Introduction to nuclear reaction models

- Compound and direct reactions
- Excitation functions
- Resonance theory
- Pre-equilibrium reactions
- Time-scales
- Coulomb effects
The compound reaction mechanism

There are two extreme mechanisms for describing the interaction of two nuclear systems:

• *compound nucleus reactions*, \( a + A \rightarrow C^* \rightarrow b + B^* \):
  - the projectile is ‘captured’ by the target and the two systems coalesce to form a highly excited state (the ‘compound nucleus’)
  - the excitation energy is shared uniformly among the constituent nucleons
  - a nucleon or group of nucleons can be given enough energy to escape (random process): decay of \( C^* \)
  - if \( B^* \) has enough energy, more emissions can occur, otherwise it will de-excite by \( \beta \)- or \( \gamma \)-decay
The direct reaction mechanism

• *direct reactions* (or *peripheral reactions*):
  - the reaction proceeds ‘directly’ from the entrance channel to the exit channel without the formation of an intermediate state (short glancing collision)
  - if the internal states of the colliding systems do not change, we have elastic scattering
  - if one or both systems are excited in the exit channel, we have inelastic scattering
  - if one or more nucleons are transferred from one nucleus to the other, we have a transfer reaction
  - if a nucleon or light nucleus is ejected from the target while the projectile continues free, we have quasi-elastic scattering
Compound nucleus and liquid drop model

The mechanism of compound nucleus formation and decay can be visualized in the liquid drop model:

- the two colliding droplets combine to form a single drop (the compound nucleus)
- since the compound nucleus is excited, it has a high temperature
- the decay of the compound nucleus is thought of as the cooling of this drop, which takes place by evaporation of one or more of the constituent particles
The independence hypothesis

In a compound nucleus reaction

\[ a + A \rightarrow C^* \rightarrow b + B^* \]

there is a considerable delay between the formation and the decay of the compound nucleus and many complicated nucleon motions take place. This results in that the nucleus \( C^* \) has lost memory of the entrance channel \( a + A \):

\textit{the probabilities of the various exit channels are independent of the entrance channel (the independence hypothesis)}

\[ \sigma = \sigma_{Aa}^{C}(E) \cdot G_{Bb}^{C}(E) \]
The independence hypothesis (cont’d)

The reaction cross-section factorizes into

$$\sigma_{AA}^C(E) = \text{the probability to form the compound nucleus } C^* \text{ from } a + A \text{ at a total excitation energy } E$$

and

$$G_{BB}^C(E) = \text{the relative probability for } C^* \text{ at energy } E \text{ to decay to } b + B$$

Assume the reaction $$d + D \rightarrow C^* \rightarrow b + B^*$$ where the compound nucleus is formed at energy $$E$$. We then have:

$$\sigma = \sigma_{DD}^C(E) \cdot G_{BB}^C(E)$$

The independence hypothesis does not hold for direct reactions.
**The multiple-scattering picture**

We can consider the interaction of e.g. a nucleon projectile with a target nucleus as consisting in a number of successive two-nucleon collisions between the projectile and the nucleons in the target:

- the projectile does not usually lose much energy in a single collision
- after one nucleon-nucleon collision, the target nucleus will not be left in a highly excited state
- after one collision:
  - if the projectile escapes: *direct inelastic scattering*
  - if projectile and struck nucleon escape: *direct quasi-elastic scattering*
  - if projectile and struck nucleon emerge as one: *direct transfer reaction*
- if projectile suffers more collisions, it loses too much energy to be able to escape, its energy is shared among many target nucleons, the compound nucleus is formed: *compound nucleus reaction*
Statistical mechanics and compound nuclei

The compound nucleus can be described by statistical mechanics as being in a state of statistical equilibrium. The energy distribution of the components of the system is given by a Maxwellian distribution.

A component of this system may receive a large amount of energy as the result of a statistical fluctuation. This amount of energy can be enough to cause the decay of the compound nucleus.

Being a statistical process, the evaporation of particles favours the escape of nucleons near the smallest possible energy. In the case of charged particles, this minimum energy is the Coulomb barrier of the compound nucleus. When the compound nucleus has reached statistical equilibrium, we say that it has been ‘thermalized’.
Excitation functions

Direct and compound nucleus reactions are not mutually exclusive. Their contribution to a process depends on the energy of the beam. The distribution of the cross-section as a function of the beam energy is called the ‘excitation function’.

The compound nucleus component is dominant at low energies.

The direct contribution increases with energy as more channels become available.

Direct becomes dominant at ~10 MeV.
Example: low-energy p and n interactions

The contributions of the direct and compound nucleus mechanisms in a reaction can be exemplified by the low-energy elastic scattering of protons and neutrons by a nucleus.

• p: they are repelled by the electrostatic field of the nucleus and are scattered with a cross-section given by Rutherford’s formula (direct reaction)

• n: may be scattered by the nuclear field, which is a potential scattering, i.e. a direct process (shape elastic scattering) or

• may be captured to form a compound nucleus and then be emitted with the same energy (compound elastic scattering)

• p can also induce compound elastic on light nuclei (because of Cb barrier). Then: $\sigma = f(\sigma_{Cb}, \sigma_{shape}, \sigma_{compound})$.
Low-energy neutron capture

The capture of a neutron by a nucleus will give the target an excitation energy equal to the sum of the channel energy and the neutron binding energy (8 MeV for most nuclei).

At about 8 MeV, nuclei have a high density of states but the neutron energy can be varied by few eV. This resolves the individual excited states of the target.

As the neutron energy goes through the energy of an excited state of the compound nucleus, a resonance occurs. The cross-section goes through a series of maxima.
Resonances at neutron capture

![Graph showing resonances in neutron capture](image)

**Graph**: Total cross-section (b) vs. neutron energy (eV)

- Resonances are observed at specific neutron energies, indicating interruptions in the continuous energy loss spectrum.
- The peaks and dips correspond to the energy levels at which resonant capture occurs.
- The peaks represent higher cross-sections, indicating more likely capture events at these energies.
Resonance theory

The compound nucleus exhibits discrete energy states both below and above the threshold for particle emission. Such states have e.g. definite spin and parity (quasi-stationary quantum states).

The instability of the compound nucleus results in an uncertainty in the energy of these states, i.e. the possible energy values follow a probability distribution. The energy uncertainty is given by the width of the resonance, $\Gamma$. The width and lifetime, $\tau$, of the state are related by Heisenberg’s uncertainty principle:

$$\tau = \frac{\hbar}{2\pi \Gamma}$$

The requirement $\tau \gg 10^{-22} \text{s}$ gives $\Gamma \ll 1 \text{ MeV}$. 
The Breit-Wigner formula

The dependence of the resonance cross-section on the beam energy (excitation function) is given by the Breit-Wigner formula:

$$\sigma(E) = \sigma_0 \frac{\Gamma^2/4}{(E - E_r)^2 + \Gamma^2/4}$$
Pre-equilibrium reactions

There are some reactions that do not qualify neither as direct nor as compound nucleus. These are called ‘pre-equilibrium’ or ‘pre-compound reactions’.

The plots show the cross-section distributions for inelastic scattering of protons by $^{54}$Fe at 40° as a function of the residual energy of the beam.

The sharp peaks at high energies correspond to direct reactions. The broad peak at low energies shows compound formation. The region between the two is the pre-equilibrium region.
The mechanism of pre-equilibrium reactions

• the emission of a particle by the target nucleus happens neither immediately after the collision nor by the statistical decay of the compound nucleus

• the projectile shares its energy among a small number of nucleons in the target

• the struck nucleons initiate a cascade of reactions within the target, at the course of which a particle can be emitted (before the compound nucleus has reached a state of statistical equilibrium)
Overview of reaction mechanisms
Time scales

On nuclear scales, we judge ‘slow’ and ‘fast’ by the time it takes a constituent nucleon to orbit one period inside the nucleus. This is about $10^{-22}$ s (corresponding to 20 MeV nucleon kinetic energy).

Direct reactions last for as long as it takes the projectile to go through the target. Typical reaction times are thus of the order $10^{-22}$ s.

Compound reactions involve much longer scales, as the energy of the captured projectile has to be distributed over all nucleons in the target. The reaction duration depends on the energy of the projectile.

Typical time scales for the decay of a compound nucleus can vary between $10^{-16}$-$10^{-15}$ s (at low beam energies) and $10^{-21}$-$10^{-20}$ s (at high beam energies).
Coulomb effects in evaporation

A charged particle within the nucleus will escape with a smaller probability than a neutral particle because it will have to tunnel out through the Coulomb barrier of the nucleus (a situation similar to α-decay).

The evaporation of low-energy charged particles from a compound nucleus will be strongly inhibited relative to the emission of neutrons. The energy spectrum for charged particles will have a maximum at higher energies than this of neutrons.