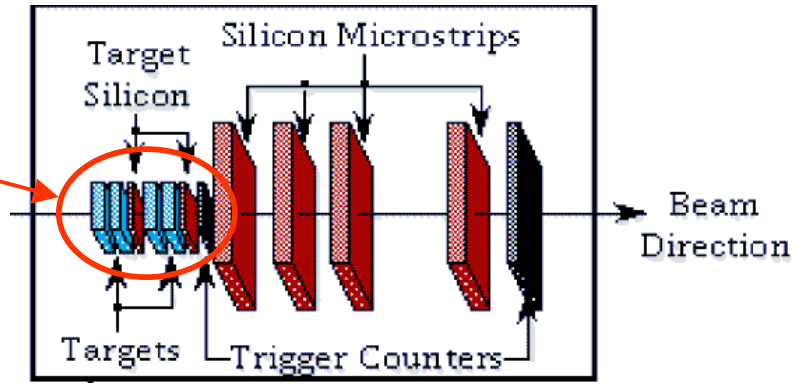
Used in the final fit.

How to reduce the background ?

- hadronic interactions and photoproduction of light mesons occur in the target material and create fake charm decay vertices

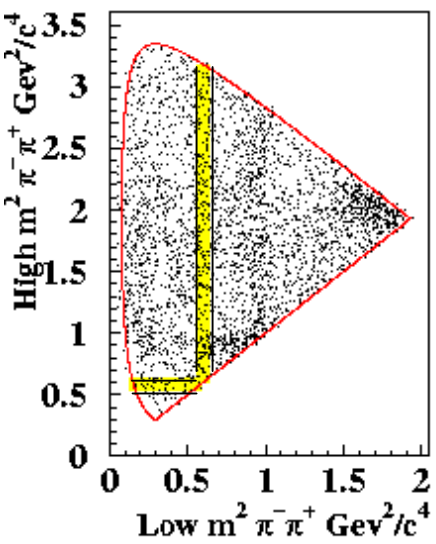
- **E687**: studies demonstrated that cutting out decay vertices that occur in the target material suppress the Bkg source due to the hadronic interaction and → S/N × 2!

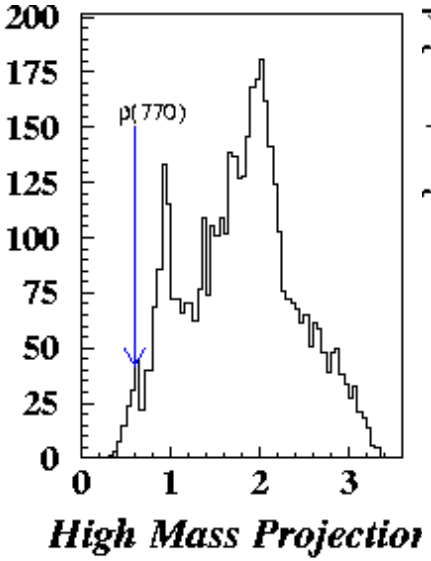
- **E831**: to improve the number of decays that occur outside the target material a segmented target has been adopted

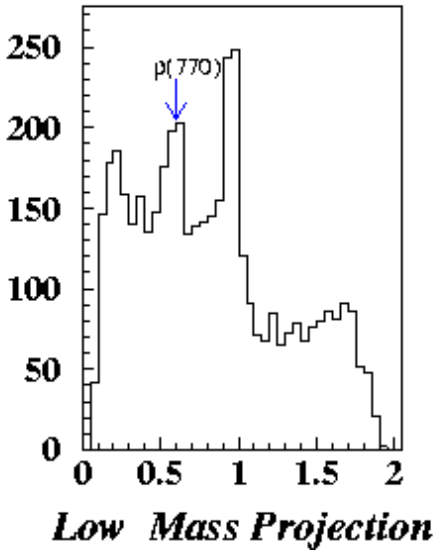


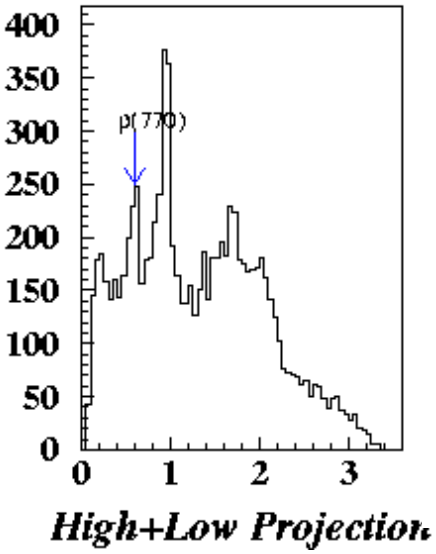
…besides, this request reduced the Bkg source due to photoproduction.
This Bkg source produces fake scalar states on the Dalitz Plot

Without Out of Material cut

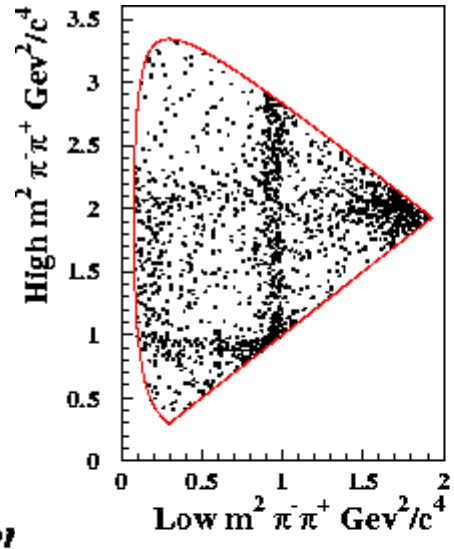


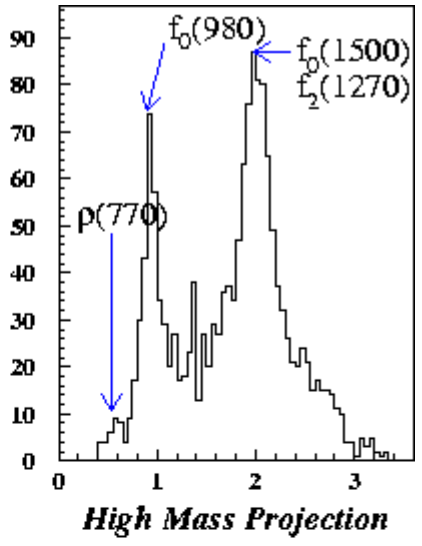


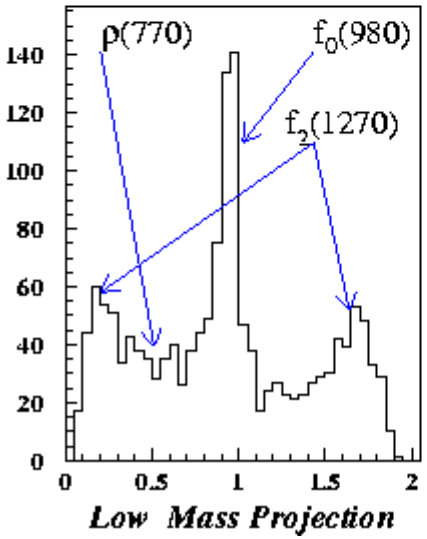


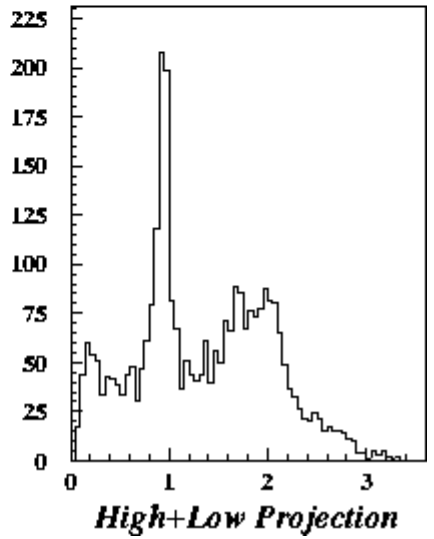


With Out of Material (OoM>3)



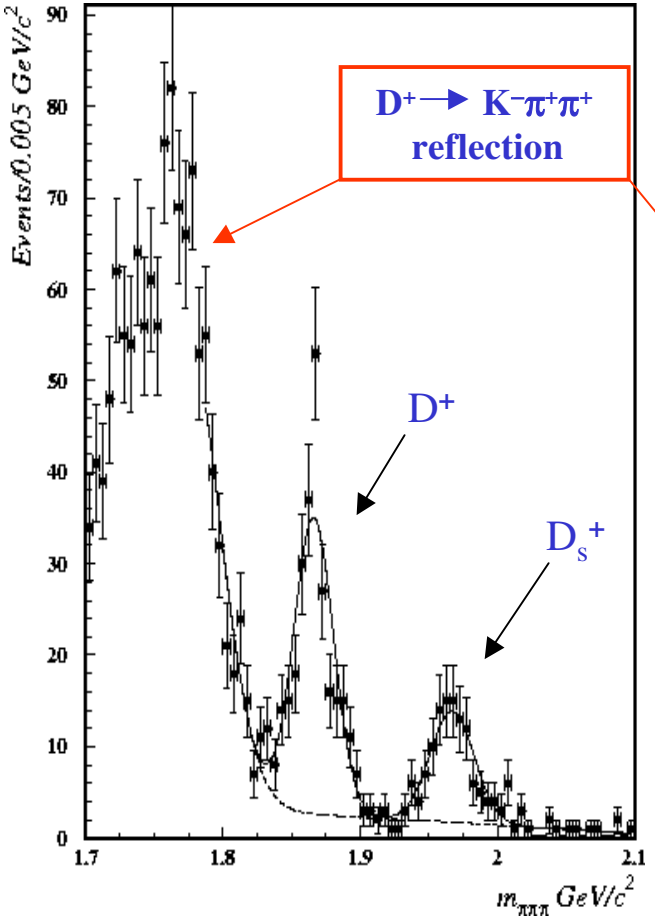




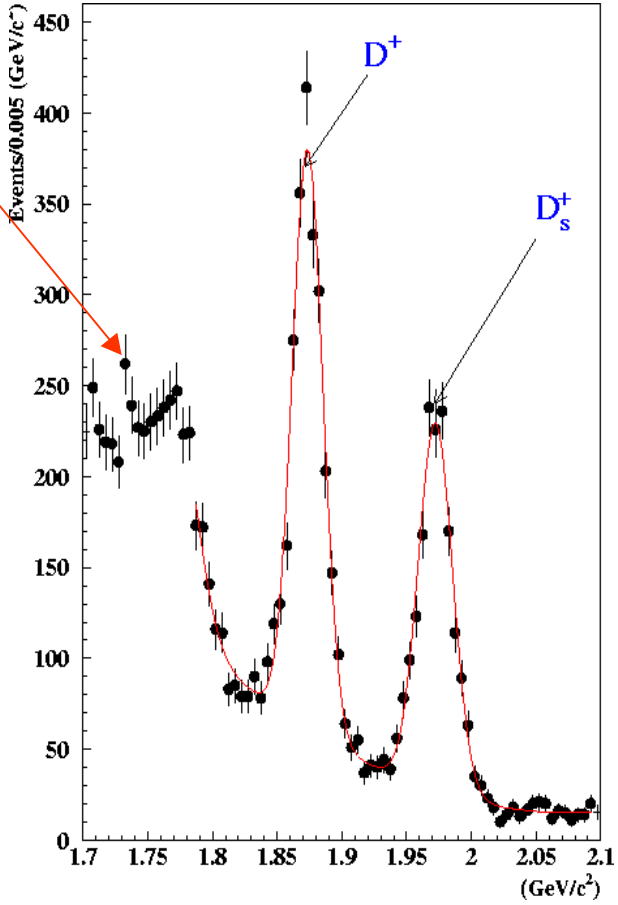


$D_s^+ \longrightarrow \pi\pi\pi$ mass spectrum: comparison between E831 and E687

E687



E831



$Y(D_s^+)_{E831} = 13 \times Y(D_s^+)_{E687}$
(same S/N)

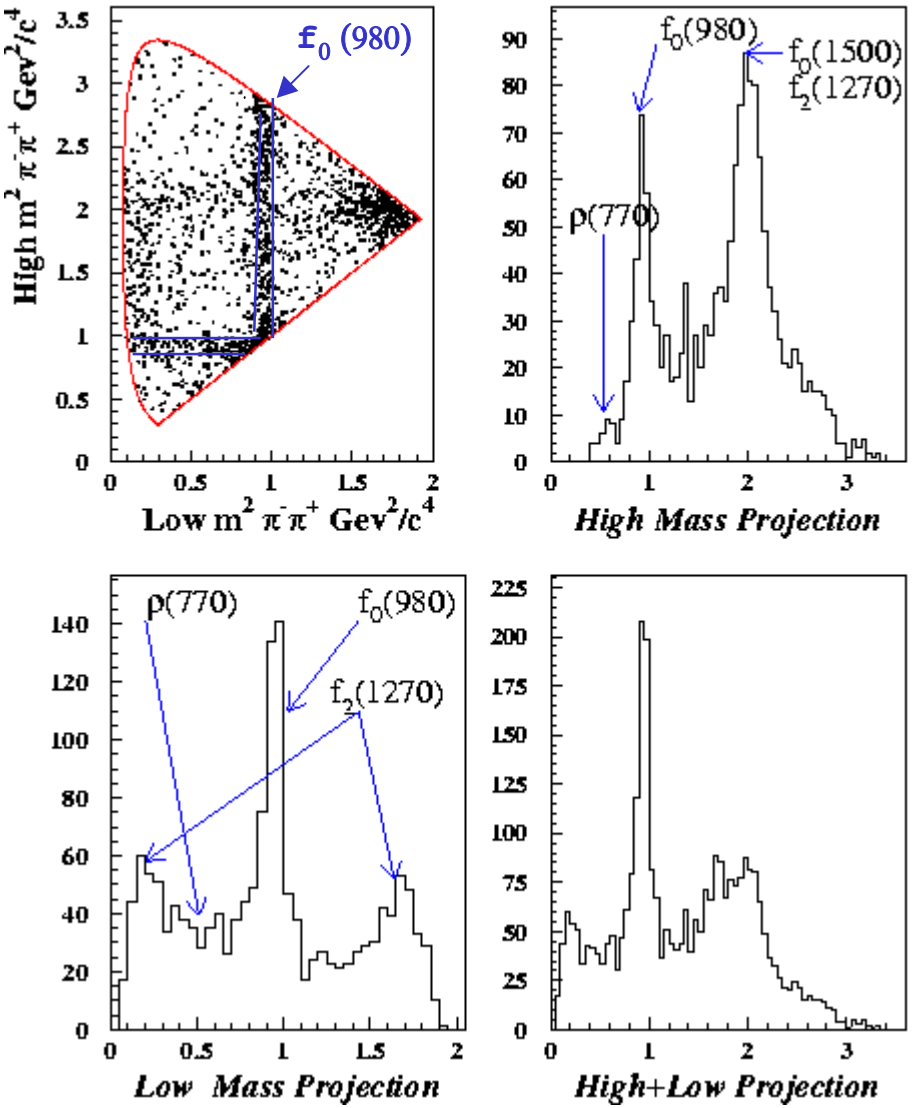
E831

D_s^+ : YIELD = 1288 ± 47
S/N = 4.34 ± 0.30
(D^+ : YIELD = 1800 ± 60
S/N = 3.08 ± 0.16)

E687

D_s^+ : YIELD = 98 ± 12
S/N = 4.22 ± 0.85
(D^+ : YIELD = 236 ± 20
S/N = 7.47 ± 1.)

$D_s^+ \rightarrow \pi^-\pi^+\pi^+$ Dalitz Plot from E831 Data (**100%** of the full sample)



Resonances included in the fit

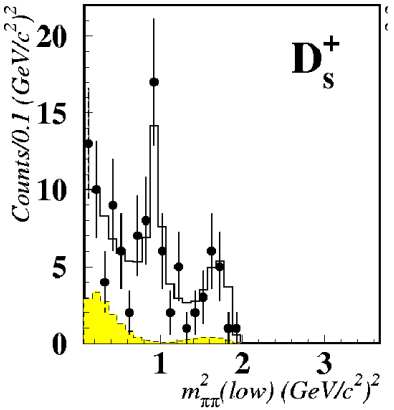
- ρ (770)
- f_0 (980)
- f_2 (1270)
- s_0 (1475)
- ρ (1450)

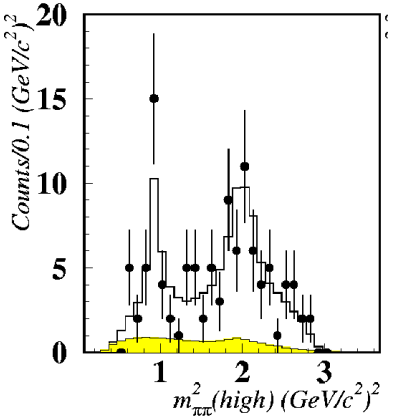
E687 published results

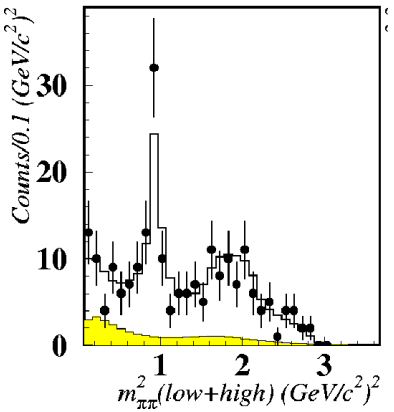
E687 published result

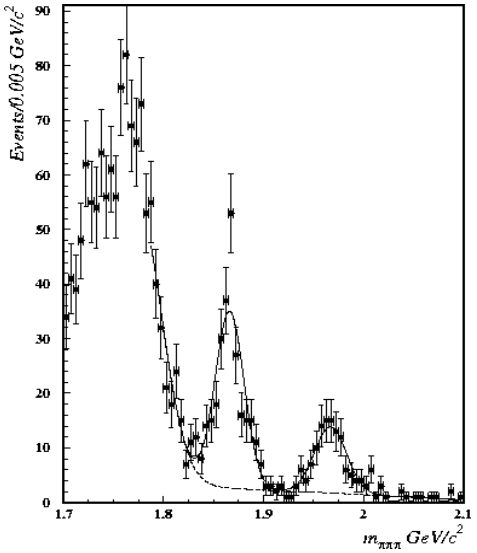
	Fit frac.	Phase
NR	0.12 ± 0.12	4.10 ± 0.38
ρ (770)	0.02 ± 0.03	0.93 ± 0.76
f_2 (1270)	0.12 ± 0.06	1.74 ± 0.31
f_0 (980)	1.07 ± 0.14	0.0 (fixed)
S_0 (1475)	0.27 ± 0.11	4.08 ± 0.26

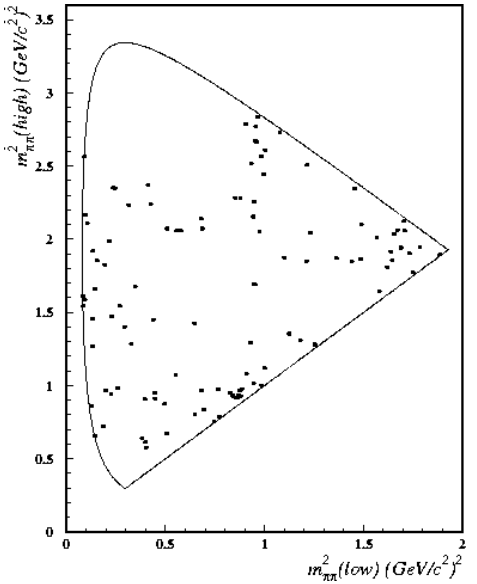
- 98 events of $D_s^+ \longrightarrow \pi^-\pi^+\pi^+$
- S/N=4.22



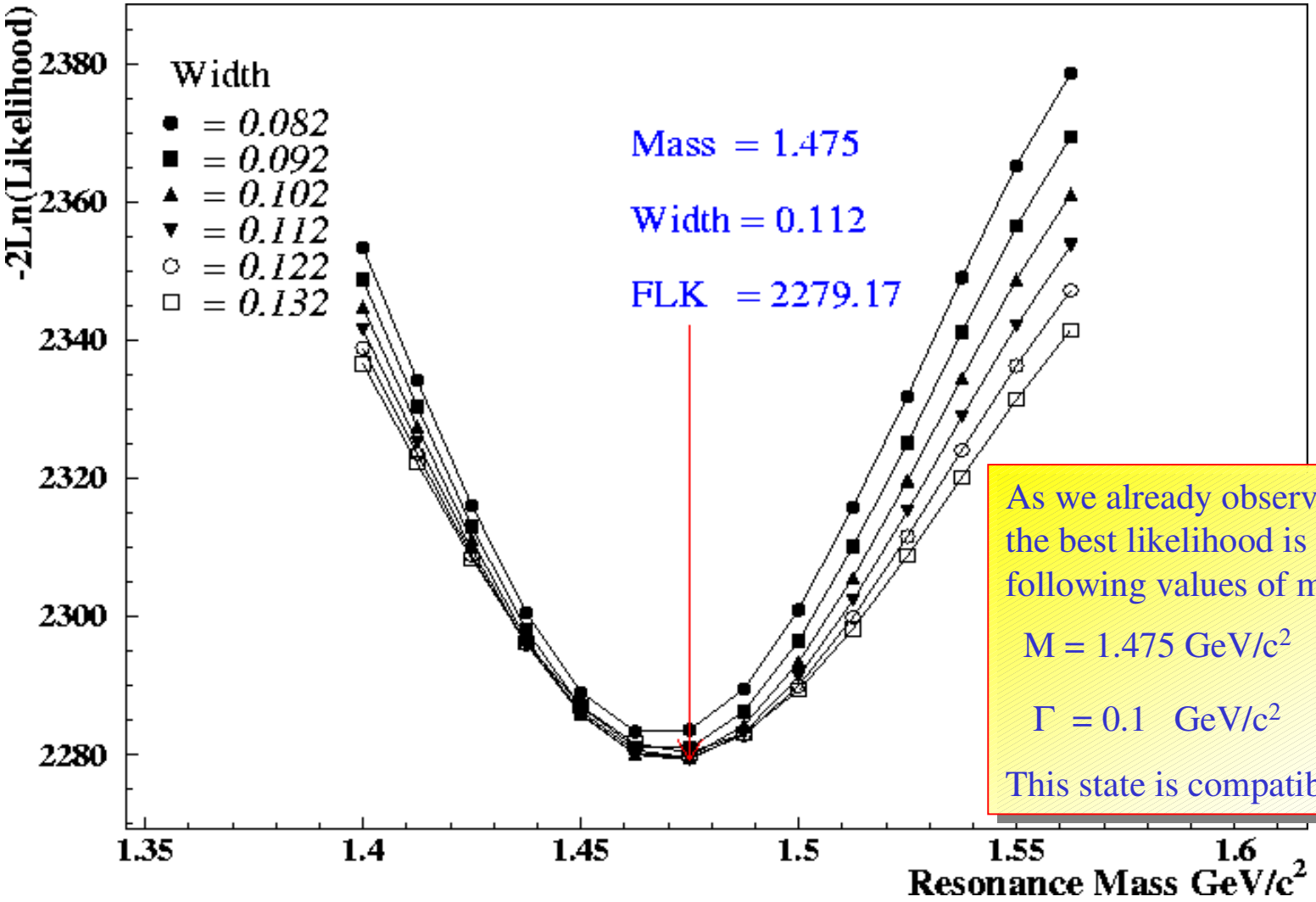








In order to establish the best parameters for the $S_0(1475)$, we performed a likelihood scan varying mass and width of this state.

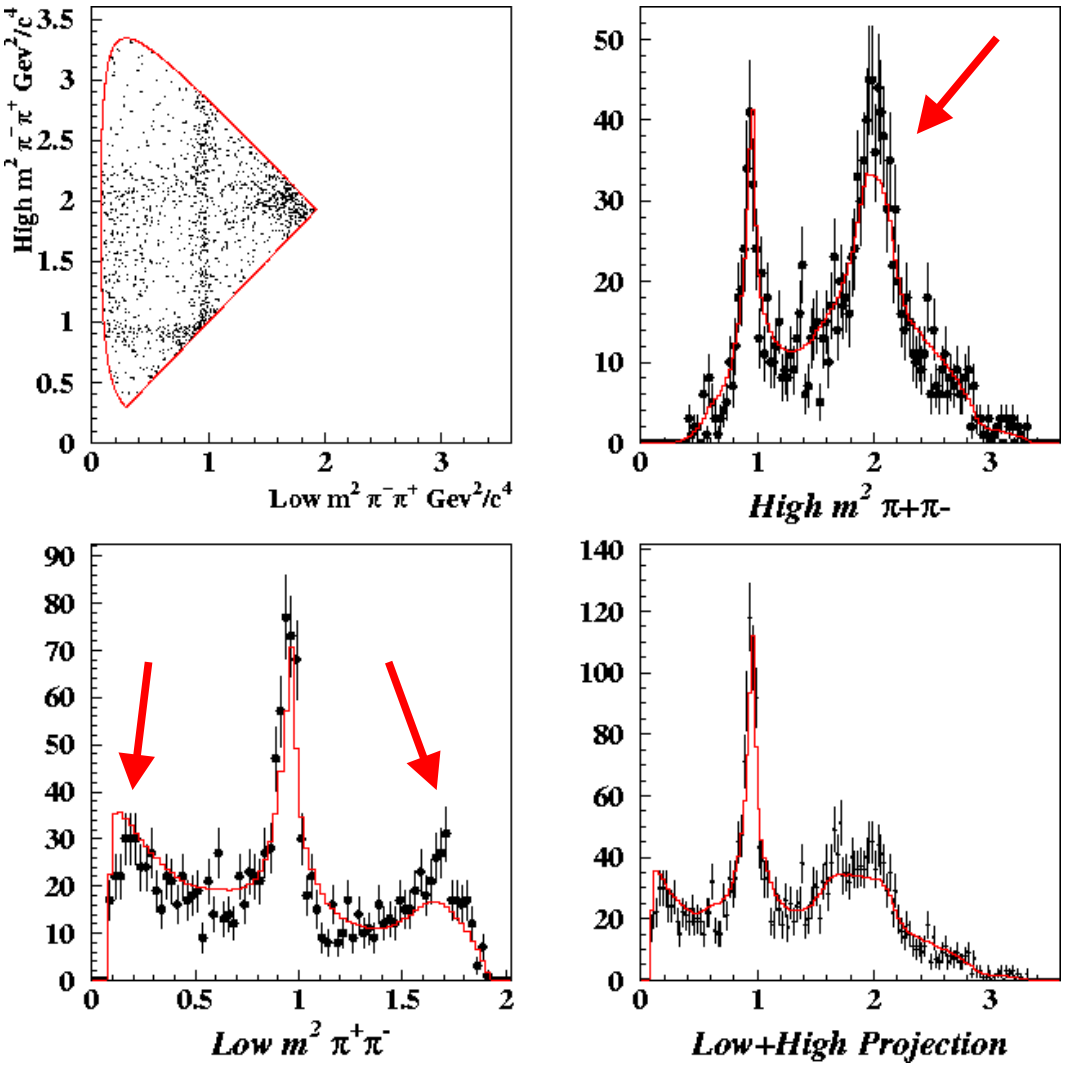


As we already observed in E687 data, the best likelihood is obtained for the following values of mass and width:

$M = 1.475$ GeV/c^2

$\Gamma = 0.1$ GeV/c^2

This state is compatible with $f_0(1500)$.



Direct comparison with E687 published results is possible

Fit results

	Fit frac.	Phase
NR	0.18 ± 0.04	241 ± 5
ρ (770)	0.03 ± 0.01	54 ± 16
f_2 (1270)	0.15 ± 0.02	116 ± 6
f_0 (980)	1.09 ± 0.04	0 (fixed)
S_0 (1475)	0.17 ± 0.03	223 ± 5

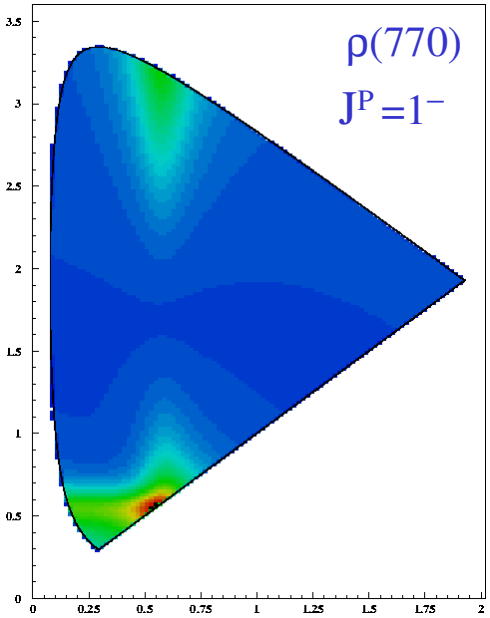
E687 published result

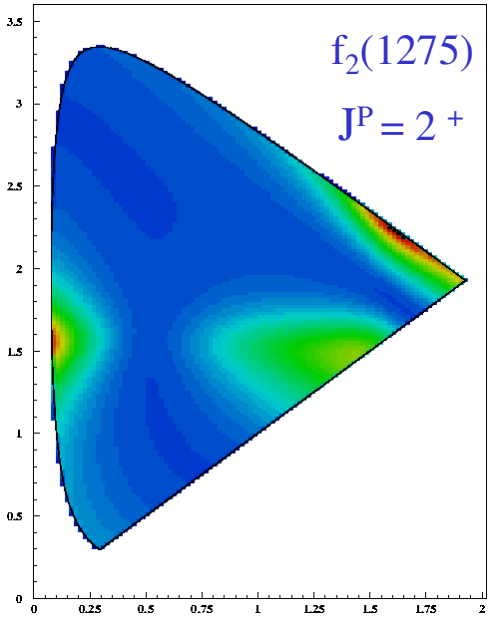
	Fit frac.	Phase
NR	0.12 ± 0.12	235 ± 22
ρ (770)	0.02 ± 0.03	53 ± 44
f_2 (1270)	0.12 ± 0.06	100 ± 18
f_0 (980)	1.07 ± 0.14	0 (fixed)
S_0 (1475)	0.27 ± 0.11	234 ± 15

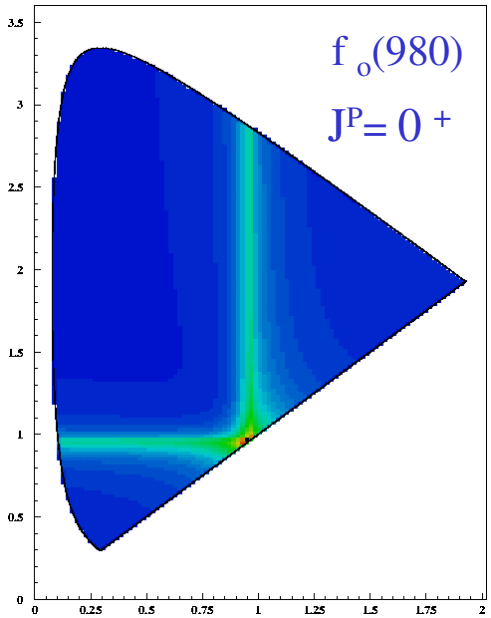
Parameterization of f_0 (980) in this fit is the canonical one by WA76 and used in the E687 analysis also.

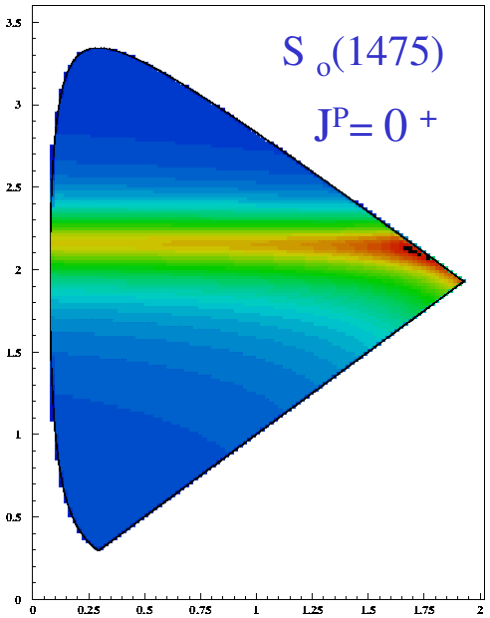
Excellent agreement!!

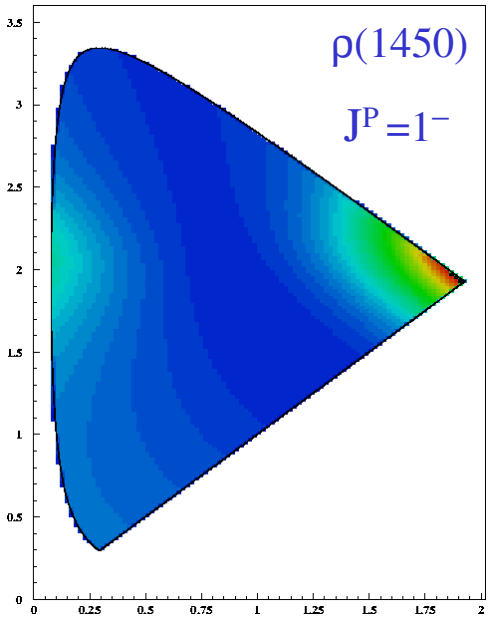
Models of the resonances

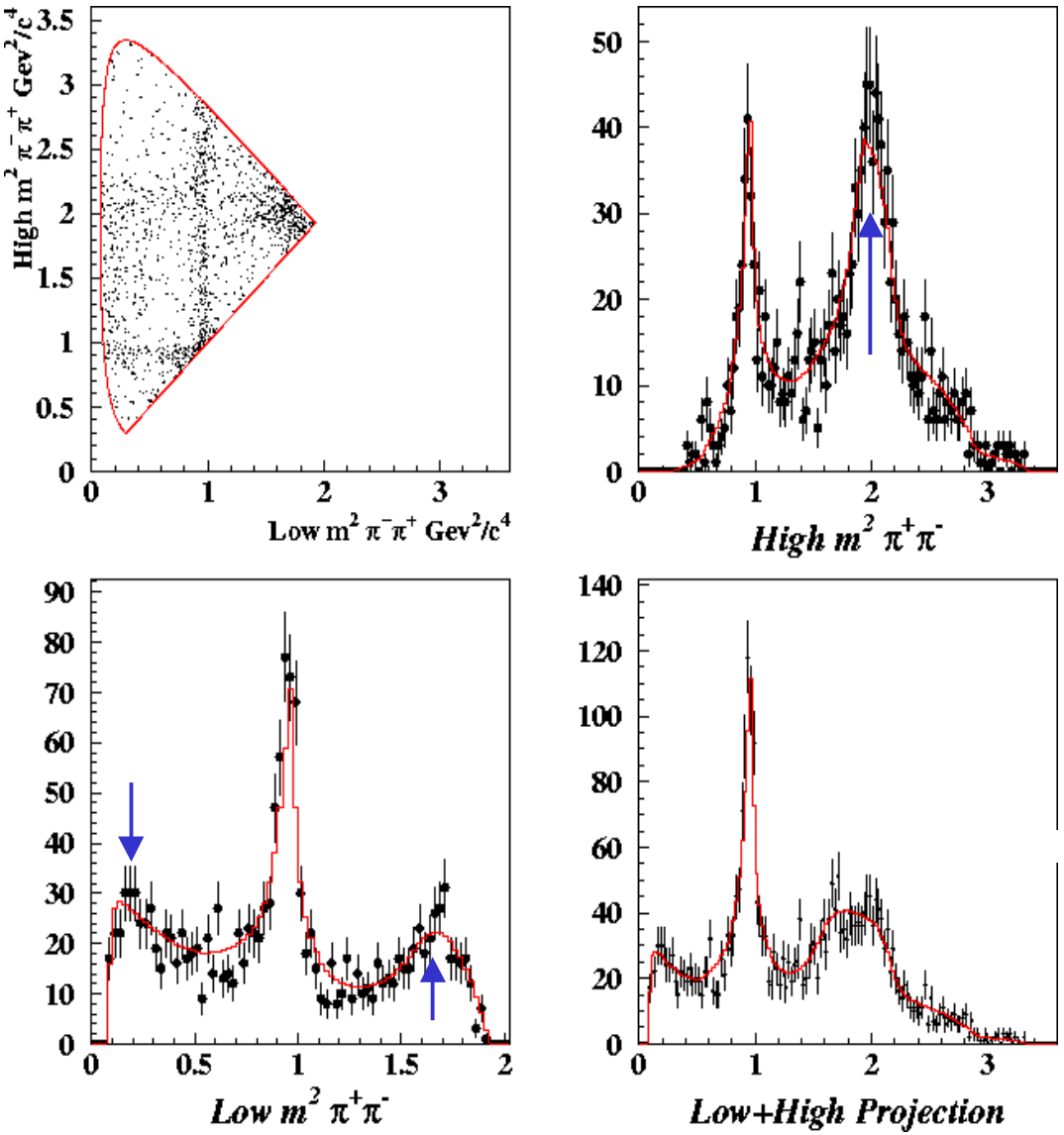








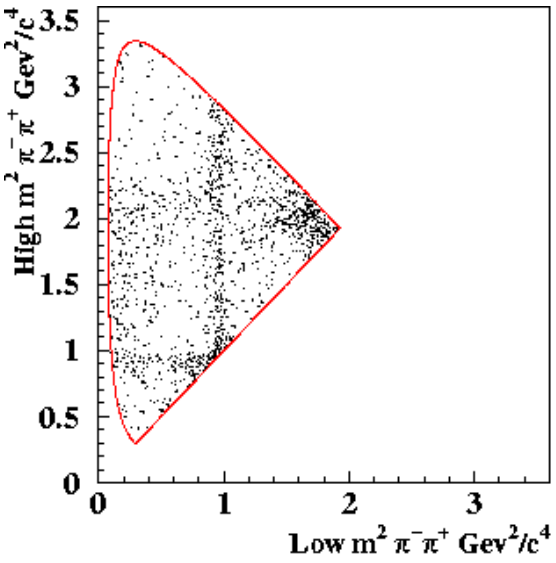


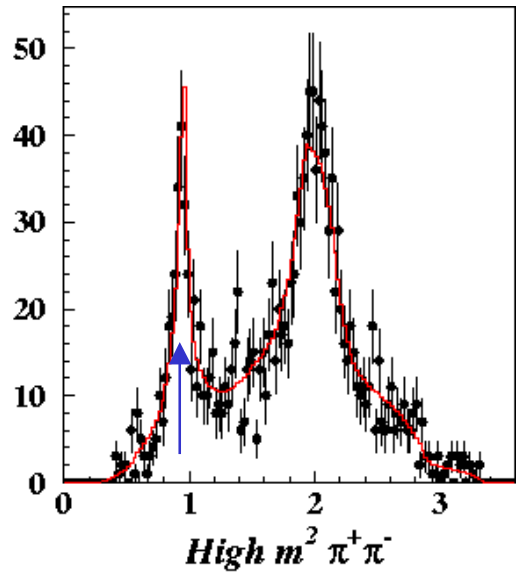


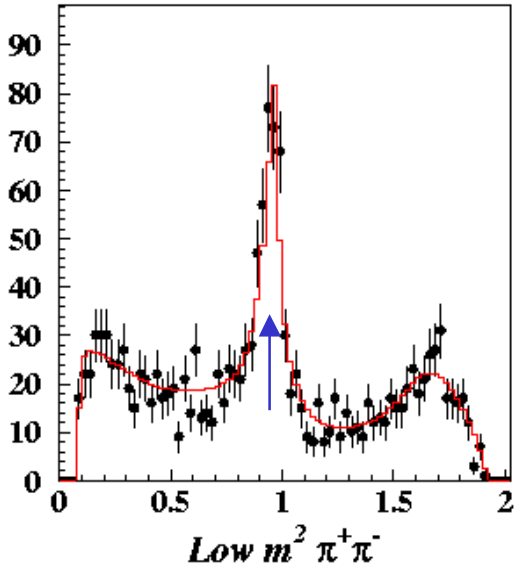
Fit results

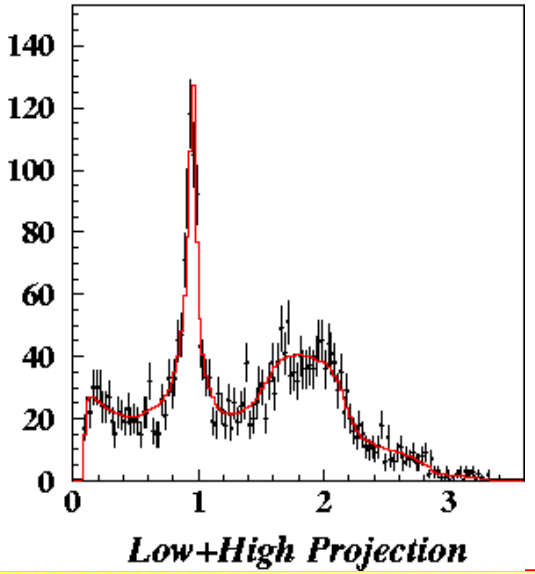
	Fit frac.	Phase
NR	0.26 ± 0.05	245 ± 4
ρ (770)	0.02 ± 0.02	-4 ± 20
f_2 (1270)	0.13 ± 0.02	131 ± 7
f_0 (980)	1.13 ± 0.06	0 (fixed)
S_0 (1475)	0.20 ± 0.03	245 ± 6
ρ (1450)	0.05 ± 0.02	211 ± 6

In this fit we have added a vector state, the **ρ(1450).**
Judging from the projections the fit quality improves significantly









The Final Results

Fit results

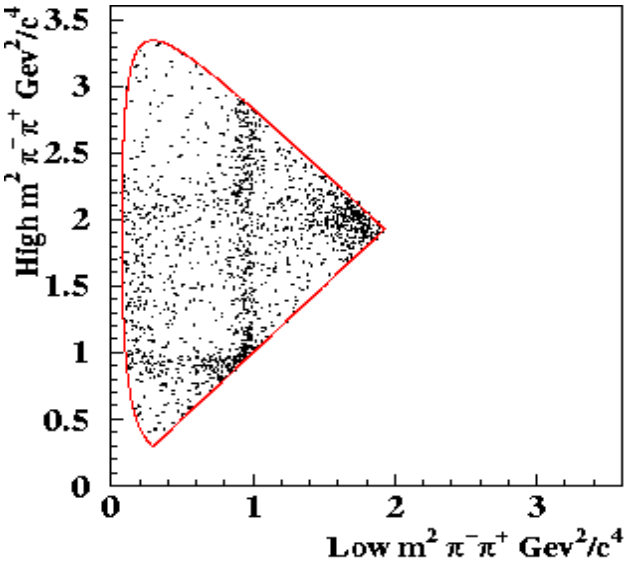
	Fit frac.	Phase (deg.)
NR	0.20 ± 0.04 ± 0.02	245 ± 5 ± 1
ρ (770)	0.02 ± 0.02 ± 0.01	11 ± 24 ± 5
f_2 (1270)	0.12 ± 0.02 ± 0.01	138 ± 7 ± 3
f_0 (980)	0.94 ± 0.05 ± 0.01	0 (fixed)
S_0 (1475)	0.21 ± 0.03 ± 0.01	245 ± 6 ± 1
ρ (1450)	0.05 ± 0.01 ± 0.01	214 ± 16 ± 6

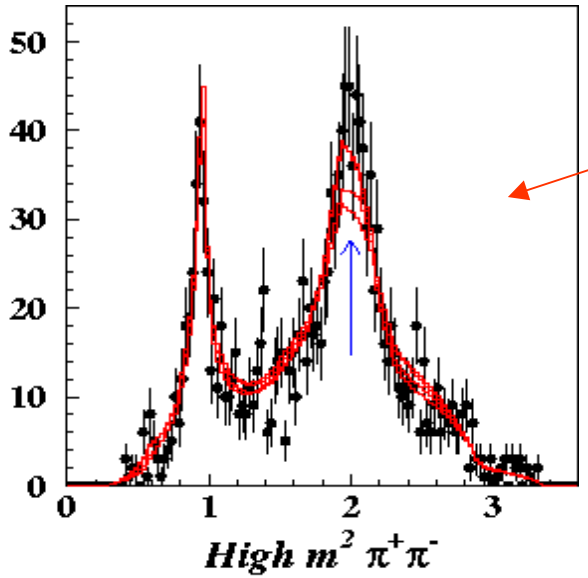
E687 published result

	Fit frac.	Phase (deg.)
NR	0.12 ± 0.12 ± 0.04	235 ± 22 ± 2
ρ (770)	0.02 ± 0.03 ± 0.01	53 ± 44 ± 10
f_2 (1270)	0.12 ± 0.06 ± 0.02	100 ± 18 ± 6
f_0 (980)	1.07 ± 0.14 ± 0.04	0 (fixed)
S_0 (1475)	0.27 ± 0.11 ± 0.02	234 ± 15 ± 4
ρ (1450)	Not considered !	

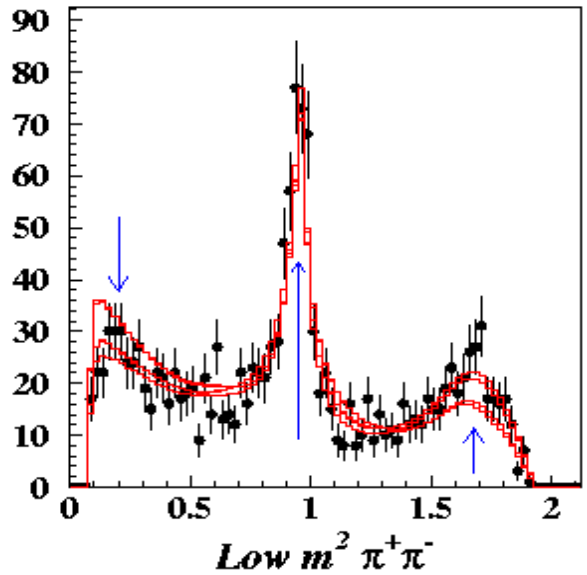
Finally, the parameterization of f_0 (980) in this fit is the one adopted by WA 102: the difference with WA 76 is the values taken by g_K and g_π

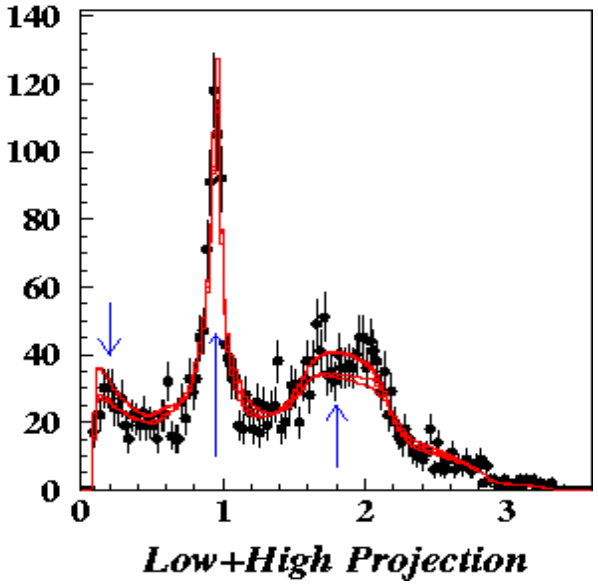
Evolution of the fit

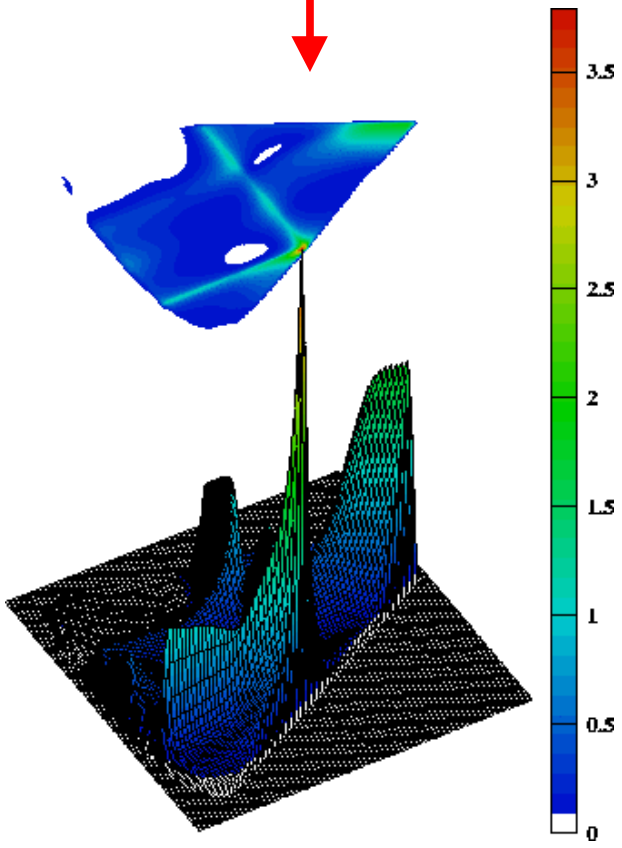




These are only the projections of the fit functions!

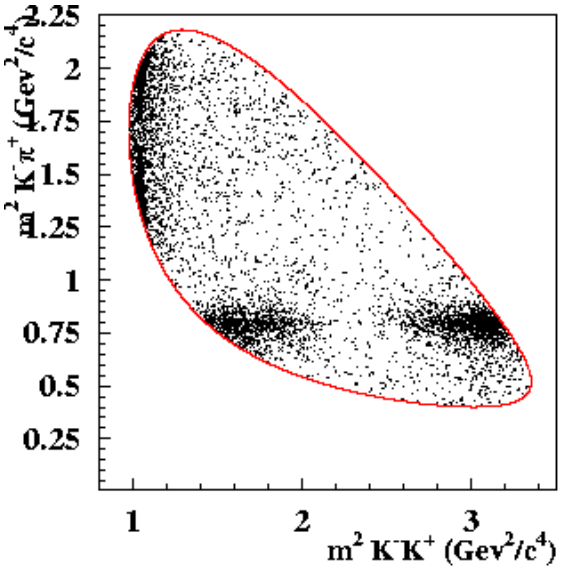


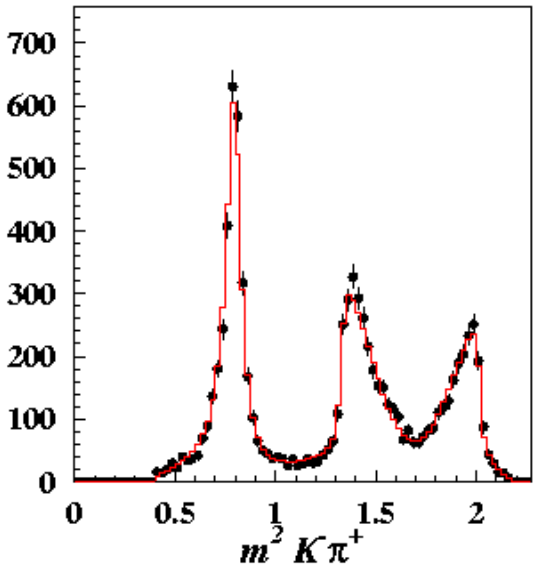


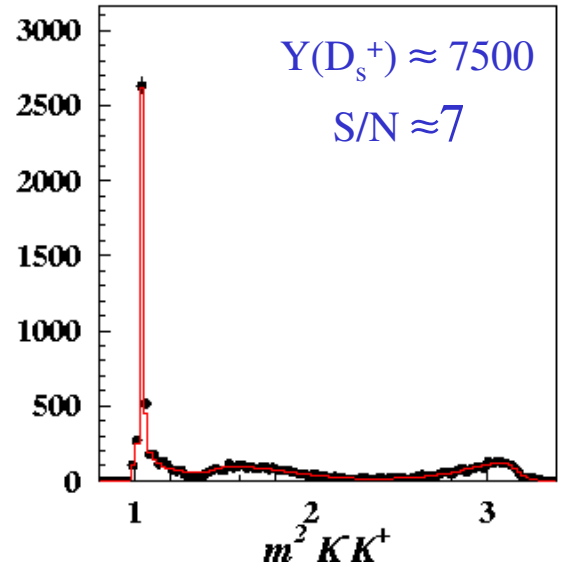


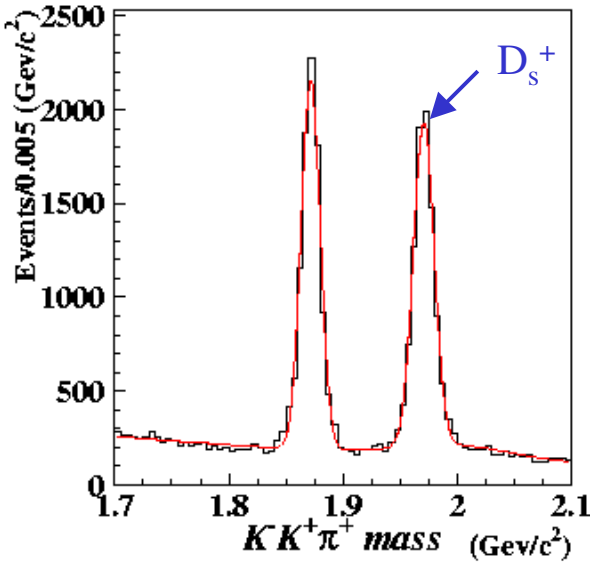
Fit Function

$D_s^+ \rightarrow K^-K^+\pi^+$ (**100%** of the full sample)









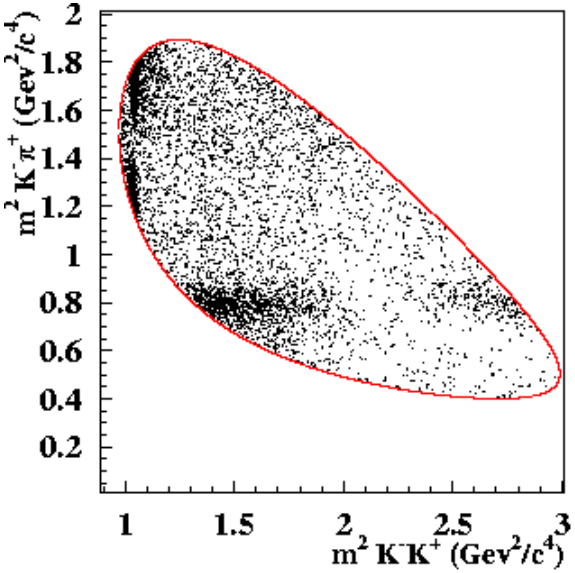
Fit results

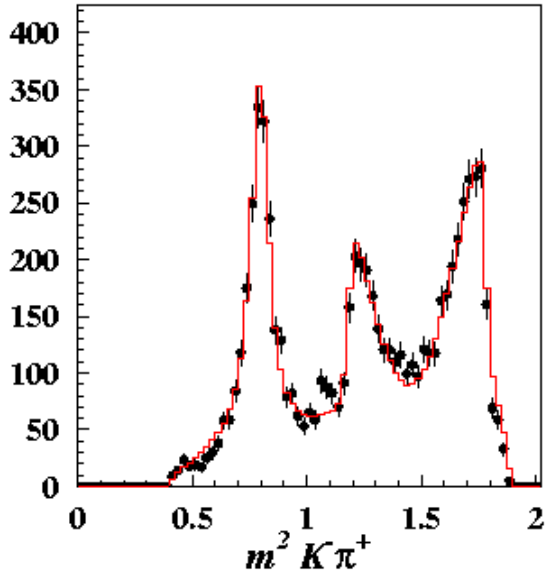
	Fit frac.	Phase (Deg)
K^{*0} (892)	0.44 ± 0.01	0.0 (fixed)
K^*_0 (1430)	0.06 ± 0.01	114 ±5
ϕ (1020)	0.45 ± 0.01	148 ± 4
f_0 (980)	0.16 ± 0.01	135 ± 4
f_j (1710)	0.04 ± 0.01	106 ± 8

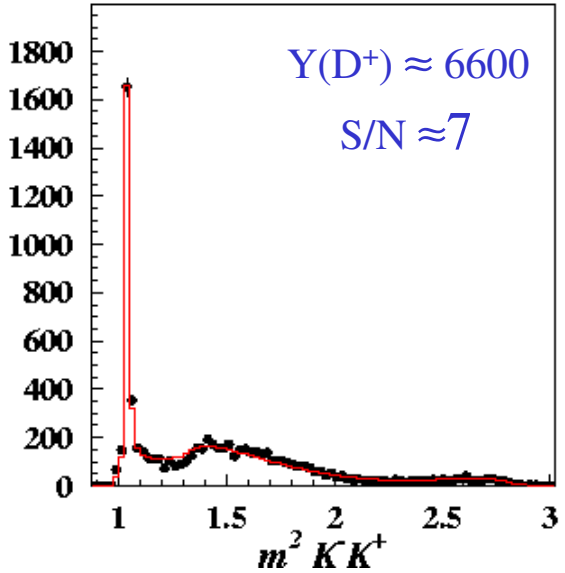
E687 published result

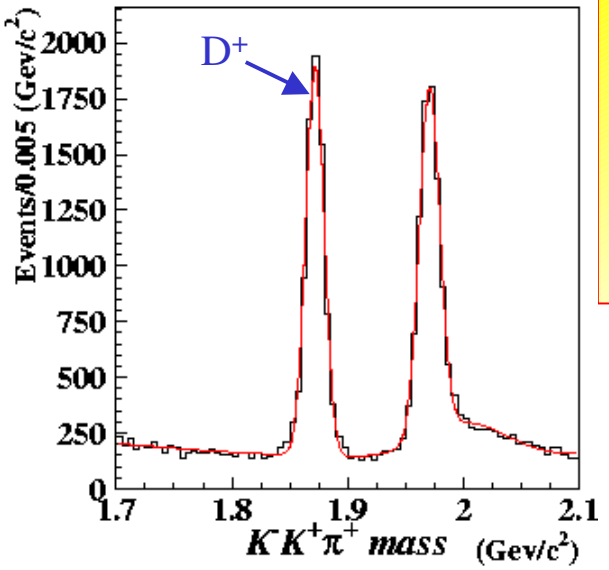
	Fit frac.	Phase (Deg)
K^{*0} (892)	0.48 ± 0.05	0.0 (fixed)
K^*_0 (1430)	0.09 ± 0.03	152 ± 40
ϕ (1020)	0.40 ± 0.03	178 ± 20
f_0 (980)	0.11 ± 0.04	159 ± 22
f_j (1710)	0.03 ± 0.02	110 ± 20

$D^+ \rightarrow K^-K^+\pi^+$ (**100%** of the full sample)









Fit results

	Fit frac.	Phase (Deg)
K^{*0} (892)	0.31 ± 0.01	0 (fixed)
K^*_0 (1430)	0.37 ± 0.01	69 ± 2
ϕ (1020)	0.29 ± 0.01	-171 ± 3

E687 published result

	Fit frac.	Phase (Deg)
K^{*0} (892)	0.30 ± 0.02	0 (fixed)
K^*_0 (1430)	0.37 ± 0.04	70 ±7
ϕ (1020)	0.29 ± 0.03	-159 ±8

PRELIMINARY $D^+ \rightarrow K^+K^-K^+$ ~100% FOCUS

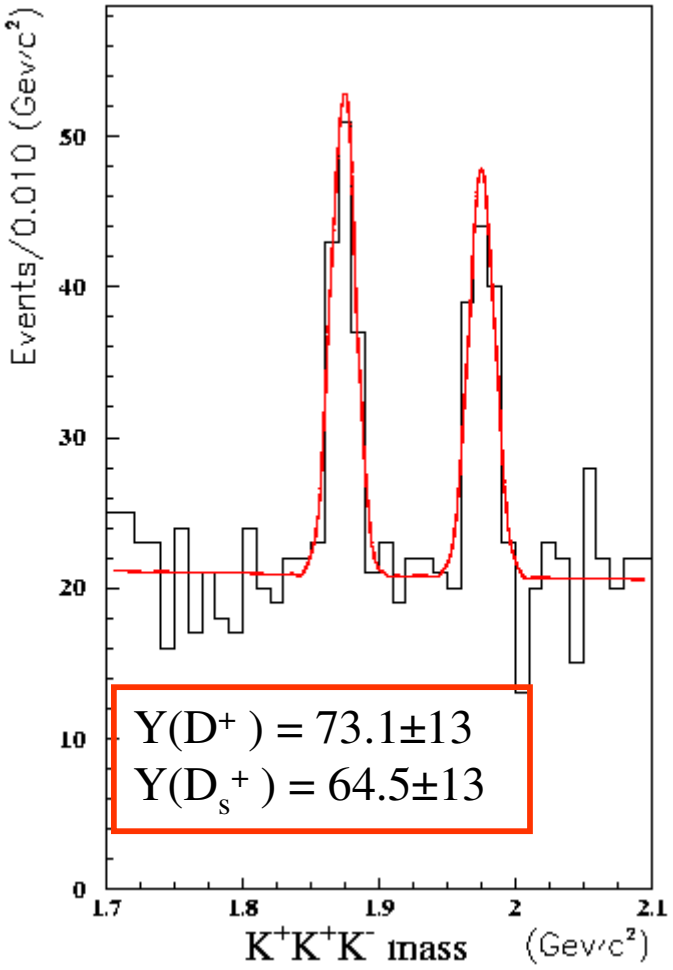
- $D_s^+ \rightarrow K^+K^-K^+$ is a SCSD
- $D^+ \rightarrow K^+K^-K^+$ is a $\Delta C = -\Delta S$ DCSD which cannot occur via a spectator diagram.

$\Gamma(D^+ \rightarrow K^+K^-K^+)/\Gamma(D^+) = (1.23 \pm 0.23) \times 10^{-4}$

Just on PDG 98 limit from E687 of
Br < 1.4×10^{-4} CL=90%

Our rarest DCSD BR yet :
$\Gamma(D^+ \rightarrow K^+K^-K^+) / \Gamma(K^- \pi^+\pi^+) \approx \tan^4\theta_C/2$

A KKK Dalitz analysis may shed light on the KKK decay mechanism.



Conclusions

What have we understood studying the dynamics of the $D_s^+ \longrightarrow \pi^-\pi^+\pi^+$?

- No evidence for annihilation diagram!
  - $\rho(770)$ is a pure combination of quarks u, d ; D_s^+ could decay in $\rho(770)\pi$ only through annihilation diagram (← strong signature):

$\Rightarrow$ but there is no evidence of the channel $\rho(770)\pi$! $(2\pm2\pm1)\%$

  - A small contribution of ρ (1450) (Fit Fraction 5%), has been measured.
    But this resonance could equally decay in KK (Crystal Barrel: Phys. Rev. D57 1998)
    $\Rightarrow$ has a small ($s\bar{s}$) contribution :

$\Rightarrow D_s^+$ could decay in $\rho(1450)\pi$ through a spectator diagram!

Conclusions

- A significant fraction of Non Resonant channel (20%) has been measured (compatible with the upper limit of 26% fixed by E687)

however this result is NOT a strong signature for annihilation diagram!

- Dynamics proceed mainly by quasi two-body, spectator diagram through resonances which can couple to KK and $\pi\pi$ states:

$\Rightarrow$ **decay is dominated by $f_0(980)\pi$**
with contributions of $f_2(1270)$ and $S_0(1475)/f_0(1500)$

and these results are compatible with E687 published results.

Conclusions

Besides the three pions analysis...

- The preliminary studies of the Dalitz Plot analyses of $D^+ \longrightarrow K^-K^+\pi^+$ and $D_s^+ \longrightarrow K^-K^+\pi^+$ give us a confirmation of the E687 published results.

  These studies show the high quality of the E831 data and give us the suggestion to perform a CP violation study in the channel $D^+ \longrightarrow K^-K^+\pi^+$ by Dalitz Plot analysis.

- I have made the first observation of the DCS decay:

$$\mathbf{D^+ \rightarrow K^+K^-K^+ : \; Y(D^+) = 73 \pm 13}$$

- I have made the first observation of the SCS decay:

$$\mathbf{D_s^+ \rightarrow K^+K^-K^+ : \; Y(D_s^+) = 65 \pm 13}$$