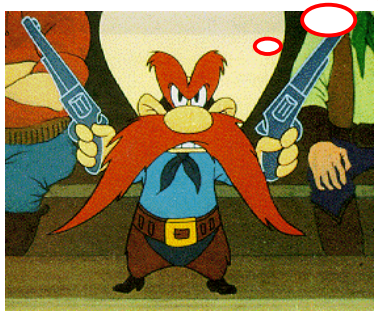
Update of the Milano Dalitz Plot Analysis

Paolo Dini
Sandra Malvezzi
and ...

Because, you know ….



...Luigi Moroni

E831-FOCUS meeting - Mar 19-20th 2000

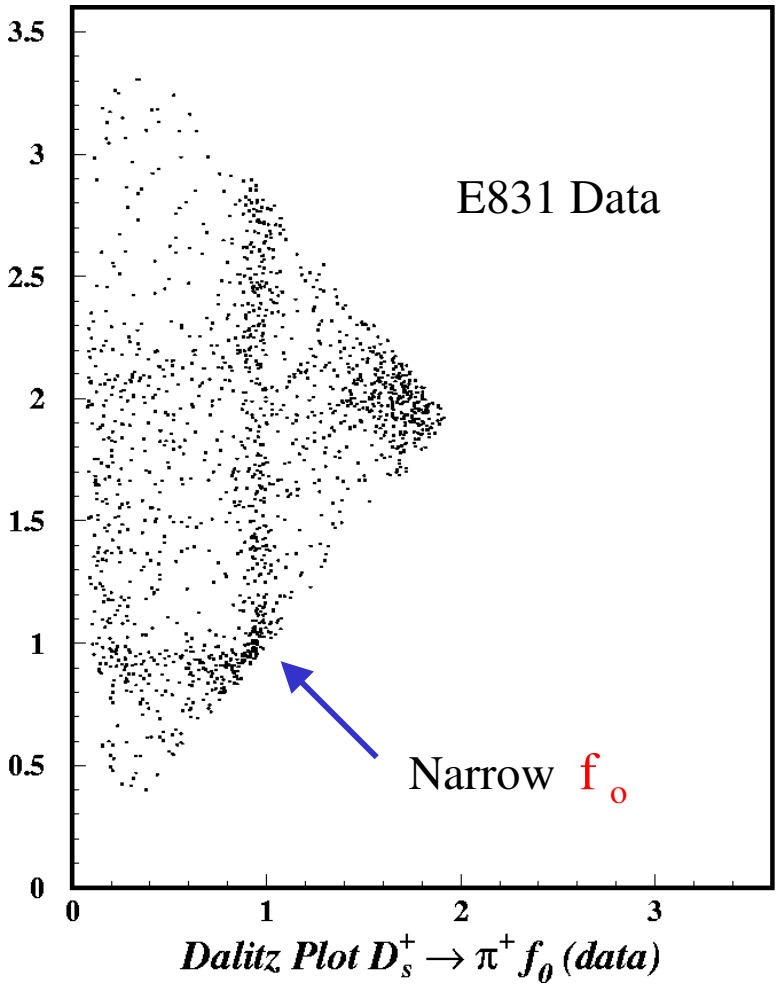
Talk outline

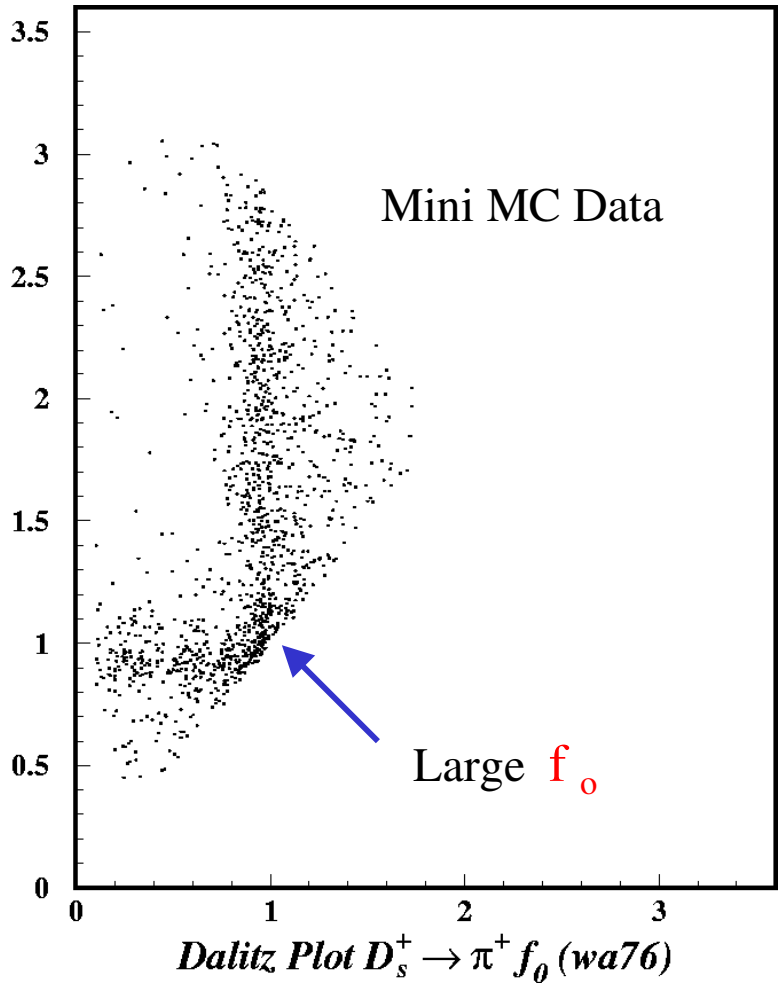
- The K-Matrix formalism for the coupled channel scalars ($f_o(980)$ $a_o(980)$).
- A better background parameterization: polynomial + Breit - Wigner
- Further generalization of the analysis code:
  - arbitrary number of resonances in the model
  - arbitrary number of resonances in the background
- Update results on :

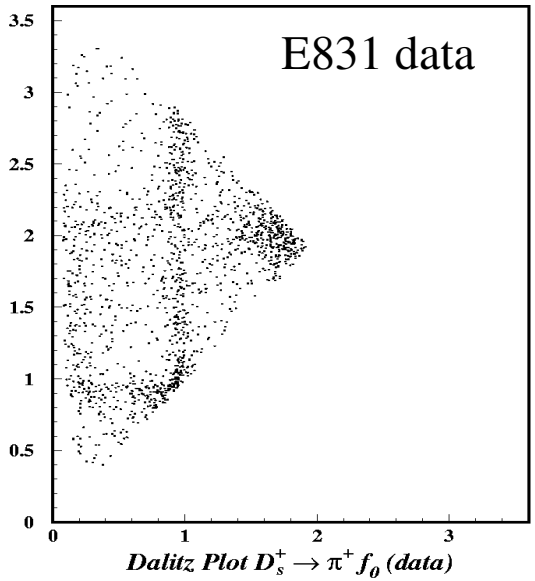
  - $D_s^+ \longrightarrow \pi^-\pi^+\pi^+$ and $D^+ \longrightarrow \pi^-\pi^+\pi^+$
  - $D_s^+ \longrightarrow K^-K^+\pi^+$ and $D^+ \longrightarrow K^-K^+\pi^+$
  - $D^+ \longrightarrow K^-\pi^+\pi^+$

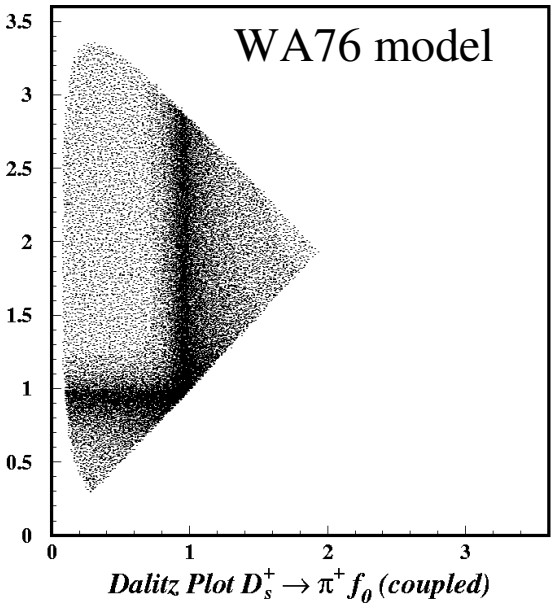
What is the $f_o(980)$ problem ?

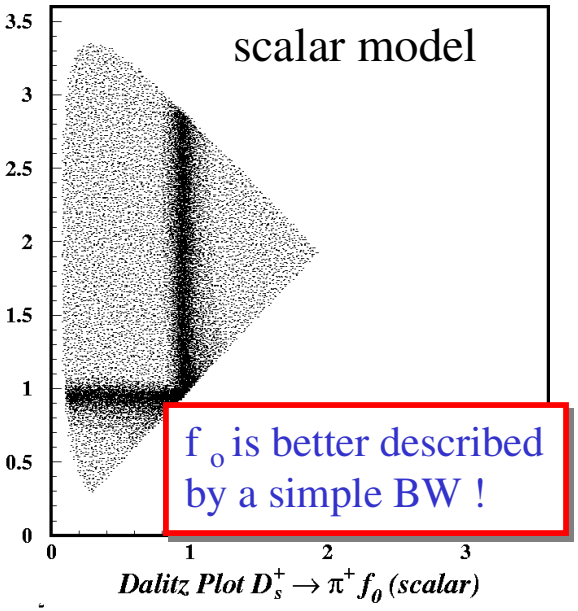
In the E687 Dalitz Plot analyses we used the $f_o(980)$ coupled channel measured by WA76. If one makes a comparison between the Dalitz Plot of $D_s^+ \rightarrow f_o \pi^+$ obtained by E831 data and Mini MC data (generated using wa76 model for the f_o) :



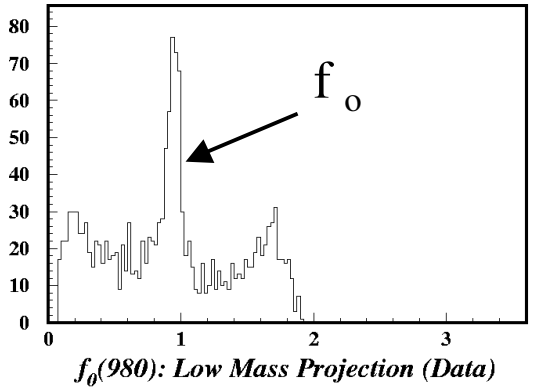


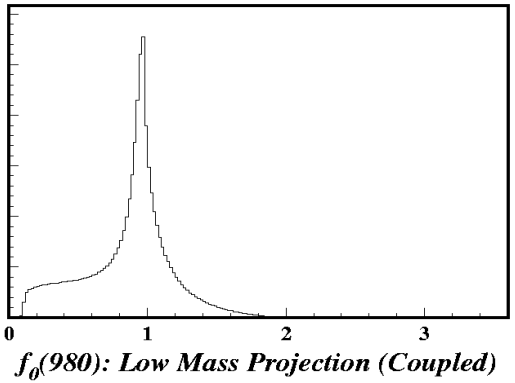


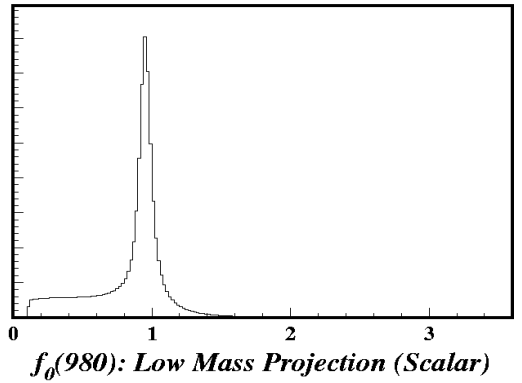


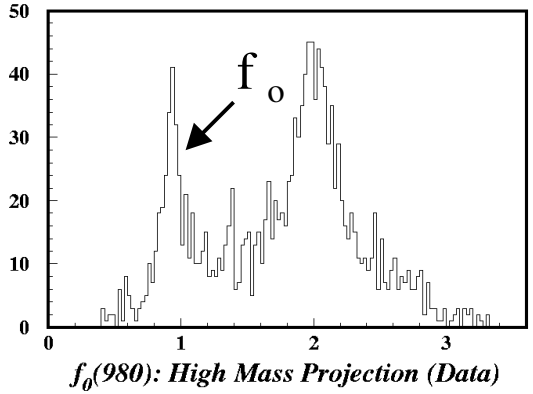


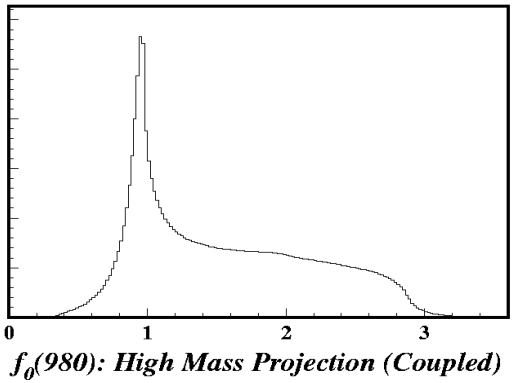
f_o is better described by a simple BW !

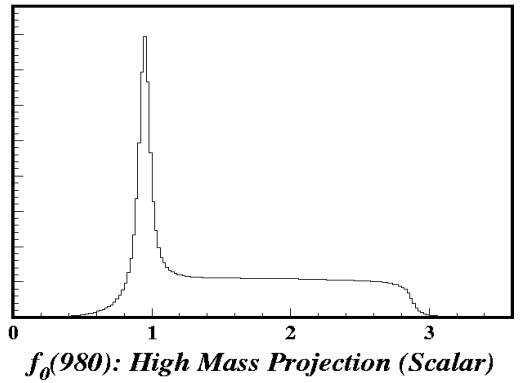


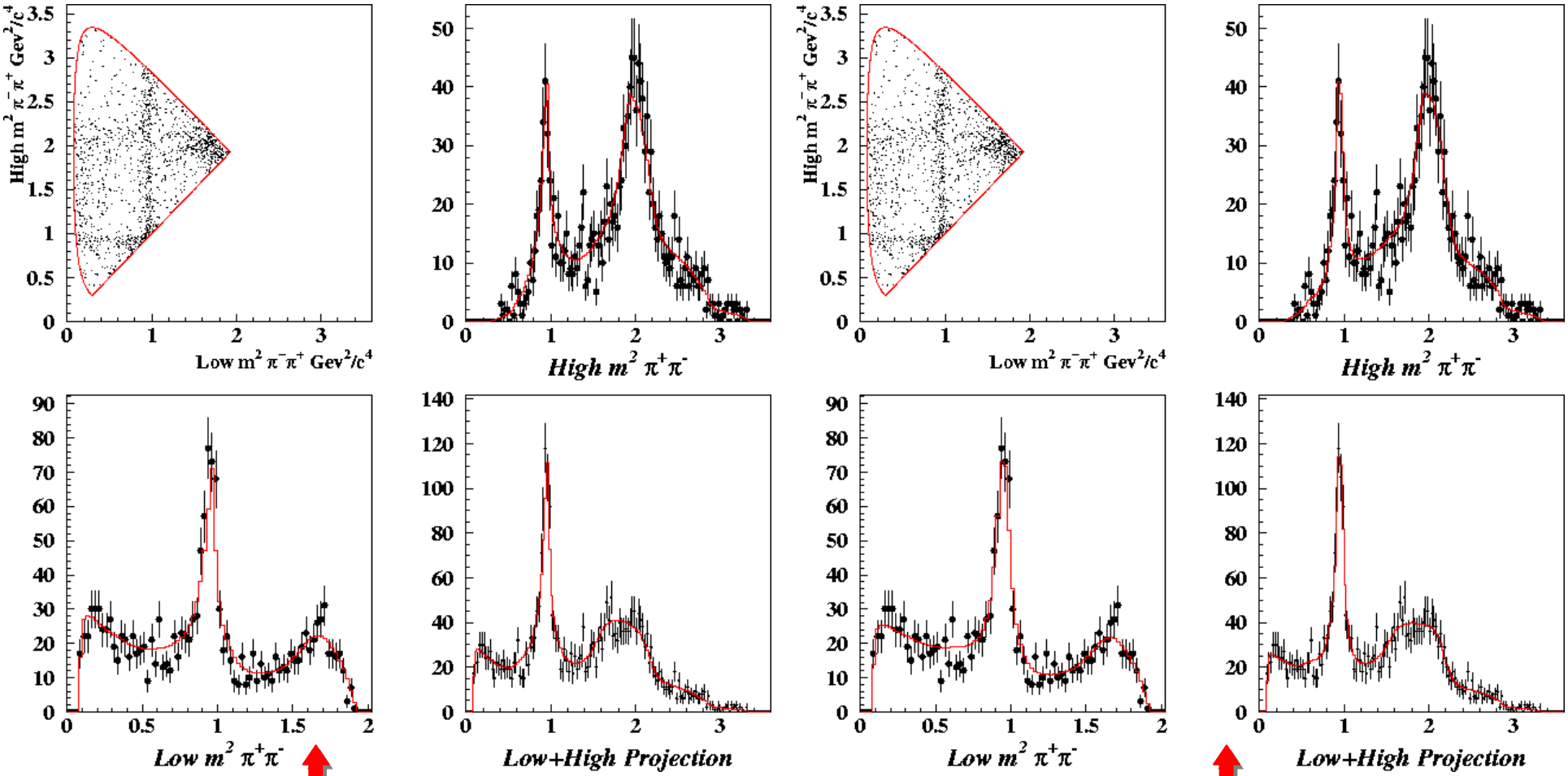












f_o described by WA76

Fit Fraction.:

NR = (26 ⊕ 5)%

f_o = (112 ⊕ 6) %

-2 *ln L* = 2295

NR seems to compensate the larger tails of the WA76 model

f_o described by a simple BW

Fit Fraction.:

NR = (15 ⊕ 4)%

f_o = (76 ⊕ 4) %

-2 *ln L* = 2275

How to solve the $f_o(980)$ problem ?

- trying to use new values for the coupling costants g_K e g_π measured by WA102 ➡ improvements in the quality of the fit and in -2 ln *L*.
  (see the next slide)

- searching a correct method that allows us to use our data to define the best $f_o(980)$ model ➡ the K-MATRIX formalism

The Dalitz Plot fit formalism: coupled channel by WA76/102.

- $$BW(1\ 2|f_0(980)) = \frac{F_D F_{f_0}}{M_{f_0}^2 - m_{12}^2 - i(\Gamma_\pi + \Gamma_K) M_{f_0}}$$

The partial widths Γ_π and Γ_K are defined as :

$$\Gamma_\pi = g_\pi \left[\frac{1}{4} m_{12}^2 - M_\pi^2\right]^{\frac{1}{2}}$$

f_0 coupled channel

$$\Gamma_K = \frac{1}{2} g_K \left[\left(\frac{1}{4} m_{12}^2 - M_{K^+}^2\right)^{\frac{1}{2}} + \left(\frac{1}{4} m_{12}^2 - M_{K^0}^2\right)^{\frac{1}{2}}\right]$$

We try to use for the coupling constants $\mathbf{g_\pi}$ and $\mathbf{g_K}$ the value measured by **WA76** (used in E687 Dalitz Plot analyses) ...

$$g_\pi = 0.28 \pm 0.04$$
$$g_K = 0.56 \pm 0.18$$

…and the most recently values measured by WA102...

$$g_\pi = 0.19 \pm 0.03 \pm 0.04$$
$$g_K = 0.40 \pm 0.04 \pm 0.04$$

The $f_0(980)$ coupled-channel resonance and the K-matrix formalism

- The K-matrix formalism provides a simple and elegant way to satisfy the constraints of unitarity in a general scattering reaction.

- The properties of the K-matrix can be established on rather general ground.

- The K-matrix is derived from the S-matrix and hence unitarity in two-body system is strictly maintained.

- The K operator is defined as

$$K^{-1} = T^{-1} + iI, \tag{1}$$

and

$$\tilde{K}^{-1} = \tilde{T}^{-1} + i\rho \tag{2}$$

in its Lorentz invariant form.

Assuming CP invariance, the K-matrix can be chosen to be real and symmetric.

The K-matrix elements can be expressed as

$$\tilde{K}_{ij} = \sum_{\alpha} \frac{g_{\alpha i}(m) g_{\alpha j}(m)}{(m_{\alpha}^2 - m^2)\sqrt{\rho_i \rho_j}} + \tilde{c}_{ij}. \tag{3}$$

- The sum on α goes over the number of poles with masses m_{α};
- The residue functions (expressed in units of energy) are given by

$$g_{\alpha i}^2(m) = m_{\alpha} \Gamma_{\alpha i}(m), \tag{4}$$

  where $g_{\alpha i}(m)$ is real above the threshold for channel i.

For **one resonance** and **one channel** the K-matrix reduces to

$$K = \frac{m_0 \Gamma(m)}{m_0^2 - m^2}, \tag{5}$$

with the mass-dependent width

$$\Gamma(m) = \tilde{\Gamma}_0 \frac{\rho(m)}{\rho(m_0)} B^2(m) \tag{6}$$

$\tilde{\Gamma}_0$ being the K-matrix width.

For this case T reduces to the relativistic Breit-Wigner

$$T = [\frac{m_0 \tilde{\Gamma}_0}{m_0^2 - m^2 - i m_0 \Gamma(m)}][B^l(m)]^2 (\frac{\rho}{\rho_0}) \tag{7}$$

For the $f_0(980)$ **case coupling to** $\pi\pi$ **(channel 1)**

and $K\bar{K}$ **(channel 2)**

$$\tilde{K}_{11} = \frac{\gamma_1^2 m_0 \Gamma_0}{m_0^2 - m^2}$$
$$\tilde{K}_{22} = \frac{\gamma_2^2 m_0 \Gamma_0}{m_0^2 - m^2}$$
$$\tilde{K}_{12} = \tilde{K}_{21} = \frac{\gamma_1 \gamma_2 m_0 \Gamma_0}{m_0^2 - m^2}$$

The normalized couplings γ_1^2, γ_2^2 are dimensionless and satisfy

$$\gamma_1^2 + \gamma_2^2 = 1 \tag{8}$$

The $\tilde{T}$ matrix is given by

$$\tilde{T} = \frac{m_0 \Gamma_0}{m_0^2 - m^2 - i m_0 \Gamma_0 (\rho_1 \gamma_1^2 + \rho_2 \gamma_2^2)} \begin{pmatrix} \gamma_1^2 & \gamma_1 \gamma_2 \\ \gamma_1 \gamma_2 & \gamma_2^2 \end{pmatrix} \tag{9}$$

If we set

$$g_i = \gamma_i \sqrt{m_0 \Gamma_0},$$

with

$$g_1^2 + g_2^2 = m_0 \Gamma_0$$

then

$$\tilde{T} = \frac{1}{m_0^2 - m^2 - i(\rho_1 g_1^2 + \rho_2 g_2^2)} \begin{pmatrix} g_1^2 & g_1 g_2 \\ g_1 g_2 & g_2^2 \end{pmatrix} \tag{10}$$

that gives the Flattè formula!

The $f_0(980)$ appears as a "regular" resonance in the $\pi\pi$ channel.

- The comparable Breit-Wigner denominator, for m near m_R is

$$m_R^2 - m^2 - im_R\Gamma_R. \tag{11}$$

- The relations between $m_0\Gamma_0$ and $m_R\Gamma_R$ are

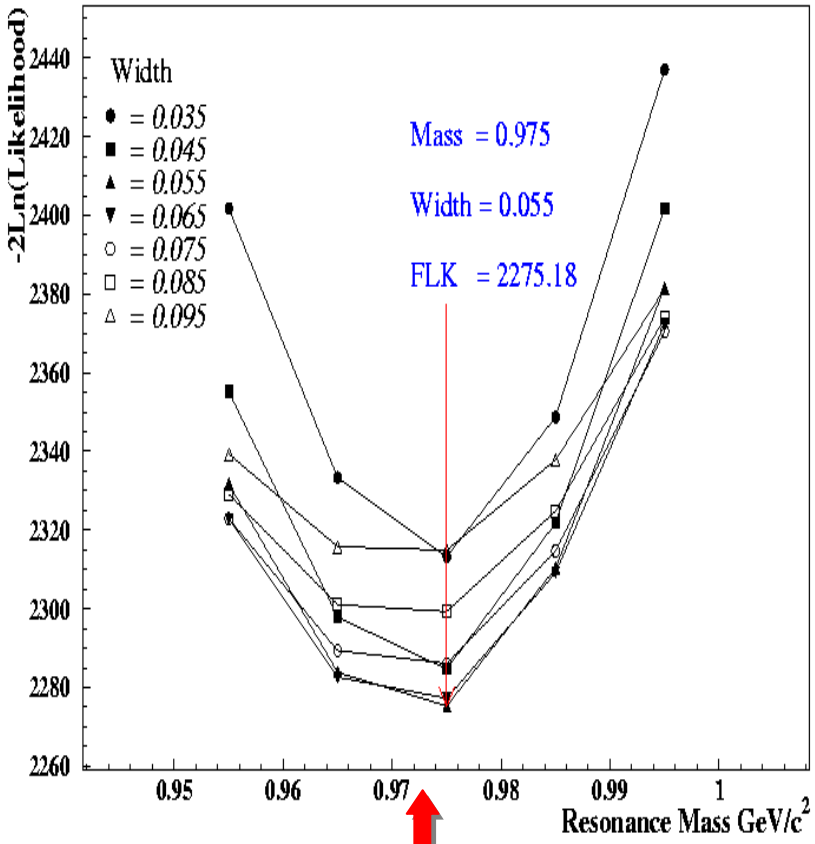
$$m_0^2 = m_R^2 + (\frac{\gamma_2^2}{\gamma_1^2})[\frac{|\rho_2(m_R|}{\rho_1(m_R)}]m_R\Gamma_R, \tag{12}$$

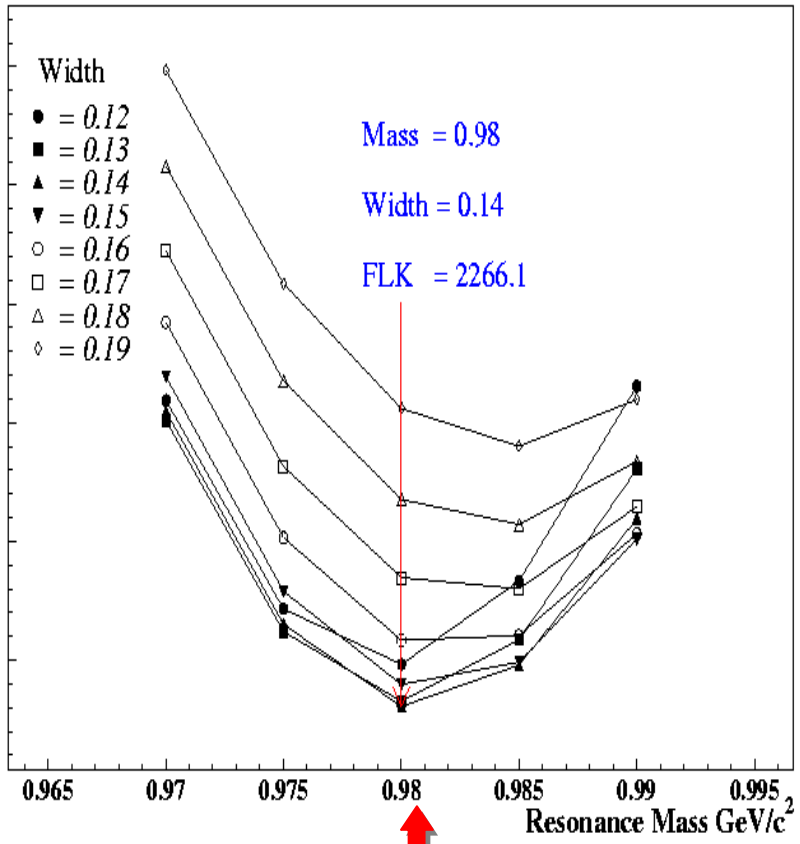
and

$$\Gamma_0 = \frac{m_R\Gamma_R}{m_0\rho_1(m_R)\gamma_1^2}, \tag{13}$$

the ρ_i are evaluated at $m = m_R$ where $\tilde{T}$ is expected to attain its maximum value.

- The above formulas are only supposed to give a good starting point in a fit procedure of the data:
  - one should vary m_0 and Γ_0 to fit the $\pi\pi$ spectrum.
  - the ratio $\frac{\gamma_2^2}{\gamma_1^2}$ is unknown but generally fixed at the SU(3) value of 1.5 on this value.





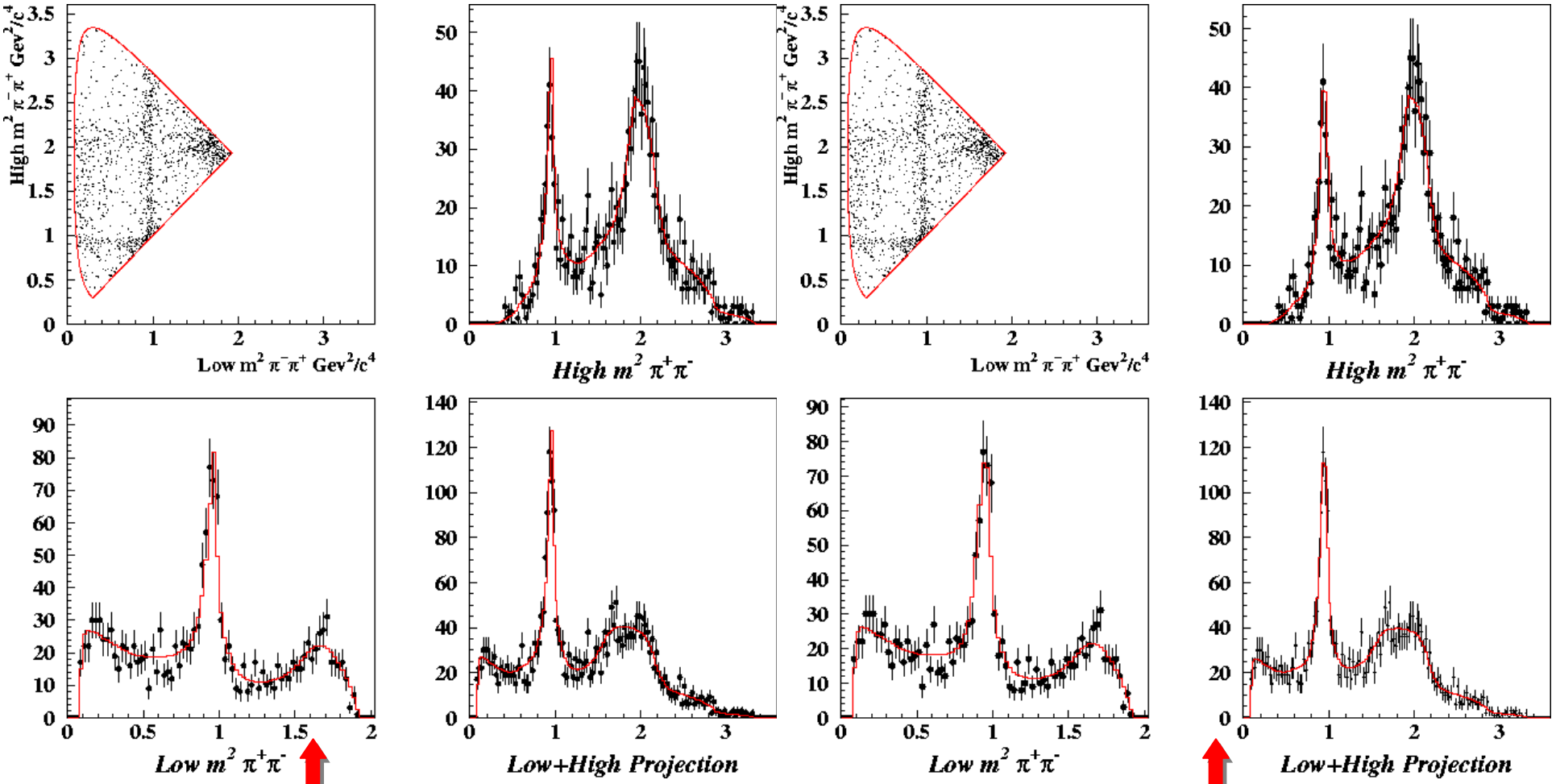
A scan of -2 *ln L* has been performed to establish the best parameters of $f_o(980)$ in the scalar model:

- M = 975 MeV
- Γ = 55 MeV

These results can be used as input in order to establish a good approximation for the K-Matrix shifted mass and width:

- M_o = 982 MeV
- Γ_o = 142 MeV

The scan of -2 *ln L* confirm these results.



f_o described by WA102

Fit Fraction.:

NR = (20 ⊕ 4)%

f_o = (93 ⊕ 5) %

-2 *ln L* = 2279

K-Matrix formalism allows us to optimize the shape of f_o on E831 data

f_o described by K-Matrix

Fit Fraction.:

NR = (19 ⊕ 4)%

f_o = (82 ⊕ 4) %

-2 *ln L* = 2266 **BEST FIT!**

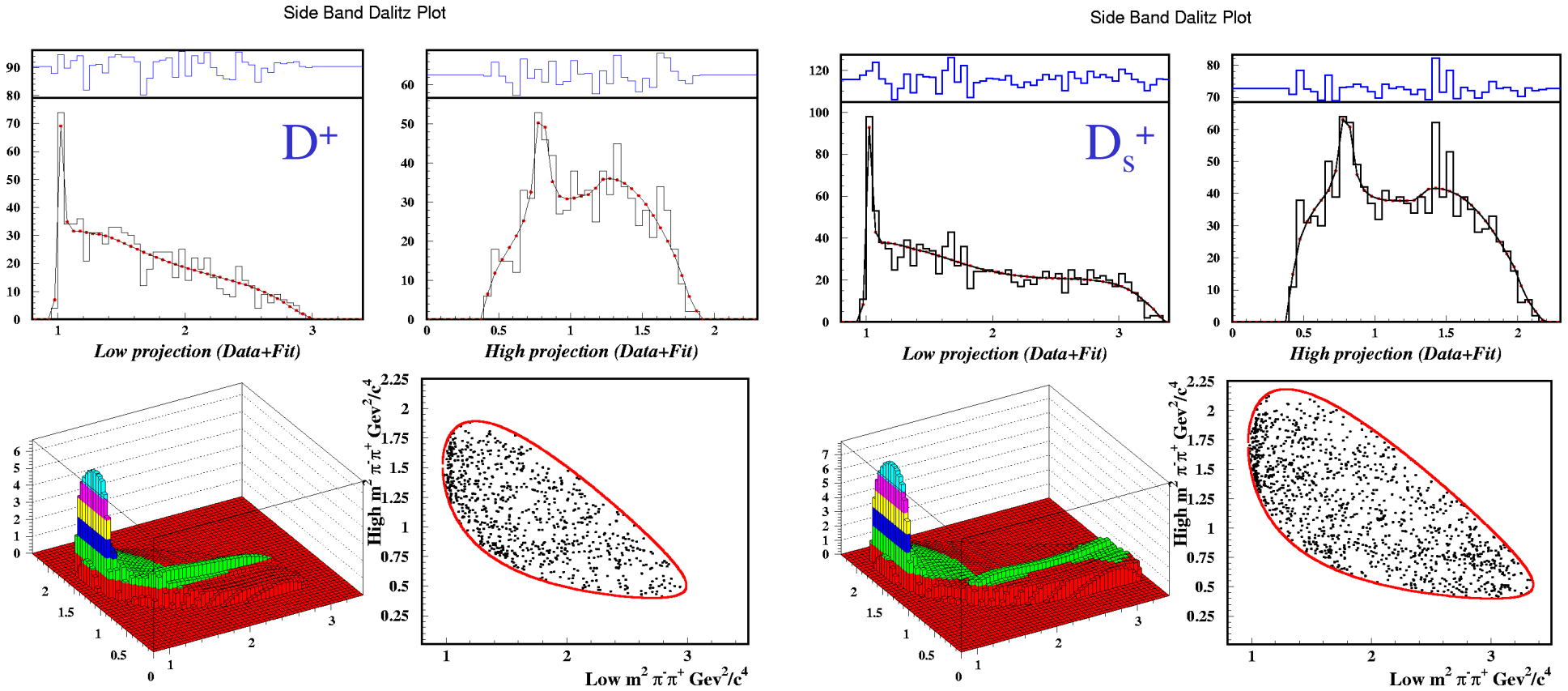
New Background parameterization in the Dalitz analyses

- There are evidences of resonant background contribution in $KK\pi$ and $\pi\pi\pi$ Dalitz Plot. Until now we have used only a polynomial of 1º order and high S/N ratio for the Dalitz Plot sample to reduce the bckg dependencies in the fit.

- Now we use a polynomial of 2º order + of Breit-Wigner (efficiency weighted) for the bckg resonances :

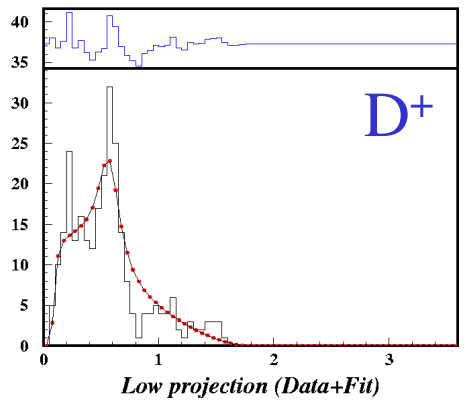
  - $K^{*0}(892)$ and $\phi(1020)$ in $KK\pi$ bckg
  - $f_o(980)$ and $\rho(770)$ in $\pi\pi\pi$ bckg

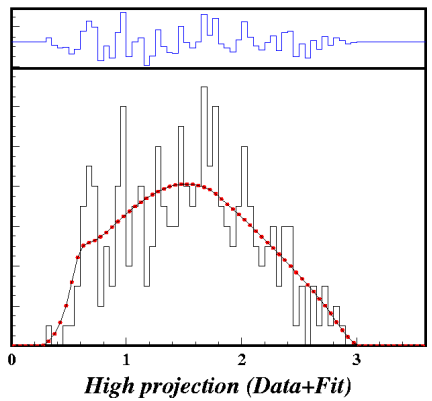
since we are working with high S/N samples we do not expect great improvements in Dalitz Plot fits.

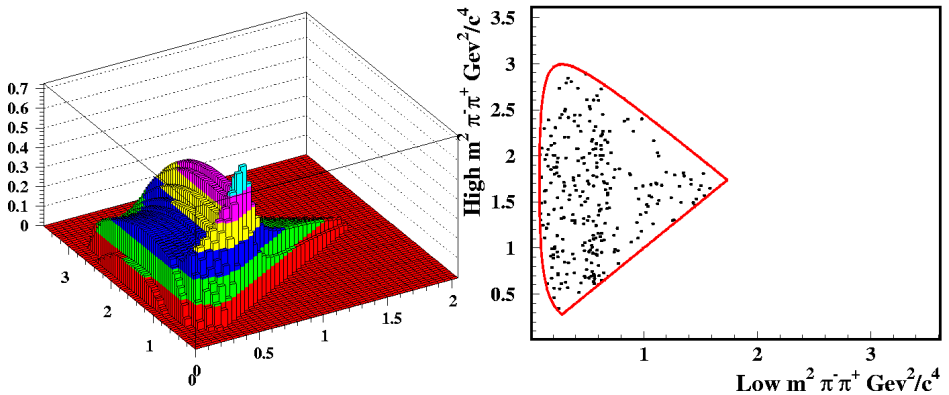


- Great improvement in -2 *ln L* in Side Band Fit for D^+ and D_s^+
  strong evidence for ϕ (1020)
  - from -2 *ln L* = 1993 to 552 for D^+
  - from -2 *ln L* = 1700 to 552 for D_s^+

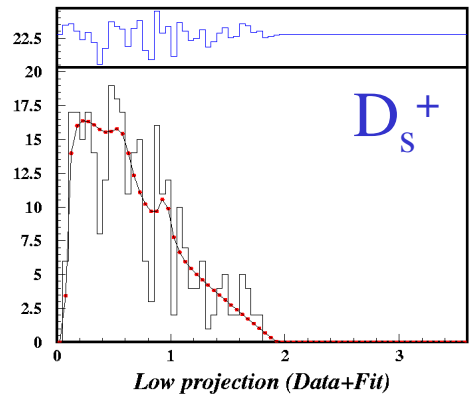
Side Band Dalitz Plot

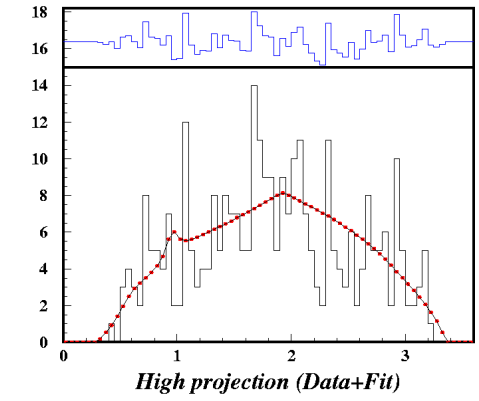


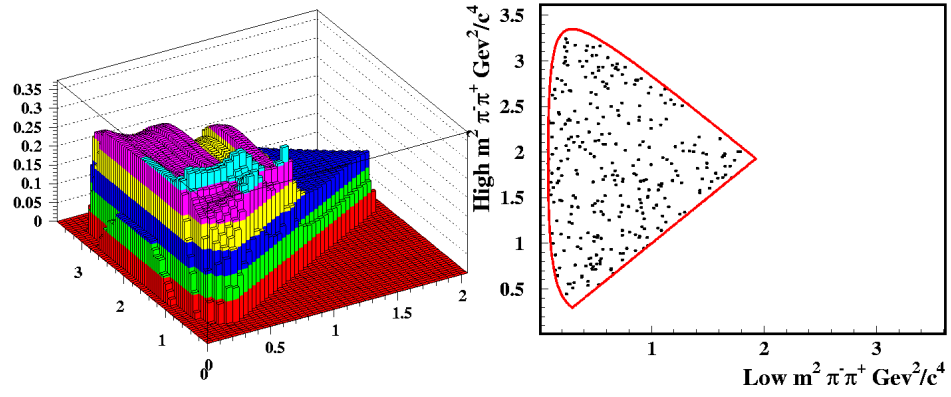




Side Band Dalitz Plot



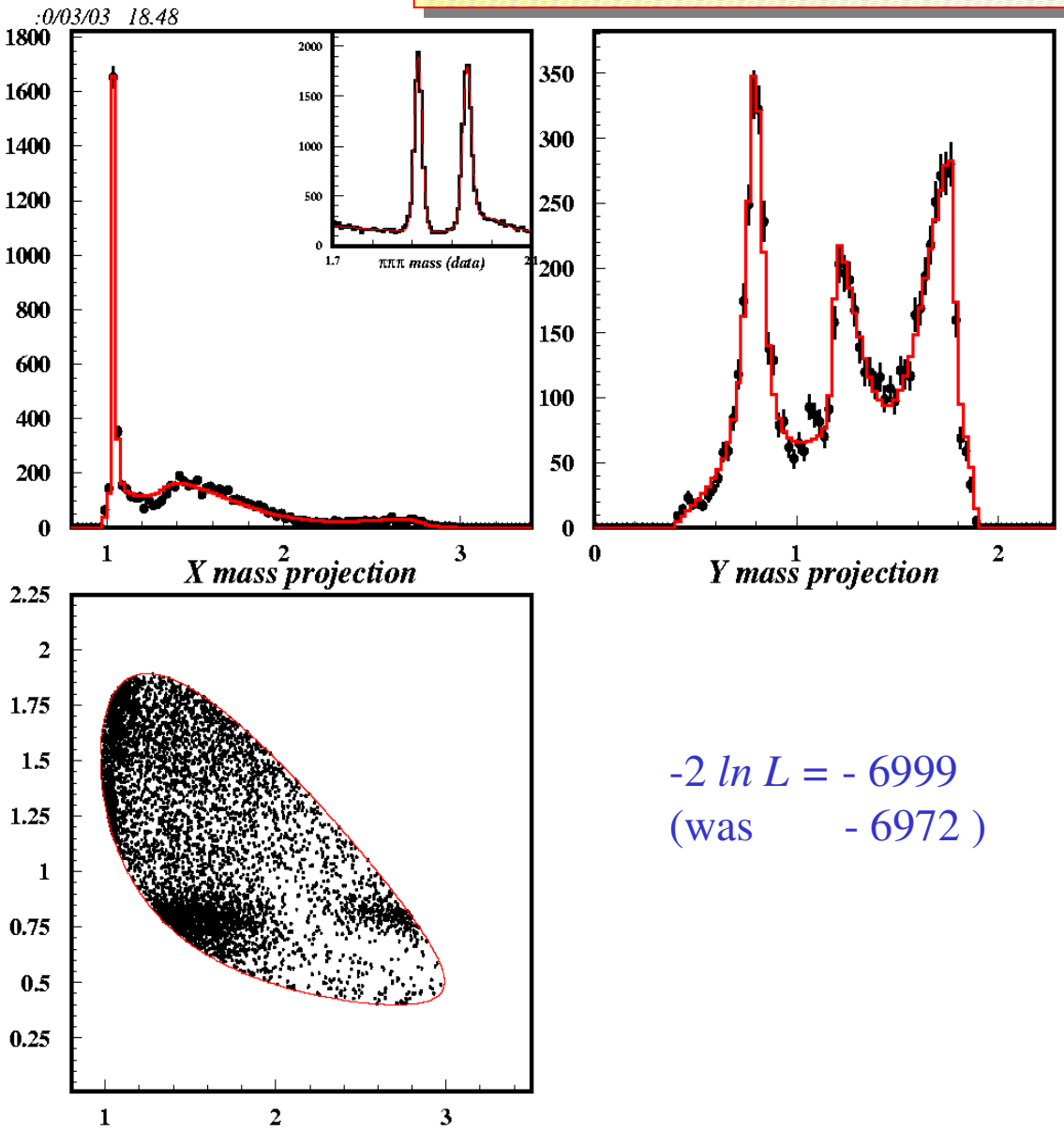




- Great improvement in -2 *ln L* in Side Band Fit for D^+ (evidence for ρ(770))
  (from -2 *ln L* = 757 to 432)

- No improvement in -2 *ln L* in Side Band Fit for D_s^+

$D^+ \rightarrow K^-K^+\pi^+$ (**100%** of the full sample)

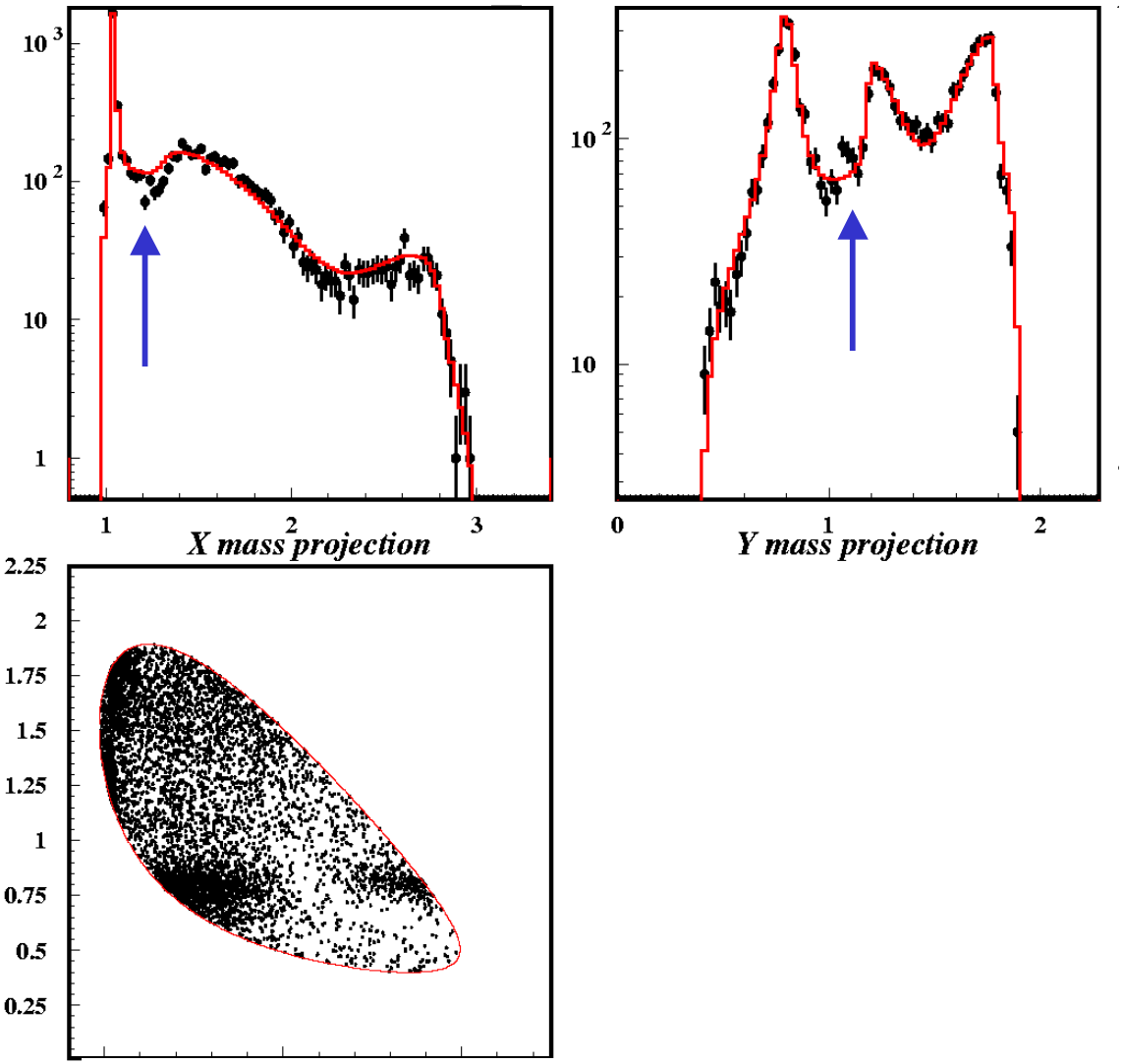


$-2\ ln\ L = -6999$
(was -6972)

Fit results

	Fit frac.	Phase (Deg)
K^{*0} (892)	0.30 ± 0.01	0 (fixed)
K^*_0 (1430)	0.38 ± 0.01	65 ± 3
ϕ (1020)	0.28 ± 0.01	-179 ± 4

E687 published result

	Fit frac.	Phase (Deg)
K^{*0} (892)	0.30 ± 0.02	0 (fixed)
K^*_0 (1430)	0.37 ± 0.04	70 ±7
ϕ (1020)	0.29 ± 0.03	-159 ±8

$D^+ \rightarrow K^-K^+\pi^+$ (semi log. scale)